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## Models and solutions for emergency logistics

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# Models and Solutions for Emergency Logistics 

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A thesis submitted to Nanyang Technological University in fulfillment of the requirement for the degree of Doctor of Philosophy


#### Abstract

Emergency logistics planning is increasingly becoming a crucial issue due to the increase in the occurrence of natural disasters and other crisis situations. An adequate level of mitigation measures and a coordinated post-disaster relief logistics management may help to reduce the loss of both human lives and economic damage. Logistics planning in emergencies involves dispatching commodities to affected areas and evacuation of wounded people to emergency units. The number of vehicles involved may be very large during on-going relief operations. Furthermore, time plays a critical role in the logistic plan, and it directly affects the survival rate in affected areas. This makes the task of logistics planning more complex than conventional distribution problems. As a result, a modeling approach that enables massive dynamic routing of people and commodities is required. In this study, a dynamic network flow model is developed and a solution framework is presented exploiting the currently efficient simplex implementation, together with a two-stage algorithm to disaggregate the flow variables and generate routes information. The efficiency of the proposed model is verified through comparison with conventional vehicle based formulation. Moreover, the dynamic application of the model is illustrated on real world scenarios.


A constructive heuristic with parallel vehicle exploration is first proposed in an attempt to produce a fast solution. Then based on a different search principle, a metaheuristic of ant colony optimization ( ACO ) is developed to improve the solution quality. The ACO approach exploits the hybrid characteristics of the problem and decomposes the original model into sub-components. It first builds stochastic vehicle paths under the guidance of heuristic information while in the second phase a successive maximum flow (SMF) algorithm is developed for the commodities
dispatch to different types of vehicle flows. Pheromone trails are updated according to the dispatch result by SMF. Thus, the two sub-problems are coordinated through trails leading to the continuous improvement of solution quality. The performances of both algorithms are tested on a number of randomly generated networks. The constructive heuristic achieves quick solutions compared to the direct model solution while the ACO algorithm results in better solution quality within shorter runtimes for larger instances. Analyzing the overall solution quality and run time consumption, one can say that ACO algorithm suits the real emergency situation where there is continuous uncertainty and information dynamism.

Logistics coordination after disaster requires selecting the sites that will provide maximum coverage of medical need in the affected areas. An important issue that arises in such emergencies is that hospital capability has to be re-distributed to achieve maximal service level. This necessitates finding optimal locations for the temporary emergency units and optimal medical personnel allocation equilibrium among them. The extended model takes this facility planning issue into consideration and the ACO meta-heuristic is also developed to include the location routing problem.

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## LIST OF NOTATIONS

T: Set of time periods in the planning horizon, $t$ (or $q$ ) denotes a specific time period in T

M: Set of vehicle types; $v$ (or $m$ ) denotes a specific vehicle type
A: Set of commodities; a denotes a specific commodity type
L: $\quad$ Set of vehicle labels; $l$ denotes a specific vehicle
H: Set of different categories of wounded people (heavy, moderate-light); $h$ denotes a specific people type

C: Set of all nodes in integrated network, o (or $p, i, j$ ) denotes a specific node
CD : Set of demand nodes including transshipment nodes, $\mathrm{CD} \subset \mathrm{C}$
CS: Set of supply nodes and vehicle depots, $\mathrm{CS} \subseteq \mathrm{C} \backslash \mathrm{CD}$
CE: Set of potential emergency center sites including existing emergency centers, $\mathrm{CE} \subseteq \mathrm{C} \backslash \mathrm{CD}$

CH : Set of available emergency centers, $\mathrm{CH} \subseteq \mathrm{CE}$
$\mathrm{d}_{\text {aot }}$ : Amount of demanded commodity type $a$ at node $o \in \mathrm{CD}$ at time $t ; \mathrm{d}_{\mathrm{aot}}=0$ for $o \in \mathrm{C} \backslash \mathrm{CD}$
sup $_{\text {aot }}$ : Amount supplied of commodity type $a$ at node $o \in \mathrm{CS}$ at time $t ; \sup _{\text {aot }}=0$ for $o \in \mathrm{C} \backslash \mathrm{CS}$
$\mathrm{d}_{\text {hot }}$ : Number of wounded people of category $h$ waiting at node $o \in \mathrm{CD}$ at time $t$; $\mathrm{d}_{\text {hot }}=0$ for $o \in \mathrm{C} \backslash \mathrm{CD}$
$\mathrm{av}_{\text {omt }}$ : Number of vehicles of type $m$ at node $o$ added to the fleet at time $t$
$\mathrm{N}_{l}: \quad$ Set of arcs for vehicle $l$
$\mathrm{t}_{\text {opl }}$ : $\quad$ Time required to traverse $\operatorname{arc}(o, p)$ for vehicle $l$
$\mathrm{av}_{\text {olt }}$ : Binary number indicating if vehicle $l$ is added to the fleet at node $o$ at time $t$
cap $_{1}$ : Load capacity of vehicle $l$
$\mathrm{P}_{\mathrm{a}}$ : $\quad$ Priority of satisfying demand of commodity type $a$
$\mathrm{P}_{\mathrm{h}}$ : Priority of serving wounded people of category $h$
$\mathrm{t}_{\mathrm{opm}}$ : Time required to traverse arc ( $o, p$ ) for vehicle type $m$
cap $_{\mathrm{m}}$ : Load capacity of vehicle type $m$
$\mathrm{w}_{\mathrm{a}}$ : Unit weight of commodity $a$
$\mathrm{w}_{\mathrm{h}}$ : Average weight of a wounded person
scap $_{\mathrm{ho}}$ : Initial per period service rate for category $h$ wounded people at hospital at node $o \in \mathrm{CH}$; scap $\mathrm{ho}_{\mathrm{o}}=0$ for $o \in \mathrm{C} \backslash \mathrm{CH}$

K: A big number
$\mathrm{K}_{\text {oqptm }}$ : Binary parameter matrix indicating if node $p$ is reachable at time $t$ from node $o$ at time $q$ using vehicle type $m$ : if $\mathrm{t}-\mathrm{q}<\mathrm{t}_{\text {opm }}$, then $\mathrm{K}_{\text {oqptm }}=0$, else $\mathrm{K}_{\text {oqptm }}=1$
$Z_{\text {aopmt: }}$ Amount of commodity type $a$ traversing arc $(o, p)$ at time $t$ using vehicle type m
$\mathrm{Y}_{\text {opmt: }}$ Integer number of vehicles of type $m$ traversing the arc $(o, p)$ at time $t$
$X_{\text {hopmt: }}$ Integer number of wounded people of category $h$ traversing arc $(o, p)$ at time $t$ using vehicle type $m$
$Z_{\text {aplt }}$ : Amount of commodity type $a$ delivered to node $p$ by vehicle $l$ at time $t$
$\mathrm{P}_{\text {aplt: }}$ : Amount of commodity type $a$ picked up at node $p$ by vehicle $l$ at time $t$
$\mathrm{L}_{\text {alt: }}$ : Amount of commodity type $a$ carried by vehicle $l$ at time $t$
$\operatorname{dev}_{\text {aot }}$ : Amount of unsatisfied demand of commodity type $a$ at node $o$ at time $t$
$\mathrm{Y}_{\text {oplt: }} \quad$ Binary variable indicating if vehicle $l$ traversing the $\operatorname{arc}(o, p)$ at time $t$
$\operatorname{dev}_{\mathrm{ht}}$ : Number of unserved wounded people of category $h$ at time $t$
$\mathrm{sp}_{\text {hot }}$ : Number of wounded people of category $h$ who are served at node $o$ at time $t$
$S_{h o}$ : Integer number of allocated service rate at an emergency facility $0 ; s_{h o}=0(\forall$ $\mathrm{o} \in \mathrm{C} \backslash \mathrm{CE})$.

## 1 Introduction

The last decade has seen a marked increase in the occurrence of natural disasters along with exposure to greater levels of loss of life, property and material damage. To elucidate a few examples: In China, the 1998 floods took 4,150 lives and made 18.39 million homeless, with direct economic damage over US\$ 26 billion. The most destructive earthquake that took place in Turkey in 1999 caused a total number of 17,727 deaths, 43,959 injuries and damaged 214,000 residential units, affecting more than 2 million people. Over 200,000 people died and more than 1.5 million people lost their homes in the 2004 tsunami disaster in the Indian Ocean, and the losses are estimated to total more than US\$ 7 billion. Furthermore, frequent bombings and other acts of terrorism aggravated concerns about the securities in many countries and regions. According to the report by the United Nations Office for the Coordination of Humanitarian Affairs, there is increasing human vulnerability in crisis situations both in natural disasters (200 million affected in 2003) and in complex emergencies (45 million in need of life-saving assistance in 2003). In all these emergency situations, the adequate preparedness and good logistics management may help to reduce the loss on both human lives and economic damage.

### 1.1 Problem Description

The basic underlying logistical planning for disaster relief management involves dispatching commodities to distribution centers in affected areas and evacuation of
wounded people to emergency service facilities. Evacuation activities take place during the initial response phase whereas logistics support operations tend to continue for a longer time for sustaining the basic needs of survivors who remain in the affected area. The timely availability of commodities such as food, shelter and medicine and effective transportation of the wounded affect the survival rate in affected areas. In disaster relief operations, various organizations often face significant problems of transporting large amounts of many different commodities including food, clothing, medicine, machinery, and people between different supply centers (emergency units) and different destinations in the disaster areas. Often, there may be many different modes of transportation available for the purpose of shipping the supplies and people. All of these modes of transportation may not be suitable for every commodity or people. Some commodities may change the type of mode while in transport from origin to destination.

Emergency logistics support and vehicle dispatch have features that are different from established dispatch settings. The goal is to minimize delay in the provision of prioritized commodities to survivors and health care services to injured people, where different types of vehicles are utilized to serve transportation needs. Some discriminating factors are highlighted as follows:
a) Supply availability is limited in the initial disaster response phases: the exact impact of the phenomenon is not known and it takes time to explore affected regions, communicate the impacts of the disaster and coordinate international and national help. Furthermore, there is always transportation delay from major support centers due to infrastructure failures.
b) Supply nodes may represent warehouses, but most often, if they are located in affected areas, they are mere tent shelters where food and other materials are distributed to survivors who are maintained at those locations. If located in unaffected zones, then they may represent district collection centers, shelters and hospitals. Thus, the number of supply nodes is large and distributed among demand nodes. This situation is different from commercial situations where 3-5 depots serve more than 100 customers.
c) The following vehicle routing and availability conditions prevail. During the first response and throughout the ongoing relief operations, a vehicle is not required to return immediately to a supply node (depot) once its current assignment is completed. It can wait for the next instruction at its last destination or may move towards a depot at the end of the shift if drivers are required to change shifts. The depot where the vehicle ends need not be the one from which its itinerary starts. Due to the latter reason and the fact that supply nodes are numerous and dispersed among demand nodes, the standard tour definition may be replaced by an itinerary that starts at the beginning of the shift and continues with any order of visits until its completion.
d) It is not logical to assume that vehicle capacity is sufficient to carry a "customer's" demand; hence, the same node has to be visited multiple times. This implies split delivery.
e) A commodity demand node may in the same time supply wounded people while commodity supply nodes may also be medical facilities. In demand nodes, commodities are delivered and wounded people are picked up and transported to hospitals (most hospitals lie at supply nodes). In supply nodes, supplies are
picked up and at times when hospitals exist at supply nodes, wounded people are delivered. Hence, simultaneous split pickup-delivery with mixed order service defines the service strategy. Furthermore, it is not necessary that the vehicle picking up some goods from a supply node should be the one to deliver them to their last destination. These may be dropped at an intermediary location and then picked up by another vehicle that takes them to their last destination. Hence, the cooperation among vehicles is possible.
f) The evacuation problem definition is also different from classical building evacuation problems. In this study, evacuation planning focuses on the rescue of wounded people where it can also be naturally extended to all affected population. Hence, the evacuation routing choice depends not only on transportation capacity constraints, but also on available medical services at the destinations.
g) Time plays a crucial role in managing the response to a particular emergency. The logistics plan involves a time horizon consisting of a given number of time periods and it deals with time-variant demand and supply. At the beginning of any planning time horizon, given a snapshot of current and future requirements/supplies, and vehicle availability, the plan generates multi-period vehicle routes/schedules along with their commodity load-unload assignments. Then it is updated at regular time intervals incorporating new information on demand, supplies and vehicle availability, and, accounting for the status of the logistics system resulting from the plan implemented previously. Since the plan has a time-dependent structure, re-planning is facilitated and is carried out repeatedly during ongoing disaster relief operations. Thus, the system is designed
to respond to time-dependent logistic needs in an adaptive manner, so that after the emergency call is announced, it responds quickly to new demand, supply and vehicle availability.

### 1.2 Research Motivation

Although the literature in logistics management is extensive, the particular problem on the emergency operations planning has received little attention: Ardekani and Hobeika (1988) studied the logistics problems in the 1985 Mexico City earthquake and identified the major problems in the relief operations management. A complicated mathematical formulation was presented in Haghani and Oh (1996) for commodity logistics problem in emergency; Özdamar et al. (2004) proposed a more compact model with a Lagrangian relaxation based iterative algorithm for small test instances. On the other hand, there has been considerable literature modeling evacuation of buildings or larger areas over the last two decades, as will be discussed in literature review chapter; however, a straightforward application of these approaches is infeasible here due to the different problem definition as depicted in previous section. Moreover, to our best knowledge, there is no research dealing with these two aspects in an integrated manner which is the subject of this study, though such plan can significantly enhance the system-wide operational efficiency.

Based on the aspects discussed above, the problem is a general dynamic routing problem that can handle various practical complexities. It integrates features of many conventional discrete optimization problems, such as vehicle routing problems, integer multi-commodity flow problems, and so on, while it does not possess those strict constraints on the solution definition; on the other hand, issues such as wounded evacuation and availability of emergency services must be integrated into the
consideration for operational efficiency. Moreover, the existence of multiple commodities, transportation modes, and large demand adds complexities to the problem and the solution would not be easy. Therefore, the study on effective modeling and the solution methodologies is of crucial importance in emergency logistics management.

### 1.3 Research Objective and Scope

This study aims at developing decision models and solution methodologies that can be potentially used by emergency response managers in planning for disaster relief operations. In particular, there are three objectives, as follows.

1) Effective formulations for the problem;

An integrated formulation should be identified to coordinate vehicles scheduling, rescue operations and commodities transportation. For practical purposes, the model must be able to produce detailed plan at operational level, while being sufficiently tractable and leading to efficient solution algorithms. Alternative models from related problems shall also be evaluated to verify the efficiency of proposed formulation.
2) Solution algorithms in the framework of both exact algorithms and heuristics;

While significant progress may be made in the formulation, specially designed solution methods will prove extremely valuable. Based on the currently powerful optimization packages, exact solution methodology can be established for the solution to the problem. On the other hand, due to the inherent hardness in routing problems, heuristic method should be designed for the treatment of large-scale problems.
3) Model extensions: facility planning;

An important extension to be addressed in this thesis is the emergency medical centers location problem, which enables the selection of the best locations among the possible proposed ones for temporary medical care units achieving an equilibrium among service capacities of medical units in order to minimize transportation delay for patients with different priorities and localities. The solution method proposed for the original emergency logistics model will also be developed to solve the locationrouting extension.

### 1.4 Organization of the Thesis

This thesis discusses various issues concerning the emergency logistics problem (ELP) and is organized as follows. In the present chapter ELP is introduced and some of its features are listed which rarely appear in the conventional routing problems, followed by the research motivation and objectives. Then, the relevant logistics models and algorithms in literature are investigated in Chapter 2 to give an overview of progress made within the area and the available techniques for this study. In Chapter 3, two formulations for ELP are proposed and the superior formulation is verified through the performance comparison. Based on that, a practical integrated model is presented in Chapter 4 for coordinating logistics support and evacuation operations in emergencies. The dynamic application of the model is illustrated by a concrete earthquake scenario, together with a brief discussion on uncertainty issues. To deal with the computational hardness resulting from the routing problem, two heuristics are designed in Chapter 5 for the treatment of large-scale problems. The methods are tested on a set of randomly generated instances and comparisons are conducted with those of the optimization package. Then, the model is extended to address the facility location problem in Chapter 6, and the solution framework based
on the proposed ACO algorithm is presented. Finally, the main conclusions and major contributions of this research are summarized and possible future extensions are discussed.

## 2 Literature Review

In this chapter, the existing literature related to emergency logistics problem will be investigated. Generally, the literature covers a wide range of different applications and methodological approaches. According to the relevancy to the problem and its importance in transportation and logistics research, this chapter is restricted to the following sub-fields: vehicle routing problem (VRP) in both static and dynamic versions, multi-commodity network flow problem, and dynamic network. A number of different papers using exact and heuristic solution approaches to these problems are also discussed as they are estimated to be of specific importance to this thesis.

### 2.1 Models and Algorithms for VRP

VRP was first introduced by Dantzig and Ramser (1959) in 1950s. A few years later, Clarke and Wright (1964) proposed a greedy heuristic that improved on the DantzigRamser approach. Following these two seminal papers, many models and exact and heuristic algorithms were proposed for the optimal and approximate solution of different versions of the VRP.

### 2.1.1 Formulations for VRP

In the literature, there are three different basic modeling approaches proposed for the VRP. The most frequently used is known as vehicle flow formulation, which uses binary variables associated with all arcs. Commodity flow models were first
introduced by Garvin et al. (1957) for the oil delivery problem and later extended by Gavish and Graves (1982) to variants of TSP and VRP. In addition to variables used by the vehicle flow formulations, these formulations require a new set of continuous variables, associated with amounts of commodity that flow over each arc. The last modeling approach is the set-partitioning problem, which was originally proposed by Balinski and Quandt (1964) and uses an exponential number of binary variables, each associated with a different feasible circuit of network.

Based on these formulations, there are several variants of the basic version of the VRP considered in the literature. The restrictive assumptions of the VRP are often relaxed to accommodate more realistic settings, among which the VRP with pickup and delivery (VRPPD) and VRP with split delivery (VRPSD) are most relevant to the ELP. In VRPPD, vehicles are not only required to deliver goods to customers but also to pick some re-cycled goods up at customer locations. Simultaneous pickups and deliveries are common in the emergency logistics setting, where some goods are delivered to affected areas from depots and injured people are picked up and transported back to medical centers. The standard definition of VRPPD necessitates that the customer is only visited once. Min (1989) solved the problem with clustering followed by TSP solutions after which infeasibilities are penalized and TSPs are resolved. Gendreau et al. (1999a) solved the TSP first and then ordered the pickups and deliveries in the TSP. Nagy and Salhi (2005) established a weakly feasible solution first (one that checks only total load delivered or picked up, but does not check vehicle capacity in-between nodes on the tour), and then removed infeasibilities by a combination of moves and an iterative procedure that reduces infeasibilities in a controlled manner. Multi-depot extension was also introduced to this problem by the authors.

A closely variation of simultaneous pickup-delivery problem is the mixed pickupdelivery problem (Golden et al. 1985, Kontoravdis and Bard 1995, Salhi and Nagy 1999). Similar to simultaneous pickup-delivery problem, maintaining the feasibility of vehicle capacity is difficult in this problem since the capacity availability fluctuates on the tour. The solution approach developed in Nagy and Salhi (2005) for simultaneous pickup-delivery problem was applied to this problem as well. Ropke and Pisinger (2004) transformed all backhaul problems into a given generic form and propose a unified heuristic based on insertion and removal moves, and Large Neighborhood Search with probabilistic move acceptance scheme.

A special case of the simultaneous pickup-delivery problem is the problem where customers are either delivery (linehaul) or collection (backhaul) nodes and linehaul customers have to be first in a tour (Deif and Bodin 1984, Yano et al. 1987, Goetschalckx and Jacobs-Blecha 1989, Toth and Vigo 1997, Osman and Wassan 2002). Proposed solution approaches include saving methods, set covering, VRP plus insertion, clustering and routing, and tabu search. A survey of the various models and techniques utilized on this problem can be found in Savelsbergh and Sol (1995). More recently, Lu and Dessouky (2004) proposed a branch and cut based algorithm for the multiple vehicle version of this problem. Bent and Henteryck (2006) proposed a simulated annealing approach for assigning customers to vehicles first and then construct feasible tours by Large Neighbourhood Search with the goals of minimizing number of routes and total travel cost.

Finally, dial-a-ride problems involve taking up on-line requests for picking up and delivering customers at their desired locations by maintaining capacity feasibility of a vehicle that is en route on cyclic trips. A survey on such dynamic routing problems
are found in Gendreau and Potvin (1998) and there are a large variety of solution techniques proposed, including insertion heuristics (Madsen et al. 1995, Diana and Dessouky 2004), local search (Healy and Moll 1995), clustering (Ioachim et al. 1995), simulated annealing (Hart 1996), branch and price (Savelsbergh and Sol 1998), and tabu search (Gendreau et al. 1999b, Cordeau and Laporte 2003). Besides these VRP variants, a dynamic VRP may include time-dependent travel times (Malandrak and Daskin, 1992). This formulation assumes that each link has a fixed travel time and includes an index that identifies the time interval the vehicle enters the link. The last line of research on dynamic VRP comes from a stochastic definition (Psaraftis, 1988; Larsen, 2000), which is outside of the scope of this study.

Similar to the standard assumption of unlimited supply quantities, the usual assumptions made in the problems discussed above are that vehicle capacity is sufficient to meet individual customer demand/supply quantities, and that vehicle availability is abundant. These assumptions are difficult to satisfy in emergencies where immediate response is required from as many as vehicles as possible. Hence, split delivery and the limitation on the number of vehicles and supplied quantities are valid in the ELP.

In split delivery a customer may be visited more than once if demand exceeds the load capacity of available vehicles. VRP with split delivery was first introduced by Dror and Trudeau (1989), where they showed that split deliveries could result in significant savings in terms of total distance and the number of vehicles. Dror et al. (1994) described an integer programming formulation of the problem and developed an exact constraint relaxation branch and bound algorithm for the VRPSD. Frizzell and Giffin (1995) extended the split delivery routing problem to the situation with time windows,
and three heuristics were implemented considering multiple time windows and grid network distances. Mullaseril et al. (1997) studied several heuristics for the splitdelivery capacitated rural postman problem with time windows. Belenguer et al. (2000) studied the polyhedron and develop a lower bound for the problem from a new class of valid inequalities. Recently, Ho and Haugland (2004) proposed a tabu search based heuristic for the problem with time windows where the split delivery options are not imposed but decided by a pool of solutions maintained in the solution process.

### 2.1.2 Heuristic Solutions for the VRP

Due to the importance of combinatorial optimization (CO) problems for the scientific as well as the industrial world, many algorithms have been developed to tackle them. These algorithms can be classified as either exact or approximate algorithms. Exact algorithms are guaranteed to find an optimal solution for every finite size instance of a CO problem in bounded time. Yet, for CO problems that are NP-hard, no such polynomial time algorithm exists. The VRP has been shown by Lenstra and Kan (1981) to be NP-hard, moreover, the largest VRP instances that can be consistently solved by the most effective exact algorithms proposed so far contain about 50 nodes, whereas larger instances may be solved to optimality only in particular cases (Toth and Vigo, 2002). Due to the limited success of exact methods, the use of approximate methods to solve VRP has received more and more attention in the last 30 years.

Several families of approximate methods have been proposed for the VRP. They can be classified into two classes: basic approximate methods and metaheuristics. Among the basic approximate methods the researchers usually distinguish between constructive methods and local search methods. Constructive algorithms are typically the fastest approximate methods. They generate solutions from scratch by adding
components to an initially empty partial solution until a solution is complete. Local search algorithms perform a relatively limited exploration of the search space. They start from some initial solution and iteratively try to find a better solution in an appropriately defined neighborhood of the current solution. Generally, these methods can produce better quality solutions within modest computing times. Moreover, as they are extendable to constraints encountered in real-life applications, they are still widely used in commercial packages. Extensive surveys on these classical heuristics are given in Golden and Assad (1988), Fisher (1995) and Toth and Vigo (2002).

In the last twenty years, a new kind of approximate algorithm has emerged, which tries to combine basic heuristic methods in higher level frameworks aiming at efficiently and effectively exploring a search space. These methods are commonly called meta-heuristics (Glover, 1986) and mainly include Ant Colony Optimization (ACO), Genetic Algorithms (GA), Simulated Annealing (SA), and Tabu Search (TS). Blum and Roli (2003) gave a survey of these meta-heuristics from a conceptual point of view, with emphasis on the analysis of their similarities and differences. SA and TS can be seen as intelligent extensions of local search algorithms. The idea of this kind of meta-heuristic is to escape from local minima and proceed in the exploration of the search space to find other hopefully better local minima. A different philosophy exists in population-based algorithms like ACO and GA. They incorporate a learning component in the sense that they implicitly or explicitly learn correlations between decision variables to identify high quality areas in the search space. Hence, this kind of meta-heuristic essentially performs a biased sampling of the search space.

In meta-heuristics, the emphasis is on performing a deep exploration of the most promising regions of the solution space. The quality of solutions is much higher than
those obtained by classical heuristics, but the price is increased computing time (Renaud et al., 1996). Moreover, the procedures usually are context dependent and require finely tuned parameters, which may make their extension to other situations difficult. Taillard et al. (2001) reviewed many meta-heuristic applications for different CO problems and proposed a unified view of adaptive memory programming to present these meta-heuristics due to some of their similarities on the way of exploiting memory during the search process.

Among the meta-heuristics that have been applied to the VRP, TS now emerges as the most effective one. Some early TS applications did not yield impressive results, but subsequent implementations were much more successful. These include Osman (1993), Taillard (1993), Gendreau (1994) and Rego (1996), among which Taillard (1993) has obtained the best known results to benchmark instances. In addition, Taillard (1993) introduced a decomposition method for the main problem, which is well suited for parallel implementation. Osman (1993) showed that applying SA does not yield competitive results against the best TS implementations. The GA applications on the basic VRP appeared in Baker and Ayechew (2003) and Prins (2004). The results show that GA is competitive with TS and SA in terms of solution time and quality. Furthermore, the latter implementation is the best algorithm for the large-scale instances generated by Golden et al. (1998). ACO algorithms have also produced quite encouraging results, which will be discussed as follows.

ACO was introduced by M. Dorigo as a novel nature-inspired meta-heuristic and first used on the traveling salesman problem (Dorigo, 1996). Then the rank-based version of the ant system was applied to VRP by Bullnheimer et al. (1999) with good results. These authors used various standard heuristics to improve the quality of VRP
solutions and modified the construction of the tabu list considering constraints on the maximum total tour length of a vehicle and on its capacity. A more recent ACO application on the VRP is that of Bell and McMullen (2004). ACO was also applied to a VRP version more close to actual logistic practice, VRP with time windows (VRPTW). Gambardella et al. (1999) proposed a multiple ant colony system for this problem. First introduced for multi-objective function minimization problem, the method coordinates the activity of different ant colonies, each of them optimizing a different objective. These colonies work by using independent pheromone trails but they collaborate by exchanging information. The approach has been experimentally proved to be more effective than the best known algorithms in the fields such as the tabu search of Rochat and Taillard (1995), the large neighbourhood search of Shaw (1998) and the genetic algorithm of Potvin and Bengio (1996).

ACO was also successfully applied to many other problems, such as the quadratic assignment problem (Gambardella et al., 1999; Maniezzo, 1999), scheduling problem (Merkle, 2000), and so on. More recently, different authors (Kaji, 2001; Tsai, 2002) have tackled the TSP with hybrid variants, mainly with tabu search, but in the case of large TSP instances, also with genetic evolution and nearest neighbor search, in order to improve both efficiency and efficacy. Detailed description of ACO's theoretical results and applications review can be found in recent papers by Dorigo and Blum (2005), and Dorigo and Stützle (2002).

According to the analysis in the previous chapter, one may see that the search space in the ELP is more relaxed than commercial routing models and the number of alternatives in a local neighborhood increases significantly. Thus, local search based methods may not be applied efficiently in this problem. Instead, the population-based
meta-heuristics seem more promising due to their highly effective exploration scheme of large search spaces. Moreover, as an extension of traditional construction heuristics, ACO solution framework is readily available for both the diversification on vehicle paths building and the efficiency on commodity dispatch to deal with the complexities of the ELP.

### 2.2 Multi-commodity Flow Problem

Network flow problems have been in the focus of interest for many years and they represent a very successful area of combinatorial optimization. According to Kennington and Helgason (1980), specialized network simplex algorithms can solve minimum cost linear programming problems with pure network structure from 50 to over 100 times faster than the general linear programming algorithm. Due to the good solvability of network model, it has been accepted by researchers that methods based on the exploitation of the embedded network structure can solve problems faster than otherwise possible with the standard linear programming algorithm, when a very high proportion of the rows of the problem form a pure network (Aderohunmu and Aroson, 1993). A comprehensive survey of classic network flow problems and solution methods can be found in Ahuja et al. (1993).

Multi-commodity networks arise in practice when more than one type of commodity must share arc capacities in a network. In some applications, the flow variable in the model can be fractional; in other contexts, however, the variables must be integers. The latter instances form integer multi-commodity network flow problems that remain a challenge and active area of research. Since the linear programming model might either be a good approximation of the integer programming model, or the linear programming model can be commonly used as a relaxation of the integer
programming and embedded within branch-and-bound or some other approaches, the literature on linear multi-commodity flow problems is discussed first, the integer counterparts next.

Several categories of methods have been considered for the multi-commodity flow problem. Direct approaches solve the problem by exploiting the special block-network structure of the constraint matrix. The solver can be either simplex-based or use interior point methodology; the other popular approach is based on decomposition, i.e., Lagrangian relaxation approach, and column generation scheme. Other solution methods proposed for the model include primal-dual heuristics (Barnhart and Sheffi, 1993), and approximation algorithms (Bienstock, 1999; Goldberg et al., 1998), and so on. In addition, due to the progress made in the simplex implementations during recent years, the solvability of the linear multi-commodity flow problem has been largely improved. A commercial package such as CPLEX 6.5 can solve the instance up to nearly 1 million of variables and 60,000 constraints in less than half an hour on a midrange workstation (Castro, 2003).

Simplex-based methods rely on primal partitioning techniques that exploit the special structure of the basis (Ahuja et al. 1993). Two codes of this type have been developed during the last years: EMNET (McBride, 1998; Mamer and McBride, 2000) and PPRN (Castro and Nabona, 1996). The efficiency of pricing strategy in simplex-based method was also demonstrated in Mamer and McBride (2000), where a new decomposition based pricing procedure results in enhanced performance on both the message routing problem and the PDS problems. According to Castro (2003), the computational results presented in McBride (1998) in the solution of the PDS (Patient Distribution System) problems with EMNET have similar performances with those
obtained with CPLEX (using the network optimizer followed by dual simplex, which is known as the most efficient combination for most multi-commodity problems), which means the highly efficient general linear programming solver might be a good replacement for current primal partitioning multi-commodity codes.

Recent research into interior point methods produced substantial advances for solving large multi-commodity flow problems directly. Application of interior point methods to multi-commodity flow problems was studied by Kamath et al. (1993). Network specializations of interior point methods were also presented by Resende and Veiga (1993), Resende and Pardalos (1996). The more recent specialized interior-point multi-commodity algorithm of Castro (2000) has shown to be the most efficient interior-point approach (Castro, 2003); however, the comparison between interiorpoint algorithm and simplex-based solvers shows that one can outperform each other for only some particular problems.

The last line of research on multi-commodity flow problem comes from the decomposition approaches, on both linear and integer problems. Early implementation of Dantzig-Wolfe decomposition scheme is not efficient (Ahuja et al., 1993), however, good computational results on some problem sets were observed recently by Chardaire and Lisser (2002) and Larsson and Yuan (2004). Lagrangian decomposition methods are more widely applied in literature, which place Lagrangian multipliers (or prices) on the bundle constraints and bring them into the objective function so that the resulting problem decomposes into a separate minimum cost flow problem for each commodity. A traditional Lagrangian heuristic method has been applied to the linear multi-commodity flow problem in Holmberg (1996), while the performance does not outperform the general purpose solver such as CPLEX. Frangioni and Gallo (1999)
proposed a cost decomposition approach to the problem based on dualizing the mutual capacity constraints and solving the resulting Lagrangian dual with a dual-ascent algorithm. Their algorithm provides excellent results for the Mnetgen instances, especially on problems where the number of commodities is relatively large with respect to the size of the graph. In a later indirect comparison of the results of Frangioni and Gallo (1999) with those of Castro (2003), CPLEX (version 6.5) seems to provide similar performances to that of the bundle-method-based algorithm. However, according to the computational work in Larsson and Yuan (2004), the two specialized price-directive decomposition methods outperformed the general-purpose solver CPLEX (version 5.0) and the specialized partitioning code PPRN on several problem sets. Thus, the comparison between the simplex-based solvers and decomposition methods depends on the implementation and the problem sets. More recently, Babonneau et al. (2006) applied the analytic center cutting-plane method to solve the Lagrangian dual problem, where an active set strategy was applied and resulted in acceleration on the large problem instances in comparison with the augmented Lagrangian algorithm proposed in Larsson and Yuan (2004).

The ability to solve large linear multi-commodity flow problems allows us to consider the solution of integer models that have been adopted in a wide variety of important large-scale applications, such as fleet management problems (Cheung and Powell, 1996; Powell and Carvalho, 1997), network design problems (Lamar et al., 1990; Holmberge and Yuan, 2000), capacity expansion (Chang and Gavish, 1995), and so on. The special structure of these problems makes decomposition an attractive solution method. Generally speaking, it was often combined with heuristic and embedded in the branch and bound tree. In Holmberg and Hellstrand (1998) an efficient solution method based on a Lagrangian heuristic and branch and bound was
developed for solving the uncapacitated network design problem formulated in integer multi-commodity flow problem. A similar approach was adapted to the capacitated version of the same problem (Holmberge and Yuan, 2000). In Gendron and Crainic (1994) different relaxation schemes were studied and discussed with heuristics for yielding feasible solutions for multi-commodity network design problems. A detailed survey of Lagrangian relaxation was given in Guignard (2003).

Dantzig-Wolfe decomposition and the related column generation is another approach for finding the correct prices that exploits the network structure of the subproblems. This approach has been receiving more and more attention in literature of multicommodity flow and other difficult combinatorial problems (Desrochers et al. 1995; Barnhart et al., 1998; Desaulniers et al., 2005). Jones et al. (1993) investigated the impact of problem formulation on Dantzig-Wolfe decomposition for the multicommodity network flow problem. They showed that the path-based formulations by decomposition outperform the equivalent tree-based formulation. Barnhart et al. (2000) presented a column-generation model and branch-and-price-and-cut algorithm for origin-destination integer multi-commodity flow problems. More recently, Holmberg and Yuan (2003) extended the basic multi-commodity flow model to include side constraints on communication paths to handle the time-delay or reliability requirements on a communication pair in the telecommunication applications, and a column generation method showed efficiency on solving the model.

Besides the readily available simplex-based methods and price-directive decomposition methods discussed above, there is another type of approach- resourcedirective decomposition (Kennington and Shalaby, 1977). Compared with the previous two methods, resource-directive decomposition method begins by allocating
the mutual capacity among the commodities and then use information gained from the solution to the resulting single-commodity problems to reallocate the capacity so as to improve the overall system cost. According to the computational comparisons (Ali et al., 1980) among these three methods, resource-directive decomposition is not quite as efficient as the other two.

### 2.3 Dynamic Network Flows

In addition to the flow problems in static networks discussed in the previous section, there are problems where time must be taken into consideration, for example, the effect that flow values on arcs may change over time, or in some applications flow traveling time through each arc plays an essential role. The latter is the case in the emergency logistics problem, where the objective is to minimize delay of services. The needs for more realistic network models lead to the development of dynamic network flow.

The use of dynamic networks was introduced in 1958 by Ford and Fulkerson to dynamic maximum flow problem. Since then, several problems have been analyzed, such as the quickest flows, dynamic minimum cost flows and so on. Quickest flow problem can be reduced to the maximum flow problem by binary search. Burkard et al. (1993) gave strongly polynomial algorithms for this problem based on Newton's method. The generalization of the quickest flow problem with several sources and sinks, the quickest transshipment problem, was studied by Hoppe and Tardos (2000) and the first polynomial algorithm for this problem was proposed. Previous surveys on general dynamic flow problems include those by Aronson (1989), Powell et al. (1995). Aronson (1989) covered extensive dynamic applications and mainly concentrated on the maximum flow and transshipment problems in discrete time.

Powell et al. (1995) focused on dynamic modeling issues, which deals with discrete time settings as well as problems where the parameters are stochastic. More recently, Kotnyek (2003) gave an annotated overview of dynamic flow problems and solution techniques.

### 2.3.1 Discrete vs. Continuous Modeling

The research on dynamic network flow has two main directions with respect to the time modeling, namely continuous and discrete time. The continuous approach models time continuously and the other models time discretely. Research using the first approach has considered networks with time-varying capacities and costs, and has focused on proving the existence of optimal solutions while further generalizing the model (Fleischer and Tardos, 1998; Hall et.al., 2003). In general, for continuoustime problems one can often find only theoretical results, whereas there are more practical solutions for discrete time model (Kotnyek, 2003). In fact, the usual solution approach for a continuous time network problem is to reduce it to discrete time. Research of the discrete type typically uses the time-expanded network (Ford and Fulkerson, 1962), either explicitly in the algorithms, or implicitly in the proofs, to produce theoretically or practically efficient algorithms. As a consequence, optimal dynamic flows can be obtained by applying static network flow optimization techniques to a time-expanded network. Hence, for the sake of existing static network flow optimization techniques, this research focuses on the discrete time model for the dynamic emergency logistical planning.

### 2.3.2 Evacuation Models

Dynamic network models have proved to be an effective modeling framework for a range of planning problems that arise in logistics. The quickest flow problem with
multiple sources and single sink is commonly used to model evacuation problems. Evacuation planning is critical for numerous important applications, such as emergency operations management and homeland defense preparation. Over the last two decades there has been considerable interest in modeling evacuation of buildings. The methods of evacuation planning can be divided into two categories, namely network flow model approach and traffic assignment-simulation approach. Network flow models fall into the category of macroscopic models, which do not consider individual differences. The evacuees are treated as groups where only common characteristics are taken into account. The earliest research on building evacuation using dynamic model was done by Chalmet et al. (1982). In the same issue, Jarvis and Ratliff (1982) proved several solution properties of this maximal dynamic network flow problem for evacuation. Hamacher and Tufekci (1987) developed additional properties of flows for evacuation process. Choi et al. (1988) extended model in Chalmet et al. (1982) by considering flow dependent capacities on arcs and presented algorithms to handle the problem with side constraints. Hoppe and Tardos (1994) gave a polynomial time algorithm for the evacuation problem with a fixed number of sources and sinks. Lovas (1998) discussed the importance of different network components, as well as population characteristics affecting evacuation performance. Simulation approach, in which the individual parameters and the interaction among evacuees may be taken into consideration, is out of the scope of this study, the interested may refer to the review given in Church and Sexton (2002).

### 2.4 Summary

A brief summary of the literature reviewed in this chapter is given here. This chapter reviews the studies in the most closely related areas, where one can see there is no
readily available model dealing with all the issues in the ELP settings. The conventional static VRP formulation has evolved itself to address various issues in practical applications; however, these variations do not seem to be applicable in a real-world ELP context, since they are static by nature and cannot address all the dynamic issues arising in emergency logistics. Dynamic VRP and network flow based models are promising for dealing with the dynamism under consideration while their historical applications are restricted only on relatively simple problem settings. Hence, the formulation development shall be conducted with careful evaluation and comparison, with particular emphasis on computational efficiency. On the other hand, heuristic methods usually dominate the exact solution methods for the VRPs as well as for many other NP-hard combinatorial optimization problems, because using exact methodologies for large scale applications in a dynamic environment would lead to very high computation times. The latter poses a new challenge in designing heuristics for complex dynamic problems.

## 3 Evaluation and Comparison of ELP Formulations

Most integer programming problems may be formulated in several ways. But in contrast to linear programming, a good formulation is of crucial importance for solving the integer programming model. A model is specified by the variables, objective function, and constraints. Typically, the variables are chosen from the definition of a solution, and once the variables and an objective function have been defined, one can speak of an implicit representation of the problem. In general, when there is a valid formulation, there are many choices of constraints, but an obvious choice may not be a good one when it comes to solving the problem. In this chapter, two formulations are presented for the ELP and compared in terms of solvability.

### 3.1 Vehicle Routing Based Dynamic Formulation

As described before, the ELP is closely related to the VRPs. However it represents quite a different setting from the VRP in the following respects. In this problem, unlike the VRP where supply is assumed to be abundant, supply is available in limited quantities and its availability varies over the planning horizon. Predictions for future demand of certain commodities are also known and a multi-period planning horizon prevails. The objective is also different. The goal is to minimize the delay in the arrival of commodities from aid centers. In other words, requirements of aid distribution centers should be met at the requested times. Hence, it is necessary to define a time-dependent logistics plan and dispatch available vehicles dispersed
throughout the logistics network so as to optimize the timing and quantity of commodities transported to demand nodes. Due to these reasons, the classical VRP formulations cannot be adapted to the logistical problem.

Another feature in disaster settings is that vehicles may execute mixed delivery trips where commodities are picked up and delivered in an arbitrary sequence that maximizes service level. There are no restrictions on the number of "customers" visited nor do the vehicles belong to given facilities where they have to return. A split delivery system is utilized since supplies and demands are far beyond individual vehicle capacities.

Here a new kind of dynamic formulation with a time index T is proposed. It can be classified as a partial delivery multi tour VRP with dynamic demand and limited supply and an objective of minimizing total delay in deliveries. The notation and the mathematical formulation of the problem are given below. For simplicity of the comparison, the formulations in this chapter do not include the evacuation problem.

Sets and Parameters:
T: Set of time periods in the planning horizon, $t$ (or $q$ ) denotes a specific time period in T

A: Set of commodities; a denotes a specific commodity type
L: Set of vehicle labels; $l$ denotes a specific vehicle
C: Set of all nodes in the network; o(or $p, i, j$ ) denotes a specific node
CD: Set of demand nodes, $\mathrm{CD} \subset \mathrm{C}$
CS : Set of supply nodes and vehicle depots, $\mathrm{CS} \subseteq \mathrm{C} \backslash \mathrm{CD}$
$\mathrm{N}_{l}: \quad$ Set of arcs for vehicle $l$
$\mathrm{t}_{\text {opl }}$ : $\quad$ Time required to traverse $\operatorname{arc}(o, p)$ for vehicle $l$
$\mathrm{av}_{\text {olt }}$ : Binary number indicating if vehicle $l$ is added to the fleet at node $o$ at time $t$
$\mathrm{d}_{\text {aot }}$ : Amount of demanded commodity type $a$ at node $o \in \mathrm{CD}$ at time $t ; \mathrm{d}_{\text {aot }}=0$ for $o \in \mathrm{C} \backslash \mathrm{CD}$
sup $_{\text {aot }}$ : Amount supplied of commodity type $a$ at node $o \in \mathrm{CS}$ at time $t ; \sup _{\mathrm{aot}}=0$ for $o \in \mathrm{C} \backslash \mathrm{CS}$
$\mathrm{w}_{\mathrm{a}}$ : Unit weight of commodity $a$
cap 1 : Load capacity of vehicle $l$
K: A big number
$\mathrm{P}_{\mathrm{a}}$ : $\quad$ Priority of satisfying demand of commodity type $a$

Decision Variables:
$Z_{\text {aplt }}$ : Amount of commodity type $a$ delivered to node $p$ by vehicle $l$ at time $t$
$\mathrm{P}_{\text {aplt: }} \quad$ Amount of commodity type $a$ picked up at node $p$ by vehicle $l$ at time $t$
$\mathrm{L}_{\text {alt }}$ : Amount of commodity type $a$ carried by vehicle $l$ at time $t$
$\operatorname{dev}_{\text {aot }}$ : Amount of unsatisfied demand of commodity type $a$ at node $o$ at time $t$
$\mathrm{Y}_{\text {oplt: }} \quad$ Binary variable indicating if vehicle $l$ is traversing the $\operatorname{arc}(o, p)$ at time $t$.

## Model E (I):

Minimize $\quad \Sigma_{\mathrm{a} \in \mathrm{A}} \Sigma_{\mathrm{o} \in \mathrm{CD}} \Sigma_{\mathrm{t}}\left(\mathrm{P}_{\mathrm{a}} \operatorname{dev}_{\text {aot }}\right)$

Subject to
$\Sigma_{\mathrm{p} \in \mathrm{C}} \mathrm{Y}_{\text {oplt }} \leq 1 \quad(\forall \mathrm{o} \in \mathrm{C}, \mathrm{t} \in \mathrm{T}, \mathrm{l} \in \mathrm{L})$
$\Sigma_{\mathrm{p} \in \mathrm{C}} \mathrm{Y}_{\text {polt }} \leq 1 \quad(\forall \mathrm{o} \in \mathrm{C}, \mathrm{t} \in \mathrm{T}, \mathrm{l} \in \mathrm{L})$
$\Sigma^{\mathrm{t}+1-\mathrm{t}} \mathrm{pol} \sum_{\mathrm{q}=1} \Sigma_{\mathrm{p} \in \mathrm{C}} \mathrm{Y}_{\mathrm{polq}}+\Sigma_{\mathrm{q}=1}^{\mathrm{t}} \mathrm{av}_{\mathrm{olq}} \geq \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{\mathrm{p} \in \mathrm{C}} \mathrm{Y}_{\mathrm{oplq}} \quad(\forall \mathrm{o} \in \mathrm{C}, \mathrm{t} \in \mathrm{T}, \mathrm{l} \in \mathrm{L})$

$$
\begin{align*}
& \Sigma_{\mathrm{a} \in \mathrm{~A}} \mathrm{P}_{\text {aolt }} \leq \Sigma_{\mathrm{p} \in \mathrm{C}} \mathrm{Y}_{\text {oplt }} * \operatorname{cap}_{\mathrm{l}}(\forall \mathrm{o} \in \mathrm{C}, \mathrm{t} \in \mathrm{~T}, \mathrm{l} \in \mathrm{~L})  \tag{3-5}\\
& \mathrm{L}_{\text {alt }}=\Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{\mathrm{o} \in \mathrm{C}} \mathrm{P}_{\text {aolq }}-\Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{\mathrm{p} \in \mathrm{C}^{2}} \mathrm{Z}_{\text {aplq }} \quad(\forall \mathrm{a} \in \mathrm{~A}, \mathrm{t} \in \mathrm{~T}, 1 \in \mathrm{~L})  \tag{3-6}\\
& \Sigma_{\mathrm{a}} \in \mathrm{~A}^{\mathrm{W}_{\mathrm{a}}}{ }^{*} \mathrm{~L}_{\text {alt }} \leq \operatorname{cap}_{\mathrm{i}} \quad(\forall \mathrm{t} \in \mathrm{~T}, \mathrm{l} \in \mathrm{~L})  \tag{3-7}\\
& \mathrm{Z}_{\text {aolt }} \leq \Sigma_{\mathrm{pl}}^{\mathrm{t}+\mathrm{l} \mathrm{t}} \mathrm{q}_{\mathrm{F}=1} \sum_{\mathrm{p} \in \mathrm{C}} \mathrm{Y}_{\mathrm{polq}} * \mathrm{~K}(\forall \mathrm{t} \in \mathrm{~T}, \mathrm{a} \in \mathrm{~A}, \mathrm{o} \in \mathrm{C}, l \in \mathrm{~L})  \tag{3-8}\\
& \operatorname{dev}_{\text {aot }}=\Sigma_{q-1}^{\mathrm{t}} \mathrm{~d}_{\text {aoq }}-\Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{1 \in \mathrm{~L}} \mathrm{Z}_{\text {aolq }} \quad(\forall \mathrm{a} \in \mathrm{~A}, \mathrm{t} \in \mathrm{~T}, \mathrm{o} \in \mathrm{CD})  \tag{3-9}\\
& \operatorname{dev}_{\mathrm{aot}}=\sum_{\mathrm{q}=1}^{\mathrm{t}} \sup _{\mathrm{aoq}}-\Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{1 \in \mathrm{~L}} \mathrm{P}_{\text {aolq }} \quad(\forall \mathrm{a} \in \mathrm{~A}, \mathrm{t} \in \mathrm{~T}, \mathrm{o} \in \mathrm{CS})  \tag{3-10}\\
& \mathrm{Z}_{\text {aolt }}, \operatorname{dev}_{\text {aot }}, \mathrm{L}_{\text {alt }}, \mathrm{P}_{\text {aolt }} \geq 0 \quad(\forall \mathrm{a} \in \mathrm{~A}, \mathrm{t} \in \mathrm{~T}, \mathrm{o} \in \mathrm{C}, 1 \in \mathrm{~L}) \tag{3-11}
\end{align*}
$$

The problem modeled above is a multi-period planning problem where demands in the future time periods are indicated by the parameter $\mathrm{d}_{\text {aot }}$. In emergency situations knowledge on future demand is scarce except for some commodities, but the disaster coordination center frequently acknowledges supply that will be available in future time periods. So, it is possible to plan ahead and take future supply into account while preparing the plans.

The objective aims at minimizing the weighted sum of unsatisfied demand over all commodities. This objective is compatible with the goal mentioned previously and commodities are transported, upon request, as soon as possible to demand centers, according to their priorities. Constraint sets (3-2) to (3-4) are vehicle flow balance constraints on the nodes in each time period. Unlike their counterparts in classical VRP formulations, the vehicles can stay and wait at any node in its route. Constraint sets (3-5) to (3-8) guarantee that the vehicle's load and delivery at each node in each
time period is consistent with the capacity. Finally, constraint sets (3-9) and (3-10) balance material flow on demand nodes and transshipment nodes and explicitly report the quantity of unsatisfied demand, $\operatorname{dev}_{\text {aot }}$, in each time period.

With the time index included in the formulation, the precedence of the nodes visited in a route is naturally determined, and there remains no need for sequencing constraints to construct tours. Due to the latter structure and the pick up/delivery restrictions, a vehicle goes to any depot automatically to pick up load as the need arises and, subtour elimination constraints (an important difficulty in the classical VRP formulation) are disposed of. However, due to the requirements of both vehicles and time indices, the dimension of the problem may quickly get out of hand as the length of the planning horizon is increased or more vehicles become added to the fleet.

In the following sections a different modeling strategy based on the article Özdamar et al. (2004) is presented, which eliminates the need for vehicle indices by taking the multi-commodity flow perspective where some of the commodities (vehicle types) are integral valued. This formulation represents vehicles by general integer variables rather than binary variables and obtains an aggregate solution in terms of vehicle logistics. Detailed vehicle instructions are obtained using a simple vehicle splitting algorithm that converts integer vehicle flows into binary vehicle itineraries and then by solving a set of linear equations to assign a loading/unloading schedule to each such itinerary. Only the transportation of commodities is taken into consideration so that a one-to-one model comparison is made in coherence with the vehicle routing based formulation $\mathrm{E}(\mathrm{I})$, while the extended model including wounded people evacuation and medical service will be given in the next chapter.

### 3.2 Formulation Based on Multi-commodity Flow Problem

The multi-commodity flow problem arises in a wide variety of important applications, and due to the better solvability of network flow problems, many practical applications are often modeled in this context. To address the high dimensionality found in the ELP and to treat transportation of bulk demand and supply, the emergency logistical problem is formulated here as a dynamic, multi-commodity, mixed integer network flow problem with side constraints. The mathematical formulation $\mathrm{E}(\mathrm{II})$ and additional notations are given below.

Additional Sets and Parameters:
M: Set of vehicle types; $m$ (or $v$ ) denotes a specific type
$\mathrm{t}_{\mathrm{opm}}$ : Time required to traverse arc $(o, p)$ for vehicle type $m$
$\mathrm{av}_{\text {omt }}$ : Number of vehicles of type $m$ at node $o$ added to the fleet at time $t$
cap $_{\mathrm{m}}$ : Load capacity of vehicle type $m$
$\mathrm{K}_{\text {oqptm }}$ : Binary parameter matrix indicating if node $p$ is reachable at time $t$ from node $o$ at time $q$ using vehicle type $m$ : if $\mathrm{t}-\mathrm{q}<\mathrm{t}_{\mathrm{opm}}$, then $\mathrm{K}_{\text {oqptm }}=0$, else $\mathrm{K}_{\text {oqptm }}=1$.

Additional Decision Variables:
$Z_{\text {aopmt: }}$ Amount of commodity type $a$ traversing arc $(o, p)$ at time $t$ by vehicle type $m$ $\mathrm{Y}_{\mathrm{opmt}}:$ Integer number of vehicles of type $m$ traversing the $\operatorname{arc}(o, p)$ at time $t$.

## Model E (II):

Minimize $\quad \Sigma_{\mathrm{a} \in \mathrm{A}} \Sigma_{\mathrm{o} \in \mathrm{CD}} \Sigma_{\mathrm{t}}\left(\mathrm{Pa}_{\mathrm{a}} \operatorname{dev}_{\mathrm{aot}}\right)$

Subject to

$$
\begin{align*}
& \sum_{\mathrm{q}=1}^{\mathrm{t}} \mathrm{~d}_{\mathrm{aoq}}-\sum_{\mathrm{m} \in \mathrm{M}} \sum_{\mathrm{q}=1}^{\mathrm{t}} \sum_{\mathrm{p} \in \mathrm{C}}\left[\mathrm{~K}_{\mathrm{pqotm}} \mathrm{Z}_{\text {aporqq}}-\mathrm{Z}_{\text {aopmq }}\right]=\operatorname{dev}_{\text {aot }}(\forall \mathrm{a} \in \mathrm{~A}, \mathrm{o} \in \mathrm{CD}, \mathrm{t} \in \mathrm{~T})  \tag{3-13}\\
& \Sigma_{\mathrm{m} \in \mathrm{M}} \sum_{\mathrm{q}=1}^{\mathrm{t}} \sum_{\mathrm{p} \in \mathrm{C}}\left[-\mathrm{K}_{\mathrm{pqotm}} \mathrm{Z}_{\mathrm{apomq}}+\mathrm{Z}_{\mathrm{aopmq}}\right] \leq \sum_{\mathrm{q}=1}^{\mathrm{t}} \sup _{\mathrm{aoq}} \quad(\forall \mathrm{a} \in \mathrm{~A}, \mathrm{o} \in \mathrm{C} \backslash \mathrm{CD}, \mathrm{t} \in \mathrm{~T})  \tag{3-14}\\
& \sum_{\mathrm{q}=1}^{\mathrm{t}} \sum_{\mathrm{p} \in \mathrm{C}}\left[\mathrm{Y}_{\mathrm{opmq}}-\mathrm{K}_{\mathrm{pqotm}} \mathrm{Y}_{\mathrm{pomq}}\right] \leq \sum_{\mathrm{q}=1}^{\mathrm{t}} \mathrm{av}_{\mathrm{omq}} \quad(\forall \mathrm{o} \in \mathrm{C}, \mathrm{~m} \in \mathrm{M}, \mathrm{t} \in \mathrm{~T})  \tag{3-15}\\
& \mathrm{Y}_{\mathrm{opmt}} * \operatorname{cap}_{\mathrm{m}} \geq \Sigma_{\mathrm{a} \in \mathrm{~A}} \mathrm{w}_{\mathrm{a}} * \mathrm{Z}_{\mathrm{aopmt}} \quad(\forall \mathrm{o}, \mathrm{p} \in \mathrm{C}, \mathrm{t} \in \mathrm{~T}, \mathrm{~m} \in \mathrm{M})  \tag{3-16}\\
& \mathrm{Y}_{\mathrm{opmt}} \leq \mathrm{K} * \sum_{\mathrm{q}=\mathrm{t}}^{T \mid} \mathrm{K}_{\text {otpqm }} \quad(\forall \mathrm{o} \in \mathrm{C}, \mathrm{p} \in \mathrm{C}, \mathrm{~m} \in \mathrm{M}, \mathrm{t} \in \mathrm{~T})  \tag{3-17}\\
& Y_{\text {opmt }} \geq 0, Z_{\text {aopmt }} \geq 0, \operatorname{dev}_{\text {aot }} \geq 0 \quad(\forall \mathrm{a} \in \mathrm{~A}, \mathrm{o} \in \mathrm{C}, \mathrm{p} \in \mathrm{C}, \mathrm{~m} \in \mathrm{M}, \mathrm{t} \in \mathrm{~T}) \tag{3-18}
\end{align*}
$$

Both in $\mathrm{E}(\mathrm{I})$ and in $\mathrm{E}(\mathrm{II})$ a heterogeneous fleet and multiple transportation modes can be modeled. In $\mathrm{E}(\mathrm{I})$, each vehicle has its own capacity and arc distance that enables a superimposed transportation network integrating different modes, such as air, land, etc. In E(II) "vehicle types" are defined rather than vehicle labels implying vehicles of different capacities and/or transportation modes. Thereby, an integer number of vehicles of the same type are aggregated in E (II) and treated as an integer valued commodity, removing the requirement of tracking vehicles individually on a route basis. Tracking aggregate vehicle flows on a time basis also facilitates multi-period representation of demand and supply. Furthermore, the embodied network structure enhances problem solvability when compared with model $E$ (I). One may note that both formulations are built on the time-expanded network whose size is larger than the original network; however, it has been made tractable by the fact that an arc is effective in the model only when it is compatible with any vehicle flow.

The objective in E (II) is the same as in $\mathrm{E}(\mathrm{I})$. Constraint set (3-13) balances material flow on the demand nodes and explicitly reports the quantity of unsatisfied demand,
dev $_{\text {aot }}$, in each time period. Especially, at supply nodes, constraints (3-14) enable the material flow and keep available supply non-negative.

The set of constraints (3-15) balances the flow of vehicles over nodes. This set of constraints enables vehicles to wait at their last stop rather than returning immediately to supply nodes once the delivery is completed. The constraint restricts the number of vehicles moving through the network by their cumulative availability over time, thereby making it possible to plan ahead with the known current and future vehicle availability. Constraint set (3-16) enables commodity flow over arcs as long as there is sufficient vehicle flow coupling other commodities. Constraint set (3-17) restricts arc traversal to corresponding networks of given vehicle types.

### 3.3 A Two-Stage Algorithm for Disaggregating Solution Flow

In E (II) formulation, vehicles are treated as commodities, and it is not required to track vehicles individually on a route basis, which results in a higher efficiency especially as the number of vehicles utilized in disasters tends to be quite large. However, the aggregated flow solution cannot be directly used in emergency logistics planning. To obtain a dispatch plan for vehicles at operational level, a two-stage algorithm is proposed to generate vehicle routes and load/unload instructions. In the first stage the procedure reads the optimal solution and generates routing schedule for each vehicle, and then loaded/unloaded quantities of commodities are calculated by solving a system of linear equations established in the second stage.

### 3.3.1 Stage 1: Algorithm for Generating Vehicle Routes

Once Model E(II) is solved and the optimal values $\mathrm{Y}^{*}{ }_{\text {opmt }}$ are determined, an algorithm called Route is implemented to determine the route of each vehicle in the
system. According to the multi-depot, split-delivery multiple tour VRP model, a tour is defined as consecutive arcs traversed by the vehicle between two pick-up actions from a supply node. A complete route of a vehicle is composed of all its consecutive tours. However, a route here is assumed to consist of a single tour that starts from any supply node at the beginning of the panning horizon and ends at its completion. This generalized definition is much more efficient.

The solution $\mathrm{Y}^{*}{ }_{\text {opmt }}$ is defined as the non-empty set of vehicles traversing arc $(o, p, m)$ at time $t$. These values are read from a file sorted in ascending order of $t$. Route picks a starting node $o$ with the minimum time index $t$ and identifies the set $\mathrm{V}_{\text {omt }}$, the union of all non-empty departing subsets, where $\mathrm{V}_{\mathrm{omt}}=\cup_{\mathrm{p}} \mathrm{Y}^{*}{ }_{\text {opmt. }}$. Given the $o, m, t$, a nonempty subset $\mathrm{Y}^{*}$ opmt corresponding to an arbitrary $p$ is taken and decomposed into a set of singular unlabelled vehicles $v^{1}$. Here $l$ is an index for counting vehicles in type $m$ whose routes have been traced completely. The maximum value that index $l$ can take is equal to the total number of vehicles utilized in the optimal solution, $\kappa(\kappa=$ $\Sigma^{\mathrm{T}}{ }_{\mathrm{q}=1} \Sigma_{\mathrm{m}} \Sigma_{\mathrm{o}, \mathrm{p} \in \mathrm{C}}\left|\mathrm{Y}_{\text {opmq }}^{*}\right|$ ). However, since each route consists of more than one arc, the number of vehicles to be traced is much smaller than $\kappa$. Each $v^{1}$ is traced to the end of its itinerary till the end of the planning horizon, T. The consecutive arcs over which $v^{1}$ travels are recorded on its route, r . The value of $\mathrm{Y}^{*}$ opmt is decreased whenever a new arc $(o, p, m)$ at time t is identified on the itinerary of $v^{1}$. Once the route is completed, another vehicle $v^{1}$, that is an element of $\mathrm{Y}^{*}{ }_{\text {opmt }}$ is selected and its route traced, until all elements of $\mathrm{Y}^{*}{ }_{\text {opmt }}$ are labeled. This procedure is repeated for all other subsets $\mathrm{Y}^{*}{ }_{\text {opmt }} \subseteq \mathrm{V}_{\text {omt }}$ until $\mathrm{V}_{\text {omt }}$ is exhausted and then the next non-empty $\mathrm{Y}^{*}{ }_{\text {opmt }}$ in the sorted list is selected and forms new $\mathrm{V}_{\text {omt }}$. In each iteration, there are fewer
vehicles to trace. Route has a worst case polynomial complexity of $\mathrm{O}\left(\kappa\left|\mathrm{C}^{2}\right| \mathrm{T}\right)$. The pseudocode of the algorithm is given below.

## Additional Notation:

CO: Set of nodes where all departing vehicles have not been traced
$\mathrm{Y}_{\text {opmt: }}^{*}$ Set of vehicles traversing arc $(o, p, m)$ at time $t, \mathrm{Y}^{*}{ }_{\text {opmt }} \subseteq \mathrm{V}_{\text {omt }}$
$\mathrm{V}_{\text {omt }}$ : Set of all vehicles of transportation mode $m$ outgoing from node $o$ at time $t$ :
$\mathrm{V}_{\text {omt }}=\cup_{\mathrm{p} \in \mathrm{C}} \mathrm{Y}_{\text {opmt }}^{*}$
$v^{1}: \quad l^{\text {th }}$ vehicle in $\mathrm{V}_{\text {omt }}, l=1$ to $\kappa$, where $\kappa=\Sigma^{\mathrm{T}} \sum_{\mathrm{q}=1} \Sigma_{\mathrm{m}} \Sigma_{(o, \mathrm{pm}) \in \mathrm{N}_{\mathrm{m}}}\left|\mathrm{Y}_{\text {opmq }}\right| . \kappa$ is an upper bound on $l$.
$r^{1}$ : Set of arcs traversed by $v^{1}$.

Pseudocode for Algorithm Route

```
Read from file: sorted list (in ascending order of t) Y}\mp@subsup{}{\mathrm{ opmt;}}{
Initialize: CO=C; r}=\phi\mathrm{ for l=1 ..к. Set l=0.
do
Construct set }\mp@subsup{\textrm{V}}{\mathrm{ omt }}{}=\mp@subsup{\cup}{\textrm{p}}{}\mp@subsup{\textrm{Y}}{\textrm{opmt}}{*}:o\in\mathbf{CO},\mp@subsup{\textrm{Y}}{\mathrm{ opmt }}{*}\not=\phi\mathrm{ and }t\mathrm{ is minimum on list;
Initialize: count = | V omt |; Head = TempHead =o;
While (count>0) do
    {
        Select any non-empty subset }\mp@subsup{\textrm{Y}}{\mathrm{ opmt }}{*}\subseteq\mp@subsup{\textrm{V}}{\mathrm{ omt }}{}:|\mp@subsup{\textrm{Y}}{\mathrm{ opmt }}{*}|\not=0\mathrm{ ;
        l= l+1;
        Select }\downarrow\in\mp@subsup{Y}{}{*}\mathrm{ opmt and label }\downarrow\mathrm{ ;
        Update: }\mp@subsup{\textrm{r}}{}{1}=\mp@subsup{\textrm{r}}{}{1}+{o,p,m};|\mp@subsup{\textrm{Y}}{\mathrm{ opmt }}{*}|=|\mp@subsup{\textrm{Y}}{\mathrm{ opmt }}{*}|-1;\mathrm{ count = count -1;
        Initialize: Tail = p; tl=t;
        do
            If any non-empty set }\mp@subsup{\textrm{Y}}{}{*}\mp@subsup{}{\mathrm{ Tail,k,m,t1 }}{}\mathrm{ exists for any arc {Tail, k,m}
            {
                        Update: r}\mp@subsup{}{}{1}=\mp@subsup{\textrm{r}}{}{1}+{\mathrm{ Tail, }k,m};|\mp@subsup{\textrm{Y}}{\mathrm{ Tail,k,m,tl}}{*}|=|\mp@subsup{\textrm{Y}}{\mathrm{ Tail,k,m,tl }}{*}|-1
                            Update: Head = Tail; Tail=k; tl= tl + t'Head,Tail,m;
                    }
                    else t1 = t1+1;
                }while(t1\leqT)/*enddo_t1*/
                Head = TempHead;
        }/*endwhile_count*/
        CO=CO-{o};
        } while( (\mp@subsup{V}{\mathrm{ omt }}{}\not=\phi);/*endwhile*/
```


### 3.3.2 Stage 2: Generating Vehicle Load/Unload Instructions

The second stage utilizes the routes of every labeled vehicle to determine the picked up and delivered quantities on each route. A matrix RC is defined whose binary element $\mathrm{R}[l, o, t]$ indicates that a vehicle $v^{1}$ is incident to a node $o \in \mathrm{C}$ at time period $t$. The time period is required because a node can be traversed more than once by vehicle $v^{1}$ on its route throughout the planning horizon. Another set of parameters are the optimal commodity and wounded flows over each arc and each period, obtained by solving Model $\mathrm{E}(\mathrm{II})$. Next a system of equations (P1) is formulated that are used to identify the quantity of commodities loaded and unloaded by each vehicle at every node on its route.

Parameters:
L: Total set of routes (one route per utilized vehicle) identified in the first stage
RC: Binary matrix of size $[|\mathrm{L}| \mathrm{x}|\mathrm{C}| \mathrm{x}|\mathrm{T}|]$. " $\mathrm{RC}[1, o, t]=1$ " indicates that a vehicle $v^{1}$ is incident to node $o$ in time period $t$
$\mathrm{k}_{1}$ : $\quad$ Number of nodes on route $l$
capl: Capacity of vehicle $v^{1}$
$\mathrm{Z}^{*}$ aopmt: Optimal amount of commodity type $a$ traversing $\operatorname{arc}(o, p, m)$ at time $t$ identified by Model E(II).

Decision Variables:
$L Z_{\text {aot: }}^{1}$ Quantity of commodity $a$ picked up at node $o$ in period $t$ by vehicle $l$
$U Z_{\text {aot: }}{ }^{1}$ : Quantity of commodity $a$ delivered at node $o$ in period $t$ by vehicle $l$
System of Equations and Inequalities (P1):

$$
\begin{align*}
& \Sigma_{1} \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \mathrm{RC}[1, \mathrm{o}, \mathrm{q}]\left[\mathrm{UZ} Z_{\text {aoq }}^{1}-\mathrm{LZ}_{\text {aoq }}^{1}\right]=\Sigma_{\mathrm{m}} \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{\mathrm{p}, \mathrm{o} \in \mathrm{C}} \mathrm{Z}_{\text {apomq }}^{*}-\Sigma_{\mathrm{m}} \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{\mathrm{o}, \mathrm{p} \in \mathrm{C}} \mathrm{Z}_{\text {aopmq }}^{*} \\
& (\forall \mathrm{a} \in \mathrm{~A}, \mathrm{o} \in \mathrm{C}, \mathrm{t} \in \mathrm{~T})  \tag{3-19}\\
& \Sigma_{\mathrm{o} \in \mathrm{C}} \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \mathrm{RC}[1, \mathrm{o}, \mathrm{q}]\left[\mathrm{LZ}_{\mathrm{aoq}}^{1}-\mathrm{UZ}_{\mathrm{aoq}}^{1}\right] \geq 0 \quad(\forall \mathrm{l} \in \mathrm{~L}, \mathrm{a} \in \mathrm{~A}, \mathrm{t}=1, \ldots, \mathrm{~T}-1)  \tag{3-20}\\
& \Sigma_{\mathrm{o} \in \mathrm{C}} \Sigma^{\mathrm{T}} \mathrm{q}_{\mathrm{q}=1} \mathrm{RC}[1, \mathrm{o}, \mathrm{q}]\left[\mathrm{LZ}_{\mathrm{aoq}}^{1}-\mathrm{UZ}_{\mathrm{aoq}}^{1}\right]=0 \quad(\forall \mathrm{l} \in \mathrm{~L}, \mathrm{a} \in \mathrm{~A})  \tag{3-21}\\
& \sum_{\mathrm{q}=1}^{\mathrm{t}} \sum_{\mathrm{a}} \Sigma_{\mathrm{o} \in \mathrm{C}} \mathrm{~W}_{\mathrm{a}} \mathrm{RC}[1, \mathrm{o}, \mathrm{q}]\left[\mathrm{LZ}_{\mathrm{aoq}}^{1}-\mathrm{UZ}_{\text {aoq }}^{1}\right] \leq \mathrm{cap}_{1} \quad(\forall \mathrm{l} \in \mathrm{~L}, \mathrm{t} \in \mathrm{~T}) \quad(3-22) \\
& \mathrm{LZ}_{\text {aot }}^{1}, \mathrm{UZ}_{\text {aot }}^{1} \geq 0 \quad(\forall \mathrm{l} \in \mathrm{~L}, \mathrm{a} \in \mathrm{~A}, \mathrm{o} \in \mathrm{C}, \mathrm{t} \in \mathrm{~T}) \tag{3-23}
\end{align*}
$$

Equations (3-19) state that at every time period the net cumulative number of commodities delivered at a node $o$ over all transportation modes $m$ should be equal to the net delivered number by all labeled vehicles. Inequalities (3-20) state that the cumulative net quantity picked up on the route by each labeled vehicle is non-negative for each type of commodity. Equations (3-21) are ending conditions that ensure that the quantity picked up by each vehicle is equal to the quantity delivered by the end of the planning horizon. Inequalities (3-22) restrict the net cumulative quantity picked up by the vehicle capacity, in the order of nodes traversed on the vehicle's route. Since it is generated from model $\mathrm{E}(\mathrm{II})$ 's optimal solution, any feasible solution to P1 gives a set of optimal instructions.

Although equations (3-19) are given in a cumulative form by $t$, it can be divided into a set of separate equations by deducting the previous accumulated equation up to $t$ from the accumulated equation up to $t+1$. The resulting equations are as follows:
$\Sigma_{1}\left[U Z_{\text {aoq }}^{1}-L Z_{\text {aoq }}^{1}\right]=\Sigma_{\mathrm{m}} \Sigma_{\mathrm{p}, \mathrm{o} \in \mathrm{C}} \mathrm{Z}_{\text {apomq }}^{*}-\Sigma_{\mathrm{m}} \Sigma_{\mathrm{o}, \mathrm{p} \in \mathrm{C}} \mathrm{Z}_{\text {aopmq }}^{*}(\forall \mathrm{a} \in \mathrm{A}, \mathrm{o} \in \mathrm{C}, \mathrm{q} \in \mathrm{T})$

Taking the indices $a$ and $q$ as a two-dimension type indication of commodity, and considering $l$ denoting the summed flow adjacent to node $o$, the equations (3-24) are exactly the mass balance constraints in the minimum cost network flow problem and therefore matrix given by (3-24) is totally unimodular.

Inequalities (3-20) and equations (3-21) define similar constraints in other dimensions. The number of variables is highly limited, as RC is an extremely sparse matrix. The number of constraints in the problem is $(|\mathrm{L}|+|\mathrm{C}|)|\mathrm{A}||\mathrm{T}|$. Especially, in the context of commodity logistics (as in E(II) in this chapter ), system of equations and inequalities P1 can be solved in polynomial time, which can be achieved easily in readily available linear programming packages.

### 3.4 Comparison between Formulations

The major determinant of an integer model's efficiency is the number of integer variables it defines. Additionally, if the problem has a special structure, it can facilitate solution process significantly. Hence, the models proposed in the previous sections are compared in two aspects. On the first aspect, $\mathrm{E}(\mathrm{I})$ contains $|\mathrm{C}|^{2} \times|\mathrm{L}| \times|\mathrm{T}|$ binary variables, while E (II) involves $|\mathrm{C}|^{2} \times|\mathrm{M}| \times|\mathrm{T}|$ integer variables that is normally far less than the former (vehicle number $|\mathrm{L}|$ is greater than vehicle type number $|\mathrm{M}|$ ). Hence, considerable solution advantage may be obtained through the vehicle aggregation when large number of vehicles is employed in many practical cases. Furthermore, because of the good network structure discussed in Chapter 2, it is reasonable to say that model $\mathrm{E}(\mathrm{II})$ has better efficiency than $\mathrm{E}(\mathrm{I})$. A small computational experiment is done to illustrate the impact of input change on problem size as well as solution efficiency. The experiment focuses on the observation of how the inputs $L$ and $M$ differentiate the two formulations' solution efficiency. Hence, an
identical distance matrix is applied to all the instances, which is taken from a test instance proposed for VRP by Eilon et al. (1971). Demand-supply-vehicle parameters are given arbitrarily and there exist sufficient supplies available in all instances. All instances are generated in GAMS (GAMS Development Corporation, 1998) and solved by the MIP solver CPLEX 7.5 (ILOG, 2001) on a PC with 3.2 GHz CPU speed and 512 MB RAM. The results are presented in the following table.

Table 3-1 Illustration on a small set of instances

|  | Input Size |  | ModelSize (Mb) |  | Number of Integer Variables |  | Number of Constraints |  | Computation Time (Sec.) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\underset{\times\|\mathrm{A}\|}{\|\mathrm{A}\| \mathrm{X}\|\mathrm{C}\| \times\|\mathrm{L}\| \times \mid \mathrm{I}}$ |  | E (1) | E (II) | E (I) | E (II) | E (I) | E (II) | E (I) | E (II) | TwoStage |
| 1 | 13, 6, 1, 8) | 0.867 | 7 | 3 | 8112 | 1352 | 12943 | 3522 | 11.0 | 1.3 | 0.36 |
| 2 | (2, 13, 6, 2, 8) | 0.774 | 7 | 5 | 8112 | 2704 | 12943 | 6435 | 10.8 | 4.5 | 0.3 |
| 3 | ( $2,13,9,1,8$ ) | 0.674 | 9 | 3 | 12168 | 135 | 19318 | 352 | 47.1 | 0.23 | 0.3 |
| 4 | (2, 13, 9, 3, 8) | 0.674 | 9 | 7 | 12168 | 4056 | 19318 | 9348 | 74.6 | 3.7 | 0.36 |
|  | (2, 13, 12, 3, 8) | 0.867 | 12 | 7 | 16224 | 4056 | 25693 | 9348 | 125.8 | 3.8 | 0.36 |
|  | (2, 13, 15, 3, 8) | 0.758 | 15 | 7 | 20280 | 4056 | 32068 | 9348 | 227.5 | 2.0 | 0.3 |

The first two columns give the problem number and size. The third column indicates the tightness of vehicle capacity, which is calculated as total load divided by total available vehicle capacity. The capacity tightness has been observed as a major impact factor on the solvability for VRPs, hence it is noted here. The subsequent columns indicate memory requirement, number of integer variables, number of constraints and computation time. The last column presents the time taken to disaggregate the flow solutions, which is consistently trivial for these small instances. The runtimes for both formulations get longer when model sizes increase. However, model $\mathrm{E}(\mathrm{II})$ is a more compact formulation because the number of vehicles $(|\mathrm{L}|)$ is a major concern in ELP context, and the difference among the two models' sizes grows quickly $|\mathrm{C}|^{2} \times|\mathrm{T}| \times(|\mathrm{L}|-|\mathrm{M}|)$ on $|\mathrm{L}|$. Hence, the computation times for $\mathrm{E}(\mathrm{I})$ increase significantly as the number of vehicles $(|\mathrm{L}|)$ increases (while the number of vehicle
types $(|M|)$ is preserved $)$. Although the number of vehicle types $|M|$ has an influence on E (II), its variation is quite limited in ELP practice and therefore should not pose a problem. Besides model size, tightness is observed as the second affecting factor to both formulations, and in both tight and less tight problems, the difference in computation times is consistently large.

### 3.5 Summary

Two kinds of formulations for emergency logistics problem are evaluated and compared in this chapter. Aggregate flow model E(II) shows better solvability by effective control over model size and the embodied network structure. These two advantages together make model $\mathrm{E}(\mathrm{II})$ outperform model $\mathrm{E}(\mathrm{I})$ substantially. A twostage algorithm is proposed to construct the vehicle routes and pickup/delivery instructions from the solution of $\mathrm{E}(\mathrm{II})$. Furthermore, in the context of commodity logistics this algorithm is of overall polynomial complexity. Hence, the whole set of modeling methodology simplifies the solution of vehicle routing significantly and it is generic and also applicable to commercial situations.

## 4 A Dynamic Logistics Coordination Model for Evacuation and Support in Disaster Response Activities

In the previous chapter, the efficiency of aggregate network flow formulation was validated through a comparison with vehicle routing based formulation. This modeling strategy is supported by the two-stage algorithm proposed to produce an operational logistics plan. This chapter completes the disaster relief model and algorithm by integrating the wounded people evacuation problem into the logistics planning. The model is illustrated on a real world natural disaster scenario, and its dynamic application is demonstrated by a re-planning procedure that is conducted at regular time intervals during on-going relief operations.

### 4.1 Modeling Evacuation in Emergencies

In disaster response actions, the survival rate among affected people also depends on the effectiveness of search and rescue operations and this requires, in turn, a good coordination of search and rescue activities and efficient evacuation of injured people. Furthermore, overall health conditions of everyone in the affected area depend on the timely availability of commodities such as food, shelter and medicine.

The model proposed here aims to evacuate wounded people to emergency medical units. Furthermore, coordination of the transportation of commodities from major supply centers to distribution centers in affected areas is considered in an integrated
manner. The model assumes that all emergency medical units have a certain service rate in proportion to their emergency handling capacities and that patients are discharged from the medical system with this rate. The service rates are modeled as demands so that wounded people can be treated as integer valued flow in the multicommodity flow model. The efficiency of this kind of formulation has been verified in Chapter 3. The goal of this disaster response logistics support and evacuation model is different from commercial applications in the sense that rather than minimizing fleet size and fleet operation costs (total distance traveled), it is desired to transport people and materials to reach their destinations where they can be served or delivered in the minimum possible time. Both wounded people and commodities are categorized into a priority hierarchy. Different types of vehicles with varying degrees of specialized equipment can be utilized to satisfy transportation needs of high priority wounded people.

The proposed framework is designed as a flexible dynamic (multi-period) coordination instrument that can adjust to frequent information updates, and vehicle re-routing. The planning horizon under consideration is short (days or even hours) due to the fact that information flow is continuous after disasters and initial screening cannot capture the attrition numbers accurately. Furthermore, continuity of commodity logistics is achieved by incorporating anticipated commodity demand for the next period.

Once a solution is obtained and integrated decisions related to routing and load/unload quantities are read from the output file, a simple polynomial algorithm converts the arc-based vehicle dispatch output into vehicle itineraries with their load/unload quantities (keeping track of these quantities is a must in split delivery policies). This
network flow model structure facilitates the execution of the re-planning procedure that is activated at regular time intervals during ongoing operations. Thus, an efficient response system is designed here to meet the requirements of dynamic disaster logistics support and provide fast schedule updates when new demand, supply and vehicle availability information arrives at the coordination center.

### 4.2 Mathematical Formulation

The mathematical formulation of the problem and the additional notation are given below, based on the formulation presented in Chapter 3 and embodying the wounded evacuation problem.

Additional Sets and Parameters:
$\mathrm{T}^{\prime}$ : Regular time interval for re-planning
H: Set of different categories of wounded people (heavy, moderate-light); $h$ denotes a specific category
$\mathrm{CH}: \quad$ Set of available emergency centers, $\mathrm{CH} \subset \mathrm{C} \backslash \mathrm{CD}$
$\mathrm{d}_{\text {hot }}$ : Number of wounded people of category $h$ waiting at node $o \in \mathrm{CD}$ at time $t$; $\mathrm{d}_{\text {hot }}=0$ for $o \in \mathrm{C} \backslash C D$
$\mathrm{w}_{\mathrm{h}}$ : Average weight of a wounded person
scap $_{\mathrm{ho}}$ : Initial per period service rate for category $h$ wounded people at hospital at node $o \in \mathrm{CH}$; scap $_{\text {ho }}=0$ for $o \in \mathrm{C} \backslash \mathrm{CH}$
$P_{h}$ : Priority of wounded people of category $h$

Additional Decision Variables:
$X_{\text {hopmt: }}$ Integer number of wounded people of category $h$ traversing arc $(o, p)$ at time $t$ using vehicle type $m$
$\operatorname{dev}_{\mathrm{ht}}$ : Number of unserved wounded people of category $h$ at time $t$
$\mathrm{sp}_{\text {hot }}: \quad$ Number of wounded people of category $h$ who are served at node $o \in \mathrm{C}$ at time $t ; \mathrm{sp}_{\text {hot }}=0$ for $o \in \mathrm{C} \backslash \mathrm{CH}$

## Model P:

Minimize $\quad \Sigma_{\mathrm{a} \in \mathrm{A}} \Sigma_{\mathrm{o} \in \mathrm{CD}} \Sigma_{\mathrm{t}} \mathrm{P}_{\mathrm{a}} \operatorname{dev}_{\text {aot }}+\Sigma_{\mathrm{h} \in \mathrm{H}} \Sigma_{\mathrm{t}} \mathrm{P}_{\mathrm{h}} \operatorname{dev}_{\mathrm{ht}}$

Subject to

$$
\begin{align*}
& \mathrm{Y}_{\mathrm{opmt}} * \operatorname{cap}_{\mathrm{m}} \geq \Sigma_{\mathrm{a} \in \mathrm{~A}} \mathrm{w}_{\mathrm{a}} * \mathrm{Z}_{\mathrm{aopmt}}+\Sigma_{\mathrm{h} \in \mathrm{H}} \mathrm{w}_{\mathrm{h}} * \mathrm{X}_{\mathrm{hopmt}}(\forall \mathrm{o} \in \mathrm{C}, \mathrm{p} \in \mathrm{C}, \mathrm{~m} \in \mathrm{M}, \mathrm{t} \in \mathrm{~T})  \tag{4-2}\\
& \Sigma_{\mathrm{m} \in \mathrm{M}^{\Sigma}{ }_{\mathrm{q}-1}^{\mathrm{t}} \Sigma_{\mathrm{p} \in \mathrm{C}}\left[-\mathrm{K}_{\mathrm{pqotm}} \mathrm{X}_{\mathrm{hpomq}}+\mathrm{X}_{\mathrm{hopmq}}\right] \leq \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \mathrm{~d}_{\mathrm{hoq}}(\forall \mathrm{~h} \in \mathrm{H}, \mathrm{o} \in \mathrm{CD}, \mathrm{t} \in \mathrm{~T})} \tag{4-3}
\end{align*}
$$

$$
\begin{equation*}
\Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{\mathrm{m}} \in \mathrm{M}\left[\mathrm{~K}_{\mathrm{pqotm}} \mathrm{X}_{\mathrm{hpomq}}-\mathrm{X}_{\mathrm{hopmq}}\right] \geq \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \mathrm{sp}_{\mathrm{hoq}}(\forall \mathrm{~h} \in \mathrm{H}, \mathrm{o} \in \mathrm{C} \backslash \mathrm{CD}, \mathrm{t} \in \mathrm{~T}) \tag{4-4}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{sp}_{\text {hot }} \leq \operatorname{scap}_{\mathrm{ho}} \quad(\forall \mathrm{~h} \in \mathrm{H}, \mathrm{o} \in \mathrm{C}, \mathrm{t} \in \mathrm{~T}) \tag{4-6}
\end{equation*}
$$

$$
\begin{equation*}
Y_{\text {opmt }}, X_{\text {hopmt }}, Z_{\text {aopmt }}, \operatorname{dev}_{\text {aot }}, \operatorname{dev}_{\text {hot }} \geq 0 \text { and (3-13), (3-15), (3-17), (3-18); } \tag{4-7}
\end{equation*}
$$

The objective aims at minimizing the weighted sum of unsatisfied demand over all commodities and the weighted sum of wounded people waiting at affected nodes and medical units. Heavily and lightly injured people hold the first and second priorities whereas medicine holds the highest priority among all commodities. The remaining commodities are given appropriate subjective priorities. Commodities are represented in units of people equivalent for convenience. Note that in this multi-period planning problem, knowledge on future demand can be predicted based on current demand. Additionally, confirmed arrivals represent next period's supplies, thereby enabling
continuity of routing plans over short periods of time. Constraint set (4-2) restricts transportation capacity of vehicles. It is possible to restrict the load of some vehicles to include only certain types of commodities by setting $X_{\text {hopmt }}=0$ or $Z_{\text {aopmt }}=0$. For instance, only ambulances and helicopters might be allowed to carry wounded people. In this formulation, the service rates in hospitals are modeled as demands for wounded people, while the wounded in affected nodes are modeled as supplies. Constraints (4-3) and (4-4) balance wounded people flow at all nodes and define those that are not served till time period $t$ (waiting in affected area or hospital queue; or on the way to hospital). Here, queue size depends on the number of arrivals and the service rate or capacity of that unit. It is reduced by those who have already been served and sent out of the emergency system. The distribution of wounded people is expected to achieve equilibrium and hence, maximizes the utilization of medical facilities. Constraints (4-5) define wounded people not served and (4-6) restrict the number of wounded served in each period by the service rate of a given medical center. The final sets of constraints (4-7) involve commodities flow constraints from model E(II) and impose bounds on the variables.

The aggregation of vehicle capacities saves substantial computational resources, but it might lead to a possible error when some of the commodities are integer variables (such as, people flow in this formulation). According to the bundle constraint, a unit commodity may utilize joint capacities from several vehicles, which is not feasible for an integer variable. Hence, a multiple assumption is imposed to eliminate the error in solution: the unit weights of all indivisible commodities are in integral multiple relationships and the capacities of vehicles are common multiples of the unit weights. This assumption imposes no loss of generality, since the unit weights of indivisible
commodities may be scaled so that the vehicle capacity can be presented by combinations of integral units.

In this formulation, vehicles are treated as commodities, and again it is not required to track vehicles individually on a route basis. Details of dispatch orders for vehicles are obtained by executing the two-stage algorithm proposed in Chapter 3 while adding equations listed below concerning the wounded flows in the system.

Additional Parameters:
$\mathrm{X}_{\text {hopmt: }}^{*}$ Optimal number of wounded people of category $h$ traversing arc $(o, p, m)$ at time $t$ identified by Model P.

Additional Decision Variables:
$\mathrm{LX}_{\text {hot }}^{1}$ : Integer number of wounded people of category $h$ picked up at node $o$ in period $t$ by vehicle $l$.
$\mathrm{UX}_{\text {hot }}$ : Integer number of wounded people of category $h$ delivered at node $o$ in period $t$ by vehicle $l$.

Additional Equations and Inequalities for P 1 :

$$
\begin{align*}
& \Sigma_{1} \Sigma^{\mathrm{t}}{ }_{\mathrm{q}=1} \mathrm{RC}[1, \mathrm{o}, \mathrm{q}]\left[\mathrm{UX} \mathrm{X}_{\text {hoq }}^{1}-\mathrm{LX} \mathrm{X}_{\text {hoq }}^{1}\right]=\Sigma_{\mathrm{m}} \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{\mathrm{p}, \mathrm{o} \in \mathrm{C}} \mathrm{X}_{\mathrm{hpomq}}^{*}-\Sigma_{\mathrm{m}} \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{\mathrm{o}, \mathrm{p} \in \mathrm{C}} \mathrm{X}_{\text {hopmq }}(\forall \\
& \mathrm{h} \in \mathrm{H}, \mathrm{o} \in \mathrm{C}, \mathrm{t} \in \mathrm{~T}) \tag{4-8}
\end{align*}
$$

$$
\begin{align*}
& \Sigma_{\mathrm{o} \in \mathrm{C}} \Sigma^{\mathrm{T}}{ }_{\mathrm{q}=1} \mathrm{RC}[1, \mathrm{o}, \mathrm{q}]\left[\mathrm{LX}_{\mathrm{hoq}}^{1}-\mathrm{UX}_{\mathrm{hoq}}^{1}\right]=0 \quad(\forall \mathrm{l} \in \mathrm{~L}, \mathrm{~h} \in \mathrm{H}) \tag{4-10}
\end{align*}
$$

$$
\begin{align*}
& \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{\mathrm{h}} \Sigma_{\mathrm{o} \in \mathrm{C}} \mathrm{~W}_{\mathrm{h}} \mathrm{RC}[1, \mathrm{o}, \mathrm{q}]\left[\mathrm{LX}_{\mathrm{hoq}}^{1}-\mathrm{UX}_{\mathrm{hoq}}^{1}\right]+\Sigma_{\mathrm{q}-1}^{\mathrm{t}} \Sigma_{\mathrm{a}} \Sigma_{\mathrm{o} \in \mathrm{C}} \mathrm{w}_{\mathrm{a}} \mathrm{RC}[1, \mathrm{o}, \mathrm{q}]\left[\mathrm{LZ}_{\text {aoq }}^{1}-\right. \\
& \left.\mathrm{UZ}_{\text {aoq }}^{1}\right] \leq \operatorname{cap}_{1} \quad(\forall \mathrm{l} \in \mathrm{~L}, \mathrm{t}=1 . . \mathrm{T})  \tag{4-11}\\
& \mathrm{LX}_{\mathrm{hot}}^{1}, \mathrm{UX}_{\mathrm{hot}}^{1}, \mathrm{LZ}_{\mathrm{hot}}^{1}, \mathrm{UZ}_{\text {hot }}^{1} \geq 0 \text {; (3-19) to (3-21), (3-23); } \tag{4-12}
\end{align*}
$$

Equations (4-8) state that at every time period the net cumulative number of people (category $h$ ) delivered at a node $o$ over all transportation modes $m$ should be equal to the net delivered number by all labeled vehicles. Inequalities (4-9) state that the cumulative net quantity picked up on the route by each labeled vehicle is non-negative for each category of wounded people. Equations (4-10) are ending conditions that ensure that the quantity picked up by each vehicle is equal to the quantity delivered by the end of the planning horizon. Inequalities (4-11) restrict the net cumulative quantity picked up on each vehicle's route in the order of nodes traversed by the cumulative vehicle capacity. Constraints (4-12) involve commodities flow equations from chapter 3 and impose bounds on the variables. Both parts of the equations are of the same structure. Although the integer variables regarding people flow exist, a good solvability feature of the feasibility verification is still maintained due to the successful packages designed for mixed integer programming problems.

### 4.3 Re-planning Procedure

Re-planning is a core issue in dynamic logistics activities. The importance of replanning increases in natural disasters because requirements, supplied quantities, demand and the fleet sizes change perpetually. An advantage of the proposed formulation is that the structure of the solution is convenient to use when a new plan has to be generated.

A new plan is generated at given time intervals (at every $\mathrm{T}^{\prime}$ periods, $\mathrm{T}^{\prime}<\mathrm{T}$ ) with the updated information. In the re-planning approach adopted here, vehicles already dispatched in the previous plan may be re-routed after arriving at a node at the earliest time period greater than $\mathrm{T}^{\prime}$ in order to optimize the service level. However, if their load consists of wounded people, the load can be transferred to another vehicle with a no-wait restriction. Accordingly, the following parameters in Model P are modified before re-planning takes place in period $\mathrm{T}^{\prime}$ (or its multiples).

If a vehicle is on the way between two nodes and it is expected to arrive at a node $o$ in a period $\mathrm{t}^{*}>\mathrm{T}^{\prime}$, then, it becomes available in period $\mathrm{t}^{*}$ and parameters $\mathrm{av}_{\mathrm{omt}}$ is adjusted for all $t>t^{*}$. Based on priorities of demanded commodities, the vehicle might unload its contents at node $o$ and leave for a higher priority mission while another vehicle picks up its previous load. If a vehicle has already arrived at a node and is currently waiting, then its availability is added to the system from period $\mathrm{T}^{\prime}$ onwards. Hence, all vehicle availabilities are updated appropriately.

Demand and supply quantities are adjusted as follows. Unsatisfied demand left over from the previous period, $\mathrm{T}^{\prime}-1$, is equal to the optimal quantity of unsatisfied demand $\operatorname{dev}_{\mathrm{ao}, \mathrm{T}^{\prime}-1}$. This quantity is added to $\mathrm{d}_{\mathrm{aot}^{\prime}}$ as well as additional quantities that came to be known during the current and previous re-planning times. Demand predictions for the next re-planning period are updated according to observations made during recent periods. The same procedure is carried out for adjusting parameters related to wounded people. Similarly, supplies left over from the previous plan take on the optimal values of the slack variables in period $\mathrm{T}^{\prime}-1$. Additional past and future quantities are also added to the supply parameters.

### 4.4 Illustration on the Earthquake Scenario

Model P and the two-stage vehicle instruction sheet generation procedure are implemented on a scenario that describes a possible Istanbul earthquake (main fault of Marmara Sea, Turkey). Other scenario description can also be found on 1995 Japan earthquake (Bardet et.al., 1995; Building Research Institute, Ministry of Construction, Japan, 1996). Istanbul receives a moderate magnitude earthquake every 50 years and very severe earthquakes every 300 years. An earthquake with severe damage risks is expected to take place with $65 \%$ probability within the next 30 years. The attrition numbers and structural damage to Istanbul are provided in a report prepared by a consortium of universities, municipalities, government agencies, and USGS and other foreign experts (BU Earthquake Engineering Dept., 2002). The structural risk grades are categorized as VII ( $20 \%$ of buildings-moderate damage), VIII ( $20 \%-60 \%$ of buildings-severe damage) and IX (20\%-60\% of buildings very severe damage). Based on risk grades and population density of districts, it is conjectured that $35,000-40,000$ buildings will collapse completely ( $5 \%$ of overall buildings in the city), 70000 will be severely damaged, and 200,000 will have moderate damage. Possible attrition numbers, damaged buildings, number of medical emergencies and similar statistics are provided in the report and mapped on Istanbul's district partition. Figure AI-1 illustrates districts where aid distribution centers and medical emergency units may be situated. Nodes 1-6 represent districts Kucukcekmece, Bakirkoy-Zeytinburnu, FatihEminonu, Bagcilar-Bahcelievler, Beyoglu and Kadikoy. Nodes 7-15 represent districts that have much less risk of damage and attrition. Resources such as commodities (in warehouse storage maintained by Natural Disaster Agency for a possible emergency) and medical personnel can be supplied from these low risk districts immediately. Assume that these districts represent existing hospital
emergency units in aggregate. Estimated medical provisions are based on information from local municipalities and Turkish Medical Doctors Association's statistics on regular patient at hospital and emergency capacities as well as the number of ambulances. Nodes 16-17 represent Bursa and Balikesir that are major cities situated across Marmara Sea and are accessible from Istanbul by sea as well as by land. These two can provide significant amount of aid to Istanbul. The roads drawn in Figure AI-1 represent the network of international highway TEM (Trans European Motorway (TEM) and its peripherals that are particularly dense in European side of Istanbul. The old parallel coastal highway E-5 that is reported to have a high damage risk is not considered in this scenario. All the tables and figure can be found in Appendix I.

Input data used in the scenario are provided in Table AI-1, Table AI-2, and Table AI-3. A time bucket of 1 hour is utilized, and the plan has to be revised every replanning time. The planning horizon takes the time interval $[1,8]$ and the re-planning happens at the beginning of $t=5$. Two major commodities, medicine and food are considered and their demands and supplies are provided in units of people equivalent for convenience. Table AI-1 indicates distribution of commodity demand among affected nodes in period $\mathrm{t}=1$ and anticipated distribution at the beginning of replanning period $\mathrm{t}=5$. Table AI-2 illustrates supply distribution and vehicle availability. Supplies keep on arriving by the hour. Percentages of available vehicles by type are provided in Table AI-2. These are helicopters (for wounded people and medicine), trucks (for food) and ambulances (for wounded people). There are in total 101 vehicles, about $20 \%$ of which is added to the system in second period. In Table AI-2, total transportation capability of different vehicle types is indicated for individual load types, but not their combinations. Hence, transportation capacity indicated in the table is above the actual one.

The wounded are categorized into two levels: light (L) and heavy (H). It is assumed that an ambulance can transport either up to 6 L category wounded persons or up to 2 heavily injured or a combination fitting its weight capacity. A helicopter can transport up to 5 H or 15 L categories respectively. Table AI-3 provides distribution of wounded people in affected area. The quantities given in all tables for period 5 are predicted values. In re-planning phase the actual quantities have been simulated as $10 \%$ above or below predictions. The sum of actual commodity needs and wounded people (information received at the end of $t=4$ ) are also indicated in these tables. Priority weights for heavy, light-moderate wounded, medicine and food are given as 20, 5, 2 and 1 , respectively.

The model is constructed and coded in GAMS, and re-planning, routing (Route) and the system of equations P1 are all coded into GAMS. The MIP solver used is CPLEX 7.5 on a PC with 3.2 GHz CPU and 512 MB RAM. The computation results are summarized in Tables 4.1-4.4, and the shaded cells indicate the values obtained before re-planning.

In Table 4-1, the amounts of wounded who have been served and departed the emergency system at each emergency unit in each time period are given. It is observed that emergency units (node $11,13,14$ ) are underutilized because these facilities are far away from affected districts. In fact, another study (Yi and Özdamar, 2007) concerning location analysis among emergency units shows that the service capacity is reduced in these facilities and then transferred to nodes that are closer to affected nodes, including those newly established temporary emergency units and existing hospitals $7,8,10$. During the first two hours medical facilities are mostly not utilized due to transportation delay. The latter takes place because vehicles are in
unaffected districts. Hence, we may consider the first two periods spent for stabilizing the system. The queue lengths can be observed in Table 4-2 where positive queue sizes are provided for each time period and node. The queue sizes are kept small while the facilities are utilized to maximum service rates under transportation and wounded supply constraints, which enable the vehicles to be devoted to other tasks for the enhancement of the overall efficiency.

Table 4-1 Number of served people in medical facilities

| Time | 7 |  | 8 |  | 9 |  |  |  | 10 |  |  |  | 11 |  |  | 12 |  |  |  | 13 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | H | L | H | L | H |  | L | L |  | H | L | L |  | H | L |  | H |  | L |  |  | L |
| 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 | 15 |  | 10 | 12 | 10 | 0 |  |  |  | 15 | 6 | 6 |  |  |  |  | 20 |  |  |  |  |  |
| 4 | 15 |  | 10 |  | 10 | 0 |  |  |  | 15 |  |  |  |  |  |  | 20 |  |  |  |  |  |
| 5 | 1515 | 2525 | 1010 | 2727 | 10 | 10 | 21 | 21 | 15 | 15 | 18 | 18 |  |  |  | 20 | 20 | 35 | 35 |  |  |  |
| 6 | 1515 | 22 | 1010 | 3 | 10 | 10 |  |  | 15 | 15 | 6 | 27 |  |  |  | 20 | 20 | 34 | 10 |  |  |  |
| 7 | 1515 | 2425 | 1010 | 3030 | 10 | 10 | 33 | 35 | 15 | 15 | 30 | 30 |  |  |  | 20 | 20 | 35 | 33 |  |  |  |
| 8 | 1515 | 20 | 1010 | 3 | 10 | 10 | 27 | 1 | 15 | 15 | 12 | 18 | 4 | 4 |  |  | 20 | 22 | 30 | 3 | 1 |  |

Table 4-2 Queue lengths of wounded people in medical facilities

| Time Node | H queue length | L queue length |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 3 | 7 | 5 |  |  |
| 3 | 9 | 6 |  |  |
| 3 | 10 | 5 | 2 | 2 |
| 5 | 7 |  | 1 | 1 |
| 5 | 12 | 1 |  | 2 |
| 7 | 7 |  | 3 |  |
| 7 | 8 |  |  |  |

The number of unsatisfied commodities at each period and node are presented in Table 4-3. Different from the low initial utilization of medical facilities due to vehicles delay, the commodities can be transported to affected nodes by the immediately available vehicles in the initial hours of the planning horizon. It is also observed that the nodes $1,4,5,6$ are much better served because they are closer to the
surrounding supply nodes and the commodities cannot be delivered to inner nodes 2 , 3 unless the outside demands are satisfied.

Table 4-3 Unsatisfied commodities in affected nodes in different time periods

| Time | 1 |  |  |  | 2 |  |  |  | 3 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | C1 |  | C2 |  |  |  | C2 |  | C1 |  | C2 |  |
| 1 | 8000 |  | 8000 |  | 700 |  | 700 |  | 9000 |  | 9000 |  |
| 2 | 6400 |  | 6400 |  | 700 |  | 700 |  | 9000 |  | 9000 |  |
| 3 | 5100 |  | 3400 |  | 700 |  | 700 |  | 9000 |  | 9000 |  |
| 4 | 1900 |  | 1900 |  | 700 |  | 700 |  | 9000 |  | 9000 |  |
| 5 | 1700 | 1200 | 1700 | 1200 |  |  | 900 | 1000 | 10100 | 10000 | 10100 | 10000 |
| 6 | 500 |  | 500 |  |  |  | 900 | 1000 | 10000 | 10000 | 10100 | 10000 |
| 7 | 500 |  | 500 |  |  |  | 900 | 1000 | 10000 | 10000 | 10100 | 10000 |
| 8 | 500 |  | 500 |  |  |  | 900 | 1000 | 10000 | 10000 | 10100 | 10000 |
| Time | 4 |  |  |  | 5 |  |  |  | 6 |  |  |  |
|  | C1 |  | C2 |  | C1 |  | C2 |  | C1 |  | C2 |  |
| 1 | 5000 |  | 5000 |  | 2000 |  | 2000 |  | 15000 |  | 15000 |  |
| 2 | 3400 |  | 3400 |  | 1200 |  | 1200 |  | 11600 |  | 13300 |  |
| 3 | 2100 |  | 2100 |  | 1100 |  | 1100 |  | 8400 |  | 7400 |  |
| 4 | 1100 |  | 1100 |  | 1100 |  | 1100 |  | 4200 |  | 3200 |  |
| 5 | 2900 | 3100 | 2900 | 3100 | 2300 | 2100 | 2300 | 2100 | 1350 | 1150 | 2000 | 2000 |
| 6 | 2900 | 3100 | 2900 | 3100 | 2300 | 2100 | 2300 | 2100 |  |  |  |  |
| 7 | 2900 | 3100 | 2900 | 3100 | 2300 | 2100 | 2300 | 1900 |  |  |  |  |
| 8 | 2900 | 3100 | 2900 | 3100 | 400 |  | 2300 | 1900 |  |  |  |  |

Table 4-4 Number of vehicles utilized in each time period

| Time | V1 |  | V2 |  | V3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 25 |  | 11 |  | 34 |  |
| 2 | 23 |  | 11 |  | 45 |  |
| 3 | 22 |  | 7 |  | 44 |  |
| 4 | 25 |  | 5 |  | 44 |  |
| 5 | 25 | 25 | 5 | 3 | 47 | 47 |
| 6 | 25 | 25 | 1 | 2 | 47 | 47 |
| 7 | 13 | 14 | 1 | 1 | 31 | 36 |
| 8 |  |  |  |  |  |  |

In Table 4-4, the number of vehicles utilized at every period before and after replanning can be observed. Throughout the planning horizon helicopters (V1) and ambulances (V3) have a high utilization ratio whereas trucks achieve about $50 \%$ maximal utilization rate due to supply limitation and the fact that transport capacity of
trucks is much larger than transport requirement of available supplies. Utilization rates drop to zero in period 8 when the planning horizon reaches end, which however will not happen in a real application running on a rolling horizon basis.

An overall assessment can be made for the plans generated on the scenario. The model coordinates logistics support and evacuation activities expediting high priority evacuation. Scarce resources are exploited to the full extent. Logistics plans that are generated by the model illustrate a flexible and dynamic system responding to changes effectively. Scenario plans also show the efficiency of the proposed system in terms of solvability. For this scenario, model P is solved in 3.08 secs. and 2375 iterations. The MIP that has to be solved for route construction and vehicle sheet preparation takes a total of 1.75 secs. and 1226 iterations. The re-planning MIP is solved in 0.95 secs. and 1036 iterations and the second one is solved in 1.42 secs. and 274 iterations.

### 4.5 Discussion on Uncertainty in ELP

From the very beginning of the application of optimization to various real-world problems, it was recognized that analysts of natural and technological systems are almost always confronted with uncertainty. An inevitable major issue in post disaster logistics management is the inherent uncertainty that exists in parameters such as commodity demand and number of injured people waiting to be hospitalized. Postdisaster chaos in logistics adds to existing uncertainties. Additional congestion is caused by people in other areas rushing to the region to look for their relatives and survivors trying to flee from the affected region. Lack of basic infrastructure functions and communication leads to further uncertainty for the disaster coordination center. It is also impossible to know the exact number of people in the region at the time of the
disaster; for instance, survivors might be lying under rubbles. Although risk assessment studies might have been conducted for given types of disasters, risk grades for residential damage are usually spelled out in wide intervals. Another important source of uncertainty lies in the service rate of hospital emergency units. All of these reasons create obstacles against accurate prediction of parameters to be used in the model.

Since the fifties of the last century, optimization under uncertainty has experienced rapid development in both theory and algorithms. Approaches to optimization under uncertainty can be grouped into two categories: stochastic programming (recourse models, robust stochastic programming, and probabilistic programming) and fuzzy programming.

Under the standard two-stage stochastic programming paradigm, the first-stage decision variables have to be decided before the random events present themselves and then the values of the second-stage (recourse) variables are selected to make further decision improvements after the actual information of the uncertain parameters is obtained. Hence, the second-stage variables are essentially either corrective measures that are assigned penalties against infeasibilities arising due to a particular realization of uncertainty or operational-level decisions following a first-stage plan. The objective is to minimize the sum of the first-stage costs and the expected recourse activity costs. The focus of the probabilistic or chance-constraint approach is to restrict the probability of infeasibility to be no greater than a pre-specified threshold. Mulvey et al. (1995) proposed robust programming to capture the notion of risk in stochastic programming, which modifies the objective function and integrates goal programming formulation and scenario based description of problem. Stochastic
programming model has been applied to linear, integer, and non-linear problems. An extensive discussion of these applications is given in Kall and Wallace (1994), Birge and Louveaux (1997) and Sahinidis (2004).

The above approaches of stochastic programming are through the use of probabilistic models that rely on the probability distributions of the uncertain parameters. When the probability distribution is not available (for example in disaster relief context, the tasks have never been or only rarely performed before), there is not enough information for inferring the probabilistic distribution functions. Moreover, the solution of these probabilistic models is computationally expensive because of the large number of scenarios resulting from a discrete representation of the uncertainty (Wets, 1974) or the complicated integration techniques needed when the continuous probability distributions is used (Schmidt and Grossmann, 2000). Hence, in such situations, people have to resort to an alternative treatment of uncertainty - fuzzy programming.

Fuzzy modeling takes a different approach in dealing with uncertainties. Rather than working with expected values, it assigns fuzzy numbers to uncertain parameters that are defined on intervals. However, this is still a priori optimization approach, that is, a solution is developed according to anticipation which can be revised when exact information arrives. Many of the developments in the area of fuzzy mathematical programming are based on the seminal paper by Bellman and Zadeh (1970). The field has been recently popularized by the work of Zimmermann (1991). In fuzzy programming, the membership function is used to represent the degree of satisfaction of constraints, the decision-maker's expectations about the objective function level, and the range of uncertainty of coefficients. The applications of fuzzy linear
programming have spanned many fields such as transportation (Chanas, 1998; Li and Lai, 2000), capacity expansion (Liu and Sahinidis, 1997), vehicle routing (Teodorovic and Pavkovic, 1996; Barbarosoglu and Özdamar, 2001) and so on. Verdegay (2003) presented a broad overview on real practical applications based on fuzzy sets and heuristic algorithms. Recent treatments on fuzzy integer programming include Osman et al. (1999), Yu and Li (2001).

Fuzzy modeling is applicable in an emergency context because historical data on disaster damage are inadequate for constructing probability density functions while risk maps are readily available to construct realistic membership functions. A fuzzy post disaster logistics support model was proposed in Yi and Özdamar (2004) for response activities. The aim is to maximize response activity service level by coordinating fast relief access to affected areas. The fuzzy model tries to minimize risk by maximizing satisfaction of anticipated demand within the limitations of total transportation capacity. This results in the well-known minimax type of objective function that maximizes the possibility of anticipated demand satisfaction. Similarly, membership functions are defined for supply availability and service rates of hospital emergency units and try to minimize related risks by minimizing the supply quantities and service rates. Thus, the risk of not being able to provide what was promised is reduced.

Furthermore, model parameters are adjusted by the hour according to actual occurrences and this narrows down the width of fuzzy parameter intervals. As data become more reliable, the robustness of solutions increases.

As illustrated by the scenario analysis in Yi and Özdamar (2004), the results show that the fuzzy approach is able to cope with uncertainty without requiring lengthy
simulations that would be inappropriate to use in this context due to lack of adequate past data and the need to generate fast solutions in quasi-real time. The dynamic reoptimization strategy coupled with the fuzzy model enhances the potential of its usage in post disaster response operations.

### 4.6 Summary

A practical emergency logistics problem integrating commodity delivery and evacuation of wounded people is addressed in this chapter where the service rates of emergency units are modeled as demands so that evacuated people can be modeled as flows facilitating the solution of the extended problem. The two-stage algorithm is also extended and carried out for the interpretation of the model's outputs. The dynamic relief operations are illustrated on an earthquake scenario and the results show that the model works well in terms of resource utilization. The uncertainty issue in ELP is also briefly discussed.

## 5 Heuristics for Disaster Relief Operations

In the previous chapters, the exact solution framework was proposed, and mediumsize problems can be solved with the current available packages (Yi and Özdamar, 2007). However, for larger size problems, it is difficult by the branch and bound system to find good solutions quickly, which is a major issue in achieving fast response in emergency logistics because re-planning needs to be conducted in a timely manner to account for the frequently updated information. Hence, a fast solution approach is necessary for the treatment of large scale problems.

### 5.1 Analysis of the Solution Complexity

The formulation of Model P in Chapter 4 is a network flow based model with integer commodities (injured people), i.e., a mixed integer multi-commodity network flow model where the vehicles themselves are treated as integral commodities that accompany other commodities. However, the solution for Model P is more difficult than the common integer multi-commodity flow problem due to the inherent routing sub-problem in emergency logistics planning, which has higher degrees of freedom as compared to conventional models. Capacity feasibility becomes an essential issue because it not only does the vehicle capacity fluctuate throughout the tour, but the assignment of the multiple type of load to available vehicles as the fleet is composed of vehicles with different capacity and type. Limited supply creates a new decision making problem: from which supply node should the vehicle pick up appropriate
supplies to meet demand at a requesting node. In previous multiple depots literature, the task of assigning (clustering) customers to depots is based on distance measures. However, in disaster relief operations, one has to make assignment and routing decisions simultaneously to match dynamic requests with supplies. Due to the latter issues and the fact that split delivery is a necessity, many of the improvement moves proposed in previous research become invalid. For instance, in dispatch settings found in the literature, when a node is inserted into the route of a vehicle, the corresponding load to be transported becomes known (due to the restriction that only one visit is to be paid to every demand node) and there is no issue of supply availability limits at depots. Consequently, supply-demand balance problem does not exist. However, in split delivery, the amount and type of load to be carried and the selection of the particular supply and demand node pair to be matched are important decisions to be optimized. Hence, the feasible space is much more relaxed and the number of alternatives in a local neighborhood search increases, making the problem more difficult to solve. In the following sections, two solution methods are proposed for this problem, which are developed from basic constructive method to meta-heuristic.

### 5.2 A Greedy Constructive Heuristic

A constructive heuristic, PATH_BUILDER, is first proposed to deal with the complexities of the emergency logistics support and dispatch problem. The heuristic is based on the k -neighborhood search technique and appends two-stop partial paths to vehicles in each iteration. However, the definition of neighborhood is extended to suit the complexities mentioned above. The motivation for developing a constructive algorithm for the emergency dispatch problem is to obtain fast solutions and facilitate
dealing with frequent status updates (new demand, supplies, and vehicle availability) that occur during disaster response activities.

Initially, the algorithm assigns a ready time $a t_{o l}$ to all vehicles available at node $o$ based on the parameter set $a v_{\text {omt }}(l \in m)$. The ready times (or path decision times) for each vehicle are tracked individually and they occur at the end of each recently appended partial itinerary (a sequence of nodes that includes a targeted demand and supply node pair). In this setting, nodes in the set CD are assumed to be demand nodes for commodities and supply nodes for wounded people. On the other hand, the nodes in the set CH become demand nodes for wounded people with their given service rates, scap $_{\text {ho }}$. Nodes in CS are supply nodes for commodities. The sets CH and CS intersect on many occasions and CH might be a proper set of CS.

### 5.2.1 Neighborhood Generation

For each ready vehicle $l$ at node $o$ in time $t, P A T H \_B U I L D E R$ identifies the k neighborhood of nodes $L_{\text {tol }}$, which creates positive utility for each vehicle l. For instance, if $l$ is on a demand node $o \in \mathrm{CD}$ where $\mathrm{d}_{\text {hot }}>0$ (wounded people wait to be picked up at node $o$ ), then it looks for a node $p \in \mathrm{CH}$. The service rate of $p \in \mathrm{CH}$ creates a demand and hence results in a satisfaction-based positive utility for vehicle $l$. On the other hand, if $\mathrm{d}_{\text {hot }}=0$, it looks for a node in $p \in \mathrm{CS}$ with nonzero supplies so that it can position itself at a node where positive utility can be created. When $l$ is on a supply node $o \in \mathrm{CS}$ with positive supplies, then it looks for a demand node $p \in \mathrm{CD}$ with $\mathrm{d}_{\text {aot }}>$ 0 for $\mathrm{t}^{\prime}=\mathrm{t}+\mathrm{t}_{\mathrm{op}}$. All commodity and wounded demand create positive $U_{l}$ for the vehicle as long it has (or can acquire) corresponding supplies. If there are no supplies available at node $o \in \mathrm{CS}$, then it looks for the nearest supply node where available supplies exist or a hospital node in CH hoping that it might find wounded people to
pick up on the way. On the paths leading to the k-neighbourhood nodes, it is also possible to create additional positive utility over supply-demand nodes that lie on the path other than the targeted demand-supply couple. Vehicle $l$ exploits all such opportunities.

The total utility of the first stop is always coupled with a succeeding node (second stop) selected from among the k-neighborhood of node $p$. Hence, two neighborhoods are explored in a sequence to construct k partial itineraries among which the one with maximum utility is finally selected if it beats the best paths of other competing vehicles. Following description gives how a path is constructed.

Suppose vehicle $l$ is at a node $o \in \mathrm{CS}$. Then, the heuristic conducts a local neighborhood search where the k-nearest demand nodes with positive utility are identified and their utilities are calculated. On the way to such a node $p$, there might be supply nodes or other demand nodes. The heuristic calculates the additional utility that can be obtained by picking up and delivering items on path ${ }_{o p}$. This is achieved by carrying supplies from the nearest supply node to the demand node. If there is no positive supply on node $o$, then the supply node with positive commodity supplies or wounded people is selected. Further, if it is the latter case, then the vehicle goes to the process identifying second stop; otherwise, the partial path is built. Next, a second stop, $q \in \mathrm{CH}$, is identified for delivering wounded people picked up from node $p$ if they are waiting to be transported to a hospital. Node $q$ is selected such that the utility of medical service nodes in the k -neighborhood of $p$ is maximized. If there are no wounded people, then the nearest supply node with positive supplies is selected as node $q$. Thus, a two-stop shortest path (partial itinerary) $I_{l}$ is constructed from a
sequence of three nodes $(o, p, q)$ where the temporal match among demands and supplies on the path is maximized.

The differential utility of delivering one unit of commodity or one wounded person from a node $r$ to a node $o$ is calculated in eq. (5-1) and (5-2), respectively.

$$
\begin{align*}
& U R_{\text {aro }}=\left(T-\max \left\{\lambda_{a o}, \text { at }_{o l}\right\}\right) P_{a}  \tag{5-1}\\
& U R_{\text {hro }}=\left(T-\text { at }_{o l}-\text { wait }_{h o}\right) P_{h} \tag{5-2}
\end{align*}
$$

where $a t_{o l}$ is the vehicle $l$ arrival time at node $o$, and $\lambda_{a o}$ is the time when positive unsatisfied demand exists at node $o$. In eq. (5-1), by taking the maximum of the latter two items, we account for anticipated future commodity demand. In eq. (5-2), waitho is the time that any additional wounded person would have to wait in queue at service node $o$. Since there is a queue of patients, $q u e_{h o}$, at each service node $o$, waitho is calculated as $\left\lceil\right.$ que $_{h o} /$ scap $_{\mathrm{ho}} 7^{+}$, where $\lceil\mathrm{a} / \mathrm{b}\rceil^{+}$represents division result rounded up to next higher integer.

Node $r$ should have demand-matching commodity supplies (or wounded people to be picked up) so that positive utility can be created by commodity demand (or medical facility) at node $o$. The utility, $U_{\text {Iro }}$, achieved by transporting commodities and/or wounded people from node $r$ to node $o$ is calculated by integrating the quantities $Z_{\text {arot }}$ and $X_{\text {rot }}$ with $U R_{\text {aro }}$, and $U R_{\text {wro }}$, respectively. This load is limited by available vehicle capacity throughout path ( $r, o$ ), available supplies and demands of matching commodity types and/or the number of wounded people at node $r$. It is assumed that vehicle capacity is consumed by loads of descending priority. The total utility for a partial itinerary is the sum of all such utilities created over pairs of nodes on the path.

### 5.2.2 Illustration on a Small Example



Supply node (supplyl, supply2, service rate, queue length)

Demand node (demand ${ }_{1}$, demand ${ }_{2}$, waiting wounded)

Figure 5-1 An example illustrating the neighborhood of a vehicle at node 0
An example for constructing $I_{l}$ in a given neighborhood is illustrated in Figure 5-1 where supply and hospital nodes are represented with label (supply1, supply2, service rate $(<=0)$, wounded queue length), demand nodes $(\mathrm{CD})$ with label (demand1 $(<=0)$, demand2 $(<=0)$, number of wounded waiting). Service rate is negative as it is treated as a demand rate. The vehicle is at node 0 that has both positive supplies and a medical facility. The length of the planning horizon, T, is 24 periods. The priorities, $\mathrm{P}_{\mathrm{a}}$, for the first and second commodities are $1 /$ unit and $2 /$ unit respectively, whereas the priority, $\mathrm{P}_{\mathrm{h}}$, of transporting 1 wounded person is 4/person. The vehicle has 6 units of shared capacity for the two commodities and a separate capacity of 5 wounded persons. In Table 5-1, the vehicle's neighborhood is listed with corresponding utility calculations, tentative loads (commodities + wounded persons), and tentative vehicle
capacity at each node on the path. For sake of simple presentation, it is assumed that anticipated demand does not exist in this sub-network. Commodities 1 and 2 are indicated as c 1 and c 2 , wounded persons as h in Table 5-1 and $\mathrm{r}, \mathrm{p}, \mathrm{q}$ are nodes.

Table 5-1 Illustration of utility calculations in neighborhood generation

| Source | $p$ | $\boldsymbol{q}$ | Tentative Load |  |  | Tentative Capacity |  |  | Tentative Adjusted Demand/Supply |  |  |  | Path Utility \{0-p-q\} |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $c_{1}$ | $c_{2}$ | $h$ | $r$ | $\begin{aligned} & c_{1}+ \\ & c_{2} \\ & \hline \end{aligned}$ |  | $r$ | $c_{1}$ | $c_{2}$ | que |  |
| 0 | 2 | 0 | - | 3 | 1 | $\begin{array}{\|l\|} \hline 0 \\ 2 \\ 0 \\ \hline \end{array}$ | $\begin{aligned} & 3 \\ & 6 \\ & 6 \end{aligned}$ | $\begin{array}{\|l\|} \hline 5 \\ 4 \\ 5 \\ \hline \end{array}$ | $\begin{aligned} & 0 \\ & 2 \\ & 0 \end{aligned}$ | $\begin{array}{\|l} \hline 3 \\ 0 \\ 3 \end{array}$ | $\begin{array}{\|l} \hline 2 \\ 0 \\ 2 \\ \hline \end{array}$ | $\begin{aligned} & 5 \\ & 0 \\ & 1 \\ & \hline \end{aligned}$ | $(24-7) \cdot 2.3+(24-$ <br> 14). $4.1=142$ |
|  |  | $\begin{gathered} \hline 7-8-9- \\ \mathbf{1 0} \end{gathered}$ | 5 | 4 | 3 | $\begin{array}{\|l\|} \hline 0 \\ 2 \\ 7 \\ 8 \\ 9 \\ 10 \\ \hline \end{array}$ | $\begin{aligned} & 2 \\ & 5 \\ & 0 \\ & 5 \\ & 6 \\ & 6 \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline 5 \\ 4 \\ 4 \\ 4 \\ 2 \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline 0 \\ 2 \\ 7 \\ 8 \\ 8 \\ 9 \\ 10 \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline 3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \hline \end{array}$ | $\begin{array}{\|l} \hline 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 4 \\ \hline \end{array}$ | $\begin{aligned} & 5 \\ & 5 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 3 \\ & \hline \end{aligned}$ | $\begin{aligned} & (24-7) \cdot 2 \cdot 3+(24-22) \cdot 4 \cdot 3+ \\ & (24-16) \cdot 1 \cdot 5+(24- \\ & 20) \cdot 2 \cdot 1=174 \end{aligned}$ |
|  |  | $\begin{gathered} 7-8- \\ 11-12 \end{gathered}$ | 5 | 4 | 5 | $\|l\|$ <br> 0 <br> 2 <br> 7 <br> 8 <br> 11 <br> 12 | $\begin{aligned} & 2 \\ & 5 \\ & 0 \\ & 0 \\ & 5 \\ & 6 \\ & 6 \end{aligned}$ | $\begin{array}{\|l\|} \hline 5 \\ 4 \\ 4 \\ 4 \\ 0 \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline 0 \\ 2 \\ 7 \\ 8 \\ 8 \\ 11 \\ 12 \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline 3 \\ 0 \\ 1 \\ 0 \\ -1 \\ 0 \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline 2 \\ 0 \\ 0 \\ 0 \\ -4 \\ 0 \end{array}$ | $\begin{aligned} & \hline 5 \\ & 0 \\ & 0 \\ & 0 \\ & 2 \\ & 5 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline(24-7) \cdot 2 \cdot 3+ \\ & \max \{0,(24-25)\} \cdot 4 \cdot 5+ \\ & (24-16) \cdot 1 \cdot 5+(24- \\ & 22) \cdot 2 \cdot 1=146 \end{aligned}$ |
|  | $\begin{gathered} 6- \\ \hline 7-8 \end{gathered}$ | 7 | 5 | 0 | 0 | $\begin{aligned} & \hline 0 \\ & 6 \\ & 7 \\ & 8 \\ & 7 \end{aligned}$ | $\begin{aligned} & 6 \\ & 6 \\ & 1 \\ & 6 \\ & 6 \end{aligned}$ | $\begin{aligned} & \hline 5 \\ & 5 \\ & 5 \\ & 5 \\ & 5 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 0 \\ & 6 \\ & 7 \\ & 8 \\ & 7 \end{aligned}$ | $\begin{array}{\|l\|} \hline 3 \\ 10 \\ 1 \\ 0 \\ 1 \\ \hline \end{array}$ | $\begin{aligned} & \hline 5 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 5 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | (24-10).1.5=70 |
|  |  | 9-10 | 5 | 1 | 2 | $\begin{array}{\|l} \hline 0 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array}$ | $\begin{aligned} & 5 \\ & 5 \\ & 5 \\ & 0 \\ & 5 \\ & 6 \\ & 6 \end{aligned}$ | $\begin{aligned} & \hline 5 \\ & 5 \\ & 5 \\ & 5 \\ & \hline 3 \\ & 5 \end{aligned}$ | $\begin{array}{\|l\|} \hline 0 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array}$ | $\begin{array}{\|l\|} \hline 3 \\ 10 \\ 1 \\ 0 \\ 0 \\ 0 \end{array}$ | $\begin{array}{\|l\|} \hline 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 4 \end{array}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 2 \end{aligned}$ | $\begin{aligned} & (24-10) \cdot 1 \cdot 5+(24-16) \cdot 4 \cdot 2+ \\ & (24-14) \cdot 2 \cdot 1=154 \end{aligned}$ |
|  |  | 11-12 | 5 | 1 | 5 | $\begin{array}{\|l\|} \hline 0 \\ 6 \\ 7 \\ 7 \\ 8 \\ 11 \\ \hline 12 \\ \hline \end{array}$ | $\begin{aligned} & 5 \\ & 5 \\ & 0 \\ & 0 \\ & 5 \\ & 6 \\ & 6 \end{aligned}$ | $\begin{array}{\|l\|} \hline 5 \\ 5 \\ 5 \\ 5 \\ 0 \\ 5 \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline 0 \\ 6 \\ 7 \\ 8 \\ 11 \\ 12 \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline 3 \\ 10 \\ 1 \\ 0 \\ -1 \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline 4 \\ 0 \\ 0 \\ 0 \\ -4 \\ 0 \\ \hline \end{array}$ | $\begin{aligned} & \hline 0 \\ & 0 \\ & 0 \\ & 0 \\ & 1 \\ & 5 \end{aligned}$ | $\begin{aligned} & \hline(24-10) \cdot 1 \cdot 5+(24-16) \cdot 2 \cdot 1+ \\ & (24-19) \cdot 4 \cdot 5=186 \end{aligned}$ |
|  | 1 | 0 | 2 | 1 | 3 | 1 1 0 | 3 6 6 | 5 2 5 | 0 <br> 1 <br> 0 | 1 0 1 | 4 0 4 | 5 0 3 | $\begin{aligned} & (24-11) \cdot 1 \cdot 2+(24-11) \cdot 2 \cdot 1+ \\ & (24-22) \cdot 4 \cdot 3=76 \end{aligned}$ |

The vehicle looks for the 3-nearest demand nodes with positive utility, these are: nodes 2,8 , and 1 in ascending order of distance. Node 2 is adjacent to node 0 and it needs 3 units of commodity 2 and 1 wounded person is waiting. Utility contribution
for delivering 3 units of second commodity is (24-7) $2 \times 3=102$, assuming that the vehicle starting time is zero. The next step is to construct a 3-node neighborhood for Node 2 in search of a hospital to carry the wounded person. The nearest hospital is node 0 where a waiting time of 2 hours exists. Utility of this itinerary is $102+$ (24-14$\max \{0,2-14\}) \times 4 \times 1=142$. The other two hospitals are at nodes 10 and 12 . There would not be a large utility for carrying the person from node 2 to Node 10 , because the vehicle would arrive there after 22 hours, the utility being (24-22) $\times 4 \times 1=8$. However, on the way, 5 units of first commodity at Node 8 would be supplied by node 7 with utility $=(24-16) \times 1 \times 5=40)$ and 1 unit of second commodity would be brought all the way from node 0 to be delivered at node 9 utility $=(24-20) \times 2 \times 1=8$, and finally 2 more wounded people would be picked up from node 9 to be dropped at node 10 , with utility of 16 . Utility of itinerary $(0-2-10)$ would be $102+40+8+8+16=174$. The latter is superior as compared with the itinerary $0-2-0$. The last option for taking the wounded person at node 2 to hospital is to take him/her to node 12 . The utility of this path would be $102+40+(24-22) \times 2 \times 1+\max \{0,24-25\} \times 4 \times 5=146$. The last two terms account for delivering 1 unit of second commodity from node 0 to node 11 , and taking 4 persons from node 11 to node 12 . Note that the rule "demand is met by the nearest supply node" is used as first come-first served on the path. Consequently, for the path $0-2$, the best option would be Node 10 , that is, the partial itinerary becomes 0-2-7-8-910. The other two demand nodes, 8 and 1, conduct their own exploration and their utilities are calculated so as to decide which path the vehicle should take from node 0 . For the path $0-6-7-8$, where Node 8 is the target demand node (first stop), there are no wounded people to carry, hence, the nearest supply/hospital nodes are sought (nodes 7, 10 and 12). The path from node 8 to node 12 has a demand node on the way and this creates positive utility for the vehicle as there are more wounded persons to pick
up from that node. Finally, node 1 is a dead end node, and the best option is to pick up the wounded and return to node 0 . Other options emanating from node 1 do not create positive utility because the arrival time to any node other than node 0 is larger than $T$, so these options are omitted from Table 5-1. Among all 3 paths in the neighborhood of the vehicle, the partial itinerary $0-6-7-8-11-12$ is the maximum utility path to be appended to its current partial route.

### 5.2.3 Parallel Vehicle Exploration

All $I_{l}$ are generated in parallel for each ready vehicle independently and evaluated as a candidate to be appended to each vehicle's current partial route, $R_{l}$. In this approach, all vehicles work out their paths independently in free competition for load pick up and demand satisfaction. Then, the vehicle $l^{*}$ having the itinerary $I_{l^{*}}$ with maximum utility is selected and its $R_{l}$ is updated. This completes one iteration of PATH_BUILDER. The reason a single vehicle itinerary extension is carried out in each iteration, is to avoid sub-optimal solutions as much as possible. If the best $I_{l}$ of $l \neq$ l* are appended to their itineraries all at once, one might end up with an inferior solution, because, after the assignment of $l^{*}$, load conditions of other vehicles' maximum utility might be changed and may no longer represent the best $I_{l}$ for the corresponding vehicles.

The iterations continue until either all vehicles are assigned itineraries that last until end of the planning horizon, T or, either all supplies or demanded quantities are transported. In brief, iterations stop when no positive utility can be generated for any vehicle.

In dynamic emergency settings, new information can be incorporated by stopping each vehicle at the node it arrives right after the schedule disturbance time. The partial dispatch schedule is then frozen for each vehicle, with all transported quantity updates made accordingly. New information entries are made and a new dispatch plan is obtained by constructing partial itineraries from that time period onwards.

### 5.2.4 Numerical Results

The performance of PATH_BUILDER is tested on 28 randomly generated test problems constructed on grid networks with integer arc travel times. All nodes in the network are connected first by constructing a minimum spanning tree, and then, node degrees are increased randomly by pairwise node connection. The number of arcs is limited to simulate the sparse road network in practice. The instances are generated as follows. Number of nodes range between [20, 80]. About 30\% of the nodes are allocated to supply and hospital nodes (all hospitals are assumed to be overlapped with supply nodes). The networks are generated on a $12 \times 12$ grid, where each cell is a probabilistically allocated node. The total number of vehicles range between [20, 65], and there are three types of vehicles, the first and last types having the ability to carry commodities and vehicle type 2 can carry wounded persons. The vehicle type (type 1 ) that can carry both wounded and commodities, provides a joint total capacity. Two levels of injury are categorized and there are two kinds of commodities in all instances. Demand and supply are given in number of persons. Demand/supply quantities as well as the service rate and wounded people are assigned to each corresponding node randomly according to a uniform distribution while total supply sufficiency is ensured. For each test problem, the number of vehicles lies within a given interval. Then, overall vehicle tightness with regard to commodity and people is
calculated by the total transportation quantity over the total compatible capacity. A value greater than 1.0 implies tight transportation capacity and vice versa. If the maximum value of commodity and people tightness is larger than 1.0 , the problem is classified as capacity tight. Half of the test problems are designed as capacity tight (average tightness for capacity tight problems is 2.06 ) and the remaining half are capacity loose (average tightness for loose problems is 0.82 ).

Table 5-2 Characteristics of test problems

| Problem | No. of <br> Nodes | No. of <br> Arcs | No. of <br> Vehicles | Tightness |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 20 | 351 | 20 | 2.53 |
| $1^{\prime}$ | 20 | 364 | 25 | 1.54 |
| 2 | 20 | 414 | 21 | 0.79 |
| $2^{\prime}$ | 20 | 373 | 16 | 1.23 |
| 3 | 30 | 606 | 20 | 2.30 |
| $3^{\prime}$ | 30 | 555 | 12 | 4.13 |
| 4 | 30 | 630 | 25 | 0.86 |
| $4^{\prime}$ | 30 | 536 | 18 | 1.17 |
| 5 | 40 | 871 | 26 | 1.61 |
| $5^{\prime}$ | 40 | 895 | 18 | 2.73 |
| 6 | 40 | 837 | 40 | 0.89 |
| $6^{\prime}$ | 40 | 853 | 27 | 1.39 |
| 7 | 50 | 975 | 32 | 2.46 |
| 7 | 50 | 968 | 25 | 3.30 |
| 8 | 50 | 987 | 36 | 0.80 |
| $8^{\prime}$ | 50 | 1038 | 43 | 0.62 |
| 9 | 60 | 1134 | 41 | 1.97 |
| $9^{\prime}$ | 60 | 1354 | 49 | 1.51 |
| 10 | 60 | 1014 | 38 | 0.85 |
| $10^{\prime}$ | 60 | 1166 | 46 | 0.73 |
| 11 | 70 | 1580 | 46 | 1.79 |
| $11^{\prime}$ | 70 | 1633 | 40 | 2.18 |
| 12 | 70 | 1598 | 49 | 0.79 |
| $12^{\prime}$ | 70 | 1828 | 40 | 1.00 |
| 13 | 80 | 1657 | 55 | 1.68 |
| $13^{\prime}$ | 80 | 1719 | 65 | 1.47 |
| 14 | 80 | 1686 | 65 | 0.77 |
| 14 | 80 | 1600 | 55 | 0.77 |
|  |  |  |  |  |

Details on problem characteristics are provided in Table 5-2. The second and third columns present the nodes number and actual number of arcs which consists of all the nodes pairs reachable throughout the whole planning horizon $\left(\mathrm{K}_{\mathrm{osptm}}=1, \forall \mathrm{o} \in \mathrm{C}, \mathrm{p} \in \mathrm{C}\right.$,
$\mathrm{s} \in \mathrm{T}, \mathrm{t} \in \mathrm{T}, \mathrm{m} \in \mathrm{M}$ ). The actual number of arcs is obtained by running a search algorithm that fans out from vehicle depots and identifies all the reachable arcs. To get more extensive results, two problems in the same group (for example, problem 1 and 1 ') are generated for each network structure while they have different vehicle distribution and arc connection. Consequently, the capacity tightness and actual numbers of arcs in the resulting expanded network are different. Two groups of instances are generated for each problem size (node number); hence, there are in total 4 instances for a certain node number. The aggregate number of vehicles is given for the three vehicle types. The planning horizon is set to 10 periods.

The PATH_BUILDER algorithm is implemented in C++ and all runs are made on a PC of 3.2 GHz CPU and 512 MB RAM. To evaluate the solution efficiency of the algorithm, direct solutions of model P are provided by executing the MIP solver ILOG CPLEX 7.5 on the same computer. CPLEX has been shown as a highly efficient solver for many multi-commodity flow problems (Castro, 2003) and therefore it serves as the reference solver in this study. All results are given in detail in Table 5-3. Here, the instance size is limited by the memory requirements of model P solution in CPLEX. The optimal solution value and CPU time in seconds are noted. The solution quality of the heuristic is presented by the gap between algorithm solution value and the optimal one. Model P near optima are obtained by applying the following relative gap termination criterion: $0 \%$ for problems with nodes less than 40 , and $0.5 \%$ for larger problems, except that the gap was held at $1 \%$ for problems $5^{\prime}, 9^{\prime}$, 12 and $12^{\prime}$. These very small tolerances do not weaken solution quality much while saving substantial CPU time on the branch and bound process in proving that a solution found is the best. CPU times taken to find optimum and heuristic solutions are shown in columns 3 and 5 .

Table 5-3 Detailed numerical results

| Problem | Model Solution |  | Heuristic |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Obj Value | Runtime Secs. | Obj Value | Runtime Secs. | Obj Gap |
| 1 | 51689 | 1.56 | 52151 | 0.72 | 0.89\% |
| 1 ' | 49477 | 1.52 | 50299 | 0.85 | 1.66\% |
| 2 | 19748 | 2.25 | 20976 | 2.44 | 6.22\% |
| 2' | 20362 | 36.16 | 21183 | 1.46 | 4.03\% |
| 3 | 52144 | 447.25 | 53067 | 2.86 | 1.77\% |
| 3' | 54000 | 105.31 | 54720 | 0.68 | 1.33\% |
| 4 | 17103 | 4.20 | 17594 | 2.77 | 2.87\% |
| 4' | 19475 | 5.00 | 19808 | 0.85 | 1.71\% |
| 5 | 41041 | 589.77 | 43376 | 11.17 | 5.69\% |
| 5' | 42886 | 1282.44 | 45038 | 3.50 | 5.02\% |
| 6 | 38610 | 89.52 | 41336 | 8.42 | 7.06\% |
| $6 '$ | 40091 | 36.44 | 42109 | 5.06 | 5.03\% |
| 7 | 99554 | 70.25 | 104210 | 20.73 | 4.68\% |
| $7{ }^{\prime}$ | 102745 | 733.02 | 106521 | 12.38 | 3.68\% |
| 8 | 35222 | 221.50 | 35417 | 4.44 | 0.55\% |
| 8' | 32943 | 55.41 | 34659 | 9.54 | 5.21\% |
| 9 | 90082 | 246.89 | 98591 | 20.14 | 9.45\% |
| 9' | 82851 | 1181.94 | 93226 | 51.78 | 12.52\% |
| 10 | 28643 | 149.47 | 32767 | 20.73 | 14.40\% |
| 10' | 25496 | 156.95 | 29857 | 33.04 | 17.10\% |
| 11 | 96443 | 152.67 | 100403 | 64.69 | 4.11\% |
| 11' | 90317 | 703.00 | 95374 | 31.28 | 5.60\% |
| 12 | 45655 | 1951.23 | 49098 | 51.50 | 7.54\% |
| 12' | 43604 | 4033.97 | 46832 | 52.82 | 7.40\% |
| 13 | 106694 | 367.31 | 114114 | 188.76 | 6.95\% |
| $13 '$ | 102024 | 593.86 | 108593 | 264.35 | 6.44\% |
| 14 | 56769 | 289.05 | 59958 | 126.07 | 5.62\% |
| $14^{\prime}$ | 57820 | 363.64 | 60628 | 108.12 | 4.86\% |

It is observed that the CPU times taken by the PATH_BUILDER are almost always less than the direct model solution. In Table 5-4 and Table 5-5, the summary of results is provided. The first property that significantly affects heuristic performance is the problem size, which can be observed on both the heuristic and the direct model solution. The average percentile deviation from the optimum grows from $3.61 \%$ for small instances to above $7 \%$ for larger scale problems. The heuristic is also affected by global transportation capacity tightness on both solution quality and time aspects. The heuristic produces better solutions for tight instances. This might be because the utilities distribution is relatively in an average state among both current and
subsequent partial paths so that myopic path selection is avoided in tight problems. However, this feature also results in more exploration time for tight problems while in capacity loose instances the number of partial paths with positive utilities is less through the planning horizon. This effect is not obvious on smaller size problems less than 40 nodes due to the short CPU times consumed for both instances; however, the difference grows in larger scale problems over 40 nodes.

Table 5-4 Summary of results- gap of objective value

|  | Gap of Obj Value |  |  |
| :---: | :---: | :---: | :---: |
|  | All problems | Tight | Not tight |
| All problems | $5.69 \%$ | $4.98 \%$ | $6.40 \%$ |
| Nodes<=40 | $3.61 \%$ | $2.73 \%$ | $4.49 \%$ |
| Nodes>40 | $7.26 \%$ | $6.68 \%$ | $7.83 \%$ |

Table 5-5 Summary of results- run time

|  | Runtime- Heuristic |  |  | Runtime- Model Solution |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | All problems | Tight | Not tight | All problems | Tight | Not tight |
| All problems | 39.33 | 48.13 | 30.52 | 495.41 | 462.63 | 528.20 |
| Nodes $<=40$ | 3.40 | 3.29 | 3.50 | 216.78 | 404.64 | 28.93 |
| Nodes $>40$ | 66.27 | 81.76 | 50.78 | 704.38 | 506.12 | 902.65 |

To summarize, a quick solution approach is proposed in this section for the complex logistics support and evacuation coordination problem that arises in emergencies. PATH_BUILDER constructs a feasible itinerary for available vehicles quickly with the goal of maximizing service level for survivors and injured people. The performance of PATH_BUILDER is measured on a set of randomly generated gridnetworks so as to maintain integral valued arc travel times. Analyzing the overall solution quality and times, one can say that the algorithm could be an alternative to the exact model solution by providing substantial savings on computational time.

### 5.2.5 Improvements on the Constructive Heuristic

A possible development on the construction heuristic is the local search based methods; for instance, two-stop or one-stop swaps among routes of different vehicles
might lead to an improvement. However, this might bring significant computational cost due to the load feasibility conservation calculations necessary for such swaps. For the same reason, perturbation moves found in existing routing literature would not be as easy to effectively implement in this problem. Hence, a direct enhancement on the solution quality seems difficult to achieve based on this algorithm.

Given the successful applications of meta-heuristics on combinatorial problems in the last twenty years, it is promising to develop more complex scheme to perform deep exploration of the most promising regions of the solution space. Another opportunity to enhance the overall solution efficiency is to exploit the network flow features which are neglected in vehicle-based heuristic PATH_BUILDER. Finally, although the parallel search employed in the constructive heuristic enhances the solution quality, it also leads to an increase in the runtime. Moreover, the information generated in the parallel process is discarded without any exploitation, which results in low efficiency on the computational resource utilization. Based on these observations, a meta-heuristic of ant colony optimization (ACO) is presented in the next section and compared with the constructive heuristic.

### 5.3 The ACO Meta-heuristic for Disaster Relief Operations

Given the hybrid characteristics of the ELP, it is a natural way to decompose the model into two components: the vehicle route construction, and the multi-commodity dispatch. They are solved sequentially where the first phase constructs the vehicles' route, and then the multi-commodity problem is solved based on the resulting vehicle flows. Thus a solution of the original problem is given. Generally, a one-pass process may not produce good solution; an iteration framework must be developed, which enables both the diversification on building vehicle paths and the efficiency on
multiple commodities dispatch, as well as smooth communication between the two phases for the continuous improvement of solution quality. An ACO meta-heuristic is proposed for the problem in this section. It builds vehicle paths probabilistically under the guidance of pheromone trails while in the second phase a successive maximum flow (SMF) algorithm is developed for the commodities dispatch to different types of vehicle flows. Pheromone trails are updated according to the dispatch resulting from SMF. Thus, the two sub-problems are coordinated through trails and thereby integrated into the overall solution framework. A time expanded network is employed in this algorithm due to the dynamic structure of the model. The notation is defined next:
l: A specific vehicle label
$(t, o, i):$ An arc with tail node $o$ at time $t$ and head node $i$; the arrival time at head node $i$ by vehicle type $m$ is implicitly set to $t+t_{o i m}$
$L_{\text {tom }}$ : Set of the neighborhood nodes of current node $o$ at time $t$ by vehicle in type $m$ $a t_{o l}: \quad$ Arrival time at node $o$ by vehicle $l$
$\tau_{\text {toim }}$ : Amount of accumulated pheromone trails of vehicles in type $m$ on the arc $(t, o, i) ; \tau_{\text {toim }}^{\mu}$ denotes the pheromone trail in solution $\mu$
$p_{\text {tojm }}$ : Probability for choosing the next node $j$ from node $o$ at time $t$ by vehicle in type $m$

DEM: Set of demand types defined jointly by the original commodity (or wounded) demand type and the time period it emerges, dem $_{a t}$ or $_{\text {dem }}^{h t}$ denotes a specific type in the set
$U_{\text {toil }}^{\mu}$ : Utility achieved by vehicle $l$ on arc $(t, o, i)$ in solution $\mu$
$\sigma: \quad$ Preserved set of best solutions obtained.

A diagram is given below as an overall structure preview of the algorithm while the components are described with details in the following subsections.


Figure 5-2 Structure of the ACO meta-heuristic

### 5.3.1 Route Construction

ACO is a meta-heuristic approach inspired by the pheromone trail tracking and it imitates the behavior of ants that communicate with pheromones. It is proposed for solving hard combinatorial optimization problems and was first used on the traveling salesman problem, and has since then been successfully applied to several other problems such as the vehicle routing problem (Bullnheimer et al., 1999; Bell and McMullen, 2004), the quadratic assignment problem (Maniezzo, 1999), the scheduling problem (Merkle, 2000), and so on. A detailed description of theoretical results and applications on ACO can be found in recent papers by Dorigo and Blum (2005), and Dorigo and Stützle (2002).

Artificial ants used in the ACO meta-heuristic conduct stochastic vehicle paths construction procedures that probabilistically build a route by iteratively adding arcs to partial itinerary. The addition of arcs is made by taking into account pheromone
trails that change dynamically to reflect the ants' acquired search experience. To facilitate the information sharing and cooperative behavior among vehicles in the same type, ants are also classified by vehicle types and pheromone trails are differentiated and aggregated in terms of types.

The algorithm runs on the time expanded network, which consists of all the nodes pairs reachable throughout the whole planning horizon by a vehicle type. Especially, it is assumed that a hold-over arc (the arc connecting a node from period t' to t' $+1, \forall$ $\left.\mathrm{t}^{\prime} \in \mathrm{T}\right)$ has infinite capacity. Initially, the algorithm assigns a ready time $a t_{o l}$ to all ants (vehicles) available at node $o$ based on the parameter set $a v_{\text {omt }}(l \in m)$. Each ant is tracked individually, and it selects the next customer node $j$ to visit from the list of feasible locations $L_{\text {tom }}$ and the path decision time is advanced to $a t_{j l}=a t_{o l}+t_{o j m}$ untill it reaches the end of planning horizon T . The content of $L_{\text {tom }}$ is set to all nodes adjacent to node $o$ in node set C , and it is tractable because not all pairs of nodes are adjacent to each other in a practical network. Moreover, the sparse network is represented in adjacency-list form for space efficiency. To select the next customer $j$ for vehicle $l \in m$, the ant uses the following probabilistic formula - also called the transition probability (Dorigo and Blum, 2005) - defined by eq.(5-3)(5-3):

$$
\begin{equation*}
p_{\text {tojm }}=\frac{\tau_{\text {tojm }}}{\sum_{i \in L_{\text {tom }}} \tau_{\text {toim }}} \quad \text { if } j \in L_{\text {tom }} \tag{5-3}
\end{equation*}
$$

The algorithm constructs a complete tour for the current ant and prior to the next ant starting its tour. This continues until each ant constructs a feasible route and reaches the end of planning horizon. The route selection rule in eq. (5-3) is different from the usual ACO implementation. The greedy selection of the most favorable path is observed being dominated in the computation test and therefore forbidden in this
problem. In fact, due to the very large dimensionality of the problem and the converging search space in ACO, diversification (exploration) plays a key role in solution improvement process. It is observed that a real time (or frequent) probability updating scheme (strong exploitation) usually worsens solution quality although convergence is accelerated. The results are generally better when probability updating (eq. (5-3)) is conducted regularly using a limited elites set ( $\sigma$ ) of best solutions. It is performed only if the following two conditions are satisfied: (1) new elite solution $\mu$ enters set $\sigma$; and (2) a predetermined number $\left(0.5^{*}|\sigma|\right.$ in our implementation) of iterations are performed. The condition (1) is the common idea in ACO literature, while condition (2) ensures a wide search in the current solution stage so as to reduce the chance of missing some good solution zones.

In addition, one may note that the desirability is not employed in this problem. Different from the conventional routing problem where length (cost) reveals the attractiveness of an arc, the desirability in this problem should be defined as the expected on utility to be achieved on that arc. However, these values could be drastically uneven among arcs and result in a myopic vehicle route choice. Hence, instead of using a very small influence factor to counteract this effect, the desirability is not explicitly defined but included in the pheromone trail information and a backward arc utility updating procedure (next subsection) is applied to reveal a relatively fair measure of arc attractiveness.

In this problem, pheromone trail $\tau_{\text {toiv }}^{\mu}$ from a provisional solution $\mu$ is calculated as the utilities over all vehicles of type $v$ on the $\operatorname{arc}(\mathrm{t}, \mathrm{o}, \mathrm{i})$ :

$$
\begin{equation*}
\tau_{\text {toim }}^{\mu}=\sum_{l \in m} U_{\text {toil }}^{\mu} \tag{5-4}
\end{equation*}
$$

where the vehicle utility $U_{\text {toil }}^{\mu}$ is evaluated by the contribution to the objective value and calculated by eq. (5-5) in the later commodities dispatch phase.

### 5.3.2 Commodities Dispatch and Trail Updating Strategies

After the vehicle routing is settled, the commodities dispatch to be addressed in this phase is an integer multi-commodity flow problem. It contributes to the final solution quality by directly affecting the provisional objective value and the pheromone trail updating. Compared to the vehicle routes construction in phase one, this phase poses more computational burden due to the complexity in the integer multi-commodity flow problem. Moreover, the ACO meta-heuristic may run over large numbers of iterations. As discussed before, the solution speed is a major concern in real emergency situations for the accommodation of frequent re-planning incurred by information updates. Hence, a tradeoff between dispatch quality and speed must be made.

Given these considerations, a successive maximum flow heuristic algorithm (SMF) is developed to solve the commodity dispatch problem. Initially a set DEM of demand types is constructed, which gives out at most $(|\mathrm{H}|+|\mathrm{A}|) *|\mathrm{~T}|$ kinds of demand since the service rates at medical facilities are formulated in Model P as demands for wounded people on time period basis. Then SMF decomposes the multi-commodity flow problem into maximal flow components regarding each demand type, which are sorted and solved sequentially with the following heuristic procedure:

## Procedure SMF:

(i) For each demand type $\left(\operatorname{dem}_{a t}\right.$ or $\left.\operatorname{dem}_{h t}\right)$ in DEM, calculate the unit weight utility $\left(\mathrm{uw}_{\mathrm{at}}=P_{a}{ }^{*}(\mathrm{~T}-\mathrm{t}) / w_{a} ; \mathrm{uw}_{\mathrm{ht}}=P_{h}{ }^{*}(\mathrm{~T}-\mathrm{t}) / w_{h}\right)$;
(ii) Sort DEM in descending order according to $\mathrm{uw}_{\mathrm{at}}$ or $\mathrm{uw}_{\mathrm{ht}}$;
(iii) For each demand $\operatorname{dem}_{a t}\left(\right.$ or $\left.\operatorname{dem}_{h t}\right)>0$ in DEM:
(a) Allocate the existing flow for the last $\operatorname{dem}_{a^{\prime} t}\left(\right.$ or $\operatorname{dem}_{h^{\prime} t}$ ) to vehicle flows, update vehicle capacity and utility $U_{\text {toil }}^{\mu}$ : for each vehicle $l$ and arc $(t, o, i)$ in its path $\left(t+t_{\text {oim }}=t^{\prime}, l \in m\right)$, suppose $x$ units of flow $a^{\prime}$ (or $\left.h^{\prime}\right)$ is assigned, then:

$$
\begin{equation*}
U_{\text {toil }}^{\mu}=U_{\text {toil }}^{\mu}+P_{a^{\prime}} *\left(|T|-t^{\prime}\right) * x\left(\text { or } U_{\text {toil }}^{\mu}=U_{\text {toil }}^{\mu}+P_{h^{\prime}} *\left(|T|-t^{\prime}\right) * x\right) \tag{5-5}
\end{equation*}
$$

(b) Build provisional network: identify vehicles compatible with dem $_{a t}$ (or $\operatorname{dem}_{h t}$, extract the arcs in their paths, as well as the hold-over arcs that span from the beginning of planning horizon to period $t$. Calculate the capacity of each arc with respect to the demand type $a$ (or $h$ ).
(c) Apply maximum flow algorithm to the provisional network; Update the supplies in type $a(h)$ and demand accordingly.
(d) If dem $_{h t}>0$ (surplus service rate), then the unsatisfied demand vanishes by setting $\operatorname{dem}_{h t}=0$; else if $\operatorname{dem}_{a>}>0$, then the unsatisfied demand transfers to the next period by setting $\operatorname{dem}_{a(t+1)}=\operatorname{dem}_{a(t+1)}+\operatorname{dem}_{a t}$, and $\operatorname{dem}_{a t}=0$.

The push-relabel method is employed in step (iii)(c) in SMF, which is known as the most efficient algorithm so far for the maximum flow problem (Cherkassky and Goldberg, 1997). In addition, the provisional networks constructed for each demand are quite compact due to the restriction of vehicle flows. Hence, the SMF algorithm shall not pose any problem on computation cost.

In order to improve future solutions, the pheromone trails must be updated to reflect the ant's performance and the quality of the solutions found. Promising solution spaces should be marked and favored in future iterations. The pheromone trail is
updated whenever a solution $\mu$ enters the elite set. For each vehicle type $m$ on arc ( t , $0, i)$, the following rule is applied:

$$
\begin{equation*}
\tau_{\text {toim }}=(1-\rho) \tau_{\text {toim }}+\rho \tau_{\text {toim }}^{\mu} \tag{5-6}
\end{equation*}
$$

A convex combination of the existing and added trails is employed here instead of evaporation process. By setting small parameter $\rho$ and a large initial trail value, eq.(5-6) suggests a diversified search at the initial stages of solution process (the accumulated trails on visited arcs decrease) whereas intensification is emphasized in the later as the accumulated trails on visited arcs begin increasing themselves. The balance scheme between intensification and diversification can be adjusted at ease by parameter $\rho$. In addition, the initial trail $\tau_{\text {toim }}^{0}$ is set to the maximal possible utility of vehicle type $m$ on the arc:

$$
\begin{equation*}
\tau_{\text {toim }}^{0}=\left(|T|-t-t_{\text {oim }}\right) * \max _{h \in H, a \in A}\left\{P_{h} *\left(\operatorname{cap}_{m} / w_{h}\right), P_{a} *\left(\operatorname{cap}_{m} / w_{a}\right)\right\} \tag{5-7}
\end{equation*}
$$

In emergency logistics operations, a vehicle may be fully utilized only in some stages of its route spanning the entire planning horizon, for example, an ambulance is loaded only in backhaul. Furthermore, some cooperation among vehicles could result in a higher overall performance, so it is not unusual to find in the optimal solution of the model that some vehicles are dedicated to the transfer work between other vehicles during some periods. In these situations, an arc choice based merely on the utilities achieved on the candidate arcs may lead to inferior results according to our observation, even though it is a probabilistic process. Therefore, a backward path utility updating procedure is called to address this issue before trail updating eq.(5-6), by which pheromone trails on the posterior arcs are revealed to front so that the ant could avoid myopic path choice: for each vehicle, scan the path from the end to the
beginning, record the maximal utility found so far, and use it to replace the utility on the current arc.

In addition to the ACO algorithm, a post-optimization procedure is also developed which improves existing solution by exploiting the most promising part of each elite solution and re-combining them into new solution. Because each solution may be biased on some vehicles, the combination of all those favored vehicles may produce good solution or even the best solution to replace an elite item and therefore enhance the convergence speed. The procedure is conducted whenever the probability updating is eligible to perform and at least one vehicle makes improvement on its best path. It can be described as follows:
(i) Construct a new set of vehicle paths by combining the best path of each vehicle, and
(ii) Run SMF and evaluate the solution quality: if a replacement happens, then update the elite set and trails.

### 5.3.3 Numerical Results

The ACO algorithm is implemented in C++ and all runs are made on the same set of test problems as in the previous section. In all ACO solutions, search parameters are set to the following values: $\rho=0.05,|\sigma|=20$. The algorithm terminates when there is no new solution entering the elite set in 30 consecutive iterations. All results are given in detail in Table 5-6. The optimal solution value and runtime in seconds are noted. The solution optimality quality of the ACO meta-heuristic is presented by the gap between algorithm solution value and the optima in column 4. The object value gap between the ACO algorithm and the greedy construction heuristic are also provided in column 5 to evaluate their solution qualities. CPU time is presented in column 3. In
addition, for the equitable measure of solution speed, the optimality gaps given by ACO (column 4 in Table 5-6) are fed back to CPLEX as tolerances, in which all the problems are re-solved. The runtimes of model solution with tolerance and the gaps between them and the ACO runtimes are presented in the last two columns.

Table 5-6 Numerical results

| Problem | ACO Meta-heuristic |  |  |  | Model Solution with Tolerance |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Obj Value | Runtime | Optimality Gap | Heuristic Gap | Runtime | Runtime Gap |
| 1 | 51858 | 0.97 | 0.33\% | -0.56\% | 1.38 | 42.19\% |
| 1 ' | 49683 | 0.88 | 0.42\% | -1.22\% | 1.23 | 40.71\% |
| 2 | 20272 | 2.38 | 2.65\% | -3.36\% | 1.19 | -50.15\% |
| 2' | 20877 | 2.17 | 2.53\% | -1.44\% | 2.73 | 25.93\% |
| 3 | 53307 | 6.79 | 2.23\% | 0.45\% | 12.75 | 87.78\% |
| 3' | 54568 | 2.67 | 1.05\% | -0.28\% | 31.97 | 1095.96\% |
| 4 | 17157 | 6.86 | 0.32\% | -2.48\% | 4.03 | -41.24\% |
| 4' | 19509 | 4.11 | 0.17\% | -1.51\% | 5.28 | 28.62\% |
| 5 | 42396 | 11.41 | 3.30\% | -2.26\% | 17.63 | 54.43\% |
| 5' | 45451 | 4.50 | 5.98\% | 0.92\% | 34.41 | 664.75\% |
| 6 | 39556 | 28.38 | 2.45\% | -4.31\% | 9.17 | -67.68\% |
| 6' | 41001 | 8.69 | 2.27\% | -2.63\% | 9.36 | 7.71\% |
| 7 | 102465 | 10.88 | 2.92\% | -1.67\% | 53.19 | 388.86\% |
| $7{ }^{\prime}$ | 105568 | 13.27 | 2.75\% | -0.89\% | 44.19 | 232.88\% |
| 8 | 35596 | 6.33 | 1.06\% | 0.51\% | 135.47 | 2040.43\% |
| 8' | 33363 | 11.17 | 1.28\% | -3.74\% | 24.05 | 115.27\% |
| 9 | 93501 | 28.18 | 3.79\% | -5.16\% | 96.02 | 240.74\% |
| 9' | 87490 | 30.93 | 5.60\% | -6.15\% | 99.41 | 221.43\% |
| 10 | 29604 | 10.46 | 3.35\% | -9.65\% | 74.11 | 608.57\% |
| 10' | 26759 | 11.14 | 4.95\% | -10.38\% | 85.44 | 666.94\% |
| 11 | 102107 | 24.62 | 5.87\% | 1.70\% | 110.83 | 350.15\% |
| 11' | 95699 | 21.75 | 5.96\% | 0.34\% | 117.28 | 439.35\% |
| 12 | 48408 | 38.00 | 6.03\% | -1.41\% | 126.98 | 234.21\% |
| $12^{\prime}$ | 46308 | 28.26 | 6.20\% | -1.12\% | 111.69 | 295.23\% |
| 13 | 111859 | 24.50 | 4.84\% | -1.98\% | 163.92 | 569.15\% |
| $13 '$ | 105817 | 31.84 | 3.72\% | -2.56\% | 168.42 | 428.89\% |
| 14 | 57394 | 30.82 | 1.10\% | -4.28\% | 200.48 | 550.56\% |
| $14 '$ | 59282 | 33.82 | 2.53\% | -2.22\% | 208.734 | 517.21\% |

Table 5-7 to Table 5-10 present the summary of results based on the data given in the above table. Table 5-7 measures performance robustness by the average value and standard deviation (SD) of CPU time on the problems grouped in same size. Columns 5, 8 and 11 show the relative standard deviation (RSD). The RSD values are high for
both solvers because the size of the problem affects CPU time significantly; besides, the data sets are quite different for the problems in a specific group. Nevertheless, ACO meta-heuristic shows a relatively more robust performance with lower RSD value of $76.74 \%$ than $94.12 \%$ in model solution with tolerance. This stable performance may result from the more compact representation of the network (adjacency list) in ACO implementation compared the incidence matrix in model P . The RSD value in PATH_BUILDER is the highest (159.30\%) showing that the performance of the greedy construction heuristic is highly dependent on the data setting.

Table 5-7 Summary of results- average and standard deviation of runtimes

| Group Problem | Heuristic Solution <br> Runtime |  |  | ACO <br> Meta-heuristic <br> Runtime |  |  | Model Solution with <br> Tolerance Runtime |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Average | SD | RSD | Average | SD | RSD | Average | SD | RSD |
| 1 | $1-2^{\prime}$ | 1.37 | 0.79 | $57.55 \%$ | 1.60 | 0.79 | $49.24 \%$ | 1.63 | 0.74 | $45.25 \%$ |
| 2 | $3-4^{\prime}$ | 1.79 | 1.19 | $66.38 \%$ | 5.11 | 2.07 | $40.49 \%$ | 13.51 | 12.90 | $95.47 \%$ |
| 3 | $5-6 '$ | 7.04 | 3.44 | $48.85 \%$ | 13.25 | 10.48 | $79.14 \%$ | 17.64 | 11.85 | $67.19 \%$ |
| 4 | $7-8^{\prime}$ | 11.77 | 6.82 | $57.89 \%$ | 10.41 | 2.92 | $28.08 \%$ | 64.22 | 49.03 | $76.35 \%$ |
| 5 | $9-10 '$ | 31.42 | 14.82 | $47.15 \%$ | 20.18 | 10.89 | $53.97 \%$ | 88.74 | 11.43 | $12.88 \%$ |
| 6 | $11-12^{\prime}$ | 50.07 | 13.86 | $27.68 \%$ | 28.15 | 7.08 | $25.15 \%$ | 116.70 | 7.43 | $6.37 \%$ |
| 7 | $13-14 '$ | 171.82 | 70.71 | $41.15 \%$ | 30.24 | 4.03 | $13.32 \%$ | 170.65 | 38.30 | $22.44 \%$ |
| All Problems |  | 39.33 | 62.64 | $159.30 \%$ | 15.56 | 11.94 | $76.74 \%$ | 69.69 | 65.59 | $94.12 \%$ |

The problems are classified according to the tightness and problem size in Table 5-8 and corresponding solution optimality gaps are provided. It is observed that the ACO meta-heuristic produces solutions with an average gap around $3 \%$. ACO solution quality is affected by transportation capacity tightness, where the average gaps are $3.48 \%$ and $2.64 \%$ for tight and non-tight problems, respectively. Problem size is the other impacting factor as expected, which results in a double average gap on larger problems. Furthermore, the influence of tightness increases with the problem size. For instance, considering smaller size problems with less than 40 nodes, there is only a small difference $(0.49 \%$ ) between tight and non-tight problems in the average gap
from the optimum. This difference grows to $1.12 \%$ in larger scale problems over 40 nodes. The objective value gaps from ACO to greedy heuristic are also provided in the last 3 columns in Table 5-8, from which one can see that the constructive heuristic is dominated by ACO algorithm.

Table 5-8 Summary of results- solution quality

|  | Optimality Gap of ACO |  |  | Heuristic Gap of ACO |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | All problems | Tight | Not tight | All problems | Tight | Not tight |
| All problems | $3.06 \%$ | $3.48 \%$ | $2.64 \%$ | $-2.41 \%$ | $-1.38 \%$ | $-3.43 \%$ |
| Nodes $<=40$ | $1.97 \%$ | $2.22 \%$ | $1.73 \%$ | $-1.56 \%$ | $-0.49 \%$ | $-2.62 \%$ |
| Nodes $>40$ | $3.87 \%$ | $4.43 \%$ | $3.31 \%$ | $-3.04 \%$ | $-2.05 \%$ | $-4.04 \%$ |

Solution speeds are presented in Table 5-9. The small instances with less tightness can be solved $16.13 \%$ faster with CPLEX than with the ACO meta-heuristic given the same tolerance. Although problem size contributes to the runtime increment for both methods, one can see the model P solution time (column 5) increases faster than ACO (column 2). Hence, the heuristic speed outperforms the direct model solution on larger instances. Furthermore, capacity tightness has virtually no influence on ACO algorithm speed. For the small scale instances, model solution with tolerance responses to capacity tightness while this effect diminishes on larger problems, which indicates the dominant factor affecting CPU time lies on the network configuration rather than tightness so that CPLEX is capable of finding solutions within stable CPU time for both tight (68.04 Secs.) and non-tight (71.34 Secs.) problems. Although it runs for a number of repetitive iterations, the ACO algorithm also shows faster speed than greedy heuristic except on the small instances with less than 40 nodes (Table 5-10). This is because PATH_BUILDER evaluates partial path utility for each vehicle resulting in larger resource consumption whereas the ACO meta-heuristic computes trails on the vehicle type basis and maintains only one path for each vehicle in an iteration.

Table 5-9 Summary of results- runtime

|  | Runtime-ACO Meta-heuristic |  | Runtime-Model Solution with <br> Tolerance |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | All problems | Tight | Not tight | All problems | Tight | Not tight |
| All problems | 15.56 | 15.23 | 15.90 | 69.69 | 68.04 | 71.34 |
| Nodes $<=40$ | 6.65 | 4.54 | 8.76 | 10.93 | 16.56 | 5.29 |
| Nodes $>40$ | 22.25 | 23.25 | 21.25 | 113.76 | 106.66 | 120.87 |

Table 5-10 Summary of results- runtime gap

|  | Runtime Gap- Model Solution with <br> Tolerance to ACO Meta-heuristic |  | Runtime Gap- Heuristic to ACO <br> Meta-heuristic |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | All problems | Tight | Not tight | All problems | Tight | Not tight |
| All problems | $349.60 \%$ | $346.95 \%$ | $352.26 \%$ | $77.29 \%$ | $110.28 \%$ | $44.30 \%$ |
| Nodes<=40 | $157.42 \%$ | $330.97 \%$ | $-16.13 \%$ | $-38.96 \%$ | $-31.01 \%$ | $-46.90 \%$ |
| Nodes>40 | $493.74 \%$ | $358.93 \%$ | $628.55 \%$ | $164.48 \%$ | $216.25 \%$ | $112.70 \%$ |

To summarize, due to the effective exploration scheme in ACO and exploitation of the network structure, the meta-heuristic provides substantial savings on computational time on larger scale instances and it creates a good compromise between solution quality and speed. The ACO algorithm outperforms the constructive heuristic on the solution quality; especially, it leads to lower runtime on larger instances. The result presented also suggests that this approach is promising in solving the larger emergency logistics problems.

### 5.4 Summary

In this chapter, two solution approaches are proposed for the emergency logistics problem. The greedy heuristic $P A T H \_B U I L D E R$ constructs vehicle itineraries upon a greedy utility criterion according to which a limited neighborhood around the vehicle's location is assessed. The utilities are collected on the way between origindestination nodes by exploiting any supply/demand nodes that lie on the partial path. This construction based heuristic achieves quick solution compared to the direct model solution. Based on a different search principle, the ACO meta-heuristic
decomposes the original emergency logistics problem into two sequential phases and iterates between them. The performance of the ACO algorithm is compared with the direct model solution as well as PATH_BUILDER. The ACO algorithm results in better solution quality within shorter runtimes for larger instances. Analyzing the overall solution quality and times, one can say that solution quality achieved within a minute of runtime is acceptable for the planner in real emergency situation where there is continuous uncertainty and information dynamism. The results also suggest that this decomposition approach may be efficient for other complex combinatorial problems with interdependent decision variables. In addition, the proposed approach can be the core of a suite of heuristics developed for the extended models such as location-routing problem in emergency logistics, which will be described in the next chapter.

## 6 Location Routing Problem in Disaster Response

## Activities

In the previous chapters, models and various solution methods were proposed for the ELP. This chapter extends the model by considering the emergency medical units planning problem. To provide maximum coverage of medical need in the affected areas, the capabilities of medical facilities have to be re-distributed among all medical units to achieve overall maximal service level. This necessitates finding the optimal medical personnel allocation equilibrium among available units. The extension is integrated into the model proposed in chapter 4, and the ACO meta-heuristic proposed in Chapter 5 is also developed for the solution of this location routing problem.

### 6.1 Problem Description

The extended model studied here aims to coordinate the transportation of commodities from major supply centers to distribution centers in affected areas and wounded people to temporary and permanent emergency units. The model enables selecting the best among possible proposed locations for temporary emergency units where injured people can be taken care of. The service capacities of temporary emergency centers in the affected area are supplied by major medical centers in the area. Hence, rather than having a fixed cost for establishing temporary centers in the objective function, the model seeks to achieve an equilibrium among re-allocated
service levels to minimize transportation delay for patients with different priorities and localities. Following the formulation in previous chapters, the model is an integrated capacitated location-routing model with a network flow based structure.

The integrated location-routing model (LRP) subsumes both facility location problem (FLP) and vehicle routing problem (VRP), and optimizes the locations and capacities of facilities as well as vehicle routes and schedules. Extensive survey on FLP and VRP are found in Owen and Daskin (1998), Desrochers et al. (1988), Fisher (1995b) and Toth and Vigo (2002). These sub-problems have been shown to be NP hard (Cornuejols et al., 1977; Lenstra and Rinnooy Kan, 1981), thus the LRP also belongs to the class of NP-hard problems. Due to its complexity, exact solution approaches to the LRP have been very limited. The procedures for solving the LRP generally follow the sequential methods because an incorporation of all the sub-problems together is computationally impractical. A classification of LRP models was given by Min et al. (1998) and it is based on VRP features such as ability to deliver only or deliver and pick up, single or multiple vehicles, uncapacitated or capacitated vehicles, and time windows, and FLP features such as uncapacitated or capacitated facilities and facility layers (existence of transshipment nodes-secondary facilities). Generally, the VRP assumptions incorporated into LRP describe very simple settings where a facility is served by only one capacitated vehicle that conducts a single tour and a client can only be served by one facility - non-split delivery assumption- (Tuzun and Burke, 1999; Albareda-Sambola et al., 2005). Heuristic approaches were common in solving LRP (Srivastava, 1993; Hansen et al., 1994) and recently, meta-heuristics were also proposed for this problem. Tuzun and Burke (1999) proposed a two-phase tabu search approach for the solution of LRP. Wu et al (2002) applied decomposition method to the multi-depot LRP and solved the sub-problems sequentially by simulated
annealing. Lin and Kwok (2005) made a comparison study between tabu search and simulated annealing based algorithms on a multi-objective version of LRP.

In this model supply availability is also limited whereas in conventional LRP and VRP models, supply availability is assumed to be unrestricted. The FLP sub-problem in the model involves the selection of locations for temporary emergency centers given possible sites in the affected area. Such centers are established after the disaster to provide immediate care for injured people whose conditions do not need immediate service from fully equipped hospitals. This reduces transportation requirements to major medical centers. However, especially during the initial response time, these temporary centers utilize the resources of other hospitals (medical personnel and equipment). Therefore, temporary centers reduce the service capabilities of hospitals. Depending on how fast care can be provided (based on transportation time needed to reach temporary centers and hospitals) and on the number and classification of injured people, the model formulates an equilibrium among the service levels of temporary and permanent medical centers by treating total service capacity as fixed. The FLP sub-problem proposed here does not involve any explicit coverage constraints since the locations of sites where such temporary centers can be established are rather restricted by the structural demolition rates of affected zones, and it is often difficult to satisfy distance-based location restraints. However, the objective function reflects the proximity of selected sites by penalizing the delay in the provision of care according to injury priority. The latter eliminates the need of imposing a fixed charge to established temporary facilities, since these facilities reduce the service levels of hospitals that might serve higher priority patients. This idea corresponds to a locationrouting problem where different facilities have different manufacturing capabilities and based on the routing problem and transportation lead times, overall system service
level is maximized (the latter can be interpreted as maximal coverage) by optimizing allocation of shared resources among collaborating facilities.

The notations in this chapter are defined, together with the extended mathematical formulation in Appendix II. The proposed model and the two-stage algorithm is illustrated on an earthquake scenario based on Istanbul's risk grid as well as hypothetical disaster scenarios with up to sixty nodes (Yi and Özdamar, 2007).

### 6.2 Extended ACO Solution for the Location-Routing Problem

Based on the ACO method in the last chapter, a solution framework is proposed for the location-routing problem. It integrates three levels of decision making in a computationally efficient manner, i.e., the emergency unit location-allocation problem (LAP), vehicle route construction and commodity dispatch. In the location stage of the procedure, an ACO algorithm is extended to determine a good distribution of medical equipment and personnel, followed by stochastic vehicle path construction under the guidance of heuristic information (pheromone trails), and the commodity dispatch phase pushes the multi-commodity flows on the resulting network by a successive maximum flow (SMF) algorithm. Although the sub-problems are solved sequentially, each time when a move is made at the location stage, the routing stage is performed based on the pheromone trails laid in the previous iterations but it does not start from scratch. In this way, good vehicle routes can be constructed on a localized space as opposed to a global search including the previously dominated routes; therefore a lot of unnecessary computation time is saved. Furthermore, the pheromone trails for the first two stages are synchronized by updating with the multi-commodity dispatch resulting from SMF. Thus the sub-problems are coordinated and naturally integrated in the solution framework.

According to the problem definition in section 6.1, the model does not impose an upper bound on the service rate of temporary emergency units and assumes service rates can be distributed on individual basis. The objective is to determine the optimal service rates at each potential medical unit. As an initiative of each iteration in the search process, the quick solution of the LAP is extremely important for the overall computational efficiency. A simple ACO procedure is applied to the service rates distribution. Since multiple types of wounded exist in the model, heuristic information must be shared among the same type of service rates. Hence, multi-type artificial ants are employed and each ant denotes a unit service rate of the same type. The pheromone trail $\tau{ }^{\mathrm{L}}{ }_{i h}$ is defined as aggregated service rate of type $h$ allocated to node $i$ (L serves as an indication of location sub-problem). No desirability is defined in this stage because no obvious principle exists favoring the selection of any node. Then the probability $p^{\mathrm{L}}{ }_{j h}$ for allocating the current service rate (ant) to node $j$ are defined by:

$$
\begin{equation*}
p_{j h}^{L}=\frac{\tau_{j h}^{L}}{\sum_{i \in C E} \tau_{i h}^{L}} \quad \text { if } j \in C E \tag{6-1}
\end{equation*}
$$

Using the above formula each ant randomly selects a node from a given set of potential emergency units. This continues until all the service rates are allocated.

After location stage is accomplished, the ACO meta-heuristic in chapter 5.3 is applied sequentially to solve the original emergency logistics problem, followed by trail update process, where the same trail updating strategies are employed here. Whenever a trail updating is triggered to routing ants, it is simultaneously applied to location ants by the following eq.(6-2); however, the probability updating eq.(6-1) is performed only if the other condition is also satisfied: a predetermined number
$\left(0.5^{*}|\sigma|\right.$ in this implementation) of iterations are performed. This ensures a wide search in the current solution stage to avoid myopic solution:

$$
\begin{equation*}
\tau_{i h}^{L}=(1-\rho)^{*} \tau_{i h}^{L}+\underset{\mu \in \sigma^{L}}{\arg \max }\left\{\tau_{i h}^{L \mu}\right\} \tag{6-2}
\end{equation*}
$$

where the same evaporation parameter $\rho$ is used. $\sigma^{\mathrm{L}}$ is the elite solution set with the same dimension as $\sigma$ in Chapter 5 and they are updated simultaneously whenever a solution enters the elite set. In addition, the initial trail $\tau^{\mathrm{L}}{ }_{i h}$ is set to the average number of total service rates over potential emergency units. In this way, the emergency unit location problem and vehicle route construction are coordinated and synchronized, the heuristic information obtained in the search process are fully utilized and substantial computation time is saved.

### 6.3 Numerical Results

Twenty-eight test instances are generated in a similar procedure as in the last chapter while capacity tightness is relaxed to show the enhancement on system-wide performance resulting from the re-allocation of service rates among emergency units. The total number of vehicles is increased to the range of [22, 81]. Details on problem characteristics are provided in Table 6-1. The second and third columns present the number of nodes and the number of potential emergency units to be selected. The number of actual arcs in the expanded network consists of the arcs and nodes reachable by all vehicles throughout the whole planning horizon. To get more extensive results, two problems in the same group (for example, problem 1 and $1^{\prime}$ ) are generated for each network structure and have same vehicle number and distribution while differing in potential hospitals number. Moreover, for each problem size (determined by number of nodes), we have two groups of different instances, where
the second one has double the number of potential emergency units. The number of vehicles is given in aggregate for three vehicle types. The planning horizon is set to 10 periods. Search parameters are set to the same values as in the previous chapter in all ACO runs.

Table 6-1 Characteristics of test problems

| Problem | No. of <br> Nodes | No. of <br> Potential <br> Hospital <br> Sites | No. of <br> Arcs in <br> Expanded <br> Network | No. of <br> Vehicl <br> es |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 20 | 2 | 351 | 22 |
| $1^{\prime}$ | 20 | 4 | 351 | 22 |
| 2 | 20 | 2 | 429 | 17 |
| $2^{\prime}$ | 20 | 4 | 429 | 17 |
| 3 | 30 | 3 | 606 | 20 |
| $3^{\prime}$ | 30 | 6 | 606 | 20 |
| 4 | 30 | 3 | 630 | 28 |
| $4^{\prime}$ | 30 | 6 | 630 | 28 |
| 5 | 40 | 4 | 1073 | 31 |
| $5^{\prime}$ | 40 | 8 | 1073 | 31 |
| 6 | 40 | 4 | 895 | 35 |
| $6^{\prime}$ | 40 | 8 | 895 | 35 |
| 7 | 50 | 5 | 1084 | 39 |
| $7 '$ | 50 | 12 | 1084 | 39 |
| 8 | 50 | 5 | 1077 | 33 |
| $8^{\prime}$ | 50 | 12 | 1077 | 33 |
| 9 | 60 | 7 | 1134 | 41 |
| $9^{\prime}$ | 60 | 15 | 1134 | 41 |
| 10 | 60 | 7 | 1186 | 52 |
| $10^{\prime}$ | 60 | 15 | 1186 | 52 |
| 11 | 70 | 9 | 1775 | 56 |
| $11^{\prime}$ | 70 | 18 | 1775 | 56 |
| 12 | 70 | 9 | 1748 | 66 |
| 12 | 70 | 18 | 1748 | 66 |
| 13 | 80 | 10 | 1840 | 72 |
| $13^{\prime}$ | 80 | 20 | 1840 | 72 |
| 14 | 80 | 10 | 1913 | 81 |
| $14^{\prime}$ | 80 | 20 | 1913 | 81 |
|  |  |  |  |  |

The algorithm is implemented in $\mathrm{C}++$ and all computational results are given in detail in Table 6-2. First of all, by comparing the objective values of problems within each group, one can see the re-allocation among a higher number of potential hospital sites leads to better relief performance. In the table, the direct model solution by CPLEX
and the runtime in seconds are provided. The solution quality of the ACO metaheuristic is presented by the gap between algorithm solution value and the near optima. To save the computational cost in the verification of optimality, direct model solutions are obtained by applying the following relative termination criterion: $0.5 \%$ for all the problems except that $1 \%$ for problem 6' and $10^{\prime}$. Similar to the previous computational tests, all the problems are re-solved in CPLEX with relative termination tolerance set to the identical optimality gap given by ACO meta-heuristic (column 6 in Table 6-2). The last two columns give the CPLEX runtime and the gap to ACO algorithm runtime.

Table 6-2 Detailed numerical results

| Problem | Model Solution |  | ACO Meta-heuristic |  |  | Model Solution with Tolerance |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Obj Value | Runtime Secs. | Obj Value | Runtime Secs. | $\begin{gathered} \hline \text { Gap(OV- } \\ \text { OP) } \end{gathered}$ | Runtime Secs. | Runtime Gap |
| 1 | 49002 | 1.83 | 49051 | 2.60 | 0.10\% | 2.20 | -15.14\% |
| 1' | 45427 | 3.48 | 46184 | 3.26 | 1.67\% | 1.98 | -39.07\% |
| 2 | 44232 | 41.14 | 46143 | 2.54 | 4.32\% | 4.44 | 74.41\% |
| 2' | 43267 | 41.75 | 45233 | 1.97 | 4.54\% | 4.25 | 115.85\% |
| 3 | 52709 | 75.14 | 54964 | 3.76 | 4.28\% | 14.78 | 293.64\% |
| 3' | 52327 | 110.34 | 54842 | 8.73 | 4.81\% | 13.22 | 51.41\% |
| 4 | 40609 | 20.17 | 42418 | 12.30 | 4.45\% | 5.39 | -56.17\% |
| $4 '$ | 37116 | 15.69 | 39252 | 16.80 | 5.75\% | 5.34 | -68.20\% |
| 5 | 40720 | 118.03 | 43137 | 10.63 | 5.93\% | 9.94 | -6.49\% |
| 5' | 37301 | 120.55 | 40699 | 18.70 | 9.11\% | 13.38 | -28.48\% |
| 6 | 38135 | 22.02 | 40709 | 16.25 | 6.75\% | 9.42 | -42.02\% |
| 6' | 33853 | 152.31 | 37503 | 24.52 | 10.78\% | 9.63 | -60.75\% |
| 7 | 96242 | 81.66 | 101038 | 22.35 | 4.98\% | 61.80 | 176.50\% |
| $7{ }^{\prime}$ | 88986 | 258.23 | 96978 | 39.36 | 8.98\% | 66.05 | 67.79\% |
| 8 | 100604 | 55.64 | 103074 | 31.59 | 2.46\% | 20.78 | -34.22\% |
| 8' | 96624 | 44.97 | 100570 | 28.60 | 4.08\% | 23.14 | -19.10\% |
| 9 | 90954 | 189.14 | 99294 | 18.54 | 9.17\% | 80.13 | 332.24\% |
| 9' | 87947 | 301.38 | 96953 | 34.99 | 10.24\% | 102.05 | 191.69\% |
| 10 | 76470 | 261.50 | 84871 | 34.12 | 10.99\% | 97.95 | 187.12\% |
| 10' | 66101 | 279.34 | 76032 | 63.92 | 15.02\% | 95.80 | 49.86\% |
| 11 | 87621 | 236.81 | 95510 | 49.58 | 9.00\% | 121.72 | 145.52\% |
| 11 ' | 83227 | 252.22 | 91993 | 61.10 | 10.53\% | 119.86 | 96.16\% |
| 12 | 89271 | 311.61 | 99350 | 50.80 | 11.29\% | 154.48 | 204.08\% |
| 12' | 82859 | 463.33 | 94795 | 64.13 | 14.41\% | 132.59 | 106.75\% |
| 13 | 90996 | 257.92 | 101336 | 28.72 | 11.36\% | 220.19 | 666.67\% |
| $13 '$ | 85956 | 291.06 | 98251 | 47.58 | 14.30\% | 211.91 | 345.36\% |
| 14 | 94115 | 308.06 | 100887 | 40.34 | 7.20\% | 192.55 | 377.34\% |
| 14 ' | 85792 | 215.31 | 92352 | 62.62 | 7.65\% | 195.375 | 211.99\% |

Table 6-3 to Table $6-5$ present the summary of results. In Table 6-3 performance robustness is measured by the average value and standard deviation (SD) of run time on the problems grouped in same size. Columns 5, 8 and 11 show the relative standard deviation (RSD). The RSD values are high for both CPLEX and ACO because the data sets are quite different for the problems in a specific group; moreover, the size of the problem affects CPU time significantly (Table 6-5), which results in the high RSD value across all problems (Last row, Table 6-3) in CPLEX (with same tolerance) and ACO that are $102.56 \%$ and $71.27 \%$, respectively. This indicates ACO meta-heuristic has a relatively higher robust performance.

The problems are classified according to the number of potential emergency units and problem size in Table 6-4 and corresponding average gaps are provided. Columns 2-4 are the average objective value gaps between ACO solutions and CPLEX optima. It is observed that the ACO meta-heuristic produces solutions with an average gap below $8 \%$ and it solves most problems within a minute, which is acceptable for re-planning purposes with frequent information updates. ACO solution quality is affected to a certain degree by the number of potential hospital sites, where the average gaps are $6.59 \%$ and $8.71 \%$ for the problems with less and more potential sites, respectively. Furthermore, the influence of sites number increases with the problem size. For instance, considering smaller size problems less than 40 nodes, there is a small gap difference (1.8\%). This difference grows to $2.34 \%$ in larger scale problems over 40 nodes. It also can be concluded that the problem size has a much more significant impact on the solution quality than the hospital sites number (5.21\% for small instances and $9.48 \%$ for larger ones). Columns 8-10 in Table 6-5 give average speed gaps between ACO and CPLEX with same tolerance. CPLEX runs faster on small problems but is dominated by ACO on larger problems. From Table 6-5 one can see
that sites number has virtually no influence on CPLEX CPU time while affects ACO speed constantly for all the problems, hence the runtime gap of more sites instances
(last column) are always smaller than less sites cases.

Table 6-3 Summary of results- average and standard deviation of runtimes

| Group | Problem | Model Solution Runtime |  |  | ACO Meta-heuristic Runtime |  |  | Model Solution with Tolerance Runtime |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Average | SD | RSD | Average | SD | RSD | Average | SD | RSD |
| 1 | 2' | 22.05 | 22.41 | 101.61\% | 2.59 | 0.53 | 20.32\% | 3.22 | 1.30 | 40.53\% |
| 2 | 3-4' | 55.34 | 45.56 | 82.33\% | 10.40 | 5.52 | 53.13\% | 9.68 | 5.02 | 51.89\% |
| 3 | 5-6' | 103.23 | 56.34 | 54.58\% | 17.52 | 5.76 | 32.87\% | 10.59 | 1.87 | 17.65\% |
| 4 | 7-8' | 110.12 | 99.93 | 90.75\% | 30.48 | 7.07 | 23.18\% | 42.94 | 24.31 | 56.61\% |
| 5 | 9-10' | 257.84 | 48.62 | 18.86\% | 37.89 | 18.93 | 49.96\% | 93.98 | 9.59 | 10.21\% |
| 6 | 11-12' | 315.99 | 103.38 | 32.72\% | 56.40 | 7.30 | 12.94\% | 132.16 | 15.90 | 12.03\% |
| 7 | 13-14' | 268.09 | 40.88 | 15.25\% | 44.82 | 14.19 | 31.66\% | 190.52 | 34.35 | 18.03\% |
| All P | oblems | 161.81 | 124.19 | 76.75\% | 28.59 | 20.37 | 71.27\% | 71.08 | 72.90 | 102.56\% |

Table 6-4 Summary of results- gap of objective value

|  | Gap of Obj Value |  |  |
| :---: | :---: | :---: | :---: |
|  | All problems | Less sites | More sites |
| All problems | $7.65 \%$ | $6.59 \%$ | $8.71 \%$ |
| Nodes<=40 | $5.21 \%$ | $4.31 \%$ | $6.11 \%$ |
| Nodes $>40$ | $9.48 \%$ | $8.31 \%$ | $10.65 \%$ |

Table 6-5 Summary of results- runtime

|  | ACO Meta-heuristic |  |  | Model Solution with Tolerance |  |  | Gap of Runtime |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | All problems | Less sites | More sites | All problems | Less sites | More sites | All problems | Less sites | More sites |
| All problems | 28.59 | 23.15 | 34.02 | 71.08 | 71.13 | 71.04 | 118.74\% | 164.53\% | 72.95\% |
| Nodes<=40 | 10.17 | 8.01 | 12.33 | 7.83 | 7.69 | 7.97 | 18.25\% | 41.37\% | -4.87\% |
| Nodes>40 | 42.40 | 34.50 | 50.29 | 118.52 | 118.70 | 118.35 | 194.11\% | 256.91\% | 131.31\% |

### 6.4 Summary

In this chapter, a solution framework is proposed for a complex location-routing problem that arises in emergencies. First the integrated location-routing model is introduced for coordinating logistics support and evacuation operations in response to emergencies. Next, an extended ACO solution framework is developed based on the
approach for the original emergency logistics problem, where a strategy of synchronizing location selection and route construction is employed to reduce computational burden. The numerical results show that location-routing model can result in notable enhancement of the overall disaster relief performance. Moreover, analyzing the overall solution quality and times, the extension of facilities planning does not add to computational complexity significantly. This suggests that the proposed approach of modeling the facility planning problem is suitable for the emergency logistics problem.

## 7 Conclusions and Future Research Directions

### 7.1 Conclusions

In the previous chapters, several modeling strategies were investigated and solution methods were proposed to address the ELP. The model and solution were also extended to address the medical facility planning problem. The contributions of this research can be summarized as follows:

- A network flow based formulation for the ELP is proposed and its efficiency is verified through comparison with conventional vehicle based formulation. The model presented essentially coordinates wounded people evacuation, commodity delivery, and vehicle scheduling. In addition, the location-routing problem is also addressed as an extension of the basic ELP model, which achieves equilibrium among the service levels of temporary and permanent medical centers under the constraint of limited total service capacity. This kind of modeling strategy may be employed in other logistics problems.
- Based on the solution of the proposed model, a two-stage algorithm was proposed to disaggregate the vehicle flows and generate load pickup/delivery instructions in each route. This algorithm and the optimization packages together form a solution framework to produce
operational plans for emergency logistics. The dynamic operations of the emergency relief system were illustrated based on real world scenarios, together with a succinct discussion on uncertainty in emergency logistics planning.
- A constructive heuristic based on parallel vehicle exploration is first proposed in an attempt to produce fast solutions. The heuristic PATH_BUILDER constructs vehicle itineraries using a greedy utility criterion according to which a limited neighborhood around the vehicle's location is assessed. This heuristic achieves quick solutions compared to the direct model solution. However, the utilization of the computational resource is not efficient because the large quantity of information generated in the search process is discarded without exploitation. Hence, an ACO meta-heuristic is developed to enhance the solution efficiency. The ACO algorithm decomposes the original emergency logistics problem into two sequential sub-problems, where the routing sub-problem takes advantage of the effective exploration scheme of large search spaces in this populationbased meta-heuristic, and the dispatch sub-problem employs the network flow structure in the formulation. The performance of the algorithm is tested on a number of randomly generated networks and the results indicate that this algorithm performs well in terms of solution quality and the run time consumption. The ACO solution framework is also extended for the proposed location-routing model.


### 7.2 Future Research Directions

### 7.2.1 Possible Improvements on the Solution Methods

The ACO meta-heuristic proposed in Chapter 5 is essentially an extension of the constructive heuristic, and this population-based method has produced satisfactory results by its highly effective exploration scheme of large search spaces. Given the successful applications of local search based meta-heuristics in logistics problems such as Tabu search in VRP, it seems promising to employ these advanced local search methods in the ELP. Although our preliminary computational results regarding Tabu search application on this problem is not comparable to the ACO approach, a mixed meta-heuristic based on different principles from various approximate methods discussed in Chapter 2 may help in improving solution quality. Moreover, the information on the optimality (or the quality) of the solution obtained is important in performing the emergency operation. In this thesis, the solution quality is evaluated by the gap between the meta-heuristic result and CPLEX optimality. However, as the large memory requirement of the model in CPLEX, it deserves the effort in the future to develop approaches from both meta-heuristics and conventional heuristics to obtain the lower bound or fast solution of the model, which can provide an alternative evaluation scheme for the proposed solution methods.

Mathematical programming methods are dominated by heuristics in the VRP. However, they still have many uses for some other types of NP-hard problems such as network design problems that explore the structure of the mathematical formulation. Moreover, the progress made in the simplex implementations and the specialized interior-point algorithm during recent years allows people to consider the solution of integer multi-commodity flow problems that have been adopted in a wide variety of
important large-scale applications. Given the network problem embodied in the ELP, it is observed that a medium-size instance up to 60 nodes can be solved in reasonable time by the default branch and cut routine in CPLEX.

Besides the readily available simplex-based methods, the special structure of ELP makes decomposition a possible solution method. The research on price-directive decomposition includes two categories of algorithm: Lagrangian relaxation and Dantzig-Wolfe decomposition. The Lagrangian relaxation is inclined to make a full exploitation of the formulation structure and Lagrangian heuristics have been applied widely to obtain primal solutions. The other is based on column generation approaches. Although research on column generation algorithm has been quite active, column generation based heuristic receives less attention in literature (Vanderbeck, 2005). In the latter context, the heuristics of sub-problems produce promising columns for the master problem, where the knowledge of the original problem can be fully exploited. Having been tested in a large number of implementations, these two algorithms are not always capable of producing fast solutions (Huisman et al., 2005; Guignard, 2003; Ahuja et al., 1993). Hence, the study on these methods for the ELP should integrate various heuristic principles and take the quick solution requirement into consideration. Finally, for both exact and heuristic methods, the solution of very large applications will be enabled using parallel computers.

### 7.2.2 Model Extension and Simulation

A preliminary discussion was given in chapter 4 for the possible directions of addressing uncertainties in emergency logistics problems; however, due to its practical importance, this topic is worth further investigation on both modeling and solution aspects. The proposed ACO meta-heuristic has been the core heuristic
developed for the location-routing problem. Based on its capability to address interdependent decision variables, this approach will also be developed to deal with the fuzzy model proposed in Yi and Özdamar (2004). In addition, broader solution methods based on soft computing could be considered for the uncertainty issue.

Besides the uncertain parameters related to wounded people and commodities demands discussed in the above paper, there are several other issues regarding the implementation of the proposed solutions which need to be taken into future research:

- Road congestion is a very important risk factor for ground vehicles in disaster relief practice and can be considered in different aspects. The study on contraflow network configuration can be a potential remedy to reduce road congestion during emergency operations. In addition to that, road damage induces another uncertain parameter, especially in the first few periods of the planning horizon, when the information of road damage is not clear. This uncertainty aggravates the traffic congestions and as a consequence causes major delays in the planned emergency operations. Moreover, even the road damage information is obtained in the subsequent planning periods; its impact on the vehicle traveling time is not immediately available to the logistics planner.
- Service failure may be another source of risk due to demolished hospitals and major supply centers, risk of non-usability of equipment.

While such uncertainties and traffic flow control cause fundamental difficulties in the optimization framework, microscopic traffic simulation models, which are capable of addressing complex individual parameters and their interaction among various factors, may provide an alternative solution approach. Previous research review on traffic
control can be found in Tuydes (2005). In addition, the simulation results can provide a complementary performance evaluation to the objective value of the optimization models, which is of particular value to the planner since several practical factors that are out of the reach of optimization model are taken into consideration. Hence, the traffic simulations collaborating with the proposed optimization models can serve as an insightful diagnosing tool to help improve the overall modeling and solution methods.

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## Appendix I Input of Model P

Table AI-1 Commodity demand distribution percentages

|  | Node |  |  |  |  |  | Total number <br> (people equivalent) |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |  |
| $\mathrm{~T}=1$ | 0.20 | 0.02 | 0.23 | 0.13 | 0.05 | 0.38 | 9300 |
| $\mathrm{~T}=5$ (expected) | 0.22 | 0.03 | 0.11 | 0.22 | 0.11 | 0.32 | 10000 |
| $\mathrm{~T}=5$ | 0.25 | 0.02 | 0.11 | 0.18 | 0.12 | 0.32 |  |

Table AI-2 Supply availability distribution percentages and fleet composition

|  | Node |  |  |  |  |  |  |  |  |  |  | Total number (people equivalent) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |  |
| $\mathrm{T}=1$ | 0.09 | 0.04 | 0.09 | 0.10 | 0.09 | 0.09 | 0.11 | 0.09 | 0.10 | 0.09 | 0.10 | 17950 |
| $\mathrm{T}=2$ | 0.10 | 0.01 | 0.09 | 0.11 | 0.12 | 0.10 | 0.12 | 0.10 | 0.07 | 0.10 | 0.09 | 13500 |
| $\mathrm{T}=3$ | 0.10 | 0.00 | 0.10 | 0.11 | 0.12 | 0.10 | 0.12 | 0.10 | 0.08 | 0.10 | 0.09 | 10200 |
|  | Service rates |  |  |  |  |  |  |  |  |  |  |  |
| H | 15 | 10 | 10 | 15 | 15 | 20 | 20 | 20 | 0 | 0 | 0 | 125 |
| L | 25 | 30 | 35 | 30 | 35 | 35 | 30 | 30 | 0 | 0 | 0 | 250 |
|  |  | Fleet composition |  | Unit capacity |  |  |  |  |  |  |  |  |
| Helic | ters | 25 |  | 1500 |  |  |  |  |  |  |  |  |
| Tru |  | 24 |  | 15000 |  |  |  |  |  |  |  |  |
| Ambu | ance | 52 |  | 600 |  |  |  |  |  |  |  |  |

Table AI-3 Percentages of wounded people

|  |  | Node |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | Total number |
| $\mathrm{T}=1$ |  |  |  |  |  |  |  |  | H



Figure AI-1 District based aggregate affected areas (nodes 1-6), supply distribution centers and hospitals (7-15), two aiding cities (16-17)

## Appendix II Mathematical Formulation

The mathematical formulation for the location routing problem in Chapter 6 is given below, which embodies the evacuation problem and the facility planning problem.

## Additional Sets and parameters:

CE: Set of potential emergency center sites including existing emergency centers, $\mathrm{CE} \subseteq \mathrm{C} \backslash \mathrm{CD}$

CH : Set of available emergency centers, $\mathrm{CH} \subseteq \mathrm{CE}$

Additional Decision variables:
$\mathrm{s}_{\mathrm{ho}}$ : Integer number of allocated service rate at an emergency facility 0 ; $\mathrm{s}_{\mathrm{ho}}=0(\forall$ $\mathrm{o} \in \mathrm{C} \backslash \mathrm{CE})$

## Location-Routing Model:

$$
\begin{equation*}
\text { Minimize } \quad \Sigma_{\mathrm{a} \in \mathrm{~A}} \Sigma_{\mathrm{o} \in \mathrm{CD}} \Sigma_{\mathrm{t}} \mathrm{P}_{\mathrm{a}} \operatorname{dev}_{\mathrm{aot}}+\Sigma_{\mathrm{h} \in \mathrm{H}} \Sigma_{\mathrm{t}} \mathrm{P}_{\mathrm{h}} \operatorname{dev}_{\mathrm{ht}} \tag{0}
\end{equation*}
$$

Subject to

$$
\begin{align*}
& \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \mathrm{~d}_{\mathrm{aoq}}-\Sigma_{\mathrm{m} \in \mathrm{M}} \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \sum_{\mathrm{p} \in \mathrm{C}}\left[\mathrm{~K}_{\text {pqotm }} \mathrm{Z}_{\text {apomq }}-\mathrm{Z}_{\text {aopmq }}\right]=\operatorname{dev}_{\text {aot }} \quad(\forall \mathrm{a} \in \mathrm{~A}, \mathrm{o} \in \mathrm{CD}, \mathrm{t} \in \mathrm{~T})  \tag{1}\\
& \Sigma_{\mathrm{m} \in \mathrm{M}} \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{\mathrm{p} \in \mathrm{C}}\left[-\mathrm{K}_{\text {pqotm }} \mathrm{Z}_{\text {apomq }}+\mathrm{Z}_{\text {aopmq }}\right] \leq \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \sup _{\text {aoq }}(\forall \mathrm{a} \in \mathrm{~A}, \mathrm{o} \in \mathrm{C} \backslash \mathrm{CD}, \mathrm{t} \in \mathrm{~T})  \tag{2}\\
& \mathrm{Y}_{\text {opmt }} \leq \mathrm{K} * \Sigma^{\mathrm{T}}{ }_{\mathrm{q}=\mathrm{t}} \mathrm{~K}_{\mathrm{otpqm}} \quad(\forall \mathrm{o} \in \mathrm{C}, \mathrm{p} \in \mathrm{C}, \mathrm{~m} \in \mathrm{M}, \mathrm{t} \in \mathrm{~T})  \tag{3}\\
& \mathrm{Y}_{\text {opmt }} * \operatorname{cap}_{\mathrm{m}} \geq \Sigma_{\mathrm{a} \in \mathrm{~A}} \mathrm{w}_{\mathrm{a}} * Z_{\mathrm{aopmt}}+\Sigma_{\mathrm{h} \in \mathrm{H}} \mathrm{w}_{\mathrm{h}} * X_{\mathrm{hopmt}} \quad(\forall \mathrm{o} \in \mathrm{C}, \mathrm{p} \in \mathrm{C}, \mathrm{~m} \in \mathrm{M}, \mathrm{t} \in \mathrm{~T})  \tag{4}\\
& \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{\mathrm{p} \in \mathrm{C}}\left[\mathrm{Y}_{\text {opmq }}-\mathrm{K}_{\text {pqotm }} \mathrm{Y}_{\text {pomq }}\right] \leq \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \mathrm{av}_{\text {omq }} \quad(\forall \mathrm{o} \in \mathrm{C}, \mathrm{~m} \in \mathrm{M}, \mathrm{t} \in \mathrm{~T})  \tag{5}\\
& \Sigma_{\mathrm{m}} \in \mathrm{M}^{\Sigma}{ }_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{\mathrm{p} \in \mathrm{C}}\left[-\mathrm{K}_{\mathrm{pqqotm}} \mathrm{X}_{\text {hpomq }}+\mathrm{X}_{\text {hopmq }}\right] \leq \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \mathrm{~d}_{\text {hoq }} \quad(\forall \mathrm{h} \in \mathrm{H}, \mathrm{o} \in \mathrm{CD}, \mathrm{t} \in \mathrm{~T})  \tag{6}\\
& \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{\mathrm{m}} \in \mathrm{M}\left[\mathrm{~K}_{\mathrm{pqotm}} \mathrm{X}_{\mathrm{hpomq}}-\mathrm{X}_{\mathrm{hopmq}}\right] \geq \Sigma_{\mathrm{q}=1}^{\mathrm{t}} \mathrm{sp}_{\mathrm{hoq}} \quad(\forall \mathrm{~h} \in \mathrm{H}, \mathrm{o} \in \mathrm{C} \backslash \mathrm{CD}, \mathrm{t} \in \mathrm{~T}) \tag{7}
\end{align*}
$$

$\Sigma_{\mathrm{q}=1}^{\mathrm{t}} \Sigma_{\mathrm{o} \in \mathrm{C}}\left[\mathrm{d}_{\mathrm{hoq}}-\mathrm{sp}_{\mathrm{hoq}}\right]=\operatorname{dev}_{\mathrm{ht}}$
$(\forall \mathrm{h} \in \mathrm{H}, \mathrm{t} \in \mathrm{T})$
$\Sigma_{\mathrm{o} \in \mathrm{CE}} \mathrm{Sho}=\Sigma_{\mathrm{o} \in \mathrm{CH}} \mathrm{Scap}_{\mathrm{ho}}$
( $\forall \mathrm{h} \in \mathrm{H}$ )
$\mathrm{sp}_{\text {hot }} \leq \mathrm{s}_{\mathrm{ho}}$
$(\forall \mathrm{h} \in \mathrm{H}, \mathrm{o} \in \mathrm{C}, \mathrm{t} \in \mathrm{T})$
$\mathrm{Y}_{\text {opmt }}, \mathrm{X}_{\text {hopmt }}, \mathrm{Z}_{\text {aopmt }}, \operatorname{dev}_{\text {aot }}, \operatorname{dev}_{\text {hot }}, \mathrm{s}_{\mathrm{ho}} \geq 0(\forall \mathrm{~h} \in \mathrm{H}, \mathrm{a} \in \mathrm{A}, \mathrm{o} \in \mathrm{C}, \mathrm{p} \in \mathrm{C}, \mathrm{m} \in \mathrm{M}, \mathrm{t} \in \mathrm{T})$ (11)

## Appendix III List of Publication

Wei Yi and Linet Özdamar, 2004. Fuzzy Modeling for Coordinating Logistics in Emergencies. International Scientific Journal of Methods and Models of Complexity, 7(1).

Wei Yi and Linet Özdamar, 2007. A Dynamic Logistics Coordination Model for Evacuation and Support in Disaster Response Activities. European Journal of Operational Research, 179 (3), pp. 1177-1193.

Wei Yi and Linet Özdamar, 2005. A Greedy Neighborhood Search Approach for Disaster Relief and Evacuation Problem, accepted for publication in the IEEE Intelligent Systems.

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