Modelling the propulsion characteristics of a standard wheelchair

Michael Hofstad, MS and Patrick E. Patterson, PhD
Iowa State University, College of Engineering, Ames, IA 50011

Abstract—Recent wheelchair modelling work has focused on the efficiency of the human-racing wheelchair interaction. This paper builds on this work, investigating the development of a model appropriate for those using standard wheelchairs. A wheelchair racing model was initially used as the starting point for the generation of a number of model variations. Force predictions from these variations were compared to load cell data taken from an instrumented wheelchair during propulsion. Additional models where then developed, based on the characteristics of the models that performed best, and used to predict the forces in a second group of subjects. The analysis procedure was originally based on the calculation of a model index as a mathematical estimation of the theoretical closeness each model prediction had with the observed force. Visual comparisons of the force versus time were then incorporated into the procedure for evaluating the physical appearances of the profiles. The combination of the statistical and visual analysis led to selection of the final models for estimating the starting, constant, and stopping phases of wheelchair propulsion. The resulting models provide insight into the effects of a variety of factors on efficiency during propulsion in a standard chair.

Key words: *models, wheelchair.*

INTRODUCTION

Much of the wheelchair research in recent years has dealt with various aspects of racing and compet-

itive sport wheelchairs. The results have led to the production of new materials for lightweight wheelchair construction and provided new wheelchair designs which have given racers a competitive edge. While it is important that this research continue to progress, it is also necessary to make sure that everyday wheelchair users benefit from these advancements as well. The purpose of this project was to help bridge this gap and to further develop criteria for prescribing wheelchairs by studying the dynamics of standard wheelchair propulsion. Mathematical models of racing wheelchairs, which have yet to be validated, were obtained from the literature and used as the starting point in attaining a valid model of standard wheelchair propulsion. The initial model used in this study was based on separate works done by Cooper (1–4). The model uses linear first order differential equations equating force with the acceleration of the wheelchair and the environmental factors resisting motion: air drag, rolling resistance, bearing resistance, and the slope of the terrain.

The objective of this study was to obtain a valid model for standard wheelchair propulsion as a first step toward the development of an integrated model of the wheelchair, environment, and user. Questions to be answered involved determining which environmental factors play a significant role and which factors, if any, are negligible. Immediate outcomes include an accurate description of the force required for an individual to propel a wheelchair of given specifications, allowing for more appropriate chair prescription. Once a valid model has been attained,

it will be possible to begin developing and combining a subject model with the wheelchair model.

Cooper's Wheelchair Model

In an effort to improve the performance and efficiency of racing wheelchairs, Cooper (1–4) developed computer and mathematical models of racing wheelchair propulsion. The combination of the models in these papers represents the entire human/machine system including the physiological aspects of the user, biomechanics of propulsion, wheelchair dynamics, and the influence of the environment on the system.

The wheelchair was modelled as a linear system which converts input forces at the push-rings into an output velocity at the center of mass. To simplify the model the authors assumed that the roll axis is parallel with the center line of the wheelchair (a valid assumption for a wheelchair propelled in a straight line). The resulting generated force was the total force of both the left and right sides,

$$F*(R/r_{pr}) = (M + I/R + I_f/r)*a + C*v^2 + K*v + (W_R*b_R/R + W_r*b_r/r)*\cosh(x) + W*\sinh(x) \quad [1]$$

where:

F = force tangent to push-ring
a = wheelchair acceleration
v = wheelchair velocity
x = distance traversed
R = radius of rear wheels
r = radius of front wheels
r_{pr} = radius of push-rings
I = inertia of rear wheels
I_f = inertia of front wheels
C = drag coefficient
K = coefficient of bearing resistance
b_R = coefficient of rear rolling resistance
b_r = coefficient of front rolling resistance
M = weight of individual and wheelchair
W = weight of individual and wheelchair
W_R = weight on rear wheels
W_r = weight on front wheels
h(x) = angle of inclination

Environmental factors were modelled by:

rolling
resistance $= (W_R*b_R/R + W_r*b_r/r)*\cosh(x)$ [1a]
bearing
resistance $= K*v$ [1b]
air drag $= C*v^2$ [1c]

No value for K, nor any validation for $K*v$ as an accurate representation of bearing resistance, was provided (1).

Modelling the Factors Influencing Performance

The various components of the model presented in the previous section dictate the amount of energy required to propel the wheelchair. Other approaches to modelling the mechanical and environmental factors that resist motion, rolling resistance, bearing resistance, and air drag, are found below.

Rolling Resistance

Rolling resistance is primarily a function of wheel and castor characteristics, total weight, and weight distribution. The total weight of the wheelchair and user (W) directly affects the force, F, needed to overcome rolling resistance while the radius of the wheel, R, inversely affects the resistance (Equation 2).

$$F_r = \frac{W*e}{R} \quad [2]$$

With an even distribution of the load to the front castors and the rear wheels, the force, F_r, is smaller for the rear wheels because of their larger diameter. Conventional wheelchair mass distribution is approximately 60 percent on the rear axle and 40 percent on the front castors. By shifting the weight rearward to create a 75:25 distribution, the rolling resistance decreases by 6 percent (5).

The variety of body types found among users poses problems for providing correct wheelchair mass distribution. An individual with lower limb amputation, for example, will have a different weight distribution and therefore a different center of gravity than an individual with an L5 lesion. As there is less weight on the front castors, the user's center of gravity shifts rearward, causing greater instability. As a result, when optimizing the rolling resistance, rearward stability must also be considered.

One must obtain information of the distribution over each wheel to apply Equation 4 to wheelchair use. Variations on Equation 2, Equations 3a and 3b, developed by Lemaire, et al. (6), incorporate the user/wheelchair system's center of gravity for determining the weight distribution over each wheel. Equation 3a is the force needed to

overcome rolling resistance of one front wheel, while equation 3b is for a single rear wheel.

$$F_f = e_f * (R_{cg}*W)/L_{wb} \qquad [3a]$$
$$F_r = e_r*[W - (R_{cg}*W)/L_{wb}] \qquad [3b]$$

where: e_f = coefficient of front wheel rolling resistance

e_r = coefficient of rear wheel rolling resistance

R_{cg} = distance from the rear wheel axle to the center of gravity

W = total weight of the user and wheelchair

L_{wb} = wheelbase length

Bearing Resistance

As with a bicycle wheel, the ball bearings used in wheelchairs transfer forces from the rotating wheel to the stationary chair components and permit translation of the wheelchair (7,8). The force generated by gravity and propulsion is transferred through the ball-raceway contacts causing bending and localized deflections in the bearing rings (8).

The bearing resistance term in Cooper's (1) model is defined as a function of the wheelchair/user's velocity (Equation 1c). For simplification purposes, K^*v can serve as an estimate of Equation 4, based on the assumption that the force to overcome the bearing resistance is proportional to the velocity of the wheelchair. Equation 4 considers the bearing resistance to be different for each wheel.

$$\text{bearing resistance} = M_B/r_{ax} + M_b/r_{axf} = K*v \qquad [4]$$

where: M_B = bearing resistance of the rear hub

M_b = bearing resistance of the front hubs

r_{ax} = radius of the rear axle

r_{axf} = radius of the front axle

Air Drag

The results of a study of a conventional wheelchair showed that the aerodynamic drag coefficient, C_D, of a wheelchair is approximately 0.96. Using a manikin the size of a fiftieth percentile man seated in the wheelchair, C_D rose to 1.4. During the use of a range of free-stream dynamic pressures, these drag coefficients remained essentially constant (9).

The force to overcome the effects of wind drag is typically modelled as proportional to the frontal area of the wheelchair-user system and the square of the relative air speed,

$$F_a = 1/2*D*(V_w - V_1)^2*A*C_D \qquad [5]$$

where: D = density of air (1.23 kgm^{-3} at STP)

V_w = velocity of the wheelchair

V_1 = velocity of the wind

A = frontal area of the wheelchair/user system

C_D = drag coefficient

MATERIALS AND METHODS

The project consisted of three stages: I) Comparison of obtained data using existing models and their variations; II) Development of additional model variations to better match data; and, III) Data collection and analysis using updated model from Stage II.

Stage I. Initial Evaluation of Models

Seven able-bodied subjects, ranging in age from 23 to 30 ($\mu = 27, \sigma = 2.3$) and having a mean weight of 79.5 kg ($\sigma = 16.8$), volunteered to propel the wheelchair for the experiment during Stage I. The wheelchair (a standard 24-inch folding wheelchair produced by Invacare, Inc.) was connected to a load cell through a constant resistance source (a model 500X Super2 Minigym by Mini Gym, Inc.) to collect force data. Prior to each trial, the subjects were instructed to pre-load the load cell such that any added force on the push rim caused forward movement. This procedure prevented severe jolting of the load cell and extreme spikes in the data. These pre-loaded values were used as the baseline during analysis. A video camera taped the motion of the wheelchair and user as they crossed the floor. The acceleration and velocity of the wheelchair were determined from the video tape and used as inputs to the models for producing the predicted force data. Statistical analysis of the observed and predicted data provided the means for validating a model. Nine test trials were run for each subject; three trials of each phase of motion: starting, constant velocity, and stopping.

Coordination of the observed and predicted (model generated) force profiles of wheelchair motion involved determining the starting point for each force profile and the best beta value to fit the curves. Beta was defined as the time-adjusted value needed to account for the time span difference

between the observed and predicted data sets. The predicted data time intervals were then multiplied by the beta value and the analysis done on matching points. The absolute value differences between the observed and predicted data were averaged over the entire time span of the trial. The lower this average absolute value difference (termed the *trial index value*), the better the beta value. The *mean trial index* was obtained by averaging these trial index values over all subjects. The *phase index* was obtained by averaging the three trials of each phase of motion for each subject after time lag adjustment. Averaging these phase index values over all subjects was termed the *mean phase index*.

Model Analysis

Fourteen variations of Cooper's model (1) were created by developing combinations of the effects discussed previously. These models were generated by the inclusion or exclusion of a term, and/or by a different mathematical representation of a parameter. Preliminary studies of the model variations reduced the initial number to eight (see Appendix). Analysis of the eight models demonstrated a significant difference between and within the groups based on trial and phase index values. **Table 1** shows the model groupings obtained with the mean trial index calculated from all subjects over the given phase of motion. Models within the same group are not significantly different, with the models having the lowest index value considered the best predictors. The level of significance was established by a *t*-test for the variable index.

As can be seen, model 2 was the best predictor of the observed forces for each phase of motion according to the mean trial index. The models found in group C all contained the same acceleration and rolling resistance terms, and were varied by their bearing resistance and air drag terms. That these Group C models are not significantly different from each other demonstrates that the air drag and bearing resistance terms do not play a significant role in the prediction of standard wheelchair propulsion. The inclusion of these terms neither added nor subtracted from the overall force prediction. Although the mean phase index indicated that model 3 was the best predictor for constant motion and stopping phases, there was essentially no difference between models 2 and 3. Since the model groups

Table 1.
Model groups of significance with mean trial index and mean phase index.

Group	Model	Starting Phase Mean Trial Index	Starting Phase Mean Phase Index	Constant Phase Mean Trial Index	Constant Phase Mean Phase Index	Stopping Phase Mean Trial Index	Stopping Phase Mean Phase Index
A	6	177.23	146.97	178.94	135.46	194.76	160.90
B	7	120.90	98.68	125.39	95.00	126.89	95.84
	5	128.77	98.46	125.24	94.87	126.60	95.41
C	8	103.92	86.13	106.82	86.91	101.33	86.63
	1	103.57	85.26	106.75	87.07	101.28	86.70
	4	103.94	85.66	106.68	86.86	100.27	84.87
	3	102.79	84.10	105.25	84.25	97.22	80.04
	2	102.19	83.38	105.22	84.45	97.18	80.10

were the same for both sets of indexes, the mean phase index was included for comparison reasons.

The difference between groups of models was accounted for by a combination of changes in the rolling resistance and mass acceleration term. Models 5, 6, and 7 used a value of rolling resistance as derived by LeMaire, et al. (6) and did not contain moment of inertia constants in the mass acceleration term. The exclusion of the inertia terms resulted in no significant difference among the models that included it. A graphical comparison of each trial was studied along with the individual index values. Visual analysis of the graphs involved comparing the maximum force prediction with what was observed. As can be seen in **Figure 1**, group A models predict a higher force than group B or group C, a trend which held for all subjects and all trials. **Figure 1** also demonstrates that for this particular trial, model group B was the best predictor in terms of a positive force magnitude. This visual analysis in combination with the index values led to the following three observations and the development of the Stage II group of models:

1. The model having the best positive amplitude prediction did not always have the lowest index value. This discrepancy can be explained by observing that the models displaying higher forces also had larger negative forces. With the index based on the absolute value difference between the profiles, the large negative values
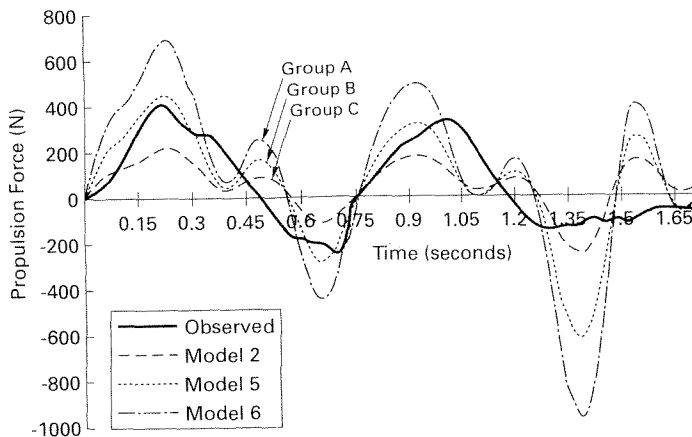
Figure 1.
Comparison of model groups for subject BF trial 5, with model 2 representing the predictions of group C, model 5 representing group B, and model 6 representing group A.

increased the average index. This trend followed for subjects whose peaks were best predicted by group A and B models for the constant motion and stopping phases. In the starting phase, the subjects attempted to accelerate continuously and thus avoided the large decelerations between strokes. In these cases, the model that predicted best visually generally had the lowest index.

2. The best model predictor of force magnitude varied within a subject trial. When the observed forces showed changes in magnitude from one peak (stroke) to the next, the theoretical profiles did not always reciprocate. Even though this observation was found for all subjects and in each phase of motion, peak-to-peak changes were predicted accurately in approximately half of the trials. The results of the second set of models will later demonstrate some conclusions that can be drawn from this seemingly random occurrence.

3. Lighter subjects had a different model group as a best predictor in terms of force amplitude than did the heavier subjects; light subjects by group A, heavy subjects by group C. Although this trend was seen in all phases of motion, it was most evident during the starting phase. The lightest and heaviest subjects had observed forces that were predicted consistently by the same group of models within a phase; however, subjects of average weight showed more fluctua-

tions among the three model groups as to which model best predicted the observed forces.

Stage II. Establishment of the Second Set of Models

The establishment of the second set of models was based on the result that model 2 had the best results in terms of the lowest index, but did not accurately predict the force amplitudes for all subjects. The approach taken to improving model 2 was derived from the three observations described above. The new model had to account for subject weight, control the negative swing of the force predictions, and match the changes in maximum force from stroke to stroke. This was achieved by establishing a linear proportionality constant, alpha, as a function of subject mass. To help control the negative swing, alpha was applied only when the acceleration of the wheelchair was positive. When acceleration was negative, model 2 was left unchanged. To account for the differences in peak force predictions, three alphas of increasing magnitude were evaluated by use in modifying model 2 (creating models 22, 23, 24). In addition, a fourth model (model 21) was developed to test the importance of controlling the negative swing. The alpha constant was generated in each case from the equation

$$\alpha = (x - W^*y) \qquad [6]$$

where W is the weight of the subject/wheelchair system. Values for x and y were derived from a sequence of two equations based on different magnitudes of alpha needed. The two extreme subject weights were used for the initial conditions of W in the calculations. For example, model 21 was derived from:

$$(x - 115^*y) = 1 \qquad [7a]$$
$$(x - 70^*y) = 2 \qquad [7b]$$

resulting in an x of 3.556 and a y of 0.0222. **Table 2** gives the values for α, x, and y for the four Stage II models.

Analysis of the Second Set of Models

Analysis of the alpha adjusted models proceeded in the same manner as the first set of original models. In this case however, the data from four subjects, chosen to reflect weights which ranged from light to heavy, were used to establish the mean trial and mean phase indexes for determining a best

Table 2.

Values for α as calculated from x and y for the second set of models.

Model	α	x	y
21[a]	$1 < \alpha < 2$	3.556	0.0222
22	$1 < \alpha < 2$	3.556	0.0222
23	$1 < \alpha < 2.5$	4.833	0.0333
24	$1 < \alpha < 3$	6.111	0.0444

[a]Model 21 differs from model 22 in that α is applied for the whole profile, not just for positive acceleration.

Table 3.

Mean trial and mean phase indexes values for the phases of motion for the second set of models.

		Starting Phase Mean Trial Index	Group	Model	Mean Phase Index
Group	Model				
A	21	96.41	A	21	73.82
B	24	93.18		22	70.83
	22	92.47		23	67.60
	23	90.33		24	67.25

		Constant Phase Mean Trial Index	Group	Model	Mean Phase Index
Group	Model				
A	24	124.14	A	24	93.94
	21	118.63		23	90.14
	23	117.88		22	88.31
	22	114.21		21	85.94

		Stopping Phase Mean Trial Index	Group	Model	Mean Phase Index
Group	Model				
A	24	114.31	A	24	99.91
	23	110.17		23	96.82
				22	94.40
B	22	107.34	B	21	84.44
	21	104.56			

model. It was again found that a different model was needed to predict each of the three phases of motion. **Table 3** shows the mean trial and mean phase index values for the corresponding phase.

The results obtained for the starting phase (see **Table 3**) indicated that a large alpha was needed to increase the magnitude of the model predictions. Although the mean phase index shows model 24 to be the best predictor, there is no significant difference between its index and model 23. For constant motion phase, there was again a discrepancy between the analysis procedures giving the best model predictors, although this difference was not significant. The lack of significance can be explained by the fact that subjects were not perfectly consistent in their propulsion efforts, leaving a different model to best predict a peak force. Analysis of the stopping phase index values showed that model 21 was the best predictor, since it also increased the magnitude of the deceleration force, it was a better predictor than model 22.

Stage III. Application of Best Models

To test the validity of the second set of models, the best models were then applied to data from three additional subjects chosen to represent the range of weights involved in the prior experiment. **Table 4** gives the resulting mean trial and mean phase indexes for each phase of motion. Despite these trials being affected by not having the proper time lag adjustment, the mean trial and mean phase index values compare favorably with the previously found indexes. **Tables 5** and **6** give the models having the lowest index value for each step of the analysis.

DISCUSSION

The current investigation led to the conclusion that, in one form or another, the models found in literature are deficient in predicting standard wheelchair motion. The initial statistical analysis of these models showed that model 2, a derivation of Cooper's model (1), was the best overall predictor of the observed forces. A second set of models was then established using model 2 as a base, adjusted by various degrees of alpha, a factor based on weight. It was found that the same model cannot be used to predict values for each of the three phases of motion because the various magnitudes of alpha demonstrate the mechanical differences between the motion phases must be considered. These differences were more important to model predictions

Table 4.
Mean trial and mean phase indexes for experimental study group.

Phase	Model	Mean Trial Index	Mean Phase Index
Starting	23	91.81	71.85
Constant	22	106.43	84.03
Stopping	21	110.52	76.73

Table 5.
Comparison of lowest mean trial index values for the three steps of analysis.

Phase	Stage I Model	Stage I Index	Stage II Model	Stage II Index	Stage III Model	Stage III Index
Starting	2	102.19	23	90.33	23	91.81
Constant	2	105.22	22	114.21	22	106.43
Stopping	2	97.18	21	104.56	21	110.52

Table 6.
Comparison of lowest mean phase index values for the three steps of analysis.

Use	Stage I Model	Stage I Index	Stage II Model	Stage II Index	Stage III Model	Stage III Index
Starting	2	83.38	24	67.25	23	71.85
Constant	3	84.25	21	85.94	22	84.03
Stopping	3	80.04	21	84.44	21	76.73

than was the inclusion of air drag or bearing resistance terms. The constant motion and stopping phases required the same alpha adjustment. The application of alpha to the deceleration values during the stopping phase helped in predicting the actual stop with a larger alpha needed for predicting starting forces.

Since alpha was based on the subject weight range between 50 and 100 kilograms (110 to 220 pounds), it is unknown how subject masses outside of this range will affect propulsion and the outcome of the models. Recommendations for further studies include tests to verify these results with larger extremes of subject weight. In addition, by controlling the speed of the subjects, the relationship between how changes in acceleration, in terms of the

different phases of motion, affect the model predicted force can be addressed. Finding this relationship may provide insight for better prediction of peak-to-peak force changes and lead to a model that can better predict the inherent inconsistencies during real-life standard wheelchair propulsion.

The improvement of the alpha adjusted models over the initial model set was demonstrated by comparable index values, better force amplitude prediction, and a more frequent occurrence where the model with the lowest index value also had the best amplitude prediction. Although further refinement of the second set of models may lead to even better results, it is believed that the models investigated provided an accurate prediction of wheelchair propulsion based on the consistency of the mean trial and mean index values generated from each analysis stage.

Recommendations for further study include testing the model for predicting wheelchair motion when on a slope and for various weight distributions. Since phase indexes show the same consistency as the trial indexes, single trials for subjects may be used. However, still better results will be obtained by averaging multiple trials. As mentioned above, further study should also include determining the relationship of acceleration with the models. A single model could be developed that would adapt to the stroke to stroke changes of real life propulsion.

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APPENDIX

Phase I Variations on Cooper's Model

Model 1
$$Z = (RPR/RR)*[(W + IR/PR^2 + IFW/RF^2)*A + C*V^2 + K*V + (WR*BRR/RR + WF*BRF/RF)*X)]$$

Model 2
$$Z = (RPR/RR)*[(W + IR/RR^2 + IFW/RF^2)*A + (0.5*D*O*C*(VW-V^2)) + (WR*BRR/RR + WF*BRF/RF)*X]$$

Model 3
$$Z = (RPR/RR)*[(W + IR/RR^2 + IFW/IF^2)*A + K*V + (0.5*D*O*C*(VW-V^2)) + (WR*BRR/RR + WF*BRF/RF)*X]$$

Model 4
$$Z = (RPR/RR)*[(W + IR/RR^2 + IFW/RF^2)*A + C*(V^2) + (WR*BRR/RR + WF*BRF/RF)*X]$$

Model 5
$$Z = (RPR/RR)*[W*A + C*V^2 + K*V + (FUR + FUF)*A]$$

Model 6
$$Z = (RPR/RR)*[W*A + C*V^2 + K*V + (2*FUR + 2*FUF)*A]$$

Model 7
$$Z = (RPR/RR)*[W*A + (FUR + FUF)*A]$$

Model 8
$$Z = (RPR/RR)*[(W + IR/RR^2 + IFW/RF^2)*A + (WR*BRR/RR + WF*BRF/RF)*X]$$

Nomenclature

A   = Acceleration of the wheelchair
BRF = Coefficient of front rolling resistance
BRR = Coefficient of rear rolling resistance
C   = Coefficient of air drag
D   = Density of air
FUF = Front wheel friction
FUR = Rear wheel friction
IFW = Inertia of the front wheels
IR = Inertia of the rear wheels
KV = Coefficient of bearing resistance
LWB = Wheelbase length
MF = Weight of one front wheel
MR = Weight of one rear wheel

O = Frontal area of the user/wheelchair system
RCG = Distance from center of gravity to rear wheel axle
RF = Radius of the front wheels
RPR = Radius of the push rings
RR = Radius of the rear wheels
V = Velocity of the wheelchair
VW = Velocity of the wind
W = Weight of the individual and the wheelchair
WF = Weight on the front wheels
WR = Weight on the rear wheels
X = Distance traveled
Z = Model predicted force

Additional equations used in models
RCG = (W − 17.73)*0.171
FUR = BRR*(W − (RCG*W)/LWB)
FUF = BRF*((RCG*W)/LWB)
where:
  17.73 = Weight of wheelchair in kilograms
  0.171 = Correction factor for subject weight differences