A&A 373, 1009–1018 (2001)
DOI: 10.1051/0004-6361:20010669

Astronomy & Astrophysics

Modelling X-ray variability in the structured atmospheres of hot stars

L. M. Oskinova[1], R. Ignace[2], J. C. Brown[1], and J. P. Cassinelli[3]

[1] Department of Physics and Astronomy, University of Glasgow, Glasgow, G12 8QQ, Scotland UK
[2] Department of Physics and Astronomy, University of Iowa, 203 Van Allen Hall, Iowa City, IA 52242, USA
[3] Department of Astronomy, University of Wisconsin, 5534 Sterling Hall, 475 N. Charter St., Madison, WI 53706-1582, USA

Received 16 February 2001 / Accepted 2 May 2001

Abstract. We describe X-ray production in the atmospheres of hot, early-type stars in the framework of a "stochastic shock model". The extended envelope of a star is assumed to possess numerous X-ray emitting "hot" zones that are produced by shocks and embedded in the ambient "cold" medium in dynamical equilibrium. It is shown that the apparent lack of X-ray variability on short (∼ hours) timescales do not contradict a shock model for X-ray production. The character of the X-ray variability is found to depend on the frequency with which hot zones are generated, the cool wind opacity to X-rays, and the wind flow parameters, such as mass loss rate and terminal speed.

Key words. stars: abundances – stars: early-type – stars: mass-loss – stars: Wolf-Rayet – X-rays: stars

1. Introduction

The X-ray emission from hot stars has proved to be an important "window" for investigating mass loss in early type stars, and the advent of the latest suite of orbital X-ray telescopes are providing new insights. Here we develop a model to explore the X-ray variability of early type stars, first to understand the near absence of significant variability observed so far, and second to determine the requisite S/N and time resolutions necessary to measure the expected X-ray variability with current and future instrumentation. The objects of interest are hot, luminous stars with supersonic winds that carry significant mass, such as OB and Wolf-Rayet (WR) stars. The main parameters characterising these stellar winds are the terminal velocity (v_∞) and mass loss rate ($\dot{M}$). Typical O star mass loss rates are $\dot{M} \approx 10^{-7}\ M_\odot\ \mathrm{yr}^{-1}$ with terminal speeds $v_\infty \approx 1000{-}3000\,\mathrm{km\,s}^{-1}$. For Wolf-Rayet (WR) stars $\dot{M}$ is larger at around $10^{-4.5}\ M_\odot\ \mathrm{yr}^{-1}$ but with terminal speeds like those of O stars. Such strong outflow results in a predominance of anomalously strong and broad emission lines in the spectra of WR stars. The high mass loss rate during the WR phase has a significant effect on its evolution and the chemical enrichment and mechanical energy input to the interstellar medium.

It is widely accepted now that stellar winds of OB stars are maintained by radiation pressure in numerous lines of heavy elements (Castor et al. 1975; Pauldrach et al. 1986). But the acceleration of high mass loss WR winds is thought to be due to multi-line scattering of photons (Lucy & Abbott 1993; Springmann 1994; Gayley et al. 1995). Non-LTE, moving atmosphere modelling codes have been developed (e.g., Hamann & Koesterke 2000; Hillier & Miller 1998), which adopt radiative and statistical equilibrium and assume a monotonic velocity law a priori. In the framework of the so-called "standard model" the emission lines are formed in a spherically symmetric, time-independent, dense, smooth stellar wind that is photoionized by the hot core. However, there has long been growing evidence that at least some assumptions of the standard model are oversimplified. A pointed example is the X-ray emission from hot star winds, first detected as discrete X-ray sources with Einstein (0.2–4.0 keV) (Harnden et al. 1979; Seward et al. 1979; Seward & Chlebowski 1982).

Early UV observations of hot stars revealed the appearance of superionisation, especially the presence of O VI which would not be expected in the winds of hot stars. Cassinelli & Olson (1979) suggested that Auger ionisation by X-rays in the context of a coronal model could explain the existence of these highly ionised species. However, the hypothesis of a hot corona in the outer parts of a stellar atmosphere was ruled out due to the fact that the soft X-ray flux is not sufficiently absorbed at low energies (e.g., Cassinelli & Swank 1983). According to a phenomenological model proposed by Lucy & White (1980) and developed by Lucy (1982), dynamical instabilities should arise in line driven stellar winds leading to shock generation throughout the flow. Cassinelli & Swank (1983) found that

Send offprint requests to: L. M. Oskinova,
e-mail: lida@astro.gla.ac.uk

the shock model of Lucy (1982) produced too low an X-ray flux and also that the X-rays were too soft to explain the EINSTEIN observations of the O stars. They also pointed out that the spherically symmetric shock model would lead to strong variability of the X-rays, which was not seen. To explain the lack of clear evidence for variability of the X-rays from hot stars they proposed that the X-rays are not from spherically symmetric shocks but instead from shock fragments in the wind. ROSAT (0.2–2.4 keV) observations have brought new insight in the properties of X-ray emission from hot stars. Kudritzki et al. (1996) have found interesting relations between the X-ray emission and the wind properties from an analysis of 42 O star spectra. For example, they confirm the relation between X-ray luminosity and stellar bolometric luminosity as $L_{\rm X} \approx 10^{-7}\, L_{\rm Bol}$ that was previously known, and also find that $L_{\rm X}$ scales with the wind kinetic luminosity $0.5\dot{M}v_\infty^2$. For the WR stars, the data is of lower quality and consists almost entirely of passband fluxes. However, based on EINSTEIN observations, Pollock (1987) was able to find that on average, N-rich WR stars (WN) tend to be more luminous than the C-rich WR stars (WC), possibly attributable to the difference in abundances of the two types. The ROSAT data have also revealed that, unlike their progenitors the O stars, the X-ray luminosities of single WR stars are *not* correlated with $L_{\rm Bol}$, wind momentum $\dot{M}v_\infty$, wind kinetic luminosity $0.5\dot{M}v_\infty^2$, or WR subtype (Wessolowski 1996; Ignace & Oskinova 1999). Recent data from CHANDRA and XMM-NEWTON of OB stars (Schulz et al. 2000; Kahn et al. 2001; Waldron & Cassinelli 2001) and WR stars (Maeda & Tsuboi 1999) are just becoming available. These newer data, mostly of O stars, are showing rich X-ray line spectra, the analysis of which are yielding some unexpected results. We refer the reader to the above mentioned papers for a description of these data and here concentrate on the topic of X-ray variability in single stars, a topic that has not much been addressed from a theoretical perspective.

Dynamical instabilities in line-driven winds have been studied extensively for OB stars (Owocki & Rybicki 1984; Owocki et al. 1988; Rybicki et al. 1990). Model computations predict shock velocity-jumps ranging from 500 $\rm km\,s^{-1}$ to 700 $\rm km\,s^{-1}$, implying post-shock temperatures and emission measures that are marginally adequate to explain the observed thermal X-ray emission (e.g. Hillier et al. 1993; Feldmeier et al. 1997a). Gayley & Owocki (1995) have shown that similar to OB stars, the instability mechanism should also operate in the denser WR winds.

In terms of observed X-ray variability, Crowther & Willis (1996) have presented a review of available data based on ROSAT observations. For ζ Ori, Berghöfer & Schmitt (1994) claimed to detect X-ray variability over a 2-day period, reaching about 30% in hard X-rays (0.64–2.38 keV), while the soft band data (0.16–0.5 keV) remained about constant. For ζ Pup, Berghöfer et al. (1996) found evidence for a modulation in the harder (0.9–2.0 keV) X-ray emission and in the optical data with a 16.7 hour period, and suggested that this reflects periodic changes in the base wind density of the O4 f star. On the other hand, the study of σ Ori showed no significant X-ray variability. Except for WR 6 (HD 50896) and WR 1 (HD 4004), no X-ray variability studies for single WR stars have been undertaken. For WR 6, Willis & Stevens (1996) reported variability at the $\leq$30% level on timescale of $\leq$1 day, together with larger epoch changes. No significant spectral shape changes were found. In the case of WR 1, no evidence of significant X-ray variability was reported by Wessolowski & Niedzielski (1996). Thus far, no short-time scale X-ray variability on the order of the wind flow time ($R_*/v_\infty \sim$ hours) has been observed in O and WR stars.

In this paper we consider the application of stochastic inhomogeneous wind models, as widely accepted necessary components to explain spectral variations at optical and UV wavelengths, to the X-ray emission from single O and WR type stars. We restrict our analysis to shock models of X-ray formation in hot stars and do not consider alternative models (see discussion in Waldron & Cassinelli 2001). In Sect. 2, we describe a stellar wind model consisting of numerous zones filled by hot X-ray emitting gas. Two following sections use two different approximations of the optical depths of the cool material, which absorbs X-ray emission. In Sect. 3 we apply the so-called exospheric approximation treating the X-ray emitting zones as spherical shells. In Sect. 4 we generalise our model by using formal integration of the radiative transfer equation for randomly distributed wind shocks. We investigate the crucial parameters of the variability and their relation to the input parameters of the model. A summary discussion of the results is presented in Sect. 5.

2. Stochastic model for the distribution of X-ray emitting material

We consider a spherically symmetric and time-independent stellar wind that is a mix of "cool" and "hot" gas in dynamical equilibrium. The minor hot gas component gives rise to X-ray emission. We assume that hot gas is present in the form of spatially separated compact volumes characterised by temperature $T_{\rm X}$ and density which are different from the temperature and density of the ambient cool wind gas.

It is common in the literature to attribute wind "clumps" to dense formations with temperatures much lower then the temperatures of the X-ray emitting material, sometimes referred to as DWEEs (discrete wind emission elements, Lépine & Moffat 1999). These clumps are presumably responsible for observed line profile variability in the optical spectra of WR stars, or the formation of DACs (discrete absorption components) observed in the optical and UV spectra of some O stars (Brown et al. 1995). Such cold clumps and hot X-ray emitting zones may well have related origins. Feldmeier et al. (1997a) considered the formation of X-ray emitting zones as a consequence of shell collisions due to the propagation of reverse shocks in the stellar wind. In their hydrodynamical simulations they demonstrated the existence of dense shells moving according to the stationary wind velocity law and small-mass, high-velocity blobs. Copious X-rays are produced when a fast blob rams into an outer shell. These X-rays thus originate over a fairly narrow range in radius. In this respect, recent observations (Schulz et al. 2000) seem to confirm such a scenario, except that the expected strong variability is not observed.

The general description of a source with a continuous spatial distribution of temperature (e.g., a radiative cooling zone behind the front of a shock wave) is given by the *differential emission measure* which includes integration over all surfaces with the same temperature, such as over multiple independent shocks in a stellar wind (Craig & Brown 1976). But it is necessary to note that an overall consistent picture of dynamical, structured, thick stellar winds is still far from being complete. Although detailed models do not predict the X-ray sources to be isothermal with unique temperature $T_{\rm X}$ amongst them, the isothermal approximation for the overall wind structure should be fairly close to the actual case because of the short radiative cooling times in these dense stellar winds (Feldmeier et al. 1997b). The two-temperature approximation with isothermal X-ray emitting zones embedded in a cooler outflowing wind is able to reasonably well reproduce the observed X-ray spectra from hot stars (Hillier et al. 1993) and might be used to place constraints on physical parameters such as the volume filling factor (e.g., Oskinova et al. 2001).

Therefore, we will consider a spherically symmetric and, on average, time independent outflow that is a homogeneous mix of "cool" and "hot" gas in dynamical equilibrium. For isothermal optically thin hot X-ray zones, the particular choice of $T_{\rm X} = 10^7$ K tends to maximise the X-ray emission in the *ROSAT* band (see Ignace et al. 2000). We use the Raymond-Smith cooling function ($\Lambda_{\rm RS}$) for optically thin plasmas (Raymond & Smith 1977). For WR stars, a rough correction of $\Lambda_{\rm RS}$ for non-solar abundances is given in Ignace et al. (2000). The ambient cool medium ($\lesssim 10^5$ K) is as described by the standard model and is optically thick to X-rays.

In the two-temperature approximation, the ratio of emission measures of hot and cool components of the wind, $EM_{\rm X}/EM_{\rm w}$, is proportional to the ratio of volumes filled by hot ($V_{\rm X}$) and cold ($V_{\rm w}$) material and to the square of their density ratio $D_{\rm s}$. Thus, the "filling factor" is

$$f_{\rm X} = \frac{EM_{\rm X}}{EM_{\rm w}} \approx D_{\rm s}^2 \left(\frac{\mu_{\rm e}^{\rm w}}{\mu_{\rm e}^{\rm X}}\right)^2 \frac{V_{\rm X}}{V_{\rm w}}, \tag{1}$$

where $\mu_{\rm e}^{\rm w}$ and $\mu_{\rm e}^{\rm X}$ are the mean mass per free electron of the cool and hot wind component. The X-ray emission arises primarily from two-body processes–collisional excitation and radiative recombination. Therefore, the volume emissivity j_ν (energy per unit time per unit volume) is proportional to the square of the density,

$$4\pi j_\nu \;=\; f_{\rm X} \rho_{\rm w}^2 \frac{\Lambda_\nu(T_{\rm X})}{\mu_{\rm e}^{\rm w}\mu_{\rm i}^{\rm w} m_{\rm H}^2} \tag{2}$$

where $\mu_{\rm i}^{\rm w}$ is the mean molecular weight per ion of the cool gas and $m_{\rm H}$ is the proton mass. Λ_ν (erg cm^3 s^{-1}) is the cooling function at the energies E concerned and $\rho_{\rm w}$ is the density of the standard wind is

$$\rho_{\rm w}(r) \;=\; \frac{\dot M}{4\pi r^2 v(r)}. \tag{3}$$

By the definition of filling factor in Eq. (1), the total specific luminosity (erg s^{-1}) emerging from the wind

$$L_{\rm X}(E) \;=\; f_{\rm X} \Lambda_\nu(T_{\rm X}) EM_{\rm w}. \tag{4}$$

This then is the monochromatic X-ray luminosity at any instant in time, since our hypothesis is essentially that the emission measure of X-ray emitting material (or correspondingly the filling factor) is time variable.

It was pointed out by Cassinelli et al. (1996) that the X-rays from a wind shock will be seen only after that shock has moved into a region where the cool wind attenuation is optically thin to the X-rays. As we shall show, the wind is quite opaque at most X-ray energies up until the flow has reached a substantial fraction ($\gtrsim$50%) of its terminal speed. Thus, we can assume that the X-rays emerge primarily from distances where $v(r) \approx v_\infty$. For the wind attenuation, K-shell absorption by metals in the cool wind is the dominant opacity source. Assuming photo-electric absorption by K-shell electrons

$$\kappa_{\rm w}(E) = \sigma_\nu(E)/\mu_i^{\rm w} m_{\rm H} = \frac{1}{\mu_i m_{\rm H}} \sum_j \frac{n_j}{n_i}\, \sigma_j(E), \tag{5}$$

where $\sigma_j(E)$ is the photo-absorption cross-section. (Actually, the appropriate molecular weight to use is that per nucleus, but in hot star winds, there is no neutral gas, and so every nucleus is an ion.) The ratio n_j/n_i defines the relative abundance by number for atomic species j, where n_i is the number density of ions. The X-ray absorption (apart from prominent K-shell edges) can be represented roughly by a power-law in energy: $\kappa_{\rm w}(E) \approx \kappa_0 E^{-\gamma}$, with γ in the range 2–3, depending on ionisation structure and chemical composition (see, e.g. Hillier et al. 1993; Cohen et al. 1996). The parameter κ_0 is a constant that depends sensitively on the abundances.

To proceed, we now develop a phenomenological model in which the wind inhomogeneities develop at random times near some inner boundary R_0. Then they propagate radially according to a monotonically increasing velocity law $v(r)$. Let us introduce dimensionless notations for velocity $u = v(r)/v_\infty$, and for distance $x = r/R_0$. Further, we normalise time to the characteristic time $\mathcal{T}_{\rm f} = R_0/v_\infty$, which is equal to the flow time in the specific case $R_0 = R_*$. Then for unitless time we have $t = \mathcal{T}/\mathcal{T}_{\rm f}$.

To avoid confusion we would like to clarify here, that further out, we shall use two descriptions for the distribution of the hot gas. In the exospheric approximation described in Sect. 3, we assume that the hot material is present in the form of spherical shells. The exospheric approach with spherical shells are simplifying assumptions, but the model does allow us to gain some insight into the nature of X-ray variability from wind shocks. In Sect. 4, we allow for randomly distributed wind shocks of arbitrary geometrical form (as long as the hot zones are not exceedingly "large", to be defined later), and we consider the full radiative transfer problem for the emergence of X-rays from the wind flow.

For the motion of the hot zones (e.g., shells), we consider an ensemble of zones filled by hot gas propagating in the radial direction with constant velocity $u = 1$. The different zones in our model are labelled by their times of appearance t at the distance $x_0 = 1$. The first zone is at distance $x_0 = 1$ at the moment of time $t_0 = 0$. At time t_k when shell number k is crossing $x_0 = 1$, the first zone is at distance $x^1(t_k) = x_0 + t_k$ with $t_k = \sum_{i=1}^{k} \delta t_i$ and zone number j is at distance $x^j(t_k) = x_0 + (t_k - t_j)$. Here, we choose to use the time intervals δt_i as drawn from uniformly random numbers in the range [0.5, 1.5] multiplied by a dimensionless parameter $\langle T \rangle$:

$$\delta t_i = \mathcal{P}([0.5, 1.5]) \cdot \langle T \rangle, \tag{6}$$

and so $\langle t_i \rangle = \langle T \rangle$. The parameter $\langle T \rangle$ is in fact an average separation in time between the appearance of subsequent zones and is measured in flow times $\mathcal{T}_{\rm f}$. Further we will explore the influence of the average time separation $\langle T \rangle$ on the character of the variability.

The parameter $\langle T \rangle$ is introduced because detailed models of X-ray production demand the presence of an initial perturbation of the stellar wind. For example, in models which depict X-ray production mechanisms in O stars (Feldmeier et al. 1997b), the existence of large scale turbulence near the wind base is postulated. As it will become clear below, the average time separation $\langle T \rangle$ is the only essential parameter of the models under consideration. We thus wish to emphasis that in principle, one can expect $\langle T \rangle$ to be an observable value, which could be inferred from the X-ray emission light curve analysis as the average separation between the peaks of intensity.

The emission measure of each zone is $\delta EM = n_{\rm e}^{\rm X} n_i^{\rm X} \delta V$. Using Eq. (3), the squared number density inversely scales with the fourth power of distance: $n_{\rm e} n_i = n_0^2 \cdot x^{-4}$, where

$$n_0^2 = D_{\rm s}^2 \left(\frac{\dot{M}}{4\pi v_\infty R_0^2} \right)^2 \frac{1}{\mu_{\rm e}^{\rm x} \mu_i^{\rm x} m_{\rm H}^2}. \tag{7}$$

On the other hand, the volume of a zone δV with fixed radial thickness and solid angle grows with the square of distance $\delta V \propto x^2$. This implies that the contribution of a zone to the total X-ray luminosity determined by its emission measure δEM has an overall decline that goes as x^{-2}. This is a plausible treatment when mass conservation for the hot shocked material is assumed so that the continuity equation Eq. (3) can be applied.

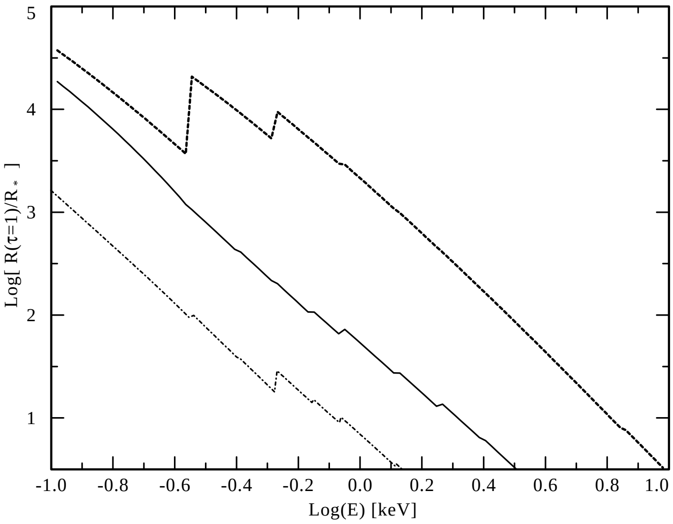

Fig. 1. Radius of optical depth unity as a function of X-ray energy. Dashed-dotted line is for typical parameters of OI stars, the solid line is for typical WN stars and the dashed line indicates WC stars, which are enhanced in C and O but deficient in N. Note the strong K shell edges due to CNO elements.

3. Exospheric model for the time dependent X-ray emission of single stars

3.1. Modelling time dependent X-ray emission

Let us assume that the observed X-ray emission arising from the hot gas emerges only from radii exterior to the optical depth unity surface of radius r_1 (exosphere), with X-rays at smaller radii completely attenuated. In this approximation the X-ray luminosity has the proper scaling with mass loss and other key wind parameters. The exospheric radius r_1 of optical depth unity is (Owocki & Cohen 1999)

$$r_1(E) = \frac{\dot{M}}{4\pi v_\infty} \kappa_{\rm w}(E). \tag{8}$$

The emission measure of the cool material outside the exosphere which is assumed to be transparent to the X-rays is given by

$$EM_{\rm w} \approx 4\pi \int_{r_1}^{\infty} n_i^{\rm w} n_{\rm e}^{\rm w} (1 - W(r_1))\, r^2 \,{\rm d}r, \tag{9}$$

where $W(r_1)$ is a dilution factor defined to be $\omega_{r_1}/4\pi$ where ω_{r_1} is the solid angle subtended by the exosphere and the parenthetical term accounts for geometric occultation by the exospheric surface of radius r_1.

Let us make the simplifying assumption that X-ray emitting material is present in the form of spherical shells. All shells have the same small fractional thickness $\mathcal{L} = \Delta R_{\rm shell}/R_0$ at a given radius in the envelope. Recall that we assume isothermal X-ray emitting zones and thus the parameter $\mathcal{L}$ is not to be misconstrued as a cooling length but rather the geometrical fractional thicknessof a shell filled with the hot gas.

In our phenomenological exospheric model, X-ray emitting shells are seen only beyond radius $r_1 = x_1 R_0$, so that effectively $x_0 \equiv x_1$. The shells start their motion outwards in the atmosphere at quasi-random times t_k. At that moment, the X-ray flux ($\delta F_{\rm X}$) is increased by the new source. Then owing to the decrease in emission measure of the zone with distance ($\delta EM_{\rm X} \propto x^{-2}$ for constant shell thickness), the incremental flux $\delta F_{\rm X}$ decreases. Thus, the total X-ray output of an ensemble of X-ray emitting zones will be observed to vary with time.

The radius x_1 has a strong dependence on mass-loss rate, chemical composition and energy (see Fig. 1). This means that in the model under consideration, the X-ray emitting zones will cross the surface of optical depth unity at different distances for different energies. Therefore their emission measures at the moment when they appear for an external observer will be different. Obviously then, one may expect the character of variability to change as a function of wavelength and for stars of different abundances.

The emission measure of an individual shell j at time t_k is

$$\delta EM_j \approx 4\pi R_0^3 n_{\rm e}^{\rm X}\, n_i^{\rm X} \cdot \mathcal{L}\, [x_1(E) + (t_k - t_j)]^{-2}. \qquad (10)$$

Combining Eqs. (7) and (10), the emission measure of the ensemble of hot shells becomes

$$EM_{\rm X}(E, t_{k_{\rm max}}) \approx 4\pi R_0^3 n_0^2 \mathcal{L} \cdot \sum_{k=1}^{k_{\rm max}} \frac{1}{[x_1(E) + t_k]^2}. \qquad (11)$$

Here the integral over the volume is replaced by summation over all X-ray emitting shells outside the sphere of $x_1(E)$, and $k_{\rm max}$ is the maximum number of shells under consideration in the wind. The value of $k_{\rm max}$ is chosen so that the most distant shells make a negligibly small contribution to the emission measure (i.e., $\delta EM_{\rm X}(x_{\rm max}) \to 0$ for $x \gg x_1$). We choose $k_{\rm max} \sim 10^5$ which allows us to account for the emission of a shell till it reaches a distance $x_{\rm max} = k_{\rm max}\langle T\rangle \sim 10^5$.

The X-ray luminosity with time is $L_{\rm X}(E,t) = EM_{\rm X}(E,t)\Lambda_\nu(T_{\rm X})$. Given that hot gas zones are a function of radius only, and not of latitude or azimuth, a relatively simple expression for the relative variability, $\sigma_{\rm L}/L_{\rm X}$, can be derived. Recognising that the X-ray emission from any given zone drops rather steeply with radius, both the emission and variability will be predominantly governed by zones that appear at the x_1 radius. Hence the fractional variability can be estimated based on Poisson statistics. The number density of zones above a given radius x scales as $1/x_1^2$; the volume in that vicinity scales as x_1^3; so the number of zones near x_1 is $N \propto x_1$. Consequently the relative variability scales as $\sigma_{\rm L}/L_{\rm X} \propto 1/\sqrt{x_1}$. Given that $x_1 \propto \kappa_\nu(E)$, we can expect the relative variability to be a function of energy. For purposes of illustration, if $\kappa_\nu(E) \propto E^{-\gamma}$, then the relative variability will be $\sigma_{\rm L}/L_{\rm X} \propto E^{\gamma/2}$, which is an increasing function of energy.

Figure 2 shows changes in the emission measure of the shell ensemble, computed using Eq. (11). We took the values of x_1 corresponding to 1.5 keV for O, WN and WC stars. As is clear from the figure, the character of variability depends on the chemical composition. As can be seen from Fig. 1, the radius of optical depth equal unity near 1.5 keV for a WC star is located about an order of magnitude further away in the wind than is the case for a WN star. This leads to an overall reduction of the X-ray emission, and some suppression of the variability in WC stars at this energy. In Sect. 4 we revisit these simulations using the formal solution of radiative transfer with angle dependent optical depth, instead of the simplified exospheric approximation. But first we make some comments on model constraints obtainable from filling factor data.

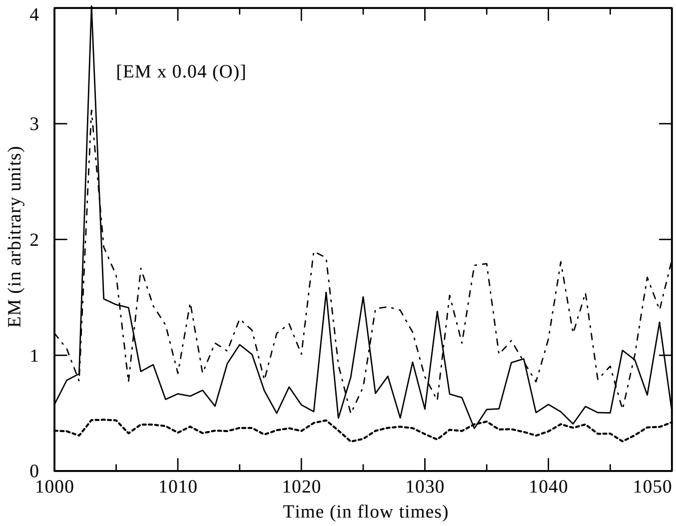

Fig. 2. Variations of emission measure with time for O, WN and WC stars. The solid line represents the variation near 1.5 keV for a star with an O star chemical composition. The dash-dotted line is for a WN star. The dotted line is for a WC star. Note that $\langle T\rangle = 10$ is choosen for these models. From the numerical simulations the ratios of standard deviation σ to the average value of emission measure are $\sigma/\langle EM\rangle = 1.0$ for the O star, $\sigma/\langle EM\rangle = 0.38$ for the WN star and $\sigma/\langle EM\rangle = 0.13$ for the WC star.

3.2. Using filling factors to constrain model parameters

In Ignace et al. (2000), lower limits to the filling factors of most putatively single Galactic WR stars were determined from an analysis of Rosat All-Sky Survey (RASS) observations. A natural question arises: Is it possible to extract information about the spatial distribution of hot, X-ray emitting gas by knowing $f_{\rm X}$ from RASS passband observations in the framework of the model under consideration? Let us again assume that the fractional thickness of the X-ray emitting layers $\mathcal{L}$ does not depend on distance. By definition of Eq. (1) and using Eqs. (9) and (11), we have:

$$f_{\rm X} = \mathcal{L} D_{\rm s}^2 \left(\frac{\mu_{\rm e}^{\rm w}}{\mu_{\rm e}^{\rm X}}\right)^2 \frac{\sum_{k=1}^{N} \int_E (x_1 + t_k)^{-2} {\rm d}E}{\int_E x_1^{-2} {\rm d}E}. \qquad (12)$$

Here, we neglect the occultation term in Eq. (9). Note that $x_1 = x_1(E)$ is implicit. One infers from Eq. (12) that in

Table 1. Typical values of $\mathcal{L}$ for WR 114 (WC5) and WR 1 (WN5)[a].

$\langle T \rangle$	WR114: $f_X = 0.175$	WR1: $f_X = 0.0831$
0.1	6.2×10^{-3}	5.1×10^{-3}
1.0	4.5×10^{-2}	2.5×10^{-2}
10.0	4.4×10^{-1}	2.0×10^{-1}

[a] Values of f_X and stellar parameters are from Ignace et al. (2000), Hamann & Koesterke (1998), and Koesterke & Hamann (1995).

fact there are two dependent parameters in the problem: $\langle T \rangle$ and $\mathcal{L}$. Let's assume that it is possible to infer $\langle T \rangle$ from observations of the variability of X-ray emission. Then making assumptions about the density ratio D_s, one may determine $\mathcal{L}$ if L_X (equivalently f_X) is known. Therefore from Eq. (12), we may estimate $\mathcal{L}$ from known values of f_X. In Table 1 we give typical values of $\mathcal{L}$ using $D_s = 4$ (the same value as in Baum et al. 1992; Hillier et al. 1993) for WN and WC stars.

So far, we have used the exospheric approximation to demonstrate that the X-ray variability is strongly influenced by the opaque cool material of the stellar wind and therefore (a) is a function of energy and (b) depends on the chemical composition of the stellar wind (see Fig. 2). Also, it was shown that lower limits to filling factors can be used to infer lower limits to the thickness of spherical X-ray emitting shells (as illustrated in Table 1), if the average time interval between subsequent shocks $\langle T \rangle$ is known from observations.

4. Formal solution for X-ray emitting zones

While it appears that the majority of WR and O stars display basically spherical winds on the large scale, there is compelling observational evidence that the winds are clumped on small scales (e.g. Lépine & Moffat 1999). In Lépine & Moffat (1999) a phenomenological model was introduced, wherein WR winds are depicted as consisting of a large number of randomly distributed, radially propagating, discrete wind emission elements. This model was used to simulate line profile variability patterns in emission lines from a clumped wind. We adopt this approximation for the X-ray producing region and consider the X-ray emission as arising from analogous optically thin zones with temperature T_X, which as before are embedded in the cooler bulk wind flow. However, the zones are no longer treated as spherical shells.

We treat these hot zones as piecewise elements, presumably produced by shocks. They expand adiabatically moving radially according to a β-law for the velocity. Zones start their motion at randomly distributed moments of time from random but uniformly distributed positions at an imaginary spherical surface of radius R_0 – the minimal predicted radius for shocks to occur.

The main difference with the exospheric model from the previous section is that the optical depth of cool material now depends not only on radius but also on the spherical polar angle as seen by an observer. Further we now compute a proper line-of-sight optical depth to each hot emitting zone, in contrast to the exospheric approximation for which hot zones are either completely attenuated, or completely unattenuated. To simplify the numerical simulation, we do require that the emission measure of every zone δEM is the same at a given radius and that the spatial size of each zone is small enough to assume the wind attenuation to every point in the zone is approximately constant (e.g., the zones cannot be hemispherical shells, since the wind attenuation would strongly vary across such a structure).

In this section we adopt the usual cylindrical coordinate system (e.g. Lamers et al. 1987). The coordinates are: distance from the star x, impact parameter for the line-of-sight p, and distance along the line of sight z, all normalised to the stellar radius. The angle between the line of sight and the radial vector is $\theta = \arccos \mu$, with $\mu = 1$ if the radial vector points to the observer. For any point in the wind, x, p and z are related by

$$z = \mu x = \mu (p^2 + z^2)^{1/2}. \tag{13}$$

With our assumptions, the emission measure changes with distance from the star as $\delta EM_X = \delta EM_0 (1/x)^2$, with δEM_0 the emission measure of a zone at distance x_0. The luminosity of each zone for which the optical depth of the ambient wind is τ_w is

$$\delta L_\nu = \Lambda_\nu(T_X)\, \delta EM_X\, \mathrm{e}^{-\tau_w} \tag{14}$$

and the luminosity of a hot gas zone occulted by the stellar core is assumed to equal zero. Applying the equation of continuity Eq. (3) with Eq. (5), we can derive the wind optical depth to be

$$\tau_w = \frac{\sigma(E)\dot{M}}{4\pi R_0 v_\infty \mu_i m_H} \int_z^\infty \frac{\mathrm{d}z}{x^2 u(x)} \equiv \tau_0(E) \int_z^\infty \frac{\mathrm{d}z}{x^2 u(x)}, \tag{15}$$

where $x = \sqrt{p^2 + z^2}$. Now the optical depth for each zone is angle dependent. For soft (0.4–2.4 keV) X-ray energies at radii where the wind acceleration takes place, the optical depth is generally quite large (see Table 2).

Although our numerical calculations can account for the X-ray emission from zones at small radii, Table 2 indicates that their contribution to the total X-ray flux will be quite negligible owing to the rather extreme attenuation. So, we simplify our calculations by considering only those zones which have already reached the constant velocity region in their motion throughout the wind, hence $u = 1$. Using Eq. (15), it is possible to obtain an analytic expression for the optical depth (MacFarlane et al. 1991):

$$\tau_w = \tau_0(E) \int_z^\infty \frac{\mathrm{d}z}{x^2} = \frac{\tau_0(E)}{x} \frac{\theta}{\sin\theta}. \tag{16}$$

So, the optical depth depends on the location of a given zone in the wind as well as on the energy.

Table 2. Representative optical depths to an X-ray emitting zone propagating radially with $\theta = 45°$ assuming a typical WN star[a] chemical composition.

u $[v(r)/\ v_\infty]$	0.1	0.9	1.0
E [keV]			
0.5	3.4×10^3	3.5×10^2	3.5×10^1
1.0	5.5×10^2	5.4×10^1	5.5
1.5	2.1×10^2	2.0×10^1	1.9

[a] With $v_\infty = 2500\,\mathrm{km\ s^{-1}}$ and $\dot{M} = 10^{-4.4}\ M_\odot\ \mathrm{yr}^{-1}$.

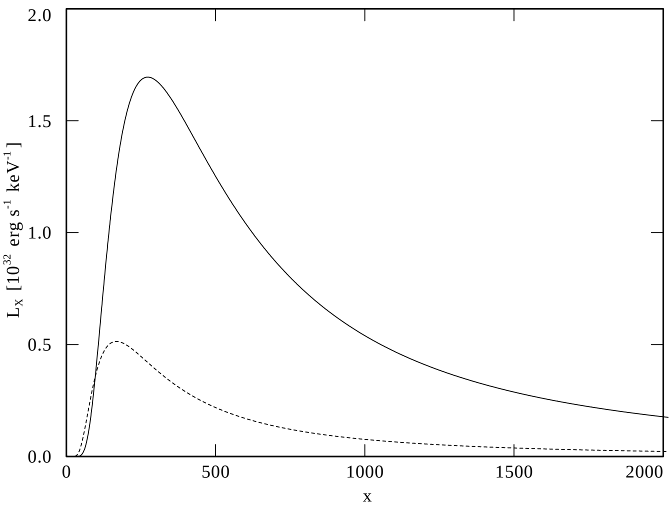

Fig. 3. Change in monochromatic X-ray luminosity L_X at 1 keV (solid line) and 1.2 keV (dashed line) of a single X-ray emitting zone with distance x in a WN star wind. The zone moves along a trajectory at $\theta = 45°$ in the envelope.

In Fig. 3 the change of X-ray flux for a single X-ray emitting zone along with its motion through the flow is shown. The luminosity grows rapidly with distance owing to the decrease in optical depth by the overlying wind. In general the zones which are propagating in directions close to $\cos\theta = 1$ have higher luminosity due to the fact that the optical depth is smaller in this direction. The emission rises to a peak followed by a decline. At this point, the attenuation is negligible, and the decreasing emission is a consequence of the inverse square fall-off of the emission measure with distance. The distance x_peak at which an X-ray emitting zone yields peak emission depends strongly on the angle θ. Further, noting that $\delta EM_\mathrm{X} \sim x^{-2}$ and $\tau_\mathrm{w} \sim x^{-1}$ it can be shown (from an analysis of Eq. (14)) that the maximum of the curve occurs when $\tau_\mathrm{w}(E, \theta) = 2$.

Let us assume that a stellar wind contains numerous X-ray emitting zones. Clearly an ensemble of such zones will also produce time variable X-ray emission. The rise in luminosity of a single zone is much sharper than its decrease after peaking. Suppose that at a given time and a given energy, the X-ray luminosity of only one zone could reach its maximum. Subsequently, the emission from this zone will be decreasing, and until the next zone reaches its maximum, the emission of the whole ensemble will drop. Thus, in the regime of constant expansion the separation between maxima of emission reflects the average time separation $\langle T\rangle$. We may expect larger levels of variability on a short time scale (order of flow times) for larger values of $\mathcal{L}$. So, we may conclude that the observed lack of variability suggests values of $\langle T\rangle \sim 1$ and therefore values of $\mathcal{L} \sim 10^{-2}$, which are marginally consistent with estimations of cooling lengths (e.g. Hillier et al. 1993). It is necessary to point out here that the available observations so far have not been capable of detecting such small fluctuations of X-ray flux.

To proceed further, let us recall that the total X-ray luminosity depends on the random variable t_i. For the angular distribution of hot zones, we select $\cos\theta$ as a uniform random variable in interval $[-1, 1]$, and for the azimuth, ϕ_k is uniform random in the interval $[0, 2\pi]$. It should be noted that although $\cos\theta_k$ is uniform, $\tau_{\mathrm{w},k}$ is not. The total emission for an ensemble of zones is a summation over all contributors with the appropriate attenuation:

$$L_\mathrm{X}^\mathrm{tot}(E) = \Lambda_\nu(T_\mathrm{X})\delta EM_0 \sum_k^{k_\mathrm{max}} \frac{1}{(1+t_\mathrm{k})^2} \mathrm{e}^{-\tau_{\mathrm{w},k}(E)}. \qquad (17)$$

The wind will be more opaque to X-rays at soft energies than to those at hard energies. Therefore, we expect the phenomenological picture wherein an X-ray emitting zone has peak flux at higher energy earlier than it has peak flux at softer energy.

Using δEM_0 as a parameter of the model, we avoid direct references to the density of hot material or volume of the zones filled by this material. In our model at each moment of time, only one X-ray emitting zone is at a given distance from the inner boundary R_0. This is similar to the propagation of subsequent spherical shells which was described in the previous section in the framework of the exospheric approximation. In Sect. 3.2 we have shown that the fractional thickness of a spherical layer $\mathcal{L}$ and $\langle T\rangle$ are not independent parameters but coupled with each other (see Eq. (12)). To place constraints on δEM_0, let us assume now that δEM_0 is the same as the emission measure of a spherical shell with thickness $\mathcal{L}$ located at distance R_0 with corresponding density n_0 as in Eq. (7).

The number of hot gas zones and their initial emission measure δEM_0 is set by the average separation in time $\langle T\rangle$. The assumption of the constant filling factor leads to the correlation between average time interval and initial emission measures of hot zones. This means that small hot zones with small separation in time should produce small changes in X-ray flux. On the other hand, large sized hot zones produce changes in X-ray flux with a more significant amplitude. To complete the picture, it is necessary to bear in mind that the degree of variability also depends on chemical composition.

Figure 4 presents numerical simulations of short-scale time variability of the monochromatic X-ray luminosity at 1 keV for two stars with different cool wind opacities, namely O stars and WN stars. Clearly, the character of variability strongly depends on the average time separation $\langle T\rangle$ and the spectral type of the star. The mass loss

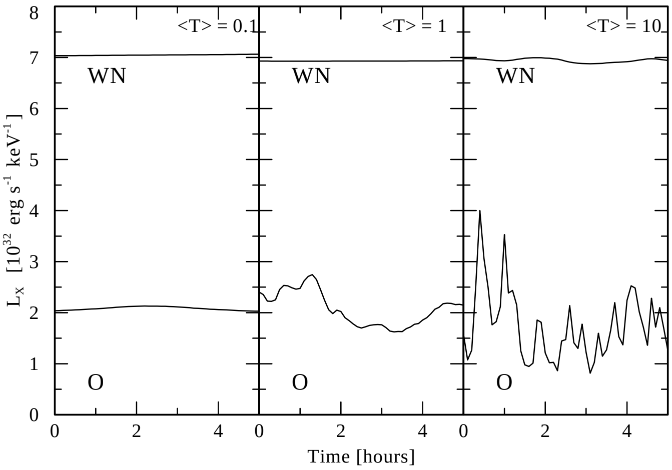

Fig. 4. Change in narrow band X-ray luminosity $L_{\rm X}$ at 1 keV with time for WR1 (WN5) (upper curve) and for ζ Pup (O4If) (lower curve). The average separation between X-ray emitting zones $\langle T\rangle$ grows from the left panel to the right $\langle T\rangle$ = 0.1, 1, 10. The flow time for the WN type star is $t_{\rm f} \approx 0.15$ h and for the O star is $t_{\rm f} \approx 0.9$ h.

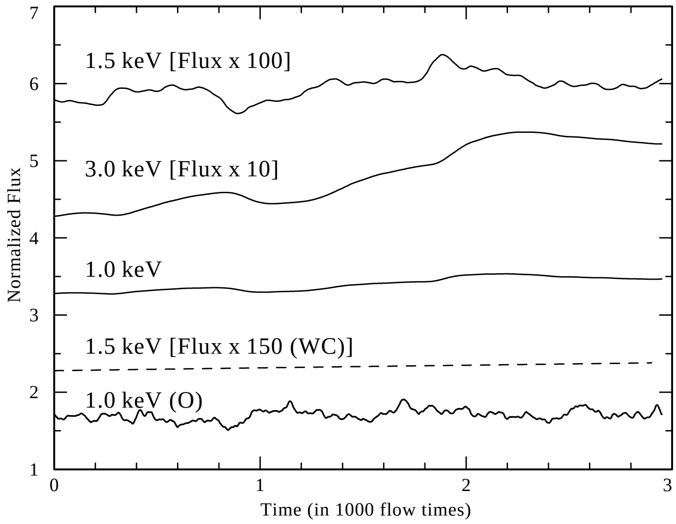

Fig. 5. Change in monochromatic X-ray flux at 1 keV, 1.5 keV and 3 keV with time for WR1 (WN5) (solid lines), for ζ Pup (O5Ia) (lowest curve) and a typical WC star (dashed line). The average separation between X-ray emitting zones is $\langle T\rangle = 1$.

rates of WN stars are much higher then those of O stars; therefore, we attribute the difference in attenuation between WR stars and O stars as the main cause for different character of variability shown in Fig. 4 owing to the fact that x_1 is much bigger for WN stars than for O stars.

The changes in luminosity seen in Fig. 4 are, in fact, due to the superposition of light curves for many zones propagating in different directions similar to those shown in Fig. 3. The average separation between subpeaks on the curves from Fig. 4 reflects the average time between two subsequent zones having peaks in their luminosity. However, when $\langle T\rangle$ is small, the flux fluctuations are negligible (order $\sim$ few per cent) and simply cannot be resolved. The interesting question to address is what is the plausible range for the parameter $\langle T\rangle$. Observed fluctuations in X-ray emission for ζ Ori (O4f) (increase in the count rate of $\approx$30% for 2 days) and ζ Pup (O9.5Ia) (modulations with a period of 16.7 hours and amplitude $\leq$10%, Berghöfer & Schmitt 1994; Berghöfer et al. 1996) suggest rather small values of $\langle T\rangle$ (≤ 1 flow time), assuming that the X-rays form in wind shocks.

As shown by our numerical simulations, on the time scale of several hours, the variability of X-ray luminosity might be negligible for stars of spectral type WN. The reasons for this apparent lack of variability are that in addition to small size and short time separation of hot zones, the total optical depth for the indicated energies is quite large in the WN star winds. As can be seen from Eq. (17), the exponential term $e^{-\tau_{{\rm w},i}}$ suppresses the diference in values of $L_{\rm X}$. Although not shown in Fig. 4, our modelling reveals that the X-ray luminosity of WC stars is almost constant due to their quite opaque stellar winds. As obvious from the figure, even the transparent winds of O stars may not demonstrate detectable levels of variability in X-rays on short time scales of about 1 hour in the case of small $\langle T\rangle$.

Although on a time-scale of hours, the X-ray variability may be quite small, the dynamical flow time $r/v(r)$ for distances $r \geq 100\ R_*$ is some 10^5 s, of order a day. So we have performed simulations of changes in flux over longer time intervals. The results of this calculation are shown in Fig. 5. Surprisingly, even for $\langle T\rangle = 1$, the variability becomes significant ($\sim$10%) on time scales of thousands of hours. In this case, the separation between peaks in the light curves do not reflect the time separation between X-ray emitting zones. To understand the character of this variability let us consider Eq. (17). The time dependent luminosity is governed by two uniformly distributed random variables δt_i, and $\cos\theta_i$. Let us assume, for simplicity, that zones are launched within constant time intervals equal to unity and concentrate on random variations in $\tau_{\rm w}$. Then $\delta t_i = 1$ and from Eq. (17)

$$L_{\rm X}{}^{\rm tot} \propto \sum_k^{\infty} \frac{1}{(1+k)^2}\ {\rm e}^{-\tau_{w,k}}. \quad (18)$$

From Eq. (16), $\tau_{w,k} \propto \tau_0\theta_k/k\sin\theta_k$ and $\cos\theta_k$ is a random variable from $[-1, 1]$. Thus, when the number of realisations k is large enough, even for rather opaque winds with $\tau_0 \gg 1$, the variance of the flux is significant.

The likelihood of detecting variability of X-ray flux increases drastically at hard energies. This is due to the strong dependence of the optical depth on energy. To illustrate this, if $\kappa_{\rm w}(E) \sim E^{-\gamma}$ then using Eq. (18) and neglecting the second order terms, the standard deviation of the X-ray luminosity becomes $\sigma_{\rm L} \sim E^{\frac{\gamma}{2}}(\dot{M}/v_\infty)^{-\frac{1}{2}}$. This is an increasing function of energy and decreasing function of the stellar wind density. As numerical simulations show, in the case of random times t_i, the general trend of the dependence of variance on energy and density is an increasing function of energy. The only condition is that the variance of emission measures of the hot zones

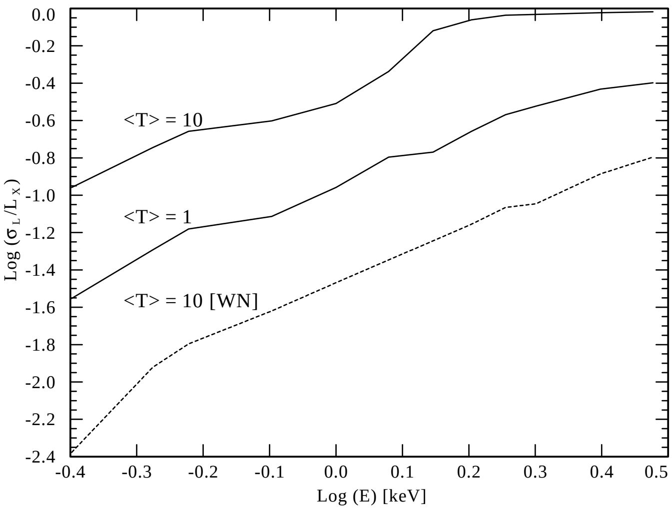

Fig. 6. Dependence of the ratio of standard deviation σ_L to the value of the X-ray luminosity$\langle L_X \rangle$ versus energy for typical abundances of O stars (solid lines) and for the average chemical composition of a typical WN star (dashed line).

should be smaller than the variance of the optical depths associated with them.

Figure 6 represents the ratio of the standard deviation σ_L to the average value of the X-ray luminosity as a function of energy, computed using Eq. (17). Apart from ionisation edges, Fig. 6 clearly shows that the relative variability is an increasing function of energy. Figure 6 confirms the basic scaling of variability with parameter x_1 that was obtained under the exospheric approximation with $\sigma_L/L_X \propto 1/\sqrt{x_1}$. Presumably, this will fail for large values of $\langle T \rangle \sim x_1$, because then there are relatively few hot zones in the outflow. At very low $\langle T \rangle$ one hits another regime where the flow approaches homogeneity.

As seen in Fig. 6 for $\langle T \rangle = 1$, the amplitude of variability for O stars can reach as much as 80% near 1.5 keV but only 10% at softer energies around 0.5 keV. At the same time, the relative variability is only a few percent near 1 keV for the WN stars. For WC stars, the level of X-ray variability is negligible. Therefore, it is not surprising that with such low sensitivity X-ray emission detectors such as ROSAT PSPC (0.2–2.4 keV) or EINSTEIN IPC (0.2–4.0 keV), the detected X-ray flux of WR stars appeared to be fairly constant X-ray sources.

5. Summary and conclusions

We present simulations of the expected X-ray variability for early-type stars in the framework of a shock model for X-ray production. We assumed that the optically thin hot X-ray emitting material is embedded in a cool X-ray absorbing stellar wind, described by the standard model. For such a medium we used the concept of filling factor, which is the ratio of emission measures of X-ray emitting and absorbing material. Assuming an isothermal X-ray emitting hot gas, the filling factor is proportional to the ratio of volumes filled by hot and cool components and to the square of the ratio of densities of hot and cool material. To examine the basic processes of X-ray emission and absorption we considered two different models.

To begin, we employed the exospheric approximation with angle independent optical depth. In this approximation we used a model of the stellar wind where hot gas is present in the form of spherical shells propagating with constant velocity. The emission of such an outflow is time dependent. This allowed us to derive a expression restricting the fractional thickness of a spherical shell emitting X-rays by using filling factors. The lower limits on filling factors for isothermal X-ray emitting material were derived in Ignace et al. (2000). Using these data, we found lower limits to the thickness of the spherical shells filled by hot gas to produce the observed level of X-ray emission. It was shown that the thickness of an isothermal spherical shell can be inferred from the analysis of the X-ray light curve and the observed filling factor.

Further, the model of an envelope consisting of a number of individual radially propagating X-ray emitting zones was developed by analogy with stochastic wind models. The total X-ray luminosity of such an atmosphere was obtained by using the formal solution of the radiation transfer equation for angle dependent optical depth. We did not place any constraints on the form, volume or density of the zones of hot gas except to demand that the total emission measure of each zone is the same as the emission measure of a spherical shell at the same distance from the stellar core, and that the zone must be relatively "small". This allows us to restrict the number of independent parameters when performing the numerical simulations. The rationale for doing this was that broadband X-ray luminosities are known for most hot stars. Therefore, any combination of parameters describing the distribution of hot gas should provide the observed level of X-ray emission for each particular star as its first priority.

The two major simplifying assumptions of our modelling are those of isothermal X-ray sources and smooth cool material. We may speculate that including radiatively cooling sources will lead to a decrease in the level of variability. On the other hand, taking into account the clumpy structure of the background wind will lead to an increase of the variability of X-ray output. A detailed consideration of these effects are a matter of future work.

Despite its appoximative nature, the analysis described here revealed the basic properties of variability of X-ray emission for early type stars. Some of the main points are:

1. The apparent lack of short time-scale variability (order of an hour) of X-ray emission cannot be considered as a deficiency of shock models for X-ray production.
2. We may expect stochastic variability on long time-scales (thousands of flow times), especially for optically thick winds.
3. The level of X-ray variability depends on chemical composition and density of the stellar wind and differs for stars of different spectral classes even for similar

mechanisms of X-ray production. It is governed by the opacity of the cool material and is substantially lower for the more opaque winds of WR stars and particularly WC stars.

4. The dependence of the wind opacity with energy means that the X-ray emission may be highly variable in the part of the spectrum, where the cool material is optically thin and practically constant in optically thick parts. Therefore the hard energies are specially apt for detection of variability of X-ray emission. Recall that existing claims of X-ray variability show differences between soft and hard passbands (e.g., possibly detected for ζ Ori by Berghöfer & Schmitt 1994).
5. Whether X-ray variability is detected or not for a given energy provides valuable information about the spatial distribution and properties of X-ray emitting material, and if variability is detected, its dependence on energy would be especially telling of the wind structure.

Acknowledgements. This research was supported by a PPARC grant (LMO, JCB) and NASA grants NAG5-9964 (RI) and NAG5-9226 (JPC). We acknowledge discussions with K. Gayley, M. Hendry and S. Rauzy, and we thank the referee L. Koesterke for comments beneficial to this paper.

References

Baum, E., Hamann, W.-R., Koesterke, L., et al. 1992, A&A, 266, 402
Berghöfer, T. W., & Schmitt, J. H. M. M. 1994, ApSS, 221, 309
Berghöfer, T. W., Baade, D., Schmitt, J. H. M. M., et al. 1996, A&A, 306, 899
Brown, J. C., Richardson, L. L., Antokhin, I., et al. 1995, A&A, 295, 725
Cassinelli, J. P., & Olson, G. L. 1979, ApJ, 229, 403
Cassinelli, J. P., & Swank, J. H. 1983, ApJ, 271, 681
Cassinelli, J. P., Cohen, D. H., & Sanders, W. T. 1996, in Proc. Throughput X-ray Spectroscopy workshop, ed. H. Tananbaum, N. White, & P. Sullivan (Harvard Press), 294
Castor, J. I., Abbott, D. C., & Klein, R. I. 1975, ApJ, 195, 157
Gayley, K. G., Owocki, S. P., & Cranmer, S. R. 1995, ApJ, 442, 296
Gayley, K. G., & Owocki, S. P. 1995, ApJ, 446, 801
Crowther, P., & Willis, A. J. 1996, in Proc. 33rd Liege Int. Astroph. Coll., Wolf-Rayet Stars in the Framework of Stellar Evolution, ed. J.-M. Vreux, A. Detal, D. Fraipont-Caro, et al., Liège, Univ. de Liège, 109
Cohen, D. H., Cooper, R. G., Macfarlane, J. J., et al. 1996, ApJ, 460, 506
Craig, I. J. D., & Brown, J. C. 1976, A&A, 49, 239
Hamann, W.-R., & Koesterke, L. 1998, A&A, 335, 1003
Hamann, W.-R., & Koesterke, L. 2000, A&A, 360, 647
Harnden, F. R., Jr., Branduardi, G., Gorenstein, P., et al. 1979, ApJ, 234, L51
Hillier, D. J., Kudritzki, R. P., Pauldrach, A. W., et al. 1993, A&A, 276, 117
Hillier, D. J., & Miller, D. L. 1998, ApJ, 496, 407
Ignace, R., & Oskinova, L. M. 1999, A&A, 348, 45L
Ignace, R., Oskinova, L. M., & Foullon, C. 2000, MNRAS, 318, 214
Feldmeier, A., Kudrizki, R. P., Palsa, R., et al. 1997a, A&A, 320, 899
Feldmeier, A., Puls, J., & Pauldrach, A. W. A. 1997b, A&A, 322, 878
Kahn, S. M., Leutenegger, M. A., Cottam, J., et al. 2001, A&A, 365L, 312
Koesterke, L., & Hamann, W.-R. 1995, A&A, 299, 503
Kudritzki, R. P., Palsa, R., Feldmeier, A., et al. 1996, in Röntgenstrahlung from the Universe, ed. H. U. Zimmerman, J. Trümper, & H. Yorke, MPE Report 263, 9
Lamers, H. J. G. L. M., & Morton, D. C. 1976, ApJS, 32, 715L
Lamers, H. J. G. L. M., Cerruti-Sola, M., & Perinotto, M. 1987, ApJ, 314, 726
Lèpine, S., & Moffat, A. F. J. 1999, ApJ, 514, 909
Long, K. S., & White, R. L. 1980, ApJ, 239, 65L
Lucy, L. B., & White, R. L. 1980, ApJ, 241, 300
Lucy, L. B. 1982, ApJ, 255, 286
Lucy, L. B., & Abbott, D. C. 1993, ApJ, 412, 771
MacFarlane, J. J., Cassinelli, J. P., Welsh, B. Y., et al. 1991, ApJ, 380, 564
Maeda, Y., & Tsuboi, Y. 1999, BAAS, 31, 1541
Oskinova, L. M., Brown, J. C., Cassinelli, J. P., et al. 2001, in Interacting Winds from Massive Stars, ed. A. F. J. Moffat, & N. St.-Louis, ASP Conf. Ser., in press
Owocki, S. P., Castor, J. I., & Rybicki, G. B. 1988, ApJ, 335, 914
Owocki, S. P., & Cohen, D. H. 1999, ApJ, 520, 833
Owocki, S. P., & Rybicki, G. B. 1984, ApJ, 284, 337
Pauldrach, A. W. A., Puls, J., & Kudritzki, R. P. 1986, A&A, 164, 86
Raymond, J. C., & Smith, B. W. 1977, ApJS, 35, 419
Pollock, A. M. T. 1987, ApJ, 320, 28
Rybicki, G. B., Owocki, S. P., & Castor, J. I. 1990, ApJ, 349, 274
Schulz, N. S., Canizares, C. R., Huenemoerder, D., et al. 2000, ApJ, 545, L135
Seward, F. D., Forman, W. R., Giaccononi, R., Griffiths, et al. 1979, ApJ, 234, L55
Seward, F. D., & Chlebowski, T. 1982, ApJ, 256, 530
Springmann, U. 1994, A&A, 289, 505
Waldron, W. L., & Cassinelli, J. P. 2001, ApJL, 548L, 45
Wessolowski, U., & Niedzielski, A. 1996, MPE Rep., 263, 73
Wessolowski, U. 1996, MPE Rep., 263, 75
Willis, A. J., & Stevens, I. R. 1996, A&A, 310, 577