



Models and Experiments for Laminar Natural Convection from Heated Bodies in Enclosures

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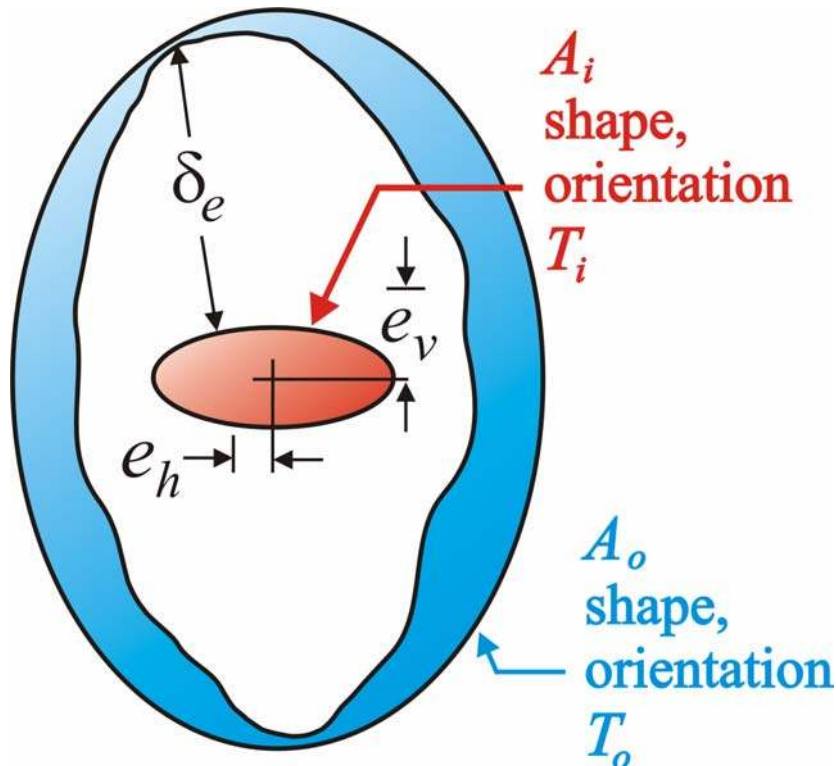
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Outline

- Introduction and problem description
- Literature review and objectives
- Experimental measurements
- Model development and validation
- Summary and conclusions

Problem Definition

- Steady state, natural convection
- Non-intersecting inner and outer boundaries
- Isothermal boundary conditions, $T_i > T_o$



Geometry:

- Relative boundary size

$$\sqrt{A_o} / \sqrt{A_i} = d_o / d_i \quad (\text{spheres})$$

- Effective gap spacing

$$\delta_e = (d_o - d_i) / 2 \quad (\text{spheres})$$

- Eccentricity

$$e_h = e_v = 0$$

Parameter Definitions

- Total heat transfer rate

$$Q = \iint_{A_i} -k \frac{\partial \theta}{\partial \vec{n}} \Big|_{A_i} dA_i, \quad \theta = T(\vec{r}) - T_b$$

- Non-dimensionalized by Nusselt number

$$Nu_{\sqrt{A_i}} = \frac{Q}{k\sqrt{A_i}(T_i - T_o)} = S_{\sqrt{A_i}}^* \quad \text{for } Ra \rightarrow 0$$

- Rayleigh number

$$Ra_{\sqrt{A_i}} = \frac{g\beta(T_i - T_o)(\sqrt{A_i})^3}{\nu\alpha}$$

Literature Review

Experimental and Numerical Studies

- Concentric spherical enclosures
 - Experimental data for high Rayleigh number, laminar boundary layer flow only
 - All other data from numerical simulations
- Other enclosure geometries
 - Spheres, cubes, cylinders
 - Experimental and numerical data
- No experimental data for full range of Rayleigh including transition and diffusive limit

Literature Review

Correlations and Models

- Warrington & Powe (1985), Warrington et al. (1988)
 - Correlation of data for variety of inner and outer shapes
 - Effective gap spacing based on equivalent spheres
 - Valid for laminar boundary layer flow only
- Raithby & Hollands (1975, 1985, 1998)
 - Analytically based model for concentric spheres
 - Series combination of resistances of conduction layers at inner and outer boundaries
 - For other geometries, effective gap spacing of Warrington & Powe (1985) recommended

Objectives

- Experimental measurements:
 - Variety of geometries, spheres, cubes, cylinder, etc.
 - Wide range of $Ra_{\sqrt{A_i}}$
 - Laminar boundary layer convection (atmospheric pressure)
 - Diffusive limit (reduced pressure)
- Analytical modeling:
 - Full range of $Ra_{\sqrt{A_i}}$ from conduction to convection
 - Applicable to wide range of geometries
 - Inner and outer boundary shapes and orientation
 - Relative boundary sizes
 - Physically based analysis

Experimental Method

- Wide range of Rayleigh number by of fluid density through reduction in gas pressure (Saunders, 1936, Hollands, 1988)

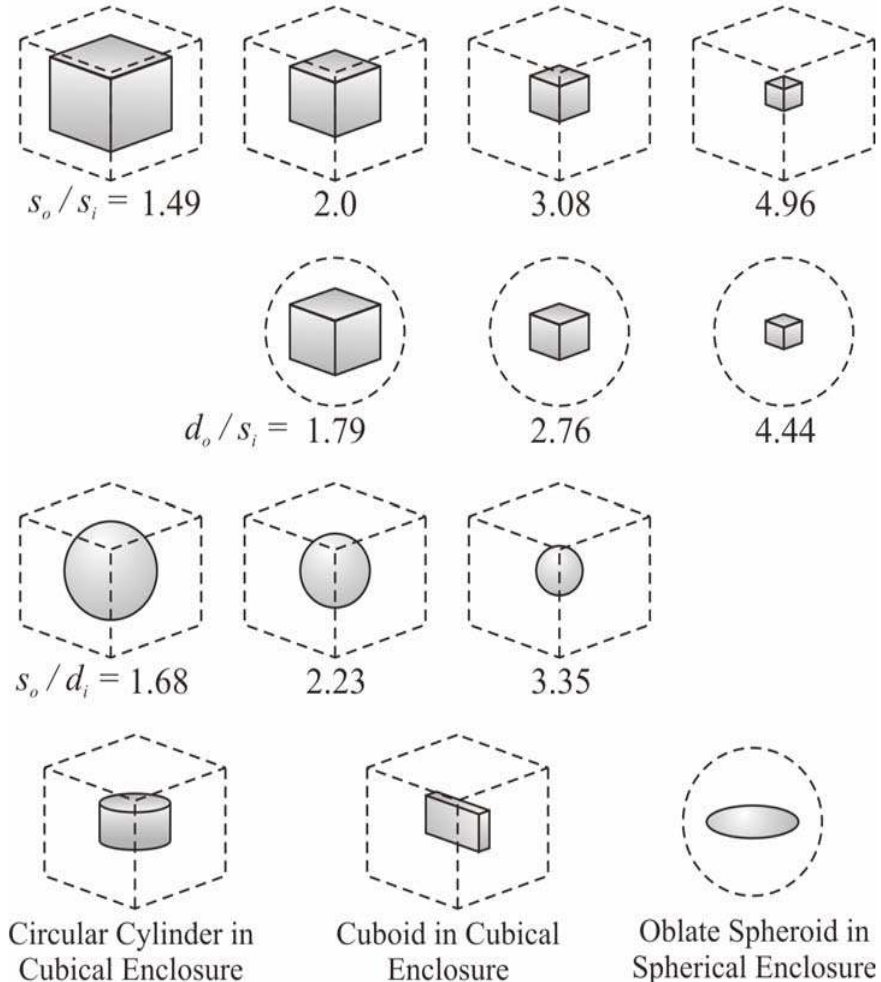
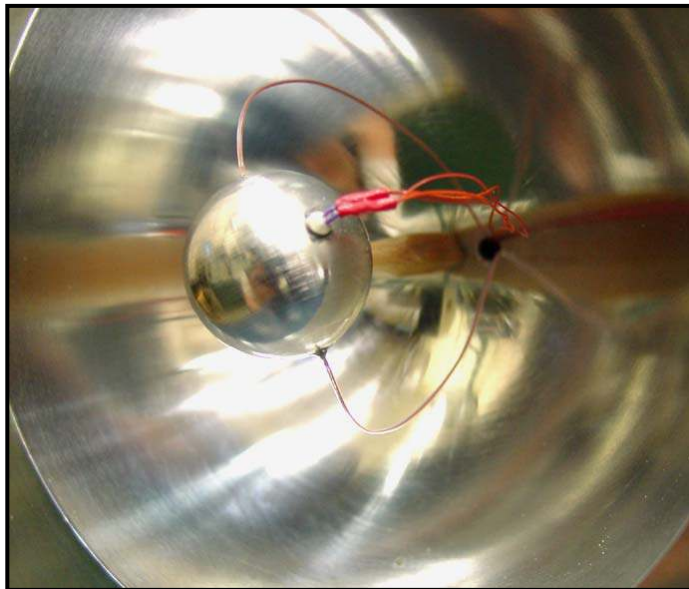
- Assume ideal gas

$$\rho = \frac{p}{R T_b Z} \Rightarrow Ra_{\sqrt{A_i}} = \frac{g \beta (T_i - T_o) (\sqrt{A_i})^3 p^2 c_p}{R^2 T_b^2 k \mu Z^2}$$

- Transient test method (Hollands, 1988)
 - Assumes “quasi” steady conditions
 - Fraction of the time required for steady state tests

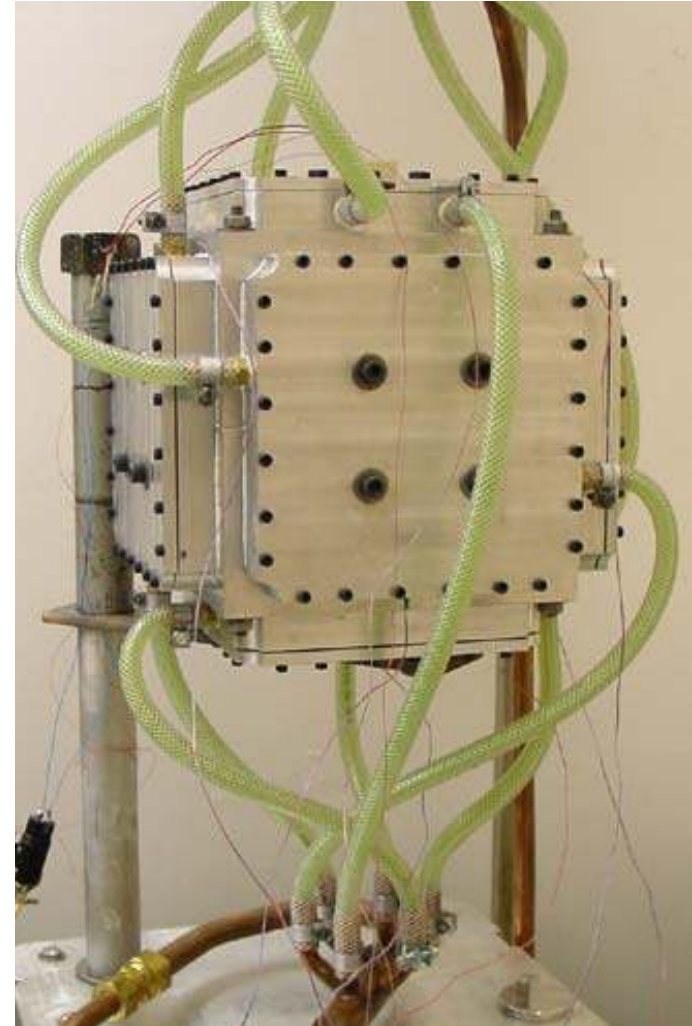
Experimental Apparatus

- Spherical and cubical outer geometries
- Eleven different inner bodies
- Temperatures measured using T-type thermocouples



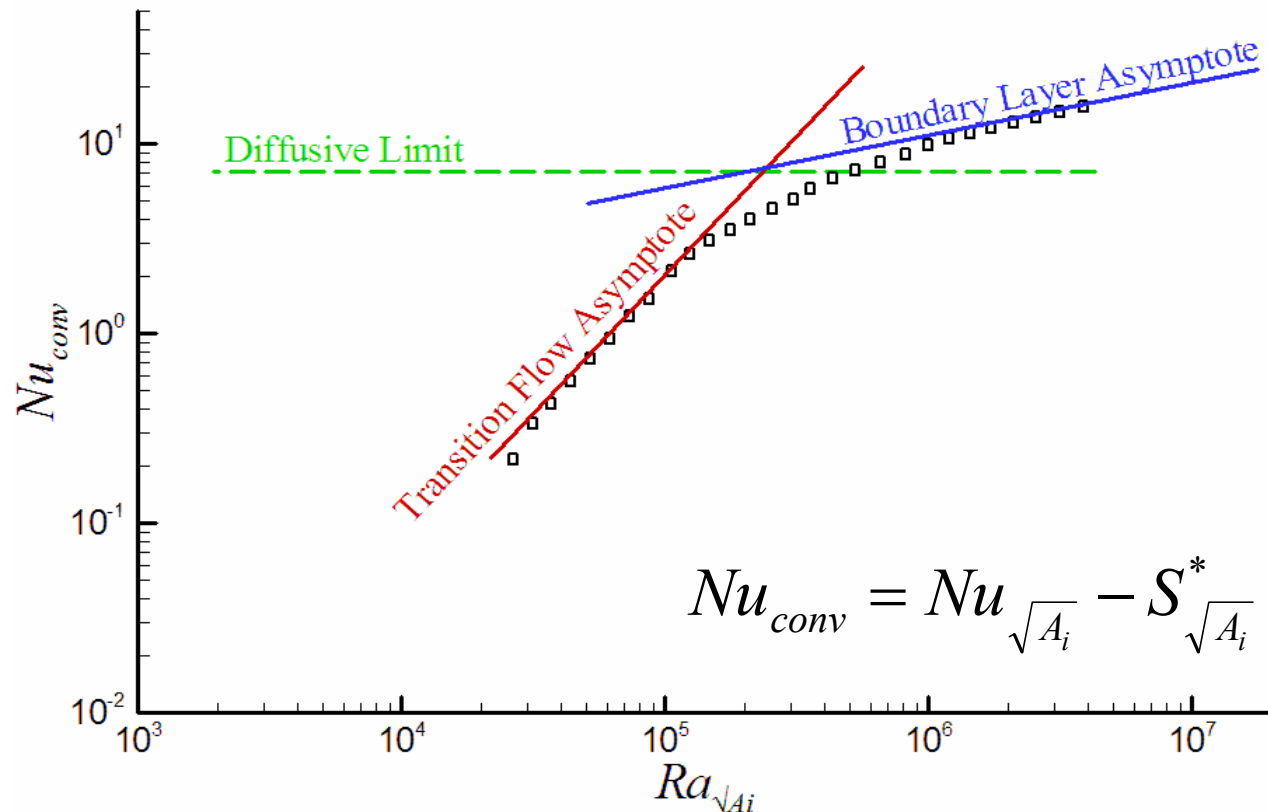
Experimental Apparatus

- All tests performed in vacuum chamber
- Enclosure walls cooled by cold plates
- Keithley 2700 data acquisition system
- Labview v.5.1 software
 - control of experiment
 - data acquisition and reduction

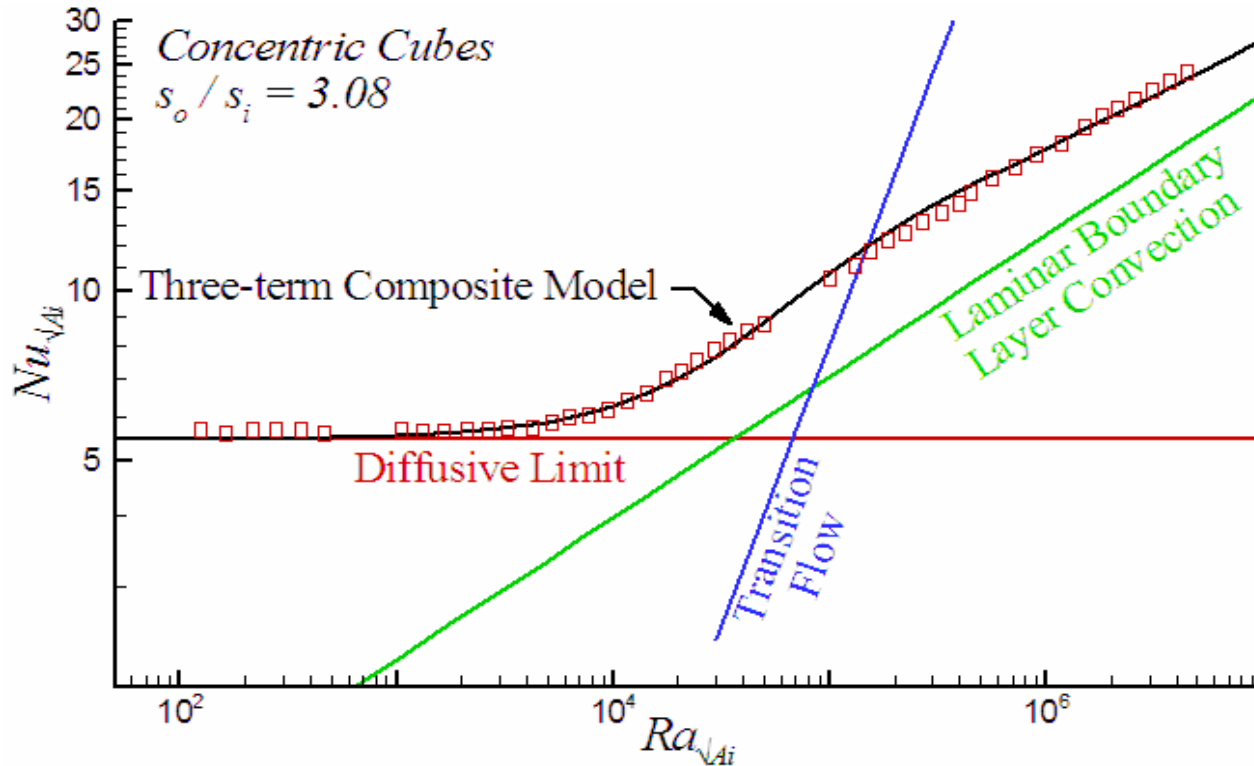


Model Development

- Assume linear superposition of diffusive and convective limits
- Convection-only data for $s_o/s_i = 2$ concentric cubes



Model Summary



- Combination of three asymptotic solutions

$$Nu_{\sqrt{A_i}} = S_{\sqrt{A_i}}^* + \left[\left(\frac{1}{Nu_{tr}} \right)^2 + \left(\frac{1}{Nu_{bl}} \right)^2 \right]^{-1/2}$$

$S_{\sqrt{A_i}}^*$ = conduction shape factor
 Nu_{tr} = transition flow convection
 Nu_{bl} = laminar boundary layer flow convection

Conduction Shape Factor

- Linear superposition of two asymptotic solutions

$$S_{\sqrt{A_i}}^* = \frac{\sqrt{A_i}}{\delta_e} + S_{\infty}^*$$

S_{∞}^* = full space diffusive limit

$\sqrt{A_i}/\delta_e$ = 1D planar resistance

- Effective gap spacing from equivalent spherical shell

$$\delta_e = \frac{d_o - d_i}{2}$$

Inner surface area $d_i = \sqrt{A_i/\pi}$

Enclosed volume $d_o = \left[6 \left(V + \frac{\pi}{6} d_i^3 \right) / \pi \right]^{1/3}$

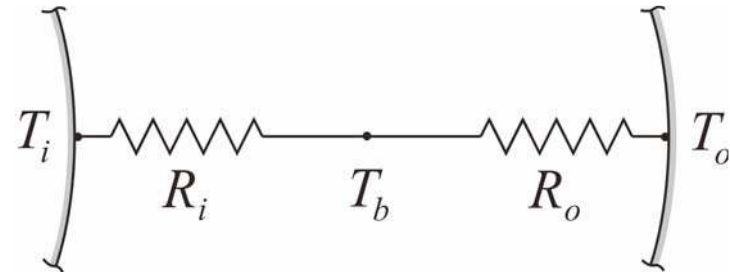
- Dimensionless conduction shape factor

$$S_{\sqrt{A_i}}^* = \frac{2\sqrt{\pi}}{\left[1 + 6\sqrt{\pi} \left(V^{1/3} / \sqrt{A_i} \right)^3 \right]^{1/3} - 1} + S_{\infty}^*$$

Boundary Layer Convection

- Assumptions

- Laminar flow
- T_b uniform
- Non-intersecting boundary layers



- Series combination of resistances

$$R_{conv} = R_i + R_o \quad R_i = \frac{T_i - T_b}{Q} \quad R_o = \frac{T_b - T_o}{Q}$$

- Non-dimensionalize using Nusselt number

$$Nu_i = \frac{1}{k\sqrt{A_i}} \frac{1}{R_i} \quad Nu_o = \frac{1}{k\sqrt{A_o}} \frac{1}{R_o} \quad Nu_{bl} = \frac{Nu_i}{1 + 1/\phi}$$

$$\phi = \frac{T_i - T_b}{T_b - T_o} = \frac{R_i}{R_o} = \frac{\sqrt{A_o}}{\sqrt{A_i}} \frac{Nu_o}{Nu_i}$$

Boundary Layer Convection

- Convection modeled using Yovanovich [34] and Jafarpur [36]

$$Nu_{\sqrt{A}} = F(\text{Pr}) G_{\sqrt{A}} Ra_{\sqrt{A}}^{1/4}$$

- Laminar boundary layer convection asymptote

$$Nu_{bl} = \frac{Nu_i}{1 + 1/\phi} = \frac{F(\text{Pr}) G_{\sqrt{A_i}} Ra_{\sqrt{A_i}}^{1/4}}{(1 + 1/\phi)^{5/4}}$$

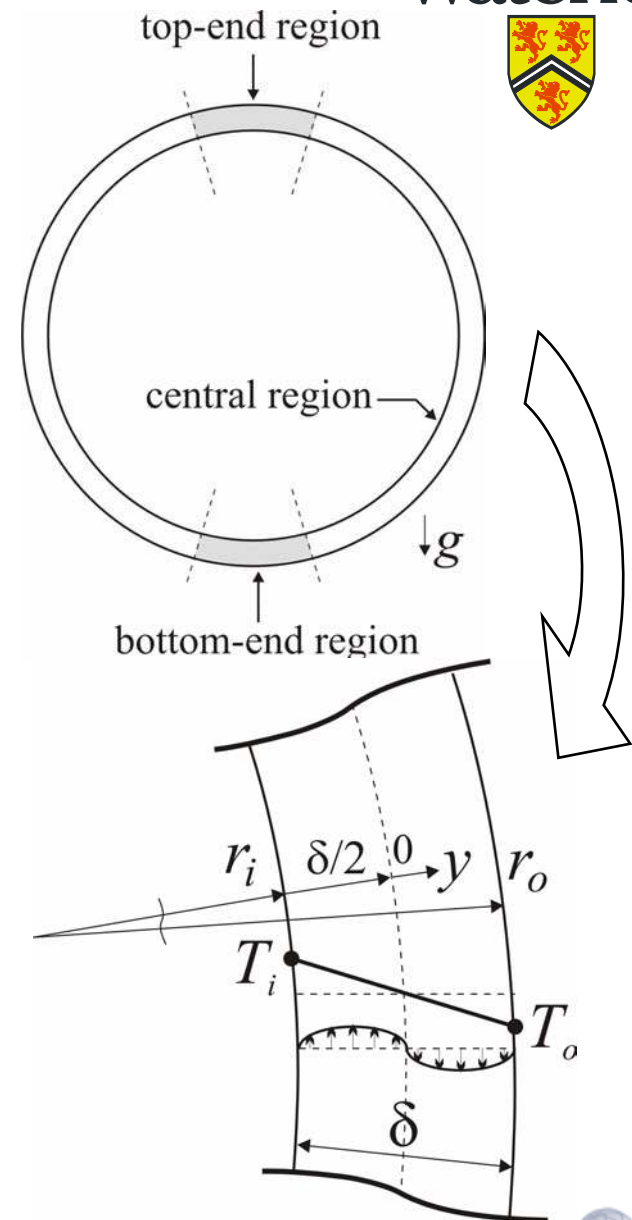
$$Nu_{bl} = \frac{F(\text{Pr}) G_{\sqrt{A_i}} Ra_{\sqrt{A_i}}^{1/4}}{\left[1 + (A_i/A_o)^{7/10} \left(G_{\sqrt{A_i}} / G_{\sqrt{A_o}} \right)^{4/5} \right]^{5/4}}$$

Transition Flow

- Boundary layers merge when $Ra < Ra_{cr}$
- Model as equivalent concentric spheres
- Three distinct regions are formed
- Central region
 - Radial conduction
 - Buoyancy induced flow
- For narrow gap spacing, $\delta \ll r_i$,
temperature, velocity in central region

$$T - T_b = -\frac{y}{\delta/2} (T_i - T_b), \quad T_b = \frac{T_i + T_o}{2}$$

$$u = \frac{g_e \beta}{12\nu} (T_i - T_o) \left(\frac{\delta}{2}\right)^2 \left[\left(\frac{y}{\delta/2}\right)^3 - \frac{y}{\delta/2} \right]$$



Transition Flow

- Enthalpy balance in top-end and bottom-end regions

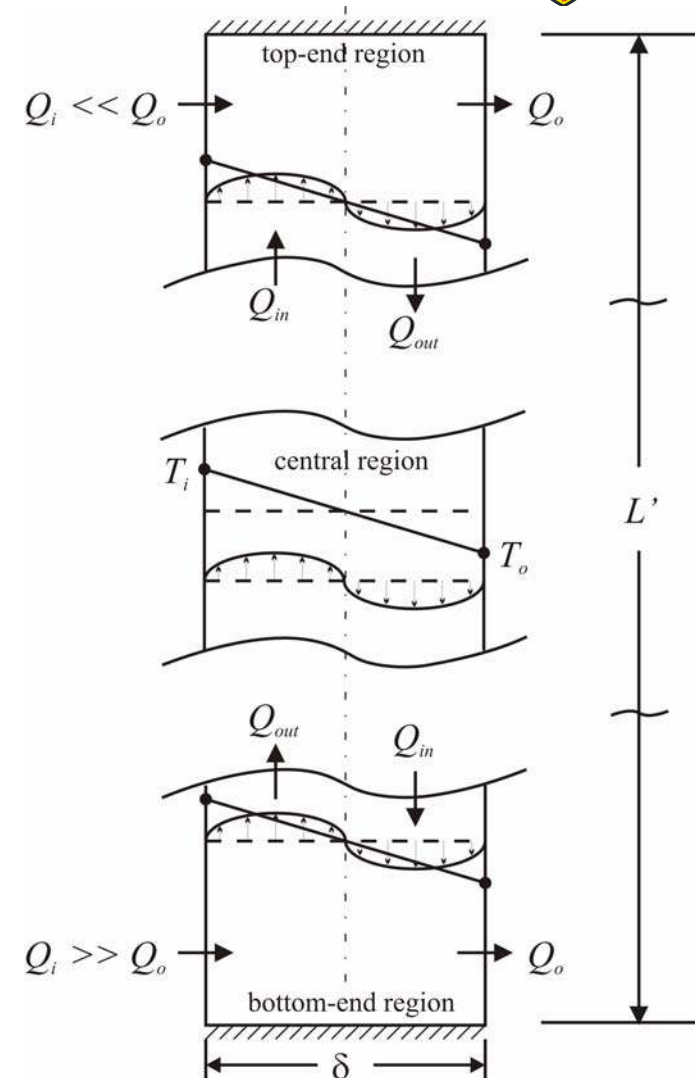
$$Q_{i,o} = \frac{\rho c_p W' g_e \beta (T_i - T_o)^2 \delta^3}{720 \nu}$$

- Transition flow asymptote

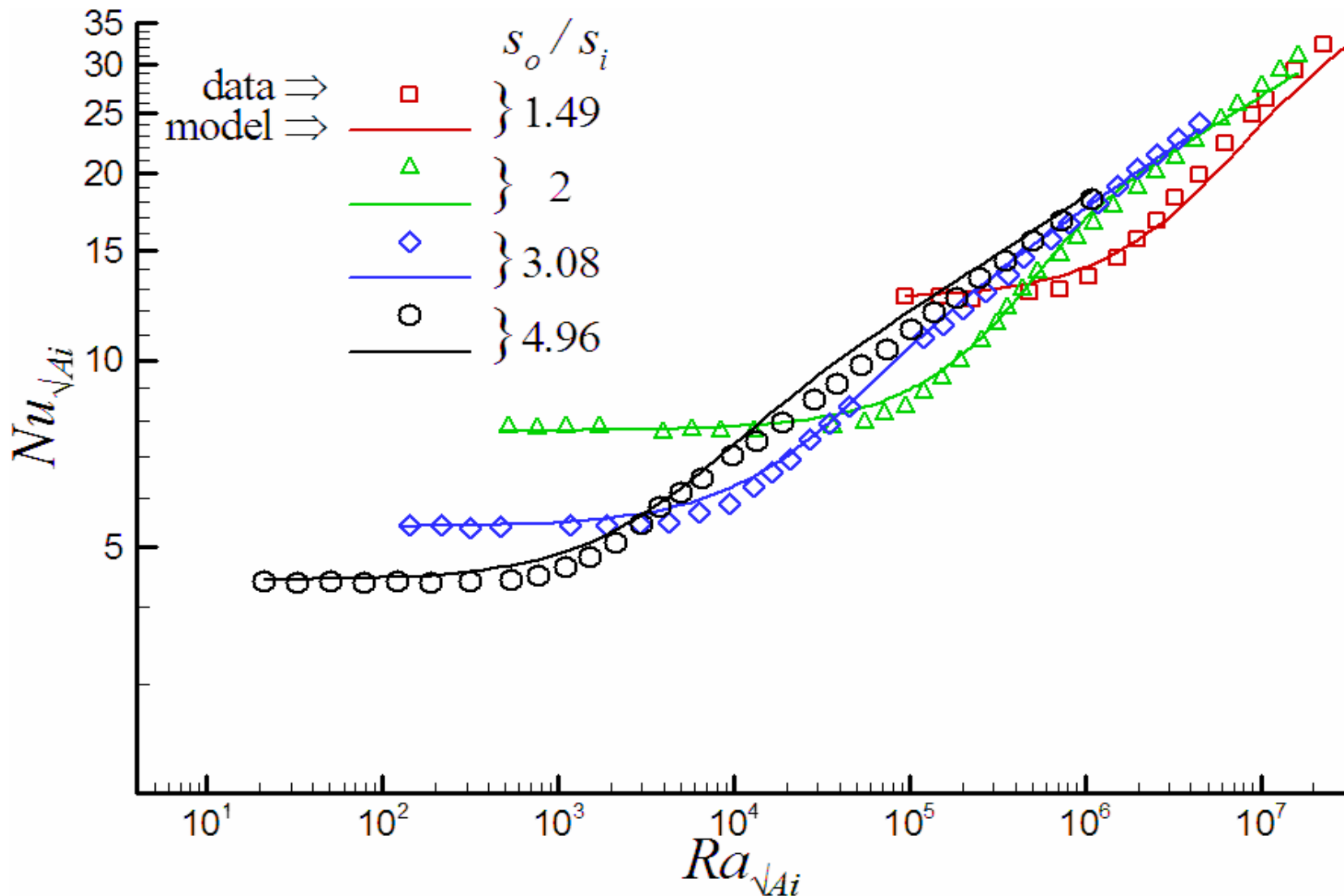
$$Nu_{tr} = \frac{\sqrt{2}}{360} \frac{\sqrt{A_i}}{L'} \left(\frac{\delta_{eff}}{\sqrt{A_i}} \right)^3 Ra \sqrt{A_i}$$

δ_{eff} = gap spacing of equivalent cavity

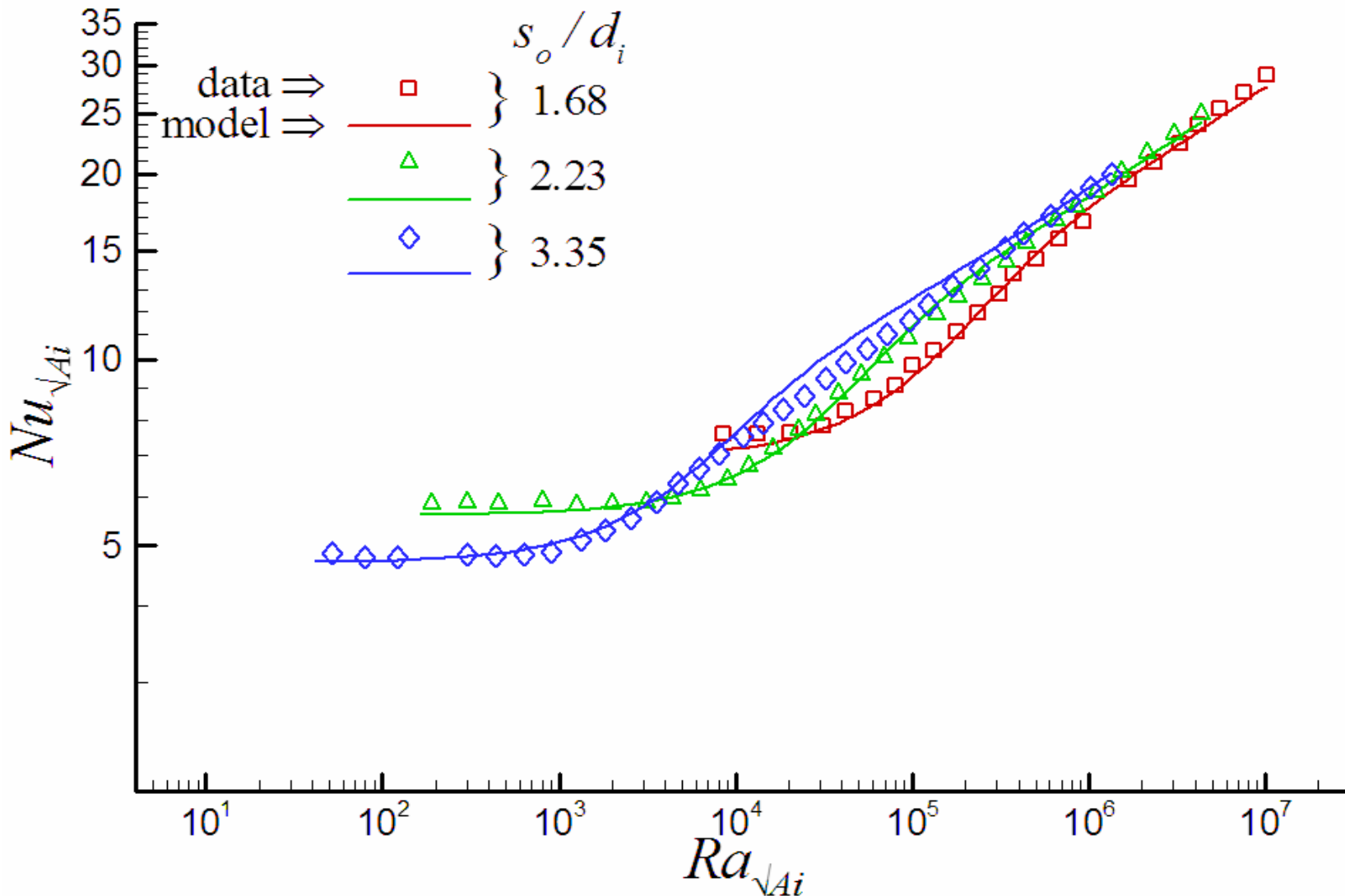
L' = effective flow length



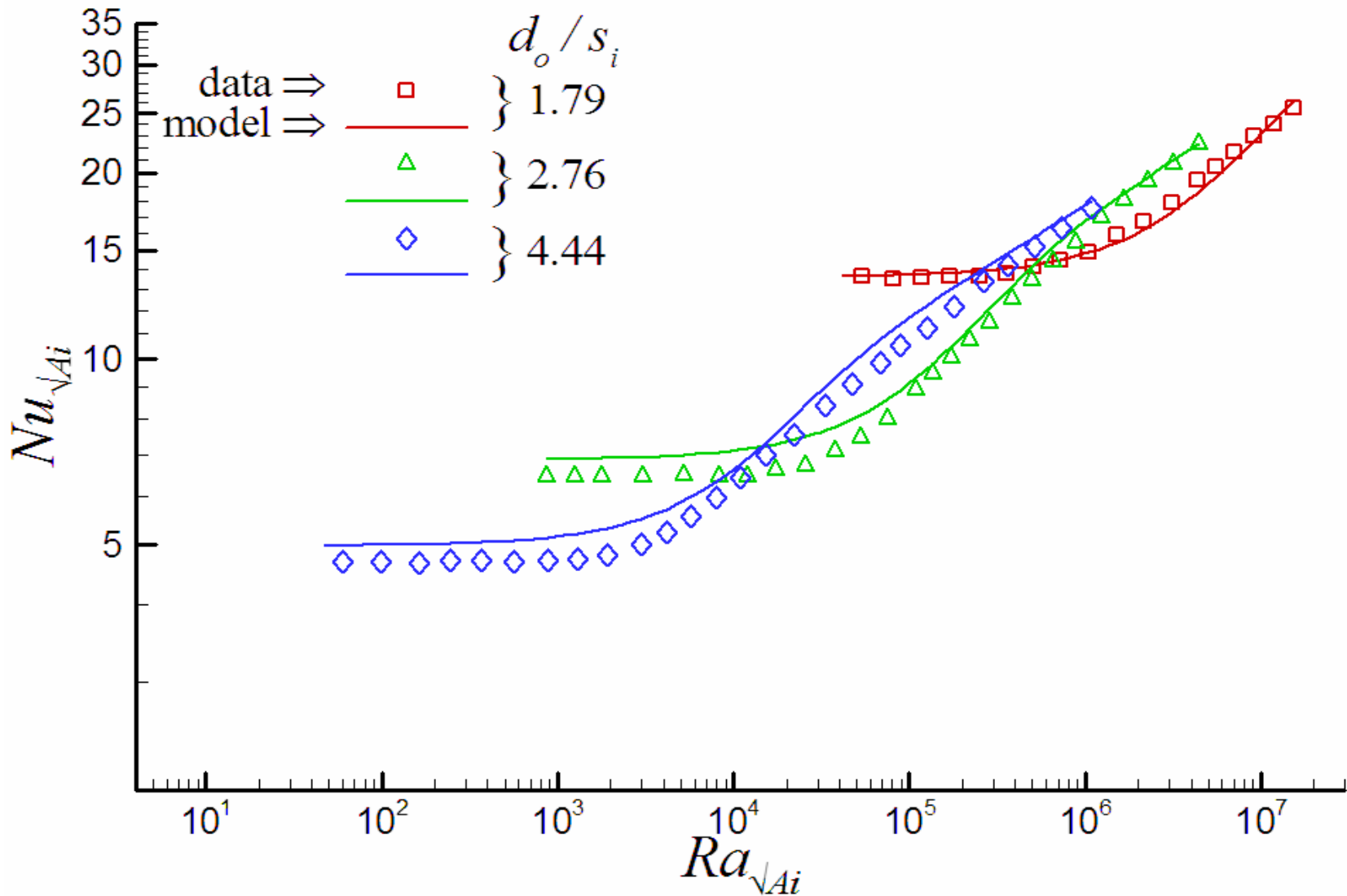
Validation: Concentric Cubes



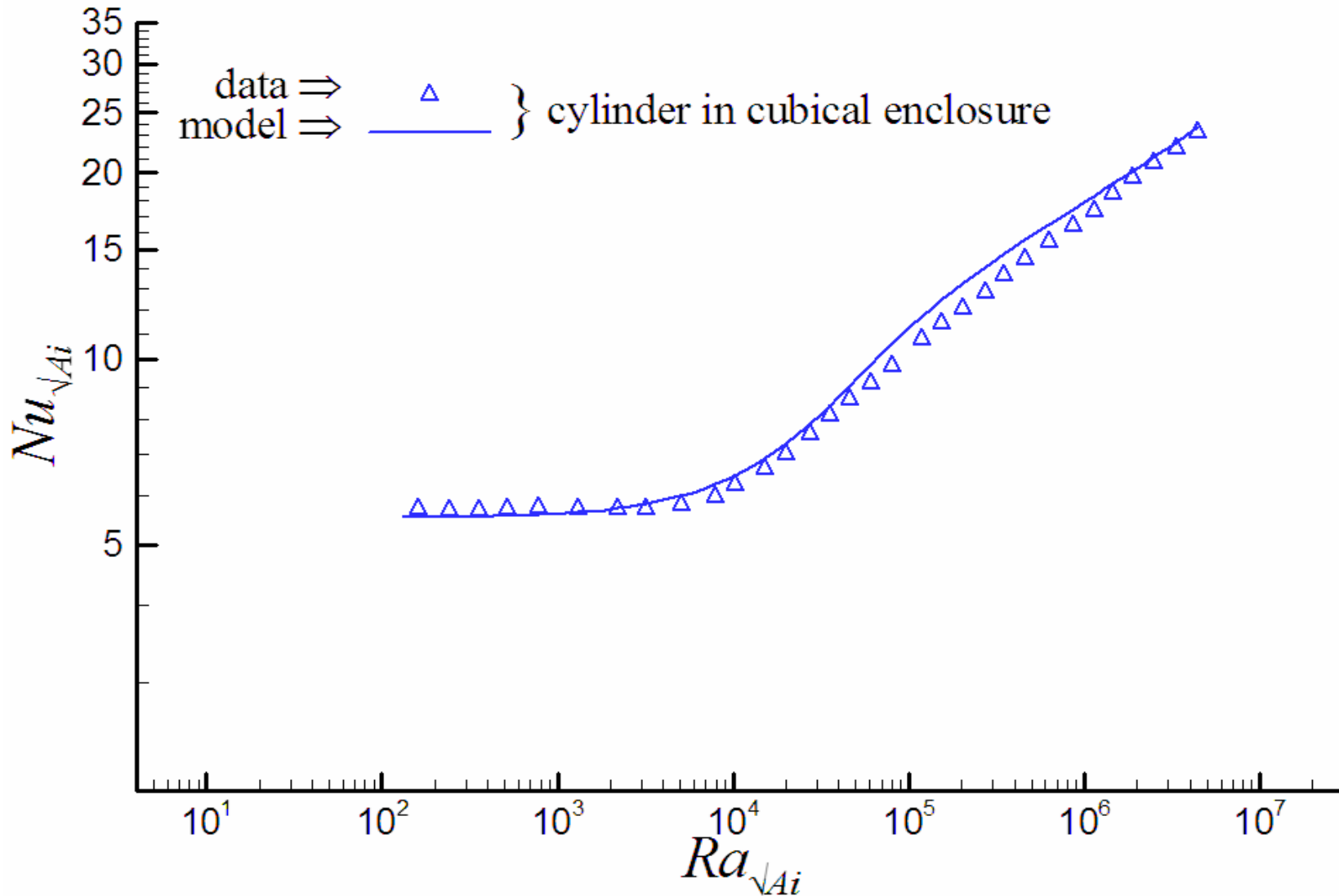
Sphere in Cubical Enclosure



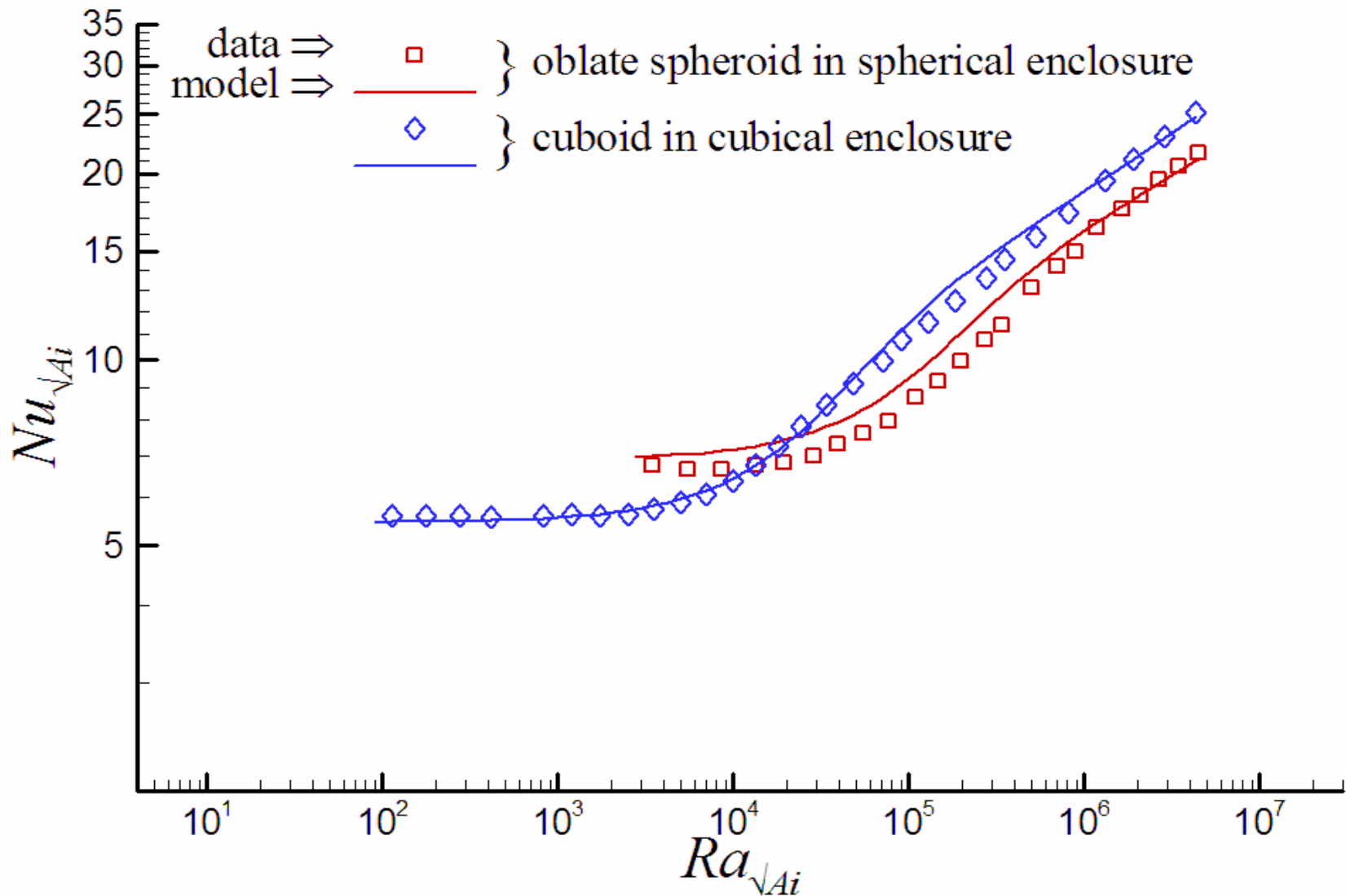
Cube in Spherical Enclosure



Cylinder in Cubical Enclosure



Other Enclosure Geometries



Summary and Conclusions

- Combined experimental / analytical study of natural convection heat transfer between heated body and cooled enclosure
- Experimental data for variety of enclosure configurations, dimensions
- Model developed based on combination of analytic, asymptotic relationships
 - Diffusive limit
 - Laminary boundary layer convection
 - Transition flow convection
- 2 – 6% RMS, 12% maximum difference between model and data