MODELLING THE DAILY BANKNOTES IN CIRCULATION IN THE CONTEXT OF THE LIQUIDITY MANAGEMENT OF THE EUROPEAN CENTRAL BANK

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Abstract

The main focus of this paper is to model the daily series of banknotes in circulation in the context of the liquidity management of the Eurosystem. The series of banknotes in circulation displays very marked seasonal patterns. To the best of our knowledge the empirical performance of two competing approaches to model seasonality in daily time series, namely the ARIMA-based approach and the Structural Time Series approach, has never been put to the test. The application presented in this paper provides valid intuition on the merits of each approach. The forecasting performance of the models is also assessed in the context of their impact on the liquidity management of the Eurosystem.

Keywords: Daily Forecast, Liquidity Management, ARIMA modelling, State Space modelling, Seasonality, Cubic Splines.

JEL: C22, C51, C53 and C59.

Non-Technical Summary

The Eurosystem has at its disposal a collection of instruments and procedures to influence interest rates and manage liquidity in the money markets. Money market liquidity refers to the balances held by banks on settlement accounts with the central bank. Generally speaking the objective of steering interest rates is achieved by managing the conditions that equilibrate supply and demand in the market for bank reserves. When assessing the liquidity needs of the banking system, it is necessary to take into account the expected value of the so-called 'autonomous liquidity factors' that affect the supply of bank reserves. These factors are called autonomous because they are beyond the control of the central bank or counterparties. Banknotes in circulation is one of the largest autonomous factors. It is a liquidity absorbing factor: cash withdrawals from banks, that translate in an increase in the level of banknotes in circulation, induce additional refinancing needs of banks which have to meet their reserve requirements with the Eurosystem.

The series of banknotes in circulation displays very marked seasonality, comprising weekly, monthly and annual patterns plus some calendar effects. The modelling of daily series that display seasonal patterns is not simple. Two major approaches for modelling seasonality in daily series have been suggested in the literature: the ARIMA-based approach of Bell and Hillmer (1983), and the structural time series (STS) model suggested by Harvey, Koopman and Riani (1997). To the best of our knowledge the empirical performance of the ARIMA-based approach and the STS model has never been compared. The application presented in this paper provides a valid comparative empirical assessment of their performance. This is particularly relevant as the nature of the STS model suggested by Harvey, Koopman and Riani (1997) incorporates the nonlinear structure of periodic cubic splines, while the ARIMA is linear in structure. Forecast combination models will also be built. These should serve to illustrate whether the models are encompassing or not.

The analysis of the performance of the models is done on the basis of their forecasting accuracy. Rather than focusing exclusively on standard statistical tests, the performance is assessed in the context of the liquidity management of the Eurosystem. The performance of the models is also compared to the performance of the current practice in the Eurosystem (referred to as AGF model in the paper). To date the forecasting of banknotes in circulation has been computed at a national level, i.e. the National Central Banks (NCBs) of the Eurosystem forecast their own respective balance sheet position and the European Central Bank (ECB) aggregates the NCBs forecasts. The quality of NCBs forecasts has been good so far. But there are two major reasons for also forecasting the volume of banknotes in circulation in the euro area directly. First, this forecast can be used to complement and

improve the national forecasts. Second, the introduction of euro banknotes in 2002 and the free movement of banknotes within the euro area may make the national forecasts less reliable.

Results presented suggest that the two major approaches, i.e. the ARIMA-based approach, and the STS approach are powerful and display a performance which is up to the standards of the current aggregated forecast approach employed by the Eurosystem. Nonetheless, the expert knowledge incorporated in the AGF model is key over certain holiday periods. The ARIMA model has the best forecasting performance over horizons of 5 days and above, while the STS is best over horizons of 1 to 4 days. The best forecasting model is a combination of the ARIMA and STS models. This may point to the fact that certain seasonal patterns may not be completely captured by a linear structure.

The assessment of the performance of the models has also been conducted in the context of the liquidity management of the Eurosystem. The error in anticipating the liquidity needs due to forecasting banknotes in circulation never exceeds ± 1 billion of euro for any of the models. A total of eight corrections to a benchnmark allotment strategy for main refinancing operations resulted from the forecasting errors of the ARIMA model, nine from the STS model and eight from the AGF model. The combination of forecasts from both the $ARIMA$ and the STS resulted in only two corrections and clearly outperform the other models.

These econometric models have been used in 'real time' by the ECB from July 2001. The role played by the models was mainly that of checking the quality of the AGF forecast, and under some circumstances, to adjust it. The 'real time' testing of the models by the liquidity management unit of the ECB showed that the models had difficulties in capturing 'exceptional' effects, such as the patterns associated with the cash-changeover process. These patterns were very pronounced towards the end of the year 2001 and first weeks of 2002. This meant that expert knowledge from NCBs played a prominent role during that phase. It seems sensible to expect the performance of the models to become better again once the cash changeover process is completed. Nevertehless, from a practitioner's viewpoint, it is necessary to undertake a thorough assessment of the quality of the model's forecasts over a period of time which also includes the cash-changeover process.

1 Introduction

The Eurosystem has at its disposal a collection of instruments and procedures to influence interest rates and manage liquidity in the money markets. Money market liquidity refers to the balances held by banks on settlement accounts with the central bank. Generally speaking the objective of steering interest rates is achieved by managing the conditions that equilibrate supply and demand in the market for bank reserves. The Eurosystem has at its disposal three different types of instruments which determine the market for bank reserves: *minimum reserves*, *standing facilities* and *open market operations*[1]. Credit institutions in the euro area are required to hold minimum reserves on accounts in the NCBs. The fulfillment of minimum reserve requirements is measured on the basis of the institutions' average daily reserve holdings over a one-month *maintenance period*. The standing facilities provide and absorb overnight liquidity. There are two standing facilities: the *marginal lending facility* and the *deposit facility*. These facilities are available to eligible counterparties. Counterparties can obtain on their own initiative unlimited overnight liquidity from the NCBs at the pre-specified interest rate of the marginal lending facility, in so far as sufficient underlying eligible assets are presented as collateral. The deposit facility allows counterparties to make 'unlimited' overnight deposits with NCBs at a pre-specified interest rate. The pre-specified interest rate on the marginal lending facility and the deposit facility define a corridor for the market overnight interest rate. The Eurosystem has at its disposal different categories of open market operations. The *main refinancing operations* are the most important open market operations conducted by the Eurosystem. Main refinancing operations are reverse transactions whereby the Eurosystem conducts credit operations with a maturity of two weeks against eligible assets that serve as collateral. These operations are executed every week in the form of tender procedures.

When making a decision on the amount alloted, or in other words when assessing the liquidity needs of the banking system, it is necessary to take into account the expected value of the so-called 'autonomous liquidity factors' that affect the supply of bank reserves. These factors are called autonomous because they are beyond the control of the central bank or counterparties. Banknotes in circulation is one of the largest autonomous factors. It is a liquidity absorbing factor: cash withdrawals from banks, that translate in an increase in the level of banknotes in circulation, induce additional refinancing needs of banks which have to meet their reserve requirements with the Eurosystem.

The series of banknotes in circulation displays very marked seasonality, comprising weekly,

[1] A detailed account of the Eurosystem's monetary policy instruments and procedures can be found in ECB (2002b). See also Borio (1997) for a survey on the implementation of monetary policy in industrial countries.

monthly and annual patterns plus some calendar effects. The modelling of daily series that display seasonal patterns is not simple. Two major approaches for modelling seasonality in daily series have been suggested in the literature: the ARIMA-based approach of Bell and Hillmer (1983), and the structural time series (STS) model suggested by Harvey, Koopman, and Riani (1997). To the best of our knowledge the empirical performance of the ARIMA-based approach and the STS model has never been compared. The application presented in this paper provides a valid comparative empirical assessment of their performance. This is particularly relevant as the nature of the STS model suggested by Harvey, Koopman, and Riani (1997) incorporates the nonlinear structure of periodic cubic splines, while the ARIMA is linear in structure. Forecast combination models will also be built. These should serve to illustrate whether the models are encompassing or not.

The analysis of the performance of the models is done on the basis of their forecasting accuracy. Rather than focusing exclusively on standard statistical tests, the performance is assessed in the context of the liquidity management of the Eurosystem. The performance of the models is also compared to the performance of the current practice in the Eurosystem. To date the forecasting of banknotes in circulation has been computed at a national level, i.e. the National Central Banks (NCBs) of the Eurosystem forecast their own respective balance sheet position and the European Central Bank (ECB) aggregates the NCBs forecasts. The quality of NCBs forecasts has been good so far. But there are two major reasons for also forecasting the volume of banknotes in circulation in the euro area directly. First, this forecast can be used to complement and improve the national forecasts. Second, the introduction of euro banknotes in 2002 and the free movement of banknotes within the euro area may make the national forecasts less reliable. The empirical results presented in this paper refer to the period January 1994 to February 2001. This means that the impact of the cash changeover process (the conversion of national coins and notes into euro coins and notes) is excluded completely from the analysis presented in this paper.

The paper is organized as follows. Section 2 provides a brief description of the Liquidity Management of the Eurosystem. Section 3 describes the series of banknotes in circulation in the euro area. The ARIMA model is described in section 4, the Structural Time Series Model in section 5, and the model-judgement approach currently employed in the Eurosystem is described in section 6. The combination of models is described in section 7. Section 8 presents standard predictive accuracy tests of the models, and also an assessment on their performance in the context of the liquidity management of the Eurosystem. Finally section 9 concludes.

2 The Liquidity Management of the Eurosystem

By definition, only transactions between a bank and the central bank can change money market liquidity. Transactions between banks can only change the individual liquidity position of those banks involved in the transaction. This means that the balance sheet of the central bank provides a daily picture of the liquidity position, i.e. the imbalance between supply and demand of reserves, see, Borio (1997) and Bindseil and Seitz (2001). Table 1 displays a very simplified balance sheet of the central bank. From a liquidity management perspective, a distinction should be made between the following three categories of balance sheet items: *autonomous factors*, *net lending to banks* and *bank reserves*. Autonomous factors are related to central bank activities or services neither determined by the central bank's liquidity management nor by counterparties. In our simple balance sheet in table 1, autonomous factors refer to: *banknotes in circulation*, *net foreign assets*, *government deposits* and *other autonomous factors*. Banknotes in circulation are one of the major autonomous factors influencing liquidity. A central bank usually has the exclusive right to issue banknotes and coins, but is not able to control the outstanding amount. The users of banknotes determine the amount they want to hold. The development of banknotes is mainly driven by demand, and therefore the volume of banknotes should be considered as an 'autonomous liquidity factor'.

Table 1: Stylized Balance Sheet of a Central Bank.

Assets	**Liabilities**
Δ Net lending to banks	Δ Banknotes in circulation
Δ Net foreign assets	Δ Bank reserves
Δ Other autonomous factors	Δ Government deposits

The item net lending to banks refers to the net liquidity created through central bank monetary policy operations, and is therefore, directly controlled by the central bank. The main components of net lending to banks are the open market operations and the standing facilities.

The *bank reserves* of counterparties with the Eurosystem can be considered a residual position which balances the balance sheet.

The main refinancing operations of the Eurosystem are geared towards the objective of steering interest rates. The amount of liquidity injected by these operations should be enough for banks to fulfill their reserve requirements without making use of the standing facilities. Otherwise, this would translate in upward pressure on the overnight interest rate. Following Bindseil and Seitz (2001) and ECB (2002a), a benchmark allotment strategy for main refinancing operations could be well described by the following equation:

$$L_t = RR^* + \frac{1}{p}\sum_{i=t}^{t+p-1} E_{t-2}\{A_i\} + \frac{1}{p}\sum_{i=1}^{t-1}(RR^* - RB_i) \quad (1)$$

where L_t denotes the amount of liquidity alloted by main refinancing operations on day t; RR^* denotes reserve requirements adjusted to take also into account the excess reserves held as a safety margin; A_i is the value of the autonomous factors on day i and E_{t-2} denotes expectations made at period $t-2$; RB_i are the reserve balances held by banks at day i; and p is an integer that equals the 'relevant' forecasting horizon as explained below. For the purposes of this paper the liquidity provided by means of past open market operations, including past main refinancing operations, together with the use of the standing facilities will be treated as an autonomous factor[2]. In the Eurosystem the period over which the reserve requirements are computed precedes the period over which they must be fulfilled. Under these circumstances the central bank knows the exact demand for reserves. Applying equation 1 requires an accurate forecast of the *autonomous factors* that affect the supply of bank reserves, the term $E_{t-2}\{A_i\}$. The amount alloted to the main refinancing operations will be computed on the basis of these forecasts. Central banks devote large resources to maintaining and improving the quality of liquidity forecasts. Accurate liquidity forecasts are of special importance for the Eurosystem due to the relatively low frequency of its main refinancing operations. Expectations are formed with information available at time $t-2$. This follows from the fact that the main refinancing operations are settled on Wednesdays, but the allotment decisions are made on Tuesdays by the ECB based on the last available information from Monday afternoon. The 'relevant' forecasting horizon should cover all days before the settlement of the next main refinancing operation. Main refinancing operations are conducted on a weekly basis, implying a need for forecasts for the autonomous factors from 1 to 7 days ahead, i.e. $p = 7$ in equation 1. The value of p is only different from 7 when the next main refinancing operation is beyond the end of the current maintenance period. The value of p is then either equal to i) the number of days remaining until the end of the maintenance period (if this occurs after the day of the settlement of the main refinancing operation), or ii) the number of days in the period that goes from the end of the maintenance period to the settlement of the next main refinancing operation (if the end of the maintenance period occurs prior to the settlement of the current main refinancing operation).

Note that in order to conduct the weekly main refinancing operations, weekly observations (rather than daily observations) of banknotes in circulation would be sufficient. This would

[2]See ECB (2002a) for a more detailed analysis on the benchmark alltment rule normally applied by the ECB in its main refinincing operations.

reduce the computational burden on the models. Notwithstanding, the daily modelling allows a closer monitoring of the liquidity situation. For example, the central bank may decide to conduct a 'fine' tuning operation if the daily monitoring warns of an extraordinary liquidity shock.

3 The Series of Banknotes in Circulation

Banknotes in circulation is the most important autonomous factors in the context of the Eurosystem liquidity management, both in terms of absolute size and in terms of volatility. It represents approximately 35% of the root mean square of total weekly changes in autonomous factors. It is a liquidity absorbing factor. Cash withdrawals from banks, that translate in an increase in the level of banknotes in circulation, induce banks to refinance those withdrawals to meet their reserve requirements with the ECB.

The log of the series of banknotes in circulation in the euro area is shown in figure 1. This series adds up the level in euros of the banknotes in circulation denominated in the 12 national currencies of the countries of the euro area. This series displays very marked seasonal patterns, which reflect certain regularities in payments and receipts as well as patterns in the consumption behavior associated with holiday periods. Weekly, monthly and annual seasonal patterns clearly appear. The amount of banknotes in circulation increases just before the weekend and decreases after the weekend (*trading day effect*). It also decreases before the middle of the month and increases towards the end as a result of the payment of salaries. The amount of banknotes in circulation rises during the summer holidays and towards the end of the year, particularly around Christmas. Public holidays other than Christmas (Easter, Ascension day, Pentecost) also have a strong impact.

The series of banknotes in circulation displays a clear trend. This trend component is not always upward. The trend mainly reflects the expansion of economic activity in nominal terms. The development of means of payment alternative to currency by financial institutions can also affect the trend pattern of the series of banknotes in circulation. This may be so insofar as they become increasingly accepted by the public. For example, the introduction of electronic means of payment tends to reduce the amount of banknotes in circulation demanded by the public. In addition, the development of an extensive network of automated teller machines (ATMs) also reduces the use of banknotes. The impact of ATMs may also be reflected in the seasonal patterns of banknotes in circulation. In particular it may change patterns associated with the trading day.

But movements in the trend component are not exclusively associated with economic activity and developments in means of payments. These could not explain the two periods over which the trend displays a negative slope coefficient: the second half of 1998 and from the 4th quarter of 2000 to the final date in the sample under study. This last negative trend pattern is associated with the preparations for the change over from national currencies to euros in January 2002. Both euro area and non-euro area residents are likely to reduce their holdings of the legacy currencies of the euro well ahead of the cash changeover. This is particularly true for those economic agents that have accumulated large amounts of banknotes in the past, e.g. through idle savings. In addition, non-euro area residents may not always be well informed about the possibilities of changing national currencies into euro. The change in behaviour in 2001, compared with preceding years, can clearly be seen in Figure 1. The models presented in this paper will be specified on the basis of their in sample performance over the period 3-Jan-1994 to 20-Feb-2000. Banknotes in circulation over the period December 1999 to January 2000 increased drastically in anticipation of potential problems related to the Y2K computer bug. The final date chosen for the in sample period (20th of February) is sufficiently distant from the 1-Jan-2000 to fit an intervention dummy variable to deal with the year 2000 effect. The remaining sample, i.e. from 21-Feb-2000 to 20-Feb-2001, will be used to assess the forecasting performance of the models.

4 ARIMA Model

The ARIMA model presented in this paper is in the spirit of that proposed by Bell and Hillmer (1983) and Box and Tiao (1975). Their model is a sum of a regression model and an ARIMA model, and can be written as:

$$y_t = D_t + \frac{\theta(B)}{\phi(B)\delta(B)}\varepsilon_t \qquad (2)$$

The regression component is defined as $D_t = \sum_{i=1}^{k} d_{t,i}$, for k equal to the number of *calendar variation* effects, and where $d_{t,i}$, is a function of a fixed vector of independent 'time dependent' variables, defined below. The second summand in the equation above provides the ARIMA component, B is the backshift operator, $\phi(B)$ and $\theta(B)$ are polynomial lag operators with all their zeros outside the unit circle, and with no common zeros, and $\delta(B)$ is a differencing operator like for example $(1 - B)$. Finally, ε_t is an *iid* stochastic process of zero mean and variance σ^2.

The regression component D_t is used to model several deterministic effects, like *calendar variation* effects, i.e. the impact that changes in the positioning of holidays in the calendar from year to year has on the series under study, y_t. A typical example is the Easter holidays, which may occur either in March or in April. The definition of calendar variation effects applies also to 'fixed holidays' such as the 1st of May. This is so because it is relevant whether the 1st of May falls on a particular day of the week, and therefore, strictly speaking its position changes also from year to year. The ARIMA component serves to model autocorrelation patterns, seasonality patters and trend patterns. For our purposes,the vector of time dependent variables $d_{t,i}$ are modelled as follows:

$$d_{t,i} = \frac{w_i(B)}{1-\rho_i B} h(\tau_i, t) \tag{3}$$

where as before, B denotes the backshift operator, $w_i(B)$ is a polynomial lag operator and ρ_i is a parameter, both associated with the i-th calendar variation effect, finally $h(\tau_i, t)$, is an indicator function that takes the value of 1 when $t = \tau_i$ and a value of zero otherwise, where τ_i is a date associated with a particular calendar variation effect, for example Easter Friday. In order to make this seasonal dummy fully compliant with a seasonal specification the values adopted by the indicator function would be $1 - f$ and $-f$, where the value of f depends on the frequency over which the dummy is defined. An alternative specification for $d_{t,i}$, could be that used by Pierce, Grupe, and Cleveland (1984) in the context of the seasonal adjustment of weekly monetary aggregates. This takes the form of a trigonometric function which fits deterministic seasonal patterns well. This alternative functional form is as follows:

$$d_{t,i} = \sum_{k=1}^{p} \left(a_j sin \frac{2\pi k m_t}{M_t} + b_j cos \frac{2\pi k m_t}{M_t} \right) \tag{4}$$

where m_k is an integer which gives the position at time t of a particular observation over a defined frequency M_t. For example, for monthly seasonal patterns m_t gives the day of the month, while M_t gives the number of days in that particular month. The value of p should be large enough for this variable to account for all the seasonality.

Bell and Hillmer (1983) provided also a model building procedure for this type of ARIMA model which follows closely the three stage strategy (identification, estimation and diagnostic checking) proposed by Box and Jenkins (1976). Under the assumption of normality of the observations, the likelihood function can be expressed in terms of the prediction errors and their corresponding variances, how to do this is well documented in Brockwell and Davis (1991). A common approach to compute the exact likelihood function is to write the ARIMA model in its state space form, then the Kalman filter recursions would provide the prediction errors, see Bell and Hillmer (1991).

4.1 An ARIMA model of banknotes in circulation

4.1.1 The deterministic structure

Table 2 shows the structure of the different deterministic components for the ARIMA model of banknotes in circulation. A similar identification strategy to that proposed by Bell and

Hillmer (1983) and Box and Tiao (1975) was followed to search for the structure of these deterministic components. The notation used in the tables is in line with that used in equation (3), and follows the explanations given above.

Fixed Festivals and Moving Festivals. The Christmas effect is the most complicated pattern to capture in this model. The Christmas and New Year effects are estimated using the same reference date, namely a dummy that takes a value of one on Christmas day, or the day before if it falls on Saturday or Sunday. The effect of increased banknote withdrawal is only significant between six and four working days before Christmas day (see table 2). This effect has also been estimated as significant in the three days following Christmas, which captures in some measure the New Year effect. The corresponding post Christmas decrease in banknotes (or end-of-year effect) lasting till mid-January, cannot be explicitly estimated by deterministic dummies. It is reasonable to suggest that this effect must be incorporated within the stochastic structure and/or the trigonometric variables. For this reason, the genuine New Year effect is incorporated in the variable reflecting the end-of-month effect. Any attempt to capture this end-of-year effect runs into multicollinearity problems.

Dummies associated with euro-area national public holidays have been tested. As it is to be expected, only those holidays common to most member countries are significant. These refer to the following: 1st May (95% of the euro area), 1st November (90%), Corpus Christi (65%), Whit Monday (65%), and Ascension (60%). Public holidays that fall on a Friday have been found to have a different effect from those associated with other days of the week. People withdraw banknotes from the system on Fridays to cover their weekend expenses. This withdrawal shifts to Thursdays whenever Friday is a public holiday. There are five such days in the period under study. Therefore, the model includes two rather than one dummy to deal with the effect of fixed holidays. Both have the structure displayed in table 2, but are estimated with different parameters. This variable is estimated to be significant and without correlations with other potentially 'conflicting' variables.

Intramonthly effect. Monthly patterns in the series of banknotes in circulation are associated with the payment of salaries in the middle and at the end of the month. This effect is captured by means of a trigonometric function like the one described in equation (4). The parameter p was fixed to 8. Figure 2 shows the intramonthly effect as a percentage of the level of the series of banknotes in circulation. The intramonthly effect fluctuates in between +1% and -1% of the level of the series.

Trading day effect. The trading day effect in which a zero-sum effect is estimated is highly significant. This shows the presence of a very robust weekly seasonal cycle. The level

Table 2: Fixed and moving holidays in models.[a]

Holiday (τ_i)	ARIMA Model $w_i(B)$	ρ_i	STS Model $w_i(B)$	ρ_i
Easter Friday	$\left(w_0 + w_1 B + \ldots + w_8 B^8\right) B^{-6}$	$\neq 0$	$\left(w_0 + w_1 B + \ldots + w_{12} B^{12}\right) B^{-4}$	0
Ascension	$\left(w_0 + w_1 B + w_2 B^2\right) B^{-3}$	$\neq 0$	$\left(w_0 + w_1 B + \ldots + w_5 B^5\right) B^{-2}$	0
Whit Monday	$\left(w_0 + w_1 B + \ldots + w_3 B^3\right) B^{-2}$	$\neq 0$	$\left(w_0 + w_1 B + \ldots + w_5 B^5\right) B^{-2}$	0
Corpus Christi	$\left(w_0 + w_1 B + \ldots + w_3 B^3\right) B^{-3}$	$\neq 0$	No	-
Christmas	$\left(w_0 + w_1 B + w_2 B^2\right) B^{-6}$	0	$\left(w_0 + w_1 B + \ldots + w_4 B^4\right) B^{-2}$	0
New year	See Text	-	$\left(w_0 + w_1 B + \ldots + w_5 B^5\right) B^{-2}$	0
Fixed Holiday	$\left(w_0 + w_1 B\right) B^{-1}$	0	$\left(w_0 + w_1 B + \ldots + w_6 B^6\right) B^{-2}$	0
Year 2K	$\left(w_0 + w_1 B + \ldots + w_4 B^4\right) B^{-4}$	$\neq 0$	See Text	-

[a]The polynomial backshift operators $w_i(B)$, have different coefficients for the different holidays displayed in the rows of the table; and the same applies to the different adjustment factors a_i. Fixed holidays correspond to the dates January 1st, August 15th and November 1st. The ARIMA model finds only a significant effect for those fixed holidays whenever they fall on a Friday.

of banknotes in circulation declines toward the middle of the week (Tuesday and Wednesday), and increases from Thursday to Friday when it peaks (when ATMs are filled for the weekend). The level of banknotes is also high on Mondays compared with Tuesdays and Wednesdays, as a result of a further withdrawal of banknotes by commercial banks.

Other deterministic variables. Other irregular phenomena have had an impact on the series of banknotes in circulation. In particular, the Y2K effect associated with the potential problems related to the Y2K computer bug led to a very strong increase in demand in the last four days of 1999 and a subsequent relatively fast run-down effect in January 2001. Figure 2 shows the estimated profile for this effect. The impact of the Y2K involved an increase of about 2.7% in the level of the series on 30th December 1999. This impact was mostly absorbed in the first few days of January and completely neutralized by mid-February. The inclusion of this variable proved to be crucial for the forecasting results presented in this paper for the period February 2000 - February 2001. If it had been left out, it would have resulted in bias in the estimation.

Finally, the model contains several outliers detected in the modelling process and identified and estimated following Chang, Tiao, and Chen (1988). The criterion adopted is to intervene only when the outliers are particularly striking. In fact, only two outliers have been considered, and both were estimated as impulse variables: December 6th 1995 and April 30th 1999. The only explanation for the negative impulse detected on 30th April (the day before the public holiday of 1st May) is that in 1999 this holiday was on a Saturday, i.e. not on a working day.

4.1.2 The stochastic structure

The parameters associated with the regular moving average structure are low in value. Those associated with seasonal frequencies are larger and therefore highlight the presence of very marked seasonality. The autoregressive structure shows a moderately significant seasonality in combination with a complex intra-year structure (month-on-month and quarterly). Likewise, the weekly and bi-weekly stochastic structure complements the trading day and intramonthly effects. The presence of correlation at a quarterly frequency is likely to be related to the payments of the Value Added Tax. The stochastic structure is described by the following equations:

$$
\begin{aligned}
\theta(B) &= \left(1 - \theta_2 B^2 - \theta_{11} B^{11} - \theta_{12} B^{12} - \theta_{17} B^{17}\right)\left(1 - \theta_{261} B^{261}\right) \\
\phi(B) &= \left(1 - \phi_1 B - \phi_3 B^3 - \phi_5 B^5 - \phi_6 B^6\right)\left(1 - \phi_{45} B^{45}\right)\left(1 - \phi_{65} B^{65} - \phi_{66} B^{66}\right) \\
\delta(B) &= (1 - B)(1 - B_{261})
\end{aligned}
$$

For analytical purposes it is also sensible to disentangle the 'stochastic' subcomponent into two components. First a non-stationary component, given by the structure of the differencing operator, $\delta(B)$, and the remaining parts of the stochastic structure. This results in three major subcomponents for the ARIMA model of banknotes in circulation: the non-stationary stochastic process, the stationary stochastic component and the deterministic component. For the series under investigation, the non-stationary component accounts for over 35% of the variance of the logarithm of the series, the stationary stochastic component for 27%, and the deterministic term approximately 30%.

The total number of parameters is 76. This figure is large, but is nonetheless necessary in order to accommodate the effect of the fixed and moving holidays. Otherwise the structure is fairly parsimonious. The specification of the model was done on the basis of the significance of the parameters and diagnostic tests on the structure of the residuals. Diagnostic tests for the final specification are reported in table 3. The tests reported are for skewness and kurtosis, for normality and the Ljung-Box statistic of serial correlation. The skewness and kurtosis statistics are normalized and therefore their probability values computed from a normal distribution. The normality test is the standard Bowman-Shenton test distributed as a chi-squared distribution with 2 degrees of freedom. The Lung-Box statistics are computed based on the first p autocorrelations, and are denoted as is standard as $Q(p)$, where the values of p chosen are related to weekly, biweekly, monthly and annual frequencies. Additionally the residual correlogram plotted in figure 4, is fairly satisfactory. The values for all autocorrelation coefficients are smaller than 0.1. There only remain a few problems, i.e. some significant (for a critical value of 5%) serial correlation coefficients at an annual frequency and bi-monthly frequency. The Ljung-Box statistics confirm this result, only for $Q(261)$ we can reject the null of no serial correlation for a critical value of 5%. Normality tests are not as good. Further improvements on the residual correlogram could come only by increasing very much the size of the deterministic component of the ARIMA model. This was decided against as it would deteriorate the forecasting performance of the model.

5 The STS Model

The STS model presented in this paper follows those proposed by Harvey, Koopman, and Riani (1997) and Groot, Koopman, and Ooms (1999). An observed univariate time series y_t is formulated in terms of components as:

$$y_t = \mu_t + \gamma_t + \varepsilon_t \tag{5}$$

where μ_t is an stochastic trend component defined below as a *local linear trend*, γ_t is an stochastic seasonal component, and the irregular component ε_t is an *iid* process with standard deviation σ_ε. The trend component μ_t has the following structure:

$$\begin{aligned} \mu_t &= \beta_{t-1} + \mu_{t-1} + \eta_t \\ \beta_t &= \beta_{t-1} + \nu_t \end{aligned}$$

where η_t and ν_t are *iid* processes normally distributed with mean zero and standard deviation σ_η and σ_ν respectively. The seasonal component is defined as the sum of k subcomponents which reflect alternative seasonality patterns, i.e. $\gamma_t = \sum_{i=1}^{k} \gamma_t^i$. Every seasonal subcomponent has the structure:

$$\gamma_t^i = \boldsymbol{z}_t^i \boldsymbol{\delta}_t^i \tag{6}$$

where $\boldsymbol{z}_t^i$ is a fixed vector of 'time dependent' variables, of dimension $g_i \times 1$. The values of $\boldsymbol{z}_t^i$ are defined over a limited range of the total sample T. For example, $\boldsymbol{z}_t^i$ is defined over the range $[1, 365]$ for annual seasonality patterns, and its values repeated periodically over the rest of the sample. To enforce that the seasonal effect adds to zero, and guarantee that γ_t^i is a truly seasonal component, the sum of $\boldsymbol{z}_t^i$ over the range it is defined should add to a vector of zeroes, i.e. when defined over an annual range $\sum_{k=1}^{365} \boldsymbol{z}_k^i = \mathbf{0}$. Finally, $\boldsymbol{\delta}_t^i$ is a time varying parameter which follows a random walk, i.e.

$$\boldsymbol{\delta}_t^i = \boldsymbol{\delta}_{t-1}^i + \boldsymbol{\chi}_t^i \tag{7}$$

where $\boldsymbol{\chi}_t^i$ is a $g_i \times 1$ vector of zero mean serially uncorrelated error processes with covariance matrix equal to $E\left(\boldsymbol{\chi}_t^i \boldsymbol{\chi}_t^{i\prime}\right) = \sigma_i^2 \boldsymbol{I}$. Not all seasonal components will be stochastic, when they are deterministic it will be assumed that $\sigma_i^2 = 0$.

Following Harvey and Koopman (1993), Harvey, Koopman, and Riani (1997) and Groot, Koopman, and Ooms (1999), some of the seasonal components γ_t^i will be modelled as *periodic seasonal cubic splines*. The use of trigonometric functions like those shown in equation (4) would have required a less parsimonious structure to accomodate the annual and monthly seasonal patterns. The cubic splines are a succession of polynomial functions of order three defined over a time range $[0, T_s]$ to approximate the seasonal pattern. For example, $T_S = 365$ for annual seasonal patterns and $T_S = 30$ for monthly seasonal patterns. The number and length of the subintervals in that range where the successive polynomial functions are defined, should be such that the seasonal pattern observed is well approximated. In the *cubic spline* terminology, the length and number of intervals is defined by the positioning of the *knots*. The *knots* provide the subintervals in which the range is divided. The cubic spline functions have certain properties imposed upon them: i) first, that the value of two consecutive polynomial functions must be the same at the coinciding *knots*, and ii) second, that the value of the first derivative of two consecutive polynomial functions must be the same at the coinciding knots. Further to these restrictions, and in order to make the cubic spline periodic, the value of the function at 0 and at T_s must be the same. Cubic spline functions admit a representation as a linear function of a vector of parameters $\boldsymbol{\delta}_t$; namely $\gamma_t^i = \boldsymbol{z}_t^i \boldsymbol{\delta}_t^i$ where $\boldsymbol{z}_t^i$ is a vector of known values of dimension $g \times 1$ (with corresponding dimensions for $\boldsymbol{\delta}_t^i$). Setting up the cubic spline requires to fix g, the number of *knots*, and the positioning of the knots in the range $[0, T_s]$. The values taken by the vector $\boldsymbol{z}_t^i$ depend on the number and positioning of the knots, and the length of the range over which the spline is defined. Once more, this type of component is made stochastic by allowing the vector of parameters $\boldsymbol{\delta}_t^i$ to follow random walks. For a more detailed analysis on cubic splines see Poirier (1976) and Harvey, Koopman, and Riani (1997).

The model can be written in State Space form and the Kalman filter implemented to extract the state component. Given that there are parameters to be estimated, Maximum Likelihood estimation in combination with the Kalman filter must be used. See Harvey (1989) for further details.

5.1 An STS Model of banknotes in circulation

The seasonal component is defined as the sum of five seasonal subcomponents which model five different patterns present in the daily data, i.e. $\gamma_t = \sum_{i=1}^{5} \gamma_t^i$. These subcomponents correspond to: i) *intra-yearly effects*, γ_t^1, ii) *intra-monthly effects*, γ_t^2, iii) *day of the week effect*, γ_t^3, iv) *moving festivals*, γ_t^4, v) *fixed festivals*, γ_t^5. The first three, intra-yearly, intra-monthly and day of the week effects are stochastic, while the last two are modelled as deterministic.

Years of 276 days are defined to accommodate leap years, and months of 23 days are used to accommodated all months. The fact that not all years are leap is not a problem, when this is the case the 29th of February is considered to be a missing value. The treatment of these days as missing values is easily handled by the Kalman filter iterations. The same applies to days such as 31st April.

Intra-yearly and intra-monthly effects The annual seasonal pattern displayed in the series will be modelled with time-varying cubic splines. The intra-monthly seasonal component will also have a similar structure. The main difference between the two is that $\boldsymbol{z}_t^1$ will enforce periodicity at an annual frequency, while $\boldsymbol{z}_t^2$ will do so at a monthly frequency. Selection of the number of knots and positioning of the splines was based on visual observation of the residual correlograms, goodness of fit performance and forecasting performance. The larger the number of knots the better the fit, but this comes at the cost of a deteriorating forecasting performance. In building the cubic splines two further issues have to be addressed. First, the model makes use of periodic cubic splines. This means that the value of the spline at the first knot and the value of the spline at the final knot should be the same (obviously this is only strictly true for deterministic rather than stochastic splines). This raises the issue of choosing the starting knot and final knot as two consecutive days of similar characteristics. Placing the first knot on 1st January and the last on 31st December proved to be a bad choice. The annual seasonal pattern of the banknotes series displays a sharp trend around Christmas, making 1st January and 31st December days with very different seasonal weights. The dates chosen as the first and final knot were the last two days of February for the annual spline. Following the same reasoning the first and last day in the month are less alike than two consecutive days in the third week of the month. Second, not all months have 23 working days, some have less, therefore there is a need to insert missing days in certain months to accommodate the splines. Therefore the issue of where to place the missing days needs to be addressed. The choice of end of month is not good because of the sharp trend of the monthly seasonal pattern towards the end of the month.

The final specification adopted for the intra-yearly spline was one of 16 knots, with knots placed at 1,23,99,121,140,166,202,207,213,220,226,231,234,239,246 and 276. But note that the knot 1 corresponds to the last day of February rather than to 1st January. For the intra-monthly spline 6 knots were sufficient with positioning at 1,6,10,14,17 and 23, and with periodicity imposed for the 17th and 18th day of the month (meaning that knot 1 corresponds to the 18th day of the month). After some experimentation, it was found best to place the missing days after the 17th day of the month. As stated above, the time varying parameters associated with these splines, $\boldsymbol{\delta}_t^1$ and $\boldsymbol{\delta}_t^2$, are modelled as random walks.

Day of the week effects Demand for banknotes displays also a day of the week pattern, there is higher demand of banknotes on Fridays in anticipation of purchases done over the weekend. To take this into account, we define $\gamma_t^3 = \boldsymbol{z}_t^3 \boldsymbol{\delta}_t^3$ where $\boldsymbol{z}_t^3$ is a vector of zeros and ones of dimension 4×1, the first element of this vector takes a value of 1 if t falls on a Monday, a value of -1 if on a Friday and 0 otherwise. Elements 2, 3 and 4 will have a similar pattern but matching Tuesday, Wednesday and Thursday respectively. This structure guarantees that the sum of the effect γ_t^3 adds to zero over one week. As above, the day of the week effect is made stochastic by modelling the 4×1 vector of parameters $\boldsymbol{\delta}_t^3$ as a random walk.

Fixed festivals and moving festivals The impact of moving festivals is modelled with deterministic dummy variables. This component of the STS model is defined as $\gamma_t^5 = \boldsymbol{z}_t^5 \boldsymbol{\delta}^5$, where the dummy elements in $\boldsymbol{z}_t$ are built according to equation (3) above, with the particularity that the parameter a_i is set to zero. Full details on the structure of the fixed and moving holiday seasonal dummies are given in table 2. Note that the parameter a_i in equation (3) plays the role of a discount factor. It would be very expensive computationally to use this discount factor in the setting of the STS model for all dummy variables. To avoid this computational burden the use of this discount factor is limited to the Y2K dummy, for all other dummies longer lags will be used. Note that according to the structures of the dummies described in table 2, this is the main only difference between the holiday dummies in the STS model and in the ARIMA model. The impact of the Y2K goes well beyond the beginning of the year and would require the use of a large number of leads. The final specification of the Y2K dummy is as follows:

$$d_{Y2K,t} = a_0 h_1(Y2K - t)\rho_1^{Y2K-t} + d_{Y2K,t-1} h_2(t - Y2K)\rho_2^{t-Y2K}$$

where a_0, ρ_1 and ρ_2 are parameters to be estimated, and $h_1(x)$ is an indicator function which takes the value of 1 for $x \geq 1$ and $h_2(x)$ is also an indicator function which takes the value of 1 for $x > 1$.

The total number of parameters of the STS model is 75. Once more this is a large figure, but necessary to deal with all holiday effects. Diagnostics test for the final specification of the STS model are reported in table 3, and the residual correlogram displayed in figure 4. Results for the normality tests on the residuals are in line with those of the ARIMA model, and therefore not entirely satisfactory. The shape of the residual correlogram is not as flat as that of the ARIMA. The main serial correlation problems for the STS are for frequencies shorter than one week, and for annual frequencies. This implies that the Ljung-box test always rejects the null of no serial correlation. The size of the correlation coefficients are nonetheless small, of 0.15 for serial correlation of order one and 0.13 for annual correlation. Following suggestions in Harvey, Koopman, and Riani (1997) we have experimented with a model that allowed for a double-variance for the knots associated with the Christmas period. This did not improve the results very much. Much more relevant for reducing the serial correlation in the residuals was the choice of the periodicity point for the specification of the splines. Improvements on the residual correlogram beyond what is reported in the tables could only be obtained by making the model much larger. This was ruled out to avoid damaging its forecasting performance.

6 The Aggregated NCBs Forecast Model (AGF)

To date the forecasting of banknotes in circulation is mainly computed at a national level, i.e. the National Central Banks (NCBs) of the Eurosystem forecast their own respective balance sheet position and the ECB aggregates the NCBs forecasts. Some NCBs are using econometric techniques while others are applying heuristic methods.

The Research Department at *Banco de España* has traditionally run and maintained an ARIMA model. The prediction results obtained from this model are not the final forecast but rather are used as a reference by the experts of the 'Liquidity Management Unit' of the Banco de España. The model chosen by *Banque de Frane* is a Structural Time Series Model. This model is similar to that described in this paper but with two major differences. First, trigonometric functions are used to model annual and monthly seasonal patterns. Second, additional structure is added to handle serial correlation and ARCH effects in the residuals. *The National Bank of Belgium* is applying an error correction model (ECM) with intervention dummies. The error correction term includes the difference between the past level of banknotes in circulation and its corresponding 'trend' component. This 'trend' component is the monthly series of banknotes in circulation interpolated to extend the series to a daily frequency. The monthly series is forecasted by expert knowledge. the intervention dummies are based on estimates for the cash transaction levels and for the distribution of withdrawals and deposits aroun transaction dates. *Deutsche Bundesbank* is applying a heuristic approach. In producing a forecast, information from three major different sources is assembled. First, the growth rates of the monthly seasonally adjusted series of banknotes in circulation. Second, expert knowledge of the impact of Easter, Christmas and other holiday periods, as well as monthly and weekly patterns as observed in the previous years. Third, the latest available data for the daily series. The other seven NCBs are mainly using expert knowledge in producing their forecast.

7 Combination of Model Forecasts

Traditional model selection methods search for an optimal model out of a set of candidate models. Optimality is defined in terms of certain statistical criteria, i.e. adjusted R^2, minimum mean forecasting square error, etc. Model selection methods rely on a number of statistical assumptions on the candidate models, i.e. linear structure, exogenous variables, endogenous variables, and so on.

But adopting one particular model and discarding the rest might not be an optimal strategy. In this paper's modelling scenario, alternative strategies are followed by the different models to specify certain seasonal patterns. Also, over certain periods, events such as the huge increase in the volume of banknotes in circulation in the weeks before 1st January 2000, are very difficult to model ex-ante, and it is sensible to say that forecasts from 'experts' should be preferred. This suggests that useful information, in addition to that in a chosen model, may be available in the discarded models.

An alternative strategy could be to combine the alternative forecasts rather than selecting an optimal model; see Clemens (1989) for a review on forecast combination methods. If our aim is to minimize the mean forecasting square error, then there are gains in combining the forecast from two models whenever their corresponding forecast errors are negatively correlated.

The forecast combination method used in this paper is the regression method suggested by Granger and Ramanathan (1984). This method is equivalent to the variance-covariance method of Bates and Granger (1969) under the specification followed in this section. For an observed series of banknotes in circulation y_t for $t = 1$ to T and two alternative forecasts f_t^a and f_t^b also for $t = 1$ to T, the optimal weights in the Mean square error sense are given by the OLS regression parameters from the equation:

$$y_t = \alpha f_t^a + (1 - \alpha) f_t^b + \varepsilon_t$$

where ε_t is an *iid* noise component. For the case under study in this note, the forecasted series displays very marked seasonality patterns. It is sensible to think that certain model might perform better over certain time periods. For example, the fluctuations around Christmas might be more difficult to forecast with the STS or ARIMA model than with the AGF model. But for 'standard' months, such as February the STS or ARIMA might be best. The forecast combination strategy adopted in this paper attempts to incorporate this argument by computing different weights (different α values) for the different months, i.e. $\alpha_{jan}, \alpha_{feb}, \ldots, \alpha_{dec}$. Different weights are also estimated for the different forecast horizons.

The empirical analysis builds upon three alternative forecasts: the ARIMA model forecast, the STS model forecast, and the AGF forecast. The results for three alternative combinations will be presented: i) combination of AGF with ARIMA and denoted as C_{dmj}^{arima} in the tables below, ii) combination of AGF with STS denoted C_{dmj}^{sts}, and iii) combination of ARIMA with STS denoted C_{arima}^{sts}. The weight parameter α for the different months has been computed with the $t+1, t+2, \ldots t+10$ 'in sample' errors for the ARIMA and STS model. Only $t+1$, $t+5$ and $t+10$ forecasting residuals over the period Feb-1999 to Feb-2000 are available forthe AGF forecasts. The weights computed for $t+1$ are used for $t+2$ and $t+3$ step ahead forecasts, the weights for $t+5$ used in 4, 5, 6 and 7 step ahead forecast and the weights estimated from the $t+10$ residuals used in the 8, 9 and 10 steps ahead forecasts.

8 Forecasting performance

The models were recursively estimated overthe forecasting period and forecasts from 1 to 10 periods ahead were computed. Forecastingperformance will be assessed on the basis of the root mean square forecast error (RMSE), and the predictive accuracy test proposed by Diebold and Mariano (1995). This test is an extension of the Central Limit Theorem to dependent processes. The test is designed to test the null of equal predictive ability between two models. Assuming a quadratic loss function for evaluatingforecasting performance we consider the mean of the differences of squared prediction errors of the two competing models. This mean, suitably normalized, has a standard normal distribution under the null. The test statistic is given by

$$S_{DM} = \frac{\bar{d}}{\sqrt{2\pi h_d(0)}} \xrightarrow{d} N(0,1)$$

where $\bar{d} = \frac{1}{N}\sum_{i=1}^{N} \hat{d}_i$, $\hat{d}_i = \hat{\eta}_{\mathrm{A},i}^2 - \hat{\eta}_{\mathrm{B},i}^2$, $i = 1, \ldots, N$, $\hat{\eta}_{\mathrm{A},i}$ are the prediction errors from model A and $\hat{\eta}_{\mathrm{B},i}$ are the prediction errors from model B; N is the number of prediction errors used; and $h_d(0)$ is the spectral density of $\hat{d}_i$ at frequency zero. The spectral density $h_d(0)$ is computed using a quadratic spectral kernel and the bandwidth is selected using the automatic criteria suggested by Andrews (1991). Strictly speaking this does not follow Diebold and Mariano (1995) formulation (they suggested the use of a bandwidth parameter equal to the forecasting horizon and weights set to unity for the sum of the autocovariances). This choice of estimate for the spectral density function is justified on the basis of the presence of some residual serial correlation in the in sample errors, and the nonlinear nature of the models. The tables below report the probability values. A probability value smaller than 0.05 allows rejection of the null of equal predictive accuracy of models A and B in favour of model A. Forecasting results are reported for the whole forecasting period and subsamples. This serves to assess the performance of a certain model over a certain period of time, particularly over critical periods such as beginning and end of year and around the Easter holidays.

Tables 4 presents the RMSE, standard deviation and Theil statistic of all the models over the whole forecasting sample. Both the ARIMA and STS model display a better forecasting performance than the AGF model. The STS model is better than the ARIMA over short run horizons, but worse over longer horizons. Disparities between the RMSE and the standard deviation point to a systematic bias in the forecasts. The AGF forecast appears unbiased, but this is not the case for the ARIMA or STS forecasts. An explanation of this bias could be found in the series of banknotes. The trend of the series of banknotes has been upwards for most of the 'in sample' period, with a minor period of negative trend for part of 1998. The slope of the trend was less pronounced by the end of 2000 and turned negative by the beginning of 2001. This is related to the preparations in anticipation of the change over of national currencies for euros at the beginning of 2002. Most of the bias occurs for the fourth subsample, clearly there is not enough information available in the 'in sample' period to forecast optimally the new pattern of the series during the year 2001. The best model is the C^{sts}_{arima} model. The other two model combinations perform also better than their individual counterparts, but gains are larger for the combination of the AGF and STS models. This result suggests that the models used in the AGF forecast being linear add little to the information already provided by the ARIMA model.

Tables 5 to 8 present the RMSE, standard deviation and Theil statistic of all the models over the four subsamples of the forecasting period. Results resemble those obtained for the full sample. The worst performance is obtained for the subsample November 2000 to February 2001. This is hardly surprising as this includes the Christmas period. It is worth noting, that while the combination model C^{arima}_{agf} is usually not a big improvement over the ARIMA for the first three subsamples, it is certainly much better for the fourth subsample, i.e. that which includes Christmas. This could be explained by the fact that the AGF model relies less on linear models over this period and, subjective (non-model based) adjustments weigh more heavily in the final forecast. This would explain why the information contained in the AGF now adds much more to the ARIMA that in other periods.

The results of the Diebold Mariano test are presented in table 9 for the whole sample and in tables 10 to 13 for the four subsamples. Results show that at a level of significance of 5% model C^{sts}_{arima} is better than all other models for all forecasting horizons, with the only exception of model C^{sts}_{agf} for 1 step ahead forecasts. Although not as clear cut, results point in the same direction when checked over subsamples.

8.1 Liquidity Forecasting

Equation (1) above described a benchmark allotment strategy for main refinancing operations. In order to assess the forecasting performance of the models presented in this paper it is fair to ask how good they can be in the context of anticipating correctly the liquidity needs of the euro area banking system. In June 2000 the ECB agreed to make publicly available its projections of the expected liquidity needs in the euro area over the frequency of the main refinancing operations. The ECB provides information on reserve requirements, RR, reserve balances, RB, the recourse to the two standing facilities, and the expected value of the autonomous factors, i.e. the second summand in equation (1). For the purposes of this paper the liquidity provided by means of past open market operations, including past main refinancing operations, together with the use of the standing facilities will be treated as an autonomous factor. Bindseil (2001) has argued that the publication of the forecasts leads to better control on steering overnight interest rates. This result is obviously dependent on the quality of the forecasts.

Table 14 presents the size of the error in anticipating the liquidity needs due to banknote forecasting errors. In order to understand those figures a few issues should be clarified. i) The amount alloted in the main refinancing operations is usually rounded to the billion by the nearest integer, say if the figure for L_t, as defined in equation (1) above, was 85.3 the allotment would be 85, and if the figure was 85.6 the allotment would be 86. ii) The figure for reserve requirements is taken as given at the time of all main refinancing operations, i.e. updates are ignored. iii) Values of forecast for all other autonomous factors are taken as their true values. iv) Furthermore, the figures of the liquidity needs, L_t, are corrected by the liquidity effects resulting from the credit institution's use of the standing facilities. As explained in section 2 above, the ECB provides liquidity on the basis of its forecast of autonomous factors and reserve requirements. If these are incorrect, counterparties have at their disposal the use of the standing facilities to adjust for the excess or lack of liquidity. Therefore, the use of the standing facilities has also a liquidity providing or absorbing effect. For the calculation of the liquidity need error, we are only interested in the effect of the autonomous factor error (in our case only the error in forecasting banknotes). Therefore the effect of the use of the standing facility has to be eliminated.

Table 14 displays details on: i) the forecasting period (usually a week), ii) the amount in billions of euro alloted in the main refinancing operations, and iii) the size of the error in billions of euro when the forecast of banknotes is computed from the alternative models, i.e. the amount by which the allotment decision deviated ex post from the correct amount as a result of forecasting errors in banknotes. The one week horizon is the relevant time period from the liquidity management perspective, because the liquidity in the money market is adjusted by the weekly *main refinancing operations*, see section 2. In total, 29 tender operations are presented.

The correct tender amount is obtained by adjusting the actual tender amount by the daily error for banknotes (liquidity change / number of days from one allotment decision day to the next one). The error in anticipating the liquidity needs never exceeds ±1 billion of euro for any of the models. With the forecast computed from the ARIMA model there is a total of 8 corrections, 9 for the STS model, 8 for the AGF and C^{arima}_{agf} models, 3 for the C^{sts}_{agf} model and 2 for the C^{sts}_{arima} model. Most of the corrections for the ARIMA and AGF have a negative sign, i.e. overestimation of liquidity need, while most of the corrections for the STS model have a positive sign. These results confirm the good forecasting performance of the C^{sts}_{arima} model.

9 Conclusion

The daily series of banknotes in circulation is one of the main autonomous factors that affect the supply of bank reserves in the euro area. The objective of steering interest rates is achieved by managing the conditions that equilibrate supply and demand in the market for bank reserves. In order to do so efficiently, the Eurosystem needs accurate forecast of certain 'autonomous factors'. Banknotes in circulation is the largest of those 'autonomous factors'. The daily series of banknotes in circulation displays very marked seasonal patterns, which reflect certain regularities in payments and receipts as well as patterns in the consumption behavior associated with holiday periods. This paper has assessed the forecasting performance of alternative approaches for modelling seasonality in daily series.

Results presented suggest that the two major approaches, i.e. the ARIMA-based approach, and the STS approach are powerful and display a performance which is up to the standards of the current aggregated forecast approach employed by the Eurosystem. Nonetheless, the expert knowledge incorporated in the AGF model is key over certain holiday periods. The ARIMA model has the best forecasting performance over horizons of 5 days and above, while the STS is best over horizons of 1 to 4 days. The best forecasting model is a combination of the ARIMA and STS models. This may point to the fact that certain seasonal patterns may not be completely captured by a linear structure.

The assessment of the performance of the models has also been conducted in the context of the liquidity management of the Eurosystem. The error in anticipating the liquidity needs due to forecasting banknotes in circulation never exceeds ±1 billion of euro for any of the models. A total of eight corrections to an allotment decision strategy described in section 2 resulted from the forecasting errors of the ARIMA model; nine from the STS model, eight from the AGF and C_{agf}^{arima} models, three for the C_{agf}^{sts} model and two for the C_{arima}^{sts} model. The combination C_{arima}^{sts} outperforms the other models.

Results presented in this paper show that the econometric models can explain a large part of the variation of banknotes in circulation. So far, forecasts of banknotes in circulation in the euro area have been computed by NCBs, i.e. each NCB computed the forecast for banknotes in circulation in its own country. The ECB would then aggregate those individual forecasts and, together with the forecasts for the remaining autonomous factors, would use this information to calculate the amount to be alloted in its weekly main refinancing operation. The introduction of the euro banknotes and the free movement of banknotes through the euro area may make the AGF forecasts less reliable. Therefore, the Eurosystem may have to rely more and more on models of the type presented in this paper.

These econometric models have been used in 'real time' by the ECB from July 2001. The role played by the models was mainly that of checking the quality of the AGF forecast, and under some circumstances, to adjust it. The 'real time' testing of the models by the liquidity management unit of the ECB showed that the models had difficulties capturing 'exceptional' effects, such as the patterns associated with the cash-changeover process. These patterns were very pronounced towards the end of the year 2001 and first weeks of 2002. This meant that expert knowledge from NCBs played a prominent role during that phase. It seems sensible to expect the performance of the models to become better again once the cash changeover process is completed. Nevertehless, from a practitioner's viewpoint, it is necessary to undertake a thorough assessment of the quality of the model's forecasts over a period of time which also includes the cash-changeover process.

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A Appendix

A.1 State Space Representation of STS Model

The State space representation of the STS model is as follows:

$$\begin{aligned} y_t &= \boldsymbol{C}_t \boldsymbol{s}_t + \boldsymbol{\varepsilon}_t \\ \boldsymbol{s}_t &= \boldsymbol{A}\boldsymbol{s}_{t-1} + \boldsymbol{e}_t \end{aligned} \tag{A-1}$$

where $\boldsymbol{C}_t$ and $\boldsymbol{A}$ are matrices of parameters, and $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{e}_t$ are two independent zero mean processes with positive definite and finite variance matrices $\Sigma_{\varepsilon\varepsilon}$ and Σ_{ee} respectively. The dependance of matrix $\boldsymbol{C}_t$ on time is due to the presence of seasonal components. Matrices $\boldsymbol{C}_t$, $\boldsymbol{A}$ and $\boldsymbol{\Sigma}_{ee}$ of the state space representation are defined as follows:

$$\boldsymbol{C}_t = \begin{bmatrix} 1 & 0 & \boldsymbol{z}_t^1 & \boldsymbol{z}_t^2 & \boldsymbol{z}_t^3 & \boldsymbol{z}_t^4 & \boldsymbol{z}_t^5 \end{bmatrix} \tag{A-2}$$

$$\boldsymbol{A} = \begin{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}$$

$$diag\left(\boldsymbol{\Sigma}_{ee}\right) = \left(\sigma_\eta^2, \sigma_\nu^2, \sigma_1^2, \ldots, \sigma_1^2, \sigma_2^2, \ldots, \sigma_2^2, \sigma_3^2, \ldots, \sigma_3^2, 0, \ldots, 0\right)$$

A.2 Kalman Filter Equations

When all parameters in the state space model (A-1) are known, and for known starting values for the unobserved component, i.e. $\boldsymbol{s}_0$ and its covariancematrix, $\boldsymbol{P}_0$, the Kalman filter equations provide and optimal estimator, in the mean square error sense, of the state s_t conditional on information up to time t, this estimator is denoted as $\boldsymbol{s}_{t|t} = E\{\boldsymbol{s}_t | y_1, \ldots, y_t\}$ and its corresponding covariance matrix is denoted as $\boldsymbol{P}_{t|t} = E\{(\boldsymbol{s}_{t|t} - \boldsymbol{s}_t)(\boldsymbol{s}_{t|t} - \boldsymbol{s}_t)' | y_1, \ldots, y_t\}$. The Kalman filter equations are given by:

$$\begin{aligned} \boldsymbol{s}_{t+1|t} &= \boldsymbol{A}\boldsymbol{s}_{t|t} \\ \boldsymbol{P}_{t+1|t} &= \boldsymbol{A}\boldsymbol{P}_{t|t}\boldsymbol{A}' + \boldsymbol{\Sigma}_{ee} \\ \nu_{t+1} &= y_{t+1} - \boldsymbol{C}_{t+1}\boldsymbol{s}_{t+1|t} \\ f_{t+1} &= \boldsymbol{C}_{t+1}\boldsymbol{P}_{t+1|t}\boldsymbol{C}'_{t+1} + \boldsymbol{\Sigma}_{\varepsilon\varepsilon} \\ \boldsymbol{s}_{t+1|t+1} &= \boldsymbol{s}_{t+1|t} + \boldsymbol{K}_{t+1}\nu_{t+1} \\ \boldsymbol{K}_{t+1} &= \boldsymbol{P}_{t+1|t}\boldsymbol{C}'_{t+1} f_{t+1}^{-1} \\ \boldsymbol{P}_{t+1|t+1} &= \boldsymbol{P}_{t+1|t} - \boldsymbol{K}_{t+1}\boldsymbol{C}_{t+1}\boldsymbol{P}_{t+1|t} \end{aligned}$$

A.3 Estimation of parameters of STS Model

When the starting values of the state, $\boldsymbol{s}_0$, $\boldsymbol{P}_0$ and the parameters in $\boldsymbol{C}_t$, $\boldsymbol{A}$, $\boldsymbol{\Sigma}_{ee}$ and $\boldsymbol{\Sigma}_{\varepsilon\varepsilon}$, which we denote as $\boldsymbol{\Psi}$ are unknown, they can be estimating by maximum likelihood estimation. Then the log likelihood based on the prediction error decomposition and assuming a Gaussian model is given by

$$\log L(\boldsymbol{s}_0, \boldsymbol{P}_0, \boldsymbol{\Psi}, y_1, \ldots, y_T) = const - \frac{1}{2}\sum_{t=1}^{T} \log f_t - \frac{1}{2}\sum_{t=1}^{T} \nu_t' f_t^{-1} \nu_t$$

Following de Jong (1988), for $\boldsymbol{P}_0 = 0$, vector $\boldsymbol{s}_0$ can be concentrated from the likelihood function above, the concentrated likelihood is then:

$$\log L^*(\boldsymbol{\Psi}, y_1, \ldots, y_T) = const - \frac{1}{2}\sum_{t=1}^{T} \log d_t - \frac{1}{2}\sum_{t=1}^{T} u_t' d_t^{-1} u_t + \boldsymbol{q}'\boldsymbol{Q}^{-1}\boldsymbol{q}$$

where u_t and d_t are the prediction error and mean square error computed from the Kalman filter equations above for starting values $\boldsymbol{s}_0 = 0$ and $\boldsymbol{P}_0 = \mathbf{0}$, and the vector $\boldsymbol{q}$ and matrix $\boldsymbol{Q}$ are computed in parallel with these Kalman filter recursions as:

$$\begin{aligned}
\boldsymbol{q} &= \boldsymbol{q} + \boldsymbol{Z}_{t-1}'\boldsymbol{C}_t' d_t^{-1} u_t \\
\boldsymbol{Q} &= \boldsymbol{Q} + \boldsymbol{Z}_{t-1}'\boldsymbol{C}_t' d_t^{-1} \boldsymbol{C}_t \boldsymbol{Z}_{t-1} \\
\boldsymbol{Z}_t &= \boldsymbol{A}(\boldsymbol{I} - \boldsymbol{K}_t \boldsymbol{C}_t)\boldsymbol{Z}_{t-1}
\end{aligned}$$

with vector $\boldsymbol{q}$ and matrix $\boldsymbol{Q}$ initialize at zero and $\boldsymbol{Z}_0$ initialize as an identity matrix. The estimated value for $\boldsymbol{s}_0$ is equal to $\boldsymbol{Q}^{-1}\boldsymbol{q}$.

T able 3:Specification Tests on in-sample residuals.

	ARIMA Model		STS Model	
	stat.	p.v.	stat.	p.v.
Skewness	2.345	0.990	1.757	0.960
Kurtosis	-4.085	0.000	-2.735	0.003
Normality	22.191	0.000	10.570	0.005
Ljung-Box on residuals				
Q(5)	1.89	0.86	73.80	0.00
Q(10)	8.38	0.59	98.30	0.00
Q(22)	17.95	0.71	124.97	0.00
Q(261)	318.37	0.01	595.06	0.00
Ljung-Box on squared residuals				
Q(5)	208.73	0.00	229.22	0.00
Q(10)	163.06	0.00	266.03	0.00
Q(22)	187.46	0.00	275.42	0.00
Q(261)	579.68	0.00	729.53	0.00

Table 4: Forecasting Performance. All sample.

Model	Stat.	1	2	3	4	5	6	7	8	9	10
		Forecast horizon									
ARIMA	RMSE	0.318	0.487	0.627	0.739	0.828	0.928	1.003	1.071	1.137	1.200
	St. Dev.	0.314	0.474	0.601	0.699	0.772	0.853	0.907	0.953	0.996	1.033
	Theil	0.300	0.261	0.251	0.245	0.234	0.224	0.214	0.209	0.208	0.208
STS	RMSE	0.281	0.461	0.605	0.735	0.847	0.955	1.050	1.140	1.230	1.319
	St. Dev.	0.276	0.448	0.583	0.703	0.802	0.896	0.976	1.050	1.122	1.193
	Theil	0.265	0.247	0.242	0.244	0.239	0.231	0.224	0.222	0.225	0.229
AGF	RMSE	0.337	-	-	-	0.999	-	-	-	-	1.521
	St. Dev.	0.334	-	-	-	0.985	-	-	-	-	1.515
	Theil	0.316	-	-	-	0.284	-	-	-	-	0.268
C_{agf}^{arima}	RMSE	0.263	-	-	-	0.702	-	-	-	-	1.015
	St. Dev.	0.259	-	-	-	0.663	-	-	-	-	0.916
	Theil	0.247	-	-	-	0.198	-	-	-	-	0.176
C_{agf}^{sts}	RMSE	0.212	-	-	-	0.673	-	-	-	-	1.076
	St. Dev.	0.212	-	-	-	0.672	-	-	-	-	1.058
	Theil	0.200	-	-	-	0.190	-	-	-	-	0.186
C_{arima}^{sts}	RMSE	0.197	0.379	0.388	0.443	0.494	0.569	0.633	0.681	0.727	0.703
	St. Dev.	0.197	0.372	0.387	0.442	0.491	0.566	0.629	0.674	0.717	0.689
	Theil	0.185	0.203	0.155	0.147	0.139	0.137	0.135	0.133	0.133	0.122

Table 5: Forecasting Performance. Subsample February - April 2000.

Model	Stat.	Forecast horizon 1	2	3	4	5	6	7	8	9	10
	RMSE	0.282	0.466	0.616	0.764	0.889	1.005	1.117	1.231	1.329	1.410
ARIMA	St. Dev.	0.282	0.465	0.615	0.762	0.886	1.002	1.112	1.224	1.318	1.391
	Theil	0.292	0.277	0.284	0.305	0.314	0.305	0.297	0.300	0.309	0.317
	RMSE	0.269	0.469	0.647	0.823	0.957	1.086	1.206	1.313	1.402	1.480
STS	St. Dev.	0.269	0.469	0.646	0.820	0.952	1.078	1.198	1.305	1.397	1.478
	Theil	0.278	0.278	0.298	0.328	0.339	0.329	0.320	0.320	0.326	0.333
	RMSE	0.347	-	-	-	0.908	-	-	-	-	1.728
AGF	St. Dev.	0.347	-	-	-	0.894	-	-	-	-	1.705
	Theil	0.356	-	-	-	0.336	-	-	-	-	0.431
	RMSE	0.271	-	-	-	0.841	-	-	-	-	1.359
C_{agf}^{arima}	St. Dev.	0.273	-	-	-	0.854	-	-	-	-	1.347
	Theil	0.281	-	-	-	0.297	-	-	-	-	0.306
	RMSE	0.181	-	-	-	0.610	-	-	-	-	0.884
C_{agf}^{sts}	St. Dev.	0.180	-	-	-	0.576	-	-	-	-	0.857
	Theil	0.188	-	-	-	0.216	-	-	-	-	0.199
	RMSE	0.130	0.348	0.269	0.307	0.337	0.397	0.423	0.405	0.397	0.381
C_{arima}^{sts}	St. Dev.	0.130	0.348	0.262	0.297	0.317	0.366	0.398	0.387	0.383	0.373
	Theil	0.135	0.207	0.124	0.122	0.119	0.120	0.112	0.098	0.092	0.085

Table 6: Forecasting Performance. Subsample May - July 2000.

		Forecast horizon									
Model	Stat.	1	2	3	4	5	6	7	8	9	10
	RMSE	0.305	0.426	0.505	0.525	0.568	0.634	0.693	0.732	0.770	0.816
ARIMA	St. Dev.	0.302	0.414	0.480	0.487	0.515	0.567	0.609	0.636	0.664	0.707
	Theil	0.369	0.304	0.284	0.263	0.253	0.242	0.234	0.230	0.232	0.236
	RMSE	0.301	0.427	0.469	0.478	0.504	0.541	0.590	0.638	0.703	0.791
STS	St. Dev.	0.300	0.424	0.460	0.461	0.478	0.505	0.550	0.597	0.665	0.759
	Theil	0.363	0.304	0.263	0.240	0.224	0.206	0.200	0.200	0.212	0.229
	RMSE	0.343	-	-	-	0.820	-	-	-	-	1.206
AGF	St. Dev.	0.338	-	-	-	0.792	-	-	-	-	1.111
	Theil	0.415	-	-	-	0.365	-	-	-	-	0.350
	RMSE	0.238	-	-	-	0.513	-	-	-	-	0.770
C_{agf}^{arima}	St. Dev.	0.233	-	-	-	0.462	-	-	-	-	0.632
	Theil	0.287	-	-	-	0.228	-	-	-	-	0.223
	RMSE	0.220	-	-	-	0.485	-	-	-	-	0.727
C_{agf}^{sts}	St. Dev.	0.220	-	-	-	0.484	-	-	-	-	0.715
	Theil	0.266	-	-	-	0.216	-	-	-	-	0.211
	RMSE	0.202	0.339	0.317	0.304	0.298	0.332	0.380	0.383	0.387	0.399
C_{arima}^{sts}	St. Dev.	0.201	0.338	0.314	0.302	0.295	0.328	0.371	0.371	0.374	0.383
	Theil	0.244	0.241	0.178	0.153	0.132	0.127	0.128	0.120	0.117	0.116

Table 7: Forecasting Performance. Subsample August - October 2000.

Model	Stat.	1	2	3	4	5	6	7	8	9	10
		Forecast horizon									
ARIMA	RMSE	0.235	0.368	0.440	0.484	0.525	0.583	0.618	0.637	0.650	0.652
	St. Dev.	0.233	0.365	0.440	0.494	0.538	0.592	0.614	0.628	0.641	0.638
	Theil	0.257	0.232	0.215	0.204	0.193	0.183	0.170	0.163	0.161	0.157
STS	RMSE	0.194	0.309	0.380	0.447	0.512	0.567	0.597	0.622	0.646	0.655
	St. Dev.	0.197	0.303	0.368	0.429	0.483	0.530	0.545	0.555	0.564	0.553
	Theil	0.219	0.196	0.184	0.182	0.180	0.172	0.161	0.156	0.156	0.154
AGF	RMSE	0.276	-	-	-	0.706	-	-	-	-	1.020
	St. Dev.	0.263	-	-	-	0.593	-	-	-	-	0.811
	Theil	0.565	-	-	-	0.471	-	-	-	-	0.506
C_{agf}^{arima}	RMSE	0.195	-	-	-	0.535	-	-	-	-	0.716
	St. Dev.	0.192	-	-	-	0.500	-	-	-	-	0.642
	Theil	0.410	-	-	-	0.359	-	-	-	-	0.353
C_{agf}^{sts}	RMSE	0.156	-	-	-	0.313	-	-	-	-	0.344
	St. Dev.	0.153	-	-	-	0.285	-	-	-	-	0.312
	Theil	0.335	-	-	-	0.269	-	-	-	-	0.251
C_{arima}^{sts}	RMSE	0.102	0.251	0.182	0.215	0.225	0.247	0.264	0.271	0.279	0.304
	St. Dev.	0.107	0.246	0.185	0.221	0.231	0.255	0.270	0.275	0.282	0.305
	Theil	0.119	0.159	0.092	0.092	0.083	0.078	0.074	0.070	0.069	0.073

Table 8: Forecasting Performance. Subsample November 2000 - February 2001.

		Forecast horizon									
Model	Stat.	1	2	3	4	5	6	7	8	9	10
	RMSE	0.398	0.615	0.819	0.992	1.112	1.248	1.342	1.432	1.522	1.616
ARIMA	St. Dev.	0.381	0.559	0.718	0.839	0.895	0.956	0.964	0.963	0.956	0.948
	Theil	0.294	0.253	0.245	0.238	0.223	0.214	0.203	0.197	0.195	0.194
	RMSE	0.326	0.572	0.796	0.998	1.165	1.323	1.463	1.597	1.733	1.868
STS	St. Dev.	0.299	0.509	0.690	0.844	0.956	1.053	1.128	1.191	1.256	1.323
	Theil	0.240	0.235	0.238	0.239	0.234	0.227	0.222	0.220	0.222	0.224
	RMSE	0.368	-	-	-	1.328	-	-	-	-	1.906
AGF	St. Dev.	0.366	-	-	-	1.327	-	-	-	-	1.806
	Theil	0.271	-	-	-	0.266	-	-	-	-	0.228
	RMSE	0.318	-	-	-	0.855	-	-	-	-	1.187
C_{agf}^{arima}	St. Dev.	0.310	-	-	-	0.758	-	-	-	-	0.993
	Theil	0.234	-	-	-	0.171	-	-	-	-	0.142
	RMSE	0.257	-	-	-	0.978	-	-	-	-	1.637
C_{agf}^{sts}	St. Dev.	0.247	-	-	-	0.904	-	-	-	-	1.343
	Theil	0.189	-	-	-	0.196	-	-	-	-	0.196
	RMSE	0.267	0.495	0.572	0.674	0.769	0.890	0.992	1.087	1.172	1.119
C_{arima}^{sts}	St. Dev.	0.267	0.457	0.567	0.658	0.729	0.840	0.926	1.000	1.058	0.983
	Theil	0.197	0.204	0.171	0.162	0.154	0.153	0.150	0.150	0.150	0.134

Table 9: Diebold Mariano Tests: All sample.

		1	2	3	4	5	6	7	8	9	10
ARIMA vs	STS	0.890	0.757	0.655	0.518	0.424	0.420	0.388	0.365	0.342	0.316
	AGF	0.241	-	-	-	0.079	-	-	-	-	0.057
	C_{agf}^{arima}	1.000	-	-	-	0.988	-	-	-	-	0.937
	C_{agf}^{sts}	1.000	-	-	-	0.904	-	-	-	-	0.680
	C_{arima}^{sts}	1.000	0.997	1.000	1.000	0.997	0.995	0.985	0.983	0.985	0.990
STS vs	AGF	0.041	-	-	-	0.096	-	-	-	-	0.150
	C_{agf}^{arima}	0.757	-	-	-	0.816	-	-	-	-	0.810
	C_{agf}^{sts}	1.000	-	-	-	0.998	-	-	-	-	0.984
	C_{arima}^{sts}	1.000	0.999	0.998	0.996	0.991	0.990	0.983	0.978	0.966	0.949
AGF vs	C_{agf}^{arima}	1.000	-	-	-	0.988	-	-	-	-	0.985
	C_{agf}^{sts}	1.000	-	-	-	0.997	-	-	-	-	0.985
	C_{arima}^{sts}	1.000	-	-	-	1.000	-	-	-	-	0.998
C_{agf}^{arima} vs	C_{agf}^{sts}	0.987	-	-	-	0.587	-	-	-	-	0.425
	C_{arima}^{sts}	0.997	-	-	-	0.937	-	-	-	-	0.901
C_{agf}^{sts} vs	C_{arima}^{sts}	0.842	-	-	-	0.993	-	-	-	-	0.905

Table 10: Diebold Mariano Tests: February 2000 to April 2000.

		1	2	3	4	5	6	7	8	9	10
ARIMA vs	STS	0.611	0.488	0.396	0.348	0.358	0.366	0.369	0.387	0.409	0.420
	AGF	0.033	-	-	-	0.403	-	-	-	-	0.180
	C_{agf}^{arima}	0.892	-	-	-	0.767	-	-	-	-	0.726
	C_{agf}^{sts}	0.982	-	-	-	0.886	-	-	-	-	0.844
	C_{arima}^{sts}	0.989	0.913	0.944	0.938	0.936	0.929	0.904	0.923	0.860	0.870
STS vs	AGF	0.005	-	-	-	0.518	-	-	-	-	0.168
	C_{agf}^{arima}	0.475	-	-	-	0.736	-	-	-	-	0.651
	C_{agf}^{sts}	0.976	-	-	-	0.918	-	-	-	-	0.859
	C_{arima}^{sts}	0.963	0.950	0.924	0.929	0.915	0.896	0.880	0.873	0.870	0.859
AGF vs	C_{agf}^{arima}	0.984	-	-	-	0.683	-	-	-	-	0.859
	C_{agf}^{sts}	0.998	-	-	-	0.837	-	-	-	-	0.888
	C_{arima}^{sts}	0.992	-	-	-	0.893	-	-	-	-	0.876
C_{agf}^{arima} vs	C_{agf}^{sts}	0.979	-	-	-	0.918	-	-	-	-	0.847
	C_{arima}^{sts}	0.991	-	-	-	0.949	-	-	-	-	0.865
C_{agf}^{sts} vs	C_{arima}^{sts}	0.925	-	-	-	0.846	-	-	-	-	0.866

Table 11: Diebold Mariano Tests: May 2000 to July 2000.

		1	2	3	4	5	6	7	8	9	10
ARIMA vs	STS	0.525	0.496	0.656	0.709	0.776	0.815	0.782	0.742	0.668	0.557
	AGF	0.331	-	-	-	0.078	-	-	-	-	0.163
	C_{agf}^{arima}	0.944	-	-	-	0.765	-	-	-	-	0.604
	C_{agf}^{sts}	0.953	-	-	-	0.837	-	-	-	-	0.723
	C_{arima}^{sts}	0.999	0.886	0.991	0.986	0.987	0.983	0.970	0.971	0.963	0.939
STS vs	AGF	0.358	-	-	-	0.043	-	-	-	-	0.194
	C_{agf}^{arima}	0.798	-	-	-	0.450	-	-	-	-	0.531
	C_{agf}^{sts}	0.928	-	-	-	0.599	-	-	-	-	0.630
	C_{arima}^{sts}	0.903	0.962	0.941	0.966	0.976	0.935	0.880	0.883	0.899	0.917
AGF vs	C_{agf}^{arima}	0.987	-	-	-	0.958	-	-	-	-	0.898
	C_{agf}^{sts}	0.993	-	-	-	0.960	-	-	-	-	0.866
	C_{arima}^{sts}	0.983	-	-	-	0.963	-	-	-	-	0.888
C_{agf}^{arima} vs	C_{agf}^{sts}	0.714	-	-	-	0.694	-	-	-	-	0.637
	C_{arima}^{sts}	0.866	-	-	-	0.995	-	-	-	-	0.900
C_{agf}^{sts} vs	C_{arima}^{sts}	0.674	-	-	-	0.979	-	-	-	-	0.960

Table 12: Diebold Mariano Tests: August 2000 to October 2000.

		1	2	3	4	5	6	7	8	9	10
ARIMA vs	STS	0.918	0.832	0.760	0.702	0.633	0.624	0.602	0.582	0.558	0.542
	AGF	0.184	-	-	-	0.115	-	-	-	-	0.098
	C_{agf}^{arima}	0.929	-	-	-	0.535	-	-	-	-	0.319
	C_{agf}^{sts}	0.984	-	-	-	0.942	-	-	-	-	0.961
	C_{arima}^{sts}	0.997	0.936	0.948	0.953	0.965	0.969	0.972	0.974	0.964	0.971
STS vs	AGF	0.061	-	-	-	0.026	-	-	-	-	0.053
	C_{agf}^{arima}	0.554	-	-	-	0.332	-	-	-	-	0.246
	C_{agf}^{sts}	0.957	-	-	-	0.916	-	-	-	-	0.934
	C_{arima}^{sts}	0.999	0.998	0.984	0.953	0.941	0.932	0.918	0.919	0.925	0.932
AGF vs	C_{agf}^{arima}	0.984	-	-	-	0.963	-	-	-	-	0.921
	C_{agf}^{sts}	0.996	-	-	-	0.949	-	-	-	-	0.912
	C_{arima}^{sts}	0.996	-	-	-	0.969	-	-	-	-	0.918
C_{agf}^{arima} vs	C_{agf}^{sts}	0.967	-	-	-	0.926	-	-	-	-	0.920
	C_{arima}^{sts}	0.999	-	-	-	0.958	-	-	-	-	0.926
C_{agf}^{sts} vs	C_{arima}^{sts}	0.982	-	-	-	0.973	-	-	-	-	0.764

Table 13: Diebold Mariano Tests: November 2000 to February 2001.

		1	2	3	4	5	6	7	8	9	10
ARIMA vs	STS	0.874	0.740	0.591	0.482	0.402	0.388	0.369	0.352	0.334	0.319
	AGF	0.743	-	-	-	0.209	-	-	-	-	0.246
	C_{agf}^{arima}	0.995	-	-	-	0.989	-	-	-	-	0.942
	C_{agf}^{sts}	0.978	-	-	-	0.722	-	-	-	-	0.484
	C_{arima}^{sts}	0.969	0.949	0.992	0.983	0.955	0.943	0.890	0.873	0.884	0.941
STS vs	AGF	0.260	-	-	-	0.264	-	-	-	-	0.461
	C_{agf}^{arima}	0.563	-	-	-	0.795	-	-	-	-	0.805
	C_{agf}^{sts}	0.993	-	-	-	0.988	-	-	-	-	0.935
	C_{arima}^{sts}	0.977	0.962	0.972	0.969	0.947	0.947	0.928	0.914	0.892	0.866
AGF vs	C_{agf}^{arima}	0.892	-	-	-	0.938	-	-	-	-	0.916
	C_{agf}^{sts}	0.966	-	-	-	0.923	-	-	-	-	0.791
	C_{arima}^{sts}	0.946	-	-	-	0.969	-	-	-	-	0.960
C_{agf}^{arima} vs	C_{agf}^{sts}	0.881	-	-	-	0.347	-	-	-	-	0.266
	C_{arima}^{sts}	0.854	-	-	-	0.629	-	-	-	-	0.564
C_{agf}^{sts}	C_{arima}^{sts}	0.336	-	-	-	0.972	-	-	-	-	0.831

T able 14:Correction of the T enderdue to banknote forecasting errors.

Maintenance P eriod	F orecasting P eriod	MP w eek	No of da ys	Actual T ender	Correction ARIMA	STS	AGF	C_{agf}^{arima}	C_{agf}^{sts}	C_{arima}^{sts}
MP 07	20/06/00 to 27/06/00	1	7							
	27/06/00 to 04/07/00	2	7	98	0	0	0	0	0	0
	04/07/00 to 11/07/00	3	7	56	0	1	-1	0	0	0
	11/07/00 to 18/07/00	4	7	100	0	0	-1	-1	-1	0
	18/07/00 to 25/07/00	5	7	52	0	0	0	0	0	0
MP 08	25/07/00 to 01/08/00	1	7	116	0	0	0	0	0	0
	01/08/00 to 08/08/00	2	7	45	0	0	0	0	0	0
	08/08/00 to 14/08/00	3	6	113	-1	0	-1	-1	0	0
	14/08/00 to 22/08/00	4	8	53	0	0	0	0	0	0
	22/08/00 to 29/08/00	5	7	115	1	0	0	0	0	0
MP 09	29/08/00 to 05/09/00	1	7	68	0	0	0	0	0	0
	05/09/00 to 12/09/00	2	7	108	0	1	-1	-1	0	0
	12/09/00 to 19/09/00	3	7	65	0	0	0	0	0	0
	19/09/00 to 26/09/00	4	7	106	0	0	0	0	0	0
MP 10	26/09/00 to 02/10/00	1	6	81	0	0	0	0	0	0
	02/10/00 to 10/10/00	2	8	99	0	0	0	0	0	0
	10/10/00 to 17/10/00	3	7	76	0	-1	0	0	0	0
	17/10/00 to 24/10/00	4	7	94	0	0	0	0	0	0
MP 11	24/10/00 to 31/10/00	1	7							
	31/10/00to 07/11/00	2	7	90	0	0	0	0	0	0
	07/11/00 to 14/11/00	3	7	96	0	0	0	0	0	0
	14/11/00 to 21/11/00	4	7	91	-1	1	0	0	0	0
	21/11/00 to 28/11/00	5	7	107	-1	0	-1	-1	0	0
MP 12	28/11/00 to 05/12/00	1	7	92	0	0	0	0	0	0
	05/12/00 to 12/12/00	2	7	127	-1	1	-1	-1	0	0
	12/12/00 to 19/12/00	3	7	90	1	-1	0	1	0	0
	19/12/00 to 22/12/00	4	3	121	-1	1	0	-1	1	-1
MP 01	22/12/00 to 02/01/01	1	11	102	0	0	0	0	0	0
	02/01/01 to 09/01/01	2	7	101	-1	0	0	-1	0	0
	09/01/00 to 16/01/01	3	7	95	0	-1	1	0	0	-1
	16/01/01 to 23/01/01	4	7	101	0	1	1	0	1	0

Figure 1: Euro Area Banknotes in Circulation (logs).

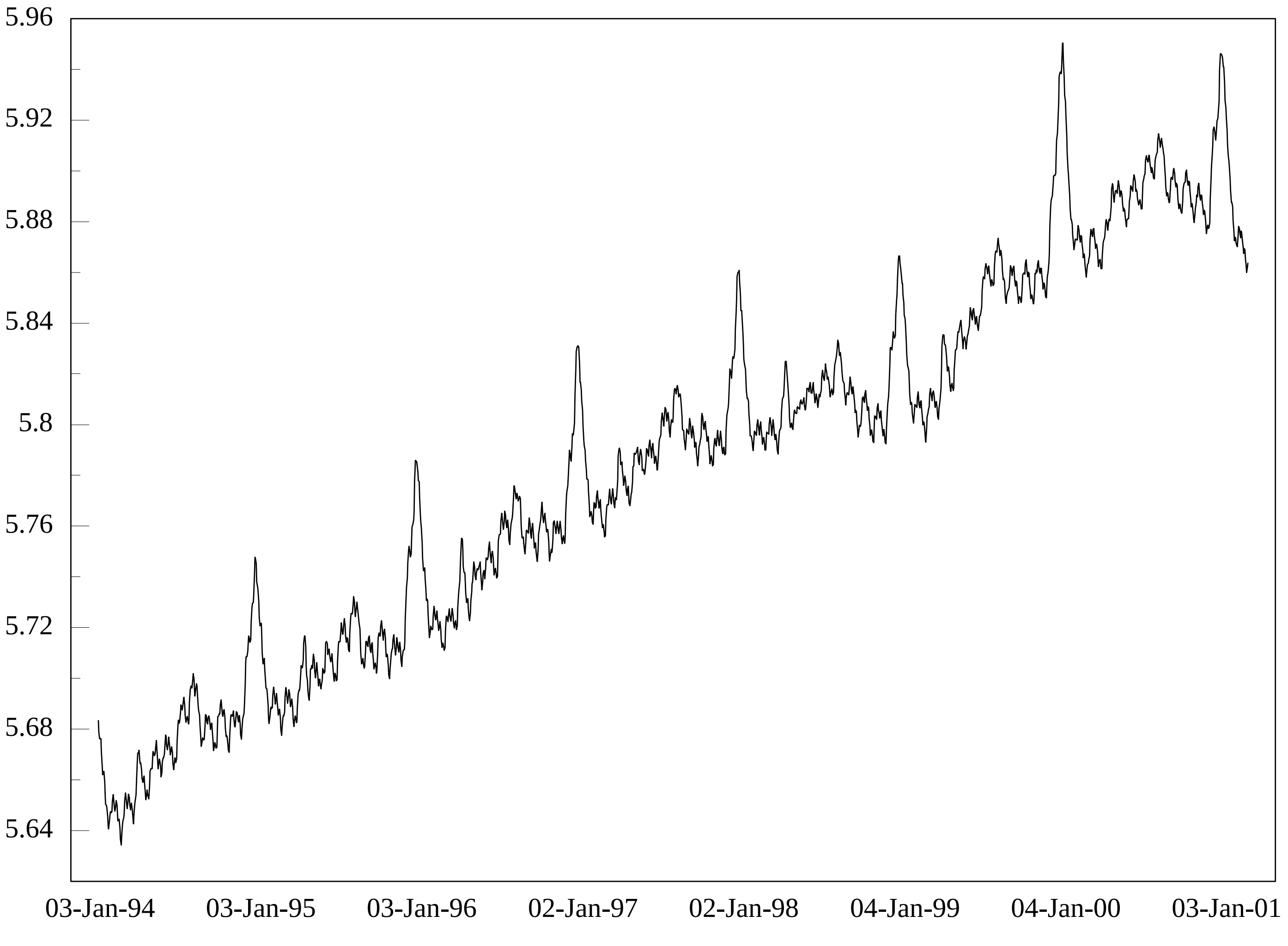

Figure 2: Calendar Variation Effects. ARIMA Model. (Values are in % of level of series).

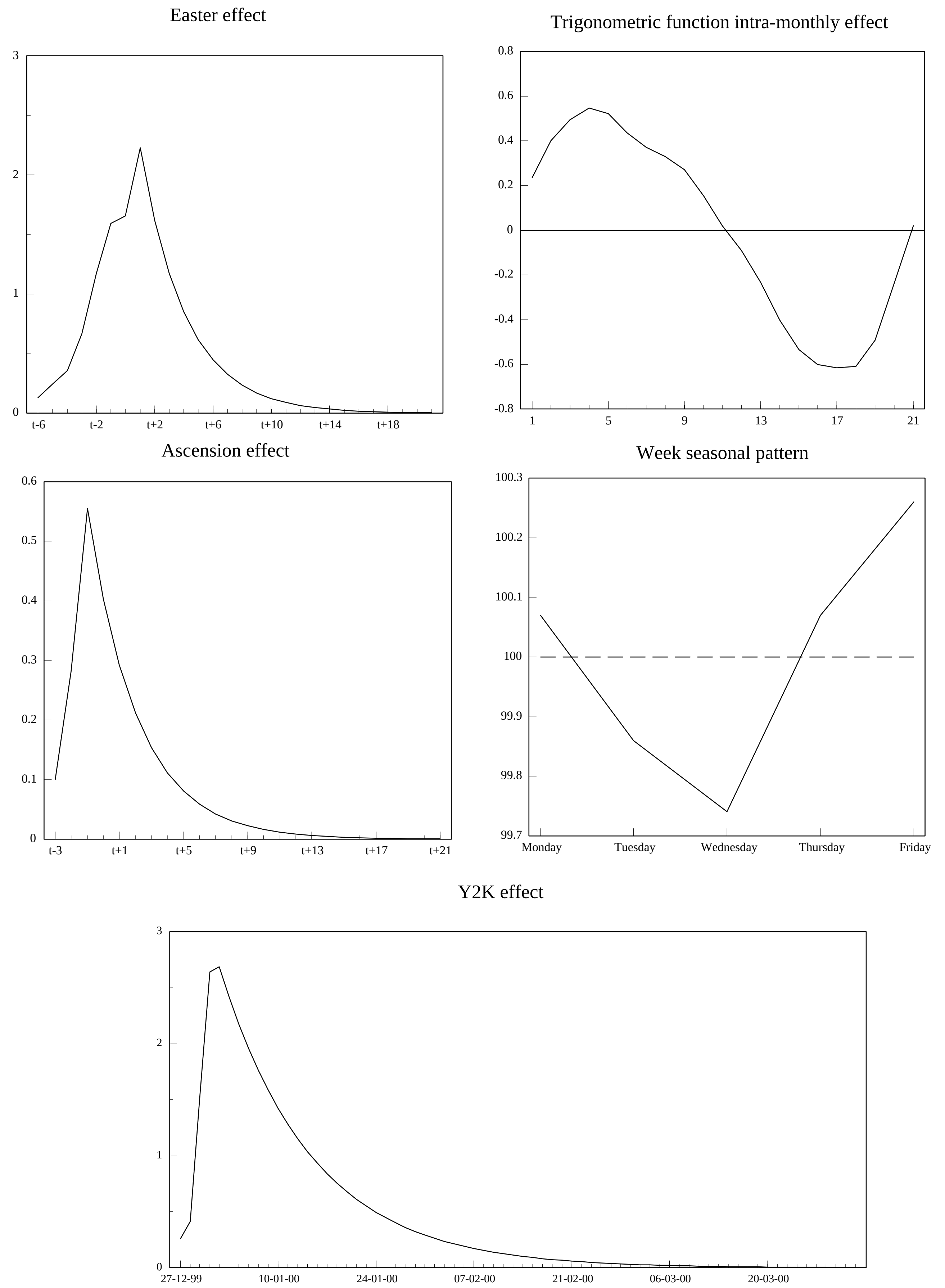

Figure 3: Calendar Variation Effects. STS Model.

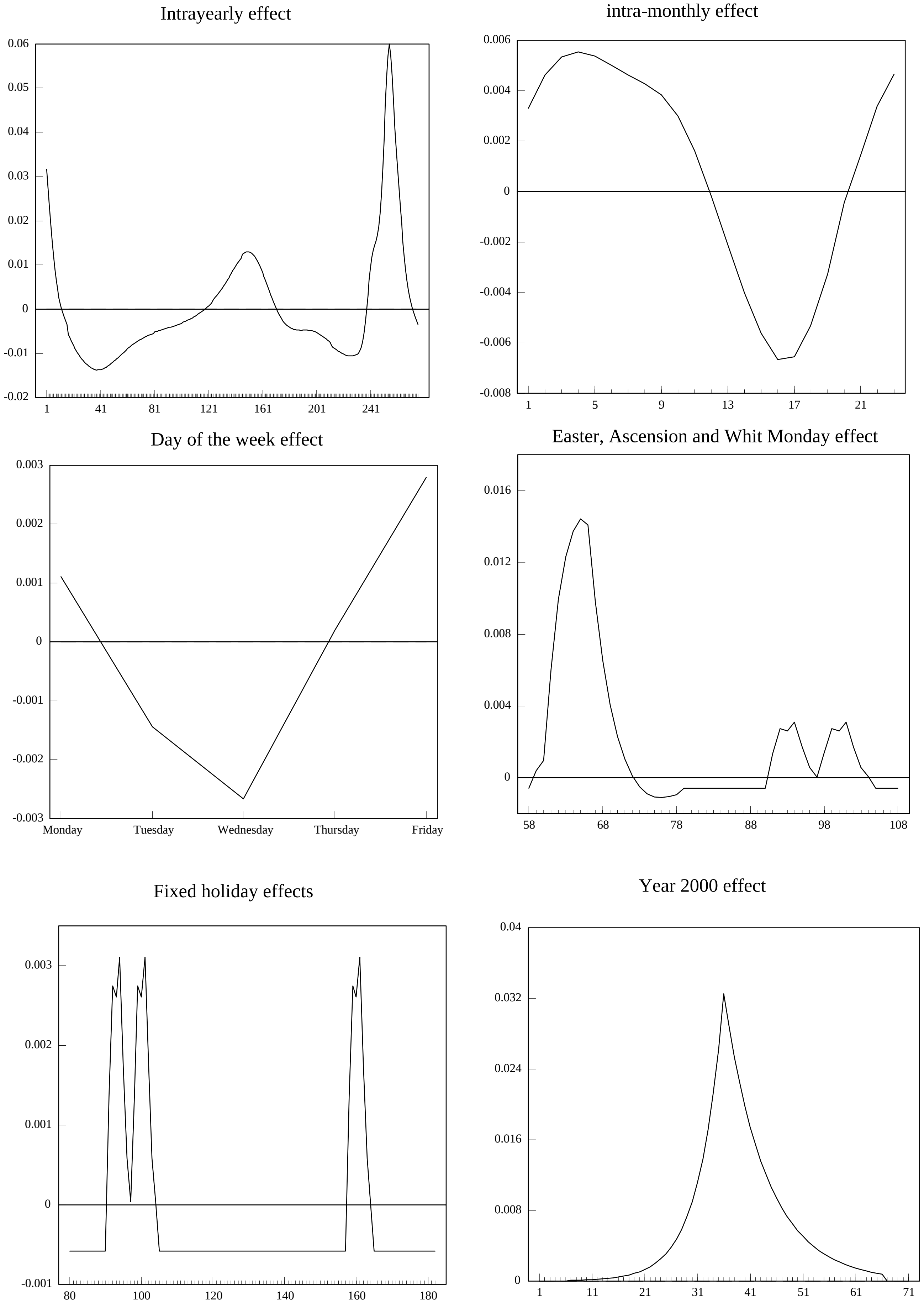

Figure 4: Residual Correlogram of Models.