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Modelling the Spatial Distribution of DEM Error

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Abstract

Assessment of a DEM's quality is usually undertaken by deriving a measure of DEM accuracy – how close the DEM's elevation values are to the true elevation. Measures such as Root Mean Squared Error and standard deviation of the error are frequently used. These measures summarise elevation errors in a DEM as a single value. A more detailed description of DEM accuracy would allow better understanding of DEM quality and the consequent uncertainty associated with using DEMs in analytical applications. The research presented addresses the limitations of using a single root mean squared error (RMSE) value to represent the uncertainty associated with a DEM by developing a new technique for creating a spatially distributed model of DEM quality – an accuracy surface. The technique is based on the hypothesis that the distribution and scale of elevation error within a DEM are at least partly related to morphometric characteristics of the terrain. The technique involves generating a set of terrain parameters to characterise terrain morphometry and developing regression models

to define the relationship between DEM error and morphometric character. The regression models form the basis for creating standard deviation surfaces to represent DEM accuracy. The hypothesis is shown to be true and reliable accuracy surfaces are successfully created. These accuracy surfaces provide more detailed information about DEM accuracy than a single global estimate of RMSE.

1.0 Introduction

Assessments of the accuracy of DEMs tend to result in a single measure of how closely the DEM's elevation values represent "reality". Measures such as Root Mean Squared Error and standard deviation of the error are frequently used (Carlisle, 2002; Day and Muller, 1988; Eklundh and Mårtensson, 1995; Fisher, 1998; Kumler, 1994; Li, 1991; Sasowsky, 1992). These measures summarise elevation errors in a DEM as a single value. There is increasing demand for more detailed description of spatial data quality (Canters *et al.*, 2002). A more detailed description of DEM accuracy would allow better understanding of DEM quality and the consequent uncertainty associated with using DEMs in analytical applications.

Anecdotal and empirical evidence shows that DEM error is spatially variable, spatially correlated and heteroscedastic, being related to the form of the terrain (Ehlschlaeger and Shortridge, 1997; Fisher, 1998; Hunter and Goodchild, 1997; Kyriakidis *et al*., 1999; Theobald, 1989; Weibel and Brändli, 1995; Wood, 1994; Zhang and Montgomery, 1994). However, very little research has attempted to model this heteroscedasticity. This paper reports on research to test the hypothesis that DEM error is related to terrain character and then to develop a more detailed description of DEM accuracy by representing the spatial variation in error across a DEM as an accuracy surface that is generated from regression modelling of the relationship between DEM error and terrain characteristics. The resulting spatially variable accuracy surfaces would provide this more detailed description and also give a better representation of DEM errors for use in uncertainty modelling using techniques such as Monte Carlo simulation.

The paper begins by reviewing the current state of knowledge of the spatial distribution of DEM error and previous work on modelling this distribution. The study areas and data sets used in the research presented here are then described. The methods used to model DEM error distributions

are then explained in three sections, each section ending with a presentation of the relevant results. The three sections are:

- Examining the relationship between DEM elevation error and terrain characteristics;

- Developing a model of the relationship between DEM error and terrain characteristics to produce spatially variable, spatially correlated, heteroscedastic error surfaces;

- Assessing the quality of the error surfaces.

The final sections of the paper discuss the findings of this research and the issues raised and draw conclusions.

2.0 DEM Error

DEM accuracy is a topic that has received considerable attention since DEMs came into widespread use in the 1980s. Many studies have sought to quantify DEM accuracy and compare the accuracy of DEMs produced using different data sources and production methods. Others seek to identify and correct major errors. Relatively little published work addresses the spatial distribution of DEM error. Previous studies into the spatial distribution of DEM error are reviewed below, followed by work to model this distribution.

2.1 The Spatial Distribution Of DEM Error

Describing elevation errors in a DEM with a single, global accuracy measure, such as standard deviation or RMSE, has advantages. The single value is relatively quick to calculate and easy to report. A single value makes comparison of DEMs a simple task. Global accuracy measures have also been used to model the influence of DEM error on uncertainty in DEM-based spatial modelling outcomes. However, a number of authors recognise that a single global accuracy measure has its limitations. Wood (1994) states that any useful study of DEM accuracy must investigate the spatial variation of error values. Kyriakidis *et al*. (1999) describe how a global

accuracy statistic does not allow identification of areas where error is greatest and additional source data would most benefit DEM quality. Theobald (1989), Zhang and Montgomery (1994) and Weibel and Brändli (1995) all recognise that appreciating the spatial variability of accuracy is critical to environmental applications. For example, small errors in relatively flat areas will have a greater impact on surface run-off and flood modelling than in steeper areas (Burrough and McDonnell, 1998). Alternatively, in viewshed analysis, errors in higher, steeper terrain will have the greatest impact on results (Fisher, 1991). Ignoring spatial non-stationarity can cause serious mis-estimation of error and uncertainty in DEM-based modelling outcomes (Canters *et al.*, 2002).

Burrough and McDonnell (1998) state that a single RMSE value implies that error is uniform across the DEM. Several authors identify this assumption of stationarity as invalid (Fisher, 1998; Kyriakidis *et al*., 1999). In their research, Ehlschlaeger and Shortridge (1997) and Hunter and Goodchild (1997) use a spatially uniform model of DEM error, but acknowledge that it is actually spatially variable.

It seems intuitive that certain types of terrain will be more suited to creation of accurate DEMs. Indeed a number of authors report that the magnitude of DEM error is related to characteristics of the terrain. Gao (1997) observes that DEM errors seem lower in less complex terrain. Hunter and Goodchild (1997) state that DEM error is probably related to slope steepness. Carrara *et al*. (1997) suggest that DEMs derived from stereo aerial photography could have greater errors on steep and shaded slopes. McDermid and Franklin (1995) state that photogrammetrically produced DEMs will be most accurate on open flat terrain and least accurate on steep, shadowed and vegetated terrain. Slope steepness and other terrain characteristics are spatially variable. It therefore follows that DEM elevation errors should also be spatially variable.

There are some examples of researchers identifying and quantifying this relationship between error and terrain characteristics. Bolstad and Stowe (1994) evaluate the accuracy of elevation values for two DEMs. They find that the largest elevation errors tended to occur in the highest and lowest parts of the study area. Fisher (1998) finds significant correlation between DEM error and slope angle. Ehlschlaeger and Shortridge (1997) report that empirical studies have shown DEM error to be related to gradient and propose that it may also be related to other elevation derivatives. Kyriakidis *et al.* (1999) find that DEM error is correlated with terrain ruggedness. Guth (1992) finds DEM error to be highly correlated with gradient, aspect and satellite image reflectance value.

2.2 Modelling the Distribution of DEM Errors

There has been little research attempting to model the spatially variable distribution of DEM error. A number of the authors mentioned above acknowledge this spatial variation, then proceed with their research assuming a uniform DEM error distribution. Ehlschlaeger and Shortridge (1997) defend this assumption by stating "*modelling elevation data uncertainty is a difficult task*". The work that has been done either models the spatial correlation of DEM errors or attempts to create an error surface as a model of the distribution of errors.

Simulated DEM error surfaces have been generated from global accuracy measures for use in Monte Carlo simulation of the uncertainty associated with using DEMs of limited quality. The initially simulated error surfaces consist of a random mixture of error values. However, DEM error is spatially correlated (Fisher, 1998; Hunter and Goodchild, 1997; Lopez, 2002). Therefore a model of DEM error should not be random, but spatially dependent. Hunter and Goodchild (1997) present a spatially autoregressive error model which switches cell values until

a spatially correlated error surface is produced, i.e. a surface in which the error values change gradually from one cell to the next. However, their work is based on the single global RMSE value and the spatially correlated error values only vary within a normal distribution with this global RMSE value.

Monckton (1994) reports on his use of Moran's I to simulate the spatial structure of elevation error. This index measures the similarity of values at specific distances (lags). A model of the change in spatial correlation with distance can be built up by calculating the index for a number of distances. Monckton finds no evidence of spatial autocorrelation in the error distribution of his study. However, the results can be considered inconclusive as the sparsity of sample points used, spot heights on Ordnance Survey maps, only allowed examination over lags of 250m or greater. There may be spatial autocorrelation of elevation error at shorter lags than those he used.

Research by Giles and Franklin (1996) used semi-variance analysis to evaluate the periodicity of DEM error in the form of random noise. This allowed an optimum sized filter to be determined so as to remove the random noise, leaving only the spatially correlated portion of the error. However, the research did not proceed to investigating the nature of the remaining error, either in terms of the level of spatial autocorrelation or the magnitude of error associated with particular terrain characteristics.

These three examples of modelling the spatial correlation of error produce error surfaces that are spatially variable and spatially correlated, but homoscedastic. This means error values vary, but not in relation to any other variable. The apparent relationship between error and terrain means that a DEM error surface should be heteroscedastic.

Burrough and McDonnell (1998) favour the use of Kriging interpolation techniques when creating a surface from point data, because a second surface is generated which represents the predicted accuracy of the interpolated values as spatially distributed standard deviation values. A linear spline fitting interpolation technique described by Wood (1994) similarly produces an RMSE surface quantifying the accuracy of the interpolated values. These surfaces are distributed error models, but they only describe uncertainty in the interpolation estimates and therefore show RMSE or standard deviation increasing with increasing distance from the original data points. They do not take into account the accuracy of the source data or the relationship of error with terrain character.

De Bruin and Bregt (2001) take high accuracy GPS measurements to determine DEM error at a set of sample locations, then use kriging to interpolate an error surface. This error surface is spatially non-stationary and spatially correlated, but unrelated to other factors such as terrain character and is, therefore, homoscedastic. Similarly, Fisher (1998) takes a geostatistical approach to modelling DEM error, using conditional simulation to derive multiple realisations of error surfaces which are spatially correlated and spatially non-stationary. However, no account is taken of the significant correlation he had found between gradient and DEM error. Kyriakidis *et al*. (1999) find a strong correlation between error and terrain ruggedness ($\rho = 0.64$), which they quantify using the standard deviation of elevation values within a 3 x 3 window of each cell. They then proceed to create higher accuracy DEMs by applying co-kriging to the original DEM, basing new elevation values on distance from high accuracy height measurements and the standard deviation of elevation values. This co-kriging is performed in a multi-Gaussian framework to create a user-specified number of realisations of this higher accuracy DEM. The standard deviation in elevation values at each cell location across the set of

multiple DEM realisations gives an estimate of the accuracy of the higher accuracy DEMs in the form of an error surface. Potential limitations to this approach can be identified. First, the error they calculate and use is the difference between a USGS DEM with a resolution of one degree and another USGS DEM with a resolution of 7.5 minutes. They are in effect analysing the difference between two models of the terrain surface rather than modelling the accuracy of a DEM in relation to the actual on-the-ground elevation. The method is based on the strong correlation between error and what they term terrain ruggedness. This correlation may be a property of the relationship between the two DEMs rather than a terrain − error relationship. This would have to be verified by repeating the technique using different DEMs and a non-DEM source of higher accuracy elevation measurements. Second, they quantify terrain ruggedness as the standard deviation of elevation, which means that it is actually a measure of relative relief (Evans, 1972). Their method does not take any account of the reported relationship between error and other terrain characteristics, such as aspect and gradient, although relative relief is related to gradient. Nonetheless, their research is the only example of the creation of a heteroscedastic, spatially correlated error surface and the approach used is worthy of further investigation.

Given the influence of terrain character on DEM error, regression modelling would seem to offer potential for creating spatially non-stationary, spatially correlated and heteroscedastic error surfaces. Ordinary least squares (OLS) regression has been widely used to represent relationships between environmental variables, whose value has been measured at a limited number of sample locations, and some other more widely measured variable (Foody, 2003). The regression model can then be used to predict values of the environmental value at unsampled locations. In the context of the work presented here, the independent and widely measured

variables are DEM-derived terrain parameters. The dependent, sparsely measured variable is DEM error.

3.0 Developing an Error Surface

The research uses DEMs and terrain parameters from two study areas. These are described below.

3.1 Study Areas and Data

Initial research was based on a 1km x 2km area of Snowdonia, North Wales. DEMs were created from Ordnance Survey Landform Profile data. The data represent 1:10 000 scale digital contour lines at a 10m vertical interval. Three DEMs were chosen from those described in Carlisle (2000; Table 1). Carlisle (2000) produced 20 DEMs using various formats of input data and interpolation procedure, then assessed their accuracy. The DEMs used here were all produced from the same source data, but the interpolation differs, as specified in Table 1. These chosen DEMs provide a range of accuracy. IDW112 is the least accurate of Carlisle's (2000) twenty DEMs. SPTEN12 is the most accurate. IDW512 is of intermediate accuracy, but the most accurate of the DEMs produced by inverse distance weighting.

<INSERT TABLE 1 APPROXIMATELY HERE>

GPS measurements of elevation, accurate to approximately 1m RMSE, were used to determine DEM error at 106 sample points (Carlisle and Heywood, 1996; Carlisle, 2000). A DEM-derived estimate of elevation at the location of the GPS sample point was derived by

inverse distance weighting interpolation of the four nearest DEM grid nodes. DEM error was measured as DEM-derived elevation minus GPS-derived elevation.

Subsequently key stages of the research were reapplied to a 23.5km x 18.1km region of Mestersvig, northeast Greenland. The Mestersvig study area has been used to validate that the methodology can be usefully applied to another mountain region using a different scale of source data and different resolution DEMs. 1:15,000 scale contour maps derived from aerial photography were the only available source of elevation data. These maps contained contour lines at a 10m vertical interval. In the low-lying coastal zone all contours from 0m to 100m above sea level and the 120m contour were manually digitised. In the steeper more mountainous areas further inland every fifth contour was digitised. This digitising strategy gave the best possible definition of the low lying areas, while avoiding excessive effort digitising the uplands. Spot heights, mainly located on summits, were also digitised. Three DEMs with a resolution of 10m were generated using the same interpolation techniques as for the three Snowdonia DEMs. GPS measurements of elevation were made at 103 sample points.

Both study areas have low arctic or sub-alpine vegetation cover which will have no effect on quality of the aerial-photography derived elevation data used to create the DEMs. Both areas are mountainous and therefore have highly heterogeneous terrain characteristics.

3.2 Terrain Parameters

In order to examine and model the relationship between DEM error and terrain morphometry, a set of terrain parameters were derived from the DEMs, which gave a comprehensive description of terrain form. Evans' (1972) five geomorphometric parameters that represent a comprehensive and non-duplicative quantification of surface form (elevation, gradient, aspect,

plan curvature and profile curvature) were derived for each DEM. The circularly scaled aspect values were transformed into east-west and north-south component vectors (termed aspect X and aspect Y respectively) to allow later manipulations such as averaging. In addition, six other terrain parameters were derived which quantify other characteristics of terrain: overall curvature, surface heterogeneity (relative relief and texture) and terrain position (mean, minimum and maximum extremity). The surface heterogeneity and terrain position calculations involved using a neighbourhood of DEM grid nodes. A circular neighbourhood of 10 cell radius was used. Relative relief was calculated as the difference between maximum and minimum elevation within the neighbourhood. Texture was calculated as the difference between maximum and minimum gradient within the neighbourhood. Mean extremity is the difference between the elevation of the grid node under consideration and the mean elevation within the neighbourhood. Minimum extremity is the difference between the elevation of the grid node under consideration and the minimum elevation within the neighbourhood, while maximum extremity is the difference with maximum elevation within the neighbourhood. The distance from a cell to the nearest contour vertex was also calculated, as it would be expected that error is higher for locations that are a greater distance from the source data. Table 2 describes the twelve parameters.

<INSERT TABLE 2 APPROXIMATELY HERE>

These terrain parameters were calculated and extracted for each DEM at every GPS sample point. The grids representing first and second order derivatives of elevation were found to be highly variable from grid cell to grid cell. Producing grids showing the average values for a grid cell and the neighbouring grid cells within a certain radius by applying a mean filter would represent the underlying trends of the terrain parameters. Terrain characteristics are

scale dependent (Wood, 1996). Mean filtering with a variety of filter kernel sizes would represent these terrain characteristics at a variety of spatial scales. Terrain parameters may have a stronger relationship with error at certain scales or the relationship may involve a combination of scales. Consequently mean values were derived for the elevation, gradient, aspect, plan curvature, profile curvature and overall curvature parameters using circular filter kernels of 5, 10 and 20 cell radii.

Relative relief, texture, and the three extremity parameters were initially produced from filtering functions, using a circular kernel of 10-cell radius. In accordance with the above consideration of scale dependency, values were derived for these parameters using circular kernels of 5 and 20 cell radii.

Evans (1972) reports that, in addition to the mean value of a parameter for an area or terrain, or in this instance a neighbourhood of grid cells, the standard deviation of values provides important information about terrain form. Therefore, the standard deviation of elevation, gradient, aspect vectors, plan curvature, profile curvature and overall curvature values within circular kernels of 5, 10 and 20 cell radii were calculated.

4.0 The Error – Terrain Character Relationship

For each DEM, correlation coefficients were calculated for each terrain parameter and elevation error to provide a first indication of any relationship between DEM error and terrain parameters. Correlation coefficients can only identify linear relationships. It was suspected that non-linear relationships between terrain parameters and DEM error might exist. To investigate this possibility the correlation between error and the squared and cubed values of each parameter were also calculated.

It was unlikely that any single terrain parameter would show a strong relationship with DEM error. It was expected that a number of terrain parameters acting in combination would influence the spatial variation in DEM error. Stepwise selection of variables to include in a multivariate ordinary least squares linear regression model was used to identify any such multivariate relationship and select the best combination of terrain parameters. Although linear regression models were employed, the use of the squared and cubed terrain parameters means that the regression models were in effect polynomial. Different regression equations were compared by looking at the regression coefficient of the step containing 20 variables. A regression equation of 20 variables does not necessarily equate to 20 terrain parameters, as some variables may be the square or cube of a parameter. Even though, beyond 20 variables, additional variables were still showing statistical significance, this cut-off point was chosen, because there appeared to be little increment in adjusted R^2 values with more than 20 variables.

4.1 Correlations

Table 3 summarises results for the correlations of error with all the terrain parameters, giving the highest correlation coefficients and the number of pairs of variables that gave a correlation that was significant at the 0.05 probability level in a two-tailed test.

<INSERT TABLE 3 APPROXIMATELY HERE>

For Snowdonia, IDW112 shows the strongest correlations with terrain parameters and a high number of significant correlations. The spline with tension DEM has the highest number of significant correlations with error. IDW512, the lower quality DEM, has weaker correlations

with terrain parameters than the other two DEMs. In contrast, of the Mestersvig DEMs, IDW512 has the greatest number of significant correlations and all three DEMs have similar strongest correlation coefficients. There are also a greater number of significant correlations than for Snowdonia. It is interesting to note that the errors in Mestersvig's two inverse distance weighted DEMs are most strongly correlated with elevation, but with opposite signs. No explanation for this has been deduced, but it does illustrate how a seemingly slight alteration to the interpolation method can cause a considerable change to the DEM's error structure.

The number of moderately strong correlations indicates that there is a relationship between DEM errors and terrain characteristics. The Mestersvig correlation coefficients are generally higher than for Snowdonia and there are a greater number of significant correlations. However, for both areas no single terrain parameter gives a good indication of the amount of error, but a combination of parameters could.

Research proceeded with all 6 DEMs. However, for simplicity, only results for the DEMs with strongest correlations are given here, i.e. IDW112 for Snowdonia and SpTen12 for Mestersvig. See Carlisle (2002) for full details.

4.2 Regression Modelling

Regression modelling for the Snowdonia data resulted in an adjusted R^2 value of 0.827 indicating that elevation error can be modelled by the DEM's terrain parameters with over 80% success. For the Mestersvig data the adjusted R^2 value of 0.902 indicates that elevation error can be modelled with a high degree of success. The regression models utilised a wide variety of the terrain parameters, including the mean and standard deviation of some

parameters, and measured over radii of 5, 10 and 20 cells. Further details of the terrain parameters used are given in Table 4. The only parameter not used in either model is the distance to the nearest contour vertex. Five parameters are used in both regression models: minimum extremity measured over a 10 cell radius neighbourhood; minimum extremity measured over a 20 cell radius neighbourhood; the standard deviation of aspect Y over a 5 cell radius neighbourhood; the average of aspect X over a 20 cell radius neighbourhood; and, profile curvature. Aspect parameters are the most common type of parameter in both models, but are particularly dominant in the Snowdonia model with 7 parameters. While there are similarities between the two models, there is clearly significant difference not only in terms of which parameters are used, but which exact form of the parameter is used and their coefficients. There is no evidence of a terrain – error relationship common to both sites. However, it does seem that a reliable site-specific error model can be constructed from regression of DEM error against terrain parameters.

<INSERT TABLE 4 APPROXIMATELY HERE>

5.0 Generating Error Surfaces

The regression equations with the highest regression coefficients were used to create error surfaces. Grids representing the required terrain parameters were derived and multiplied by the corresponding coefficients from the regression equation, then added together. This produced predicted error values for the entire extent of the DEM. In addition, a mean filter was applied to the error values and the local standard deviation of error values and the local standard deviation of mean filtered error values were calculated. These three additional grids were derived using a circular kernel of 20-cell radius. The grids representing standard

deviation of error values are produced to provide a spatially variable estimate of DEM accuracy that can be used in stochastic simulation (Carlisle, 2002).

6.0 Model Validation

The use of multivariate regression of terrain parameters to model DEM error was validated in three ways.

First, the GPS measurements were randomly split into three subsets of equal numbers of points. A regression equation was determined using two of the point subsets and the terrain parameters identified as most effective by the stepwise regression modelling. This regression equation was used to predict the elevation error for the third subset of points. This was performed three times so that elevation error for each subset of points was predicted. The predicted error values were then compared to the GPS-measured error values to assess whether the regression equation could be usefully employed to predict error values for the entire DEM.

Second, assessing the characteristics of each error surface identified spurious and extreme error values and allowed examination of the overall distribution and range of error values. This involved calculating summary statistics (minimum, maximum, mean and standard deviation) and examining the frequency distribution of error surface values.

Third, the error surfaces and standard deviation of error surfaces were visually assessed by examining 2D renderings and orthographic views. This allowed a general check for reasonableness in the scale of modelled error values and their spatial distribution.

6.1 Model Validation - Predicted Errors

Values for the minimum, maximum, range, mean and standard deviation of errors at Snowdonia's 106 sample points and Mestersvig's 103 sample points are given in Table 4. The table shows the actual errors and the corrected errors (actual minus predicted error).

<INSERT TABLE 5 APPROXIMATELY HERE>

The regression modelling can be considered successful when the corrected values are less than 100% of the actual values as this means the degree of error has been reduced. This does not occur for all summary statistics. Snowdonia's minimum corrected error exceeds the actual error. However, the range of corrected errors is lower than the range of actual errors for the Snowdonia DEM. Additionally, the standard deviation of corrected errors is lower than the standard deviation of actual errors. This indicates that there are a low number of extreme corrected errors, but overall the corrected errors are less widely dispersed than the actual errors. The spread of error values has been significantly reduced. The mean corrected error is near zero indicating that the regression modelling successfully removes a systematic bias.

The prediction success for the Mestersvig DEM is significantly better than for the Snowdonia DEM. All measures are much reduced. Mean corrected error is again near zero, indicating removal of a systematic bias. All other corrected measures are between 46% to 50% of their equivalent actual measures.

6.2 Model Validation - Error Surface Characteristics

Summary statistics for the error surfaces are shown in Table 5 with corresponding figures for the GPS sample points. The minimum and maximum values are clearly extreme. A mean

filter of circular 20-cell radius was applied to the error surfaces to reduce this occurrence of occasional extreme values. Summary statistics for these mean error surfaces are also given in Table 5.

<INSERT TABLE 6 APPROXIMATELY HERE>

The mean values for the Snowdonia surface are slightly higher than the GPS sample mean. The mean filter drastically reduces the maxima and minima and consequently brings standard deviation closer to the GPS sample standard deviation.

For the Mestersvig surface the mean value is noticeably lower than that of the GPS samples. This could be because the distribution of GPS sample points on different terrain types does not match the distribution of terrain types for the whole DEM area. It does not necessarily mean that the error surfaces are incorrect. As for Snowdonia, the mean filter drastically reduces the maxima and minima and consequently brings standard deviation closer to the GPS sample standard deviation. However, unrealistically extreme errors remain.

The summary statistics for both locations indicate that prediction of actual errors by means of regression modelling is only partially successful. There is uncertainty about the accuracy of the error surface. A spatially variable estimate of DEM accuracy could be more appropriate than predicting actual error on a cell-by-cell basis. This accuracy estimate could be a surface representing the standard deviation of estimated error for a cell and its neighbours. The neighbourhood calculations involved in producing such a surface could suppress the extreme values. Also, it is an estimate of accuracy that is required as input to models of uncertainty,

such as Monte Carlo simulation, whereas a prediction of actual error only allows a corrected DEM to be produced.

The standard deviation of error values within a 20 cell circular radius of each cell was calculated from both the error surface and the mean error surface for each DEM. Summary statistics for these accuracy surfaces are shown in Table 6.

<INSERT TABLE 7 APPROXIMATELY HERE>

The summary statistics of a high quality accuracy surface would show a minimum value close to zero, a low maximum value and a mean value similar to the GPS sample's standard deviation. The standard deviation of error surfaces appear reasonable. The standard deviation of mean error surfaces have much lower maximum values. However, the mean values are also reduced to much less than the GPS samples' standard deviations. This indicates that accuracy is generally underestimated by these surfaces.

6.3 Model Validation - Visual Assessment

Orthographic views of the standard deviation of error surfaces are shown in Fig. 1.

<INSERT FIGURE 1 APPROXIMATELY HERE>

The orthographic views clearly show that for Snowdonia standard deviation of mean error removes the more extreme values and produces a rather spatially invariable accuracy surface. The lowest accuracy is found on the central parts of the steepest slopes. Gentle and evenly sloping terrain have the highest accuracy. There is a clear variation in the distribution of cells

with a standard deviation of less than 4m, no extreme values and only a few cells with a standard deviation of above 32m.

For Mestersvig, standard deviation of mean error again removes the more extreme values and gives lower standard deviation values. Lowest accuracy is found along ridge tops and peaks, unlike Snowdonia where lowest accuracy is found on the central parts of the steepest slopes. Accuracy is also lower on southeast facing slopes. As suggested by McDermid and Franklin (1995), this could be due to these slopes being in shadow at the time of the aerial photography from which the original contour line data were created. There is a clear contrast between the accuracy of the flatter coastal areas and the more mountainous interior. Some of the low number of extreme accuracy values are found at the edge of the study area. These extreme values are probably caused by poor interpolation where there is an inadequate distribution of contour vertices and edge effects influencing the derivation of terrain parameters.

7.0 Discussion

This section considers the hypothesis that DEM error is related to terrain characteristics, examines the quality of the DEM error and accuracy surfaces and discusses issues affecting the quality of these surfaces.

7.1 The Relationship between DEM Error and Terrain Character

The research presented in this paper evaluates the hypothesis that the spatial variation in a DEM's error is related to characteristics of the terrain. This hypothesis has been developed in response to the anecdotal and empirical evidence of authors such as Guth (1992), Bolstad and Stowe (1994), Gao (1997) and Hunter and Goodchild (1997). For both study areas and all six DEMs coefficients of up to approximately $\rho = 0.5$ have been identified for the correlation

between DEM error and a number of terrain parameters. These values are lower than the $\rho = 0.64$ that Kyriakidis *et al.* (1999) computed for the correlation between error and standard deviation of elevation. Kyriakidis *et al.* (1999) consider error as the difference between two DEMs rather than the difference between a DEM and the elevation of the real terrain. This could explain their higher correlation coefficients. The lower correlation coefficients and the number of significant correlations found in the work presented here indicates that DEM error is related to terrain character, but this relationship is best quantified by multivariate modelling of a number of terrain parameters. The high adjusted regression coefficients of approximately 0.9 and predicted error statistics indicate that the spatial distribution of DEM error can be modelled from OLS regression modelling of terrain parameters.

OLS regression is a global technique in that a single regression model is created which best fits the whole data set over the entire study area. Given that DEM error is spatially non-stationary and terrain character is highly variable, most notably in mountain environments as studied here, it seems unlikely that the terrain character / DEM error relationship would be spatially stationary. The limitations of OLS regression, due to its assumption of spatial stationarity, have been identified by several authors in recent years (Cohen *et al.*, 2003; Foody, 2003; Fotheringham *et al.*, 2002). Geographically weighted regression (GWR) has been developed in response to these limitations. GWR extends OLS regression so that the regression parameters can vary locally and thus a non-stationary regression model is produced (Fotheringham *et al.*, 2000; Fotheringham *et al.*, 2002). The research presented here has shown that DEM error is related to terrain character, but this relationship is different at the two study sites, i.e. the relationship is not global, but local. Therefore, GWR is evidently a technique offering potential for further development of DEM error modelling. GWR was not used in this study for two reasons. First, a major part of the research involved initial

identification of a relationship between terrain character and DEM error. Going on to include GWR modelling would be beyond the scope of a single paper. Second, the study areas are of limited spatial extent and disjunct. Approximately 100 sample points were available at each study site. This would probably provide inadequate support for local modelling, especially in areas of high frequency change, such as the mountainous environments of the two study sites. It was felt that an investigation into the potential of GWR in DEM error modelling was better applied to a larger and contiguous study area. Further work is needed in this area.

7.2 The Quality of Error and Accuracy Surfaces

A new method for creating spatially variable, spatially correlated and heteroscedastic error surfaces has been developed and successfully applied to DEMs of the Snowdonia and Mestersvig study areas. However, the quality of these error surfaces is variable. Although the average value for a whole error surface is usually reasonable, there are extreme maximum and minimum values and figures for standard deviation of an error surface indicate that there is an unrealistically wide dispersion about the mean. Applying a 20-cell radius mean filter does reduce the standard deviation and reduces, but does not remove, the extreme values. A filter of sufficiently large kernel size would remove these extreme values. Larger filter sizes (up to 50 cell radius) were tested. However, extreme values still remained and excessive smoothing of non-extreme values was removing local variation in error values. Due to these characteristics of the error surfaces they are not suited to correcting a DEM.

Although they are of limited quality, the error surfaces can be used to generate standard deviation of error surfaces. These accuracy surfaces estimate error characteristics for neighbourhoods of cells and can be used to model the degree of uncertainty in subsequent

DEM-based analyses. The more anomalous error values are subdued by the generalisation involved in summarising error values for local 20-cell radius kernels.

For both Snowdonia and Mestersvig the accuracy surfaces can be considered of sufficient quality for further use. The technique has been demonstrated to work for two areas of mountain terrain, modelled at different resolutions using different types of source data. The indications are that the technique is broadly applicable. However, there are a number of issues to consider, which are discussed below.

7.3 GPS Sample Point Issues

The error surface technique involves estimating errors at over a million grid cells from approximately 100 GPS measurements. The number and location of these sample points will influence the quality of the accuracy surfaces.

An appropriate number of GPS measurements was determined by means of equations to estimate sample size based on a required accuracy for the mean error estimate and for the estimate of the standard deviation of the error (Li 1991). 100 GPS sample points should give a 95% reliable estimate of a DEM's accuracy. However, this relates to the reliability of a global accuracy estimate rather than local estimates. Further research is needed to investigate the influence of the number and location of sample points on the resulting accuracy surfaces.

Time, accessibility and GPS operational issues mean that the sample points give a limited representation of the true variety of terrain characteristics found in a mountain environment. There will be terrain that is inaccessible and locations where GPS surveying will not be successful due to the obstruction of the sky by the terrain. Both study areas contain steep

slopes and near vertical rock faces that cannot be sampled. The coastal region of the Mestersvig study area is characterised by near flat terrain traversed by numerous heavily braided, often deep and fast flowing channels, which are largely inaccessible. Also, the ruggedness of mountain terrain means that there is a high degree of variability in terrain character over relatively short distances. An impractically large number of GPS survey points would be required to sample all varieties of terrain character. For these reasons a derived regression model will give a limited representation of the relationship between terrain character and DEM error.

7.4 Choice of Terrain Parameters

All of the types of derived terrain parameter have been used in at least one regression model. Additional parameters that quantify the form of the terrain may be useful. Also, the values of derivatives are dependant on the algorithms used (Jones, 1998). Horn's (1982) and Zevenbergen and Thorne's (1987) algorithms have been used here to derive gradient and curvature values. Other algorithms would compute different derivative values that may give a better regression model.

The regression models' inclusion of mean and standard deviation parameters calculated using 5, 10 and 20 cell radius filter kernels indicates that error is related to terrain characteristics measured at various scales. Other filter kernel radii may be more appropriate than those chosen for use here. Processing time becomes a limiting factor for filter kernels of greater than 20-cell radius, and also when a large number of different sized filter kernels are used. However, further research would be beneficial into the influence of choice of filter kernel size on the regression modelling results.

Squared and cubed terrain parameters have been derived and selected for use in the regression models. Other transformations may also be useful. However, the logarithms and exponents of Snowdon's SpTen12 DEM terrain parameters were briefly investigated, but no significant correlations with DEM error were found and these parameters were not selected in the stepwise regression modelling routine.

Although there are a number of ways in which terrain parameters could be further explored, it seems unlikely that the benefit to the regression modelling will be great. High adjusted R^2 values of up to 0.9 have already been achieved with the terrain parameters used in this study.

7.5 Quality of Terrain Parameters

The distribution of error has been modelled from terrain parameters derived from the DEMs, rather than from on-the-ground measurement of the true terrain parameters. The DEMs contain errors and therefore, the derived terrain parameters will be subject to error. A gradient grid will not be a true representation of the real gradient. It is not possible to measure terrain parameters in the field with a sufficient degree of accuracy and consistency. This is a prime reason for the widespread use of DEMs. Terrain parameters derived from a DEM give the best available description of terrain character. It may be that terrain parameters derived from an accurate DEM may give a better model of the distribution of error in a lower quality DEM, than terrain parameters derived from the lower quality DEM. However, this is not useful because, in practice, the DEM in question is almost always the highest quality DEM available. The DEM-derived terrain parameters only give an approximation of terrain character. Therefore, a fully accurate model of the relationship between terrain character and DEM error cannot be achieved by this method.

Many of the cells with extreme error values are found towards the boundaries of the study areas. This is due to lower quality DEM interpolation and edge effects influencing derivation of terrain parameters at these locations. To mitigate these effects, the regression modelling should use a DEM that extends more than 20 cells beyond the limits of the study area. Subsequently derived error and accuracy surfaces should then be cropped to the extent of the study area.

7.6 Differences between the Error and Accuracy Surfaces

The six regression models all utilise a variety of the terrain parameter types, but they differ in the specific terrain parameters used. There is clearly no generic relationship between DEM errors and terrain character. This indicates that the nature of the terrain being modelled and the method used to generate the DEM influence the distribution of errors, the parameters that are most useful to the regression model and how well the model fits the sampled DEM errors.

The correlation coefficients, adjusted R^2 values and accuracy surface summary statistics indicate that the relationship between error and terrain character in the Mestersvig area is stronger than for Snowdonia and consequently a better accuracy surface can be created. The technique has worked better for the study area with the smaller scale, coarser resolution, and lower accuracy DEM. DEM errors can be assumed to have a heteroscedastic element and a random element. The heteroscedastic element is related to terrain character and can be modelled from DEM-derived terrain parameters. It is likely that the random element is due to small variations in the accuracy of elevation measurements and small local variations in the elevation of the terrain, for instance individual hummocks or boulders that cannot be captured at the DEM scale concerned. The difference between the quality of accuracy surfaces for the

two study areas could be because the higher resolution, higher accuracy DEMs reduce much of the heteroscedastic component leaving random errors to dominate to a greater extent.

8.0 Conclusions

The research presented in this paper has shown that the magnitude and distribution of errors in a DEM are related to the varying character of the terrain. GPS surveys of elevation error and DEM-derived terrain parameters, which quantify terrain character, can be used in regression modelling to estimate the distribution of DEM errors in the form of an error surface. The nature of the relationship between DEM error and terrain parameters varies according to the type of terrain, the resolution of the DEM and the DEM production method.

The quality of the error surface is limited, primarily due to limitations in the size and distribution of GPS sample points, the quality of the DEM-derived terrain parameters and the presence of non-systematic, non-heteroscedastic error components. A standard deviation filter can be applied to the error surface to create an accuracy surface. The filtering process absorbs the more spurious error estimates, creating an accuracy surface that gives a much more complete description of a DEM's accuracy compared to the commonly used single, global accuracy measure such as the DEM's RMSE. A future paper will report on work investigating the use of such a spatially variable, heteroscedastic representation of DEM accuracy in modelling uncertainty in DEM-based analyses.

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Table 1 Snowdonia DEMs

DEM	Accuracy (standard deviation of error)	Interpolation method
IDW112	5.11m	inverse distance weighting with a weight of 1 and a search radius of 12 points
IDW512	3.96m	inverse distance weighting with a weight of 5 and a search radius of 12 points
SpTen12	3.78m	ArcView's "spline with tension" and a 12 point search radius

Table 2 Terrain parameters.

Parameter	Description
Elevation	
Gradient	The first derivative of elevation, also known as slope angle, representing the maximum rate of change of elevation measured both as degrees and percentage rise over run .
Aspect	The direction of the maximum rate of change in elevation expressed as vectors in the X and Y directions.
Plan Curvature	The horizontal component of the second derivative of elevation representing the rate of change of aspect.
Profile Curvature	The vertical component of the second derivative of elevation representing the rate of change of gradient.
Overall Curvature	The second derivative of elevation representing the surface's degree of convexity or concavity.
Relative Relief	The range of elevation values of all grid cells within a 5-cell radius of the grid cell concerned.
Texture	The range of gradient values of all grid cells within a 5-cell radius of the grid cell concerned.
Mean Extremity	The elevation of a grid cell minus the mean elevation of all grid cells within a 5-cell radius of that grid cell. Indicates the vertical position of the grid cell relative to its neighbours.
Minimum Extremity	The elevation of a grid cell minus the lowest elevation of all grid cells within a 5-cell radius of that grid cell. A value of near zero would indicate that that grid cell is in a pit.
Maximum Extremity	The elevation of a grid cell minus the highest elevation of all grid cells within a 5-cell radius of that grid cell. A value of near zero would indicate that that grid cell is on a peak.
Vertex Distance	The distance between a grid cell and the nearest of the contour vertices from which the DEM was interpolated.

Table 3 The most significant correlations and number of significant correlations of elevation error with terrain parameters.

	SNOWDONIA			MESTERSVIG		
	SpTen12	**IDW512**	**IDW112**	**SpTen12**	**IDW512**	**IDW112**
Coefficient & parameter of strongest correlation	0.487 Profile curvature	0.458 Texture%	-0.564 Plan curvature	0.629 Aspect vector X	-0.597 Elevation	0.568 Elevation
No. of significant correlations at 0.05 level	57	42	54	103	125	92

Table 4 Regression equation variables (most significant first).

Snowdonia - IDW512	Mestersvig - SpTen12	where:
Z_SD203	Ax_AV203	• Z = elevation
MinEx10	Z_AV202	• GP = gradient measured in percent
CH_SD20	Ax_SD103	• Ax = aspect vector X
GP3	MinEx103	• Ay = aspect vector Y
GP_AV20	MaxEx10	• CH = plan (horizontal) curvature
CH	Ay_SD20	• CV = profile (vertical) curvature
Ay_AV103	CH_AV52	• C = overall curvature
Ay_SD5	MinEx203	• AvEx = average extremity
Ay_AV202	Rel20	• MaxEx = maximum extremity
Ax_AV20	MinEx202	• MinEx = minimum extremity
Ay_SD103	Ay_SD53	• Rel = relative relief
GP_SD20	AvEx53	• TextP = texture measured in percent
CV_SD53	CV	• AV = neighbourhood average
CH_SD52	Z_AV203	• SD = neighbourhood standard deviation
C3	TextP52	• 5, 10 and 20 indicate the radius of neighbourhood parameters in number of cells
CV	CV_AV10	• 2 and 3 indicate the square and cube of a parameter respectively
MinEx202	Z_AV20	
CH2	TextP20	
Ax_AV5	AvEx52	
Ax_AV52	CH_SD203	

Table 5 Distribution of actual and corrected errors.

SNOWDONIA	Minimum	Maximum	Range	Mean	Standard deviation
Actual errors	-6.19	15.33	21.52	-2.72	5.11
Corrected errors	-11.14	6.71	17.85	-0.14	3.20
Corrected as % of actual	180%	44%	83%	5%	63%
MESTERSVIG	Minimum	Maximum	Range	Mean	Standard deviation
Actual errors	-101.02	118.32	219.34	4.90	32.41
Corrected errors	-42.25	59.56	101.81	0.58	15.41
Corrected as % of actual	46%	50%	46%	12%	47%

Table 6 Summary statistics for error surfaces and mean error surfaces.

SNOWDONIA	Minimum	Maximum	Mean	SD
GPS sample	-6.19	15.33	-2.72	5.11
Error	-194	389	-2.26	7.65
Mean error	-17	66	-2.26	3.96
MESTERSVIG	Minimum	Maximum	Mean	SD
GPS sample	-101.02	118.32	4.90	32.41
Error	-166350	56915	-3.95	244.56
Mean error	-214	2057	-3.76	56.77

Table 7 Summary statistics for accuracy surfaces.

SNOWDONIA	Minimum	Maximum	Mean	SD
GPS sample	-6.19	15.33	-2.72	5.11
SD of error	0.02	71.11	2.63	3.84
SD of mean error	0.03	10.49	0.54	0.71
MESTERSVIG	Minimum	Maximum	Mean	SD
GPS sample	-101.02	118.32	4.90	32.41
SD of error	0.01	8823	32.39	181.41
SD of mean error	0.01	276	4.84	9.51

Fig. 1 Standard deviation (SD) surfaces draped over orthographic views of the corresponding DEM: **a)** Snowdonia SD of error; **b)** Snowdonia SD of mean error; **c)** Mestersvig SD of error; **d)** Mestersvig SD of mean error.

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Fig. 1 Standard deviation (SD) surfaces draped over orthographic views of the corresponding

DEM: **a)** Snowdonia SD of error; **b)** Snowdonia SD of mean error; **c)** Mestersvig SD of error;

d) Mestersvig SD of mean error