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Modelling pulsation amplitudes of ξ Hydrae

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ABSTRACT

Amplitudes of stochastically excited oscillations of models of ξ Hydrae (HR4450) are presented. The theoretical results are compared with the recent measurements by an international group of astronomers announced in the ESO press release 10/02. Using a stochastic excitation model we find fair agreement between estimated velocity amplitudes and the values quoted in the ESO press release.

Key words: convection – stars: individual: ξ Hya – stars: oscillations.

1 INTRODUCTION

With the advent of high-precision stellar spectroscopy, it has been possible to detect solar-type oscillations – i.e., a spectrum of oscillation modes that are intrinsically stable and excited by turbulent convection – in other stars, namely Procyon (Martic et al. 1999; Barban et al. 1999), β Hydri (Bedding et al. 2001; Carrier et al. 2001) and α Cen A (Bouchy & Carrier 2001). Recently, an international team of astronomers has announced in the ESO press release 10/02 the detection of solar-type oscillations in the red giant star ξ Hydrae (HR 4450). This is the first giant star in which solar-type oscillations have been detected; in some respects its structure differs more from that of the Sun than does that of any of the other stars in which such oscillations have been seen, and one might therefore expect it to provide a more stringent test of oscillation properties that have been estimated by theoretical scaling from solar conditions. We report here a comparison of the reported oscillation velocity amplitudes of ξ Hydrae with theoretical expectations.

The first computations of expected amplitudes from stochastic excitation in stars on or near the main sequence was carried out by Christensen-Dalsgaard & Frandsen (1983), who obtained the amplitudes by postulating equipartition between the energy of an oscillation mode and the kinetic energy in one convective eddy having the same turnover time as the period of the oscillation, as assumed by Goldreich & Keeley (1977). They found velocity and luminosity amplitudes to increase with both age and mass along the main sequence. Subsequent more sophisticated calculations by Houdek et al. (1999), which took some account of all the convective eddies, and which included the interaction of convection with pulsation in the linearized pulsation calculations, yielded similar results.

From the results of Christensen-Dalsgaard & Frandsen's (1983) early computations, Kjeldsen & Bedding (1995) proposed the scaling $V \propto L/M$, where V is the oscillation velocity amplitude, L is the luminosity and M is the mass of the star. The calculations by Houdek et al. (1999) more-or-less confirmed this result, within the parameter range in which the comparison could be made, although they fitted better the law $V \propto (L/M)^s$ with $s \simeq 1.5$. However, although both the simple scaling law and the computations of Houdek et al. appear to fit the observations of α Cen A rather well – although it should be appreciated that α Cen A is very similar to the Sun – and they overestimate the amplitudes of β Hyi perhaps by only an insignificant amount, they grossly overestimate the amplitudes of the relatively hot and luminous star Procyon (Kjeldsen & Bedding 1995; Houdek 2002). It appears, therefore, that there is something quite seriously wrong with the theory of either the pulsations, the convection, or their coupling.

To provide an improved empirical scaling law, not associated with theory and with no apparent justification other than to provide a vehicle to reduce the 'expected' amplitude of Procyon, Kjeldsen & Bedding (2001) proposed that V is independent of the effective temperature T_e amongst stars of given M and radius R. Thus, writing $L/M \propto R^2T_e^4/M \propto T_e^4/g$, where g is the surface gravity, they proposed that $V/V_\odot = g_\odot/g$, which for Procyon yields a velocity only 15 per cent higher than the observations. However, neither this nor the earlier simpler scaling law agrees with the new observations of ξ Hydrae: the g^{-1} scaling exceeds the observations by a factor 4, and the L/M scaling by a factor of 2. As we report below, the theoretical excitation calculation yields a somewhat better result.

In this Letter we estimate the velocity amplitudes for ξ Hydrae according to the stochastic excitation model used by Balmforth (1992b). In this approach, first linear damping rates are computed assuming a time-dependent treatment of convection to account for the linear dynamical interaction between the pulsation and the turbulent convective velocity field. The excitation (energy supply rate) of an intrinsically damped acoustic mode is obtained by assuming that most of the driving is due to the fluctuating Reynolds stresses (Lighthill mechanism). The amplitude is then obtained by balancing the driving and the damping of the mode.

A comparison between the theory and the greatest amplitudes of all the observed stellar solar-type oscillations reported prior to ξ Hydrae is presented in a review by Houdek (2002). Similar results,

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Table 1. Comparison of scaling laws and theoretical velocity estimates with the observations of solar-like oscillations. The quantity $\hat{V}_{\rm obs}$ is the typical observed maximum apparent velocity amplitude, and $\hat{V}$ is the value obtained by the excitation theory, scaled to render $\hat{V} = \hat{\rm V}_\odot$ for the Sun. The solar value was taken to be $\hat{\rm V}_\odot = 0.23$ m s^{-1}. The parameters used in the convection theory (see Gough 1976) were $\alpha = 2.03$, $\Phi = 5/3$, $a^2 = 900$ and $b^2 = 2000$.

Star	$M/{\rm M}_\odot$	$L/{\rm L}_\odot$	$T_{\rm e}/{\rm T}_\odot$	$L{\rm M}_\odot/{\rm L}_\odot M$	${\rm g}_\odot/g$	$\hat{V}/\hat{\rm V}_\odot$	$\hat{V}_{\rm obs}/\hat{\rm V}_\odot$
α Cen A	1.16	1.58	1.004	1.35	1.34	1.39	1.5[a]
β Hyi	1.11	3.50	1.004	3.15	3.11	3.25	2.2[b]
Procyon	1.46	6.62	1.107	4.53	3.02	6.47	2.6[c]
ξ Hya	3.31	60.00	0.857	18.13	33.64	9.04	8.7[d]

Notes: [a]From Bouchy & Carrier (2001), [b]From Bedding et al. (2001), [c]From Martic et al. (1999), [d]From ESO press release 10/02.

all computed with a common set of convection-theory parameters, are summarized in Table 1, together with entries for ξ Hydrae. Unlike Houdek et al. (1999), who worked with physical model amplitudes V, we here compare what we call apparent velocity amplitudes $\hat{V}$, which are simply the estimated Doppler amplitudes as observed; $\hat{V}$ is related to V via a mean observational filter factor (cf. Christensen-Dalsgaard & Gough 1982) which depends on the degree of the mode and the spectral lines used for the observations. The estimates $\hat{V}$ were obtained by normalizing the theory such that the greatest amplitudes agreed with those of the solar modes. It is noteworthy that, in the case of β Hydri, the only star for which an apparently more careful comparison between theory and observation has been made, that more careful comparison results in a greater discrepancy (Gough 2001). A continuous time-series of solar Doppler data, kindly provided by BiSON (Chaplin et al. 1998) were scaled in frequency and amplitude according to the theory to provide a time-series (assumed to be noise-free) to mimic the oscillations of β Hyi; an amplitude scaling factor Λ was then determined such that when the scaled proxy oscillations were observed with the same observing window and with the same added noise as those reported by Bedding et al. (2001) the power in a central region of the oscillation spectrum, where the amplitudes are greatest, agreed with that of the observations of β Hyi, the latter having been kindly provided by Bedding et al. (2001). That factor Λ, which has the value 1.9, is the factor by which the theory was judged to overestimate the observations. It is rather greater than the corresponding factor 1.5 which can be inferred from Table 1. We do not know which is the better estimate. Whereas there is greater uncertainty from using only the modes of greatest amplitude resulting from stochastic variation in only few data, the signal from the modes of the greatest amplitude is at least less susceptible to contamination by background and instrumental noise. In this paper we work only with what the observers have called 'greatest amplitudes', which are readily available in the literature.

It is interesting to note that the observations of β Hyi by Carrier et al. (2001), using the same instrument as was used for observing ξ Hydrae, found a maximum amplitude inferred from only those data from three consecutive nights when the weather was good that is somewhat greater than that from the entire data set, and from that reported by Bedding et al. (2001); the amplitude relative to solar is $\hat{V}_{\rm obs}/\hat{\rm V}_\odot = 2.5$, which is not significantly different from the theoretical result.

The star ξ Hydrae is reported in the ESO press release to have a mass $M = 3.31 \pm 0.17$ M$_\odot$, a luminosity $L \simeq 60$ L$_\odot$ and an effective temperature $T_{\rm e} = 4950 \pm 100$ K; the greatest velocity amplitudes are up to 2 m s^{-1}, with frequencies in the range 60–100 µHz. The velocity amplitudes are in fair agreement with the theoretical values that we have obtained, which are about 2.08 m s^{-1} $\simeq 1.04\hat{V}_{\rm obs}$ for stellar models with the most likely values of M, L and $T_{\rm e}$.

2 MODEL COMPUTATIONS

The computations that we have performed are as described by Balmforth (1992a) and Houdek et al. (1999). The turbulent fluxes are obtained from a non-local, time-dependent generalization of the mixing-length formulation of Gough (1977, 1976), with a mixing length calibrated to the Sun. In this generalization there is an additional parameter Φ which specifies the shape of the convective eddies, and there are two more parameters, a and b, which control respectively the spatial coherence of the ensemble of eddies contributing to the turbulent fluxes of heat and momentum and the degree to which the turbulent fluxes are coupled to the local stratification. Roughly speaking, the latter two parameters control the degree of 'non-locality' of convection; low values imply highly non-local solutions, and in the limit $a, b \to \infty$ the system of equations formally reduces to the local formulation (except near the boundaries of the convection zone, where the local equations are singular). Gough (1976) has suggested theoretical estimates for their values, but it is likely that the standard mixing-length assumption of assigning a unique scale to turbulent eddies at any given location causes too much smoothing; accordingly, somewhat larger values probably yield more realistic results. In this paper we adopt the values $a^2 = 900$ and $b^2 = 2000$, values that have been used by Houdek et al. (1999) in order to ensure that most of the modes in all stellar models are linearly stable.

2.1 Envelope and pulsation models

Both the envelope and pulsation calculations assumed the three-dimensional Eddington approximation to radiative transfer (Unno & Spiegel 1966). The integration was carried out inwards, starting at an optical depth of $\tau = 10^{-4}$ and ending at a radius fraction $r/R = 0.2$. The opacities were obtained from the OPAL tables (Iglesias & Rogers 1996), supplemented at low temperature by tables from Kurucz (1991). The equation of state included a detailed treatment of the ionization of C, N, and O, and a treatment of the first ionization of the next seven most abundant elements (Christensen-Dalsgaard 1982), as well as 'pressure ionization' by the method of Eggleton, Faulkner & Flannery (1973); electrons were treated with relativistic Fermi-Dirac statistics. Perfectly reflective mechanical and thermal outer boundary conditions in the pulsation calculation were applied at the temperature minimum in the manner of Baker & Kippenhahn (1965). At the base of the model envelope the conditions of adiabaticity and vanishing displacement were imposed. Only radial p modes were considered.

The mixing-length parameter α was calibrated for a solar model to obtain the helioseismically inferred depth of the convection zone (Christensen-Dalsgaard, Gough & Thompson 1991) with the value $\Phi = 5/3$ that was used by Houdek et al. (1999). The same value of α was used for all the models.

2.2 Stochastic excitation model

The noise generated by the turbulent motion and injected into the acoustic radial modes is estimated according to the formulation by Balmforth (1992b). In this paper we assume that the noise generation rate is predominantly from the fluctuating Reynolds stresses. This assumption is supported by the latest hydrodynamical simulation in the Sun by Stein & Nordlund (2001; for a recent review see Houdek 2002).

Amplitudes were estimated first for a solar model. The 'maximum' value of the theoretical solar amplitudes (defined to be $\sqrt{2}$ times the rms value) was then scaled to 0.23 m s^{-1}, a value obtained from observations by the BiSON group (Chaplin et al. 1998). The scaling factor so obtained has the value 2.0 and was used for all the models.

3 RESULTS

We computed several models for ξ Hydrae with various values of mass M, luminosity L, effective temperature $T_{\rm e}$ and chemical composition X and Z, where X and Z are respectively the abundance by mass of hydrogen and heavy elements. The models are summarized in Table 2. In Fig. 1 the computed linear damping rates η are plotted for the central model m1 (see Table 2) and joined by the solid lines. Near the frequency $\nu \simeq 110$ μHz the damping rates exhibit a deep depression. It is related to the properties of the outer superadiabatic boundary layer – in particular, its thermal relaxation time (Balmforth 1992a); the coupling between pulsation and the radiative processes in this boundary layer is very efficient, thereby promoting the depression in the damping rates (cf. Houdek et al. 1999). We moderate this sharp depression by applying a median smoothing filter which returns the median value of 7 consecutive values of η, and thereby works like a boxcar filter. The value 7 for the filter width is arbitrary; it was chosen to iron out small-scale fluctuations without obliterating the overall frequency dependence of η. Fig. 1 also includes the results of filtering with 5 and 9 modes, to provide an idea of how the filtering influences the predicted velocity amplitudes.

Table 2. Estimated model parameters and apparent oscillation velocity amplitudes for ξ Hydrae.

Model	$T_{\rm e}$ (K)	L (L$_\odot$)	M (M$_\odot$)	X	Z	$\hat{V}$ (m s^{-1})
m1	4950	60	3.31	0.70	0.030	2.08
m2	4950	60	3.60	0.70	0.030	2.10
m3	4950	60	3.00	0.70	0.030	2.78
m4	4950	70	3.31	0.70	0.030	2.90
m5	4950	50	3.31	0.70	0.030	1.82
m6	4850	60	3.00	0.70	0.030	2.47
m7	5050	60	3.00	0.70	0.030	3.00
m8	4950	60	3.31	0.70	0.024	1.98
m9	4950	60	3.31	0.70	0.036	2.22
m10	4950	60	3.31	0.72	0.030	2.25
m11	4950	60	3.31	0.68	0.030	2.11

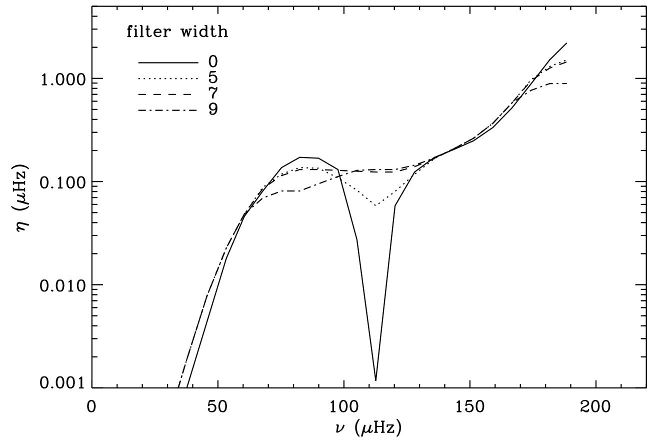

Figure 1. Theoretical damping rates η as a function of frequency for the central model m1. The solid lines join the raw damping rates; the other lines join rates smoothed over 5 (dotted), 7 (dashed) and 9 (dot–dashed) orders n respectively. Granted that the final amplitude estimates are inversely proportional to the square root of the damping rate, one can estimate from this figure the effect of the smoothing on the predicted amplitudes.

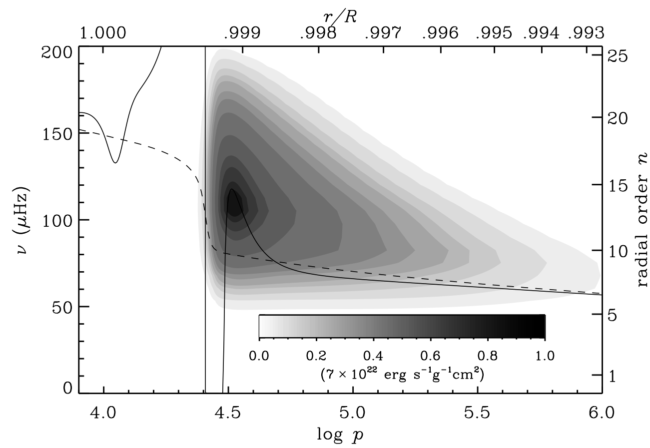

Figure 2. The contours represent the integrand of the acoustic energy supply rate, divided by the mode inertia, as a function of frequency ν and the depth variable log p (p being the total pressure) for the central model m1 computed in the manner of Balmforth (1992b). The solid and dashed curves denote the exact acoustical potential (for the Lagrangian pressure perturbation) and the corresponding potential obtained by replacing the acoustical cutoff frequency by $c/2H_p$, where c is the adiabatic sound speed and H_p is the pressure scaleheight, the latter potential being more pertinent to asymptotic representation (Gough 1993).

Fig. 2 shows the normalized energy supply rate to the acoustic oscillations from the turbulent motion per unit mass of the star (i.e., it is proportional to the quantity to be integrated over the mass of the star to obtain the total rate of energy injection P, except that it has been divided at each frequency by the total inertia I of a low-degree mode – which is essentially independent of degree (e.g., Christensen-Dalsgaard & Gough 1982) – in order that it represents the contribution to the power in the observed surface velocities: $\frac{1}{2}\hat{V}^2 = P/2\eta I$) computed in the manner of Balmforth (1992b). As is the case for the Sun, the region of greatest normalized acoustic energy supply rate to a particular mode is located near the upper turning point of that mode.

The oscillation velocity amplitudes are estimated from the computed linear damping and energy supply rates. In Fig. 3 the

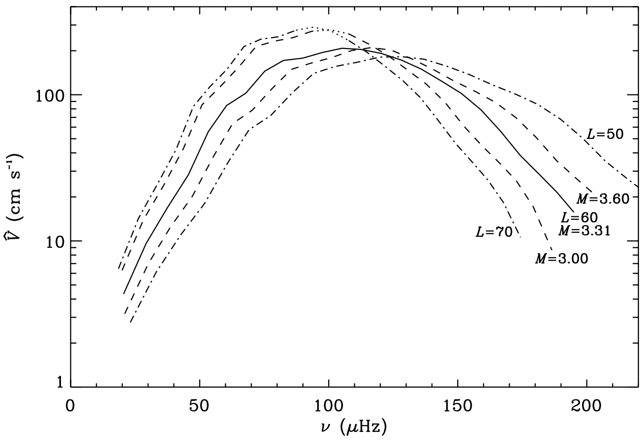

Figure 3. Estimated apparent velocity amplitudes as functions of frequency. The solid curve represents the result for the central model m1 with $L = 60\ \mathrm{L}_\odot$, $M = 3.31\ \mathrm{M}_\odot$ and $Z = 0.03$. The dashed curves are the results for models with different masses but with the same values for L, T_e and Z: $M = 3.6\,\mathrm{M}_\odot$ (m2) and $M = 3.0\,\mathrm{M}_\odot$ (m3). Results for varying luminosity L but for fixed values for M, T_e and Z are displayed by the dot–dashed curves: $L = 70\ \mathrm{L}_\odot$ (m4) and $L = 50\ \mathrm{L}_\odot$ (m5). Dotted segments of the curves indicate linearly overstable modes; the damping rates used for computing the amplitudes are all positive, and were obtained by smoothing the positive raw values, and thereby using the smoothing function as an interpolation formula to replace the negative values.

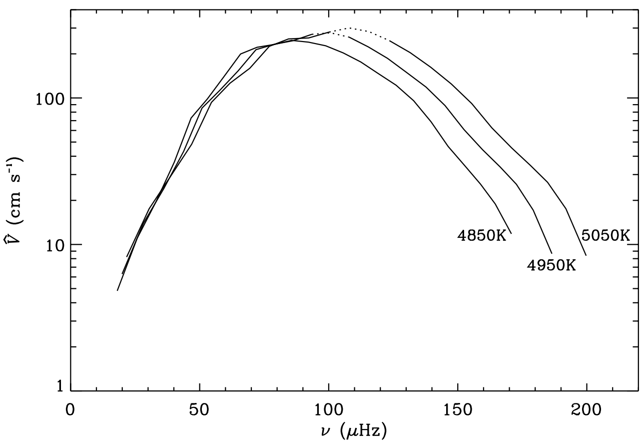

Figure 4. Estimated apparent velocity amplitudes as functions of frequency. The results are displayed for models with varying effective temperature: $T_\mathrm{e} = 4850\,\mathrm{K}$ (m6), $T_\mathrm{e} = 4950\,\mathrm{K}$ (m3) and $T_\mathrm{e} = 5050\,\mathrm{K}$ (m7). The remaining model parameters are fixed.

apparent velocity amplitudes are plotted for models with different masses (dashed lines) and different luminosities (dot–dashed lines); the solid lines join the results for the central model m1.

The dependence of the velocity amplitudes on effective temperature T_e is illustrated in Fig. 4. The amplitudes are increasing with T_e, with model m7 showing the largest amplitudes of all models considered.

The effect on the oscillation amplitudes of varying the chemical composition is shown in Fig. 5; the amplitudes increase with Z as a result of the increase in opacity which reduces the radiative flux, causing an increase in convective velocities (see also Houdek et al. 1999). Similar small changes in $\hat{V}$ are obtained by varying only the hydrogen abundance X (models m10 and m11 in Table 2).

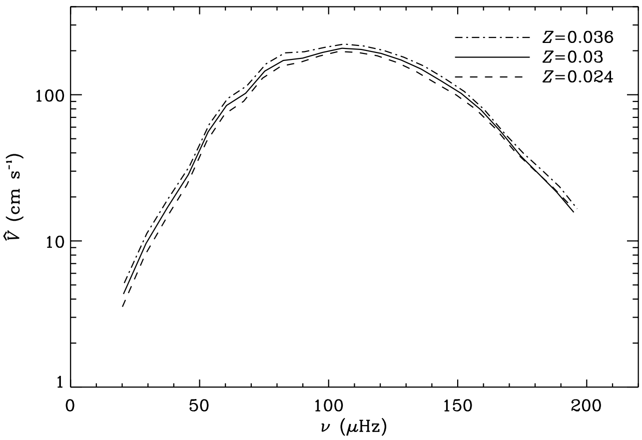

Figure 5. Estimated apparent velocity amplitudes as functions of frequency. The results are displayed for models with varying metallicity: $Z = 0.024$ (m8), $Z = 0.03$ (m1) and $Z = 0.036$ (m9). The remaining model parameters are fixed.

4 CONCLUSIONS

The excitation theory reproduces the observed amplitudes of solar-like oscillations in the red giant star ξ Hydrae tolerably well. It agrees substantially better than either of the two scaling laws suggested by Kjeldsen & Bedding. There are, of course, many uncertainties in the theory, and some uncertainty in the chemical composition and in the parameters M, L, T_e that characterize ξ Hydrae. Yet on the whole our predicted oscillation amplitudes are fairly robust against variations of the several parameters adopted in the computations. We believe, therefore, that our results are a fairly reliable representation of the theory.

One of the most important uncertain aspects of the theory lies in the values of the parameters a and b, which determine the degree of non-locality of the mixing-length model. These influence both the damping and the driving of the modes. Since the predicted oscillation amplitude is inversely proportional to the difference between damping and driving, it is particularly sensitive to uncertainty in the theory in the frequency range in which there is near cancellation. We have accordingly tried to suppress that sensitivity by smoothing the predicted net damping rates of the modes (see Fig. 1), as was done previously (Houdek et al. 1999). Without smoothing, one or more of the mode amplitudes would be anomalously high. The results are relatively insensitive to the value adopted for the shape parameter Φ.

We must point out, however, that the theory yields much too great an amplitude for the relatively hot star Procyon. We do not yet know why that is so. Inspection of the most obvious properties of the models that relate to the convection, such as $\nabla - \nabla_\mathrm{ad}$ and the Mach number of the convective flow, reveals that in the region where they are relatively large the values for the poorly modelled Procyon are greater than those of β Hydri, ξ Hydrae and the Sun (see Fig. 6). Thus we are led to suspect that the fault lies in modelling the most vigorous convection, which arises principally in stars with high effective temperature. Indeed, the assumption of a unique eddy scale at any location, which we have already noted causes too much smoothing if the theoretical estimates of the non-locality parameters a and b are adopted, also yields too high a Mach number in the highly superadiabatic boundary layer, and is thereby expected to lead to an overestimate of the oscillation amplitudes. Evidently it would be helpful for yet more sophisticated theories to have oscillation data from an even hotter star.

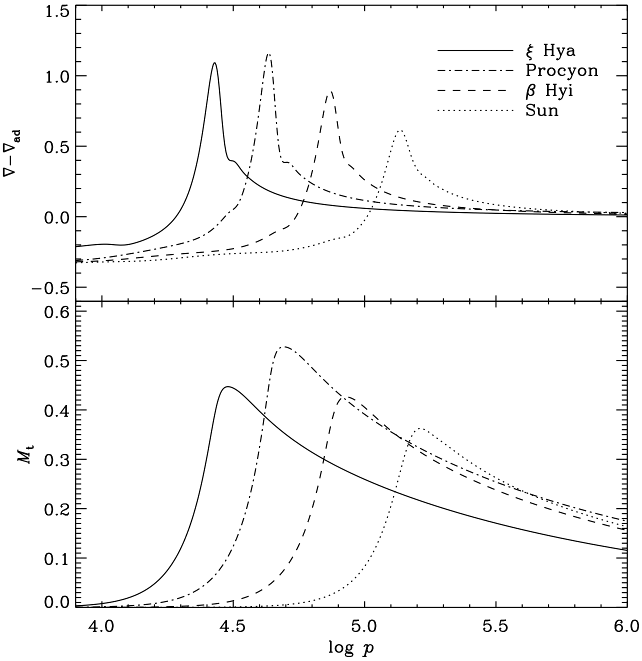

Figure 6. Superadiabatic temperature gradient (top panel) and turbulent Mach number (bottom panel). Results are plotted for envelope models of the stars ξ Hya (m1), Procyon, β Hyi and for the Sun.

Although one might take some comfort in the rough agreement between theory and the few greatest of the observed amplitudes, it would be preferable for testing the theory more carefully if it were possible to find a more robust comparison that is both less sensitive to random fluctuations and is not limited to those modes with low damping rates whose theoretical amplitudes are probably the least reliable. That comparison should take account of all the oscillation data, and not just the modes that happen to have the greatest amplitudes. It should also take due account of where in the atmosphere the modes are sampled.

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