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Models for estimating evapotranspiration using artificial neural networks, and their physical interpretation

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Abstract:

Estimation of evapotranspiration (ET) requires a knowledge of the values of many climatic variables, some of which require special equipment and careful observations. Although ET is an important component of water balance, the data required for its accurate estimation are commonly available only at widely spaced measurement stations. The major objective of this study was to estimate ET using an artificial neural network (ANN) technique and to examine if a trained neural network with limited input variables can estimate ET efficiently. The results indicate that even with limited climatic variables an ANN can estimate ET accurately. The paper also outlines a procedure to evaluate the effects of input variables on the output variable using the weight connections of ANN models. Such an analysis performed on the ANN-ET models developed was able to explain the reasons for the ANN's potential in estimating the ET effectively from limited climatic data. Copyright © 2008 John Wiley & Sons, Ltd.

KEY WORDS artificial neural network; evapotranspiration; Penman Monteith; temperature; radiation

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INTRODUCTION

Evapotranspiration (ET) is an important component of the hydrologic cycle. Correct estimation of ET is necessary in many studies, such as catchment modelling, agricultural water management, estimating the components of water balance, assessment of the impact of land use changes on the hydrologic response of a catchment, etc. In many watersheds, the return of moisture to the atmosphere through the process of ET is a large proportion of the input precipitation.

Despite widespread application of the ET concept, there has been considerable ambiguity in the use of such terms as potential ET and reference crop ET. To overcome this, the Food and Agricultural Organization (FAO) of the United Nations brought out a report, commonly referred to as FAO-56 (Allen *et al*., 1998). Among other things, it introduced uniformity and standardization in the interpretation and use of various terms, such as potential ET and reference crop ET. FAO-56 discourages the use of the term potential ET because of ambiguities in its definition. Moreover, FAO recommended that a hypothetical reference surface 'closely resembling an extensive surface of green grass of uniform height, actively growing, completely shading the ground and with adequate water' (Allen *et al*., 1998) be adopted as reference surface. In the FAO approach, the surface characteristics that influence ET are quantified in an unambiguous fashion (Itenfisu *et al*. 2003).

The ET rate from a reference surface, not short of water, is called the reference crop ET or reference evapotranspiration and is denoted as ET_0 (Allen *et al*. 1998). The reference surface is a hypothetical grass reference crop with specific characteristics. Further, crop ET under standard conditions (ET_c) refers to the evapotranspiration from excellently managed, disease-free, large, well-watered fields that achieve full production under the given climatic conditions. Further, due to suboptimal crop management and environmental constraints that affect crop growth and limit evapotranspiration, ET_c under non-standard conditions generally requires a correction.

To estimate ET from a well-watered agricultural crop, reference evapotranspiration from a standard surface (ET_0) is first estimated. This value is multiplied by an empirical crop coefficient to obtain the ET from the crop (ET_c). The crop coefficient accounts for the difference between the standard surface and the crop. Reference ET is expressed in units of depth time^{-1}, e.g. mm day^{-1}. It is a climatic parameter expressing the evaporative power of the atmosphere at the given space and time coordinates. Crop and soil features are not involved in its computation.

Evapotranspiration can be measured with a lysimeter or water balance approach, or estimated from climatological data. Measurement of ET with a lysimeter is time-consuming and needs careful planning. Installation and maintenance of a lysimeter requires skilled manpower, instruments, and finances. For these reasons, indirect

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methods based on climatological data are frequently used for estimation of ET_0.

Numerous reference ET equations have been developed and are being used, depending upon the availability of weather data. These equations range in sophistication from empirical solar radiation- or temperature-based equations, to complex methods based on physical processes such as the combination method of Penman (1948). The combination approach links evaporation dynamics with the flux of net radiation and aerodynamic transport characteristics of a natural surface. Based on the observations that latent heat transfer in plant stems is influenced not only by these abiotic factors, Monteith (1965) introduced a surface conductance term that accounted for the response of leaf stomata to its hydrologic environment. This modified form of the Penman equation is widely known as the Penman–Monteith (PM) equation.

The PM equation is physically based because it attempts to incorporate the physiological and aerodynamic characteristics of the reference surface. While the use of the modified Penman method (Doorenbos and Pruit, 1977) was recommended by FAO, recent studies have suggested that this method overestimates ET (Sudheer *et al.*, 2003). FAO has now recommended the use of the PM method to compute reference ET from a grass surface and has specified a grass reference ET equation (Allen *et al.* 1998). Recent studies by Allen *et al.* (1994, 1998), Ventura *et al.* (1999), Howell *et al.* (2000) and Wright *et al.* (2000) have shown that the reference ET computed using the PM equation yields estimates that are close to observed reference ET values.

FAO-56 Penman–Monteith Method

As described in the Irrigation and Drainage Paper 56 (Allen *et al.*, 1998), the FAO has adopted the PM equation (named here FAO56-PM) as the standard technique to compute reference ET. The FAO56-PM can be used for hourly or daily time steps. For hourly time steps, the equation is stated as (Allen *et al.*, 1998):

$$ET_0 = \frac{0{\cdot}408\Delta(R_n - G) + \gamma \dfrac{37}{T_{hr} + 273} u_2[e^0(T_{hr}) - e_a]}{\Delta + \gamma(1 + 0{\cdot}34u_2)} \tag{1}$$

where ET_0 is grass reference ET in mm h^{-1}, R_n is the net radiation at the grass surface in MJ m^{-2} h^{-1}, G is the soil heat flux density in MJ m^{-2} h^{-1}, T is the mean hourly air temperature in °C, u_2 is the mean hourly wind speed at 2 m height in m s^{-1}, $e^0(T_{hr})$ is the saturation vapour pressure in kPa at air temperature T_{hr}, e_a is the actual hourly vapour pressure in kPa, Δ is the slope of vapour pressure versus temperature curve in kPa °C^{-1}, and γ is the psychrometric constant in kPa °C^{-1}. Allen *et al.* (1998) described the procedure and steps for the application of the PM equation for various time step sizes.

Many scientists have studied the reliability of the PM method for estimating ET_0 (Allen *et al.* 1989; De Souza and Yoder 1994; Chiew *et al.* 1995). Jensen *et al.* (1990) analysed the performance of 20 different methods against lysimeter-measured ET for 11 stations located in different climatic zones around the world. The PM method was ranked as the best method for all climatic conditions. Allen *et al.* (1994) further state that the FAO56-PM equation should be considered superior to most lysimeter-measured ET_0 data during calibration of empirical ET_0 equations.

Computational aspects

An application of the FAO56-PM equation requires data on solar radiation, wind speed, air temperature, vapour pressure, and humidity. However, all these input variables may not be easily available at a given location. In developing countries in particular, difficulties are often faced in collecting accurate data on all the necessary climatic variables, and this can be a serious handicap in applying the FAO56-PM equation. Among the inputs needed, temperature data are routinely measured and solar radiation can be estimated with sufficient accuracy. But the other variables are generally measured at only a few locations.

Automatic weather stations (AWS), which are commonly used these days in developed countries to measure climatic variables, are rare in many other countries. Often there may not be even a single AWS over an area of thousands of square kilometres. In such circumstances, one may be forced to use data from the 'nearest' station, which may in fact be far away, often in completely different hydrometeorological settings. Experience shows that extrapolation of wind speed from one site to another is subject to large errors (Irmak *et al.*, 2003). In addition, wind speeds measured over non-agricultural, arid terrain may be much faster than those measured over agricultural crops (Burman *et al.*, 1975). Extrapolation of other climatic variables is equally questionable because of their unique behaviour.

In view of the above, it is necessary to develop techniques that can be employed to estimate accurately ET_0 for situations where values of some of the potential influencing variables are not available. An artificial neural networks (ANN), which is a modern data-driven technique, may be well suited for this purpose, since ANNs have proven to be efficient in approximating functions to an arbitrary degree of accuracy (Sudheer *et al.*, 2003). So far, this empirical technique has been applied successfully to a wide range of problems in hydrology. Here, it is proposed to explore the application of this technique to the estimation of ET using climatic data. The objective of this paper is to use ANNs with various combinations of inputs to compute ET. The focus of the paper is on the identification of a combination of inputs that are easily available in a given situation, and yield a reasonably accurate estimate of ET. Also examined is the physical process representation of the input combination in these ANN ET models by studying the relative importance of the input variables in estimating the ET. This is achieved with the help of connection weights of the trained ANN models.

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The paper sectioned as follows. First, a brief review of the theory of ANNs and relevant applications in water resources, and ET estimation in particular, is presented. Following this, a description of the methodology employed for constructing the input variable combinations and ANN model building is discussed. Results are presented and discussed in detail in the succeeding sections.

ARTIFICIAL NEURAL NETWORKS

ANNs are analogous to biological neural networks, and are highly simplified mathematical models of their biological counterparts. They include the ability to learn and generalize from examples to produce meaningful solutions to problems even when input data contain errors or are incomplete, and to adapt solutions over time to compensate for changing circumstances and to process information rapidly.

A system may be nonlinear and multivariate, and the variables involved may have complex interrelationships. ANNs are capable of adapting their complexity, and their accuracy increases as more and more input data are made available to them. They are capable of extracting the relationship between the input and output of a process without any knowledge of the underlying principles. Because of the generalizing capabilities of the activation function, one need not make any assumption about the relationship (linear or nonlinear) between input and output.

The ANN approach is faster than to its conventional counterparts, robust in noisy environments, and flexible in the range of problems it can solve. An ANN has the ability to learn from examples, to recognize a pattern in the data, to adapt solutions, and to process information rapidly. Owing to these advantages, ANNs have been used in numerous real-world applications. All these properties make ANNs an attractive tool for water resources practitioners. Applications of ANNs to hydrology are rapidly gaining popularity because of their power and potential in mapping nonlinear system data.

ANN applications in water resources

In the past decade, and particularly in the past five years, extensive attention has been focused by scientists on applying ANNs in such diverse fields as system modelling, system diagnosis and control, medicine, pattern recognition, forecasting, and water resources.

In the field of water resources, ANNs have been used for flow predictions, flow/pollution simulation, parameter identification, and to model complex nonlinear input–output time series. Hsu *et al*. (1995) have shown that the ANN approach can provide a better representation of the rainfall–runoff relationship of a medium sized basin than does the ARMAX approach or the Sacramento soil moisture model. Recent studies on ANN applications in the area of hydrology include rainfall–runoff modelling (Cigizoglu, 2003; Wilby *et al*. 2003; Lin and Chen, 2004); river stage forecasting (Imrie *et al*., 2000; Lekkas *et al*. 2001; Campolo *et al*. 2003); reservoir operation (Jain *et al*., 1999); land drainage design (Shukla *et al*., 1996; Yang *et al*., 1998); aquifer parameter estimation (Srinivasa, 1998); describing soil water retention curve (Jain *et al*., 2004) and optimization or control problems (Wen and Lee, 1998; Bhattacharya *et al*., 2003). Some of the studies (Zealand *et al*., 1999; Yang *et al*., 1996) have also shown that ANN is more accurate than conventional methods in flow forecasting and drainage design.

A set of two papers published by the ASCE task committee on application of ANNs in hydrology (ASCE, 2000a, 2000b) contains a detailed review of the theory and applications of ANNs in water resources. Maier and Dandy (2000) have also provided a review of modelling issues and applications of neural networks for the prediction and forecasting of water resources variables. Govindaraju and Rao (2000) have described many applications of ANNs to water resources.

Evapotranspiration is a complex and nonlinear phenomenon because it depends on several interacting climatological factors, such as temperature, humidity, wind speed, and radiation. Kumar *et al*. (2002) found that an ANN model can be trained to predict lysimeter ET_0 values better than the standard PM method. Sudheer *et al*. (2002) and Keskin and Terzi (2006) tried to compute pan evaporation using temperature data with the help of an ANN. Sudheer *et al*. (2003) employed a radial-basis function ANN to compute the daily values of ET for rice crop. It is evident from the literature that very few studies have been carried out to utilize the input–output mapping capability of an ANN in the prediction of ET.

Despite their numerous advantages, such as universal function approximation, robustness, and ability to learn, one of the major criticisms of ANN hydrologic models is that they do not consider/explain the underlying physical processes in mapping the relationship, resulting in them being labelled 'black box' models. This criticism stems mainly from the fact that no satisfactory explanation of their internal behaviour has been offered yet. This is a significant weakness, for without the ability to produce comprehensible decisions it is hard to trust the reliability of networks addressing real-world problems. Extracting the knowledge embedded within trained ANNs is still an active and evolving discipline (Sudheer and Jain, 2004). In view of this, the current study also aims at interpreting the physical inference of the trained ANN model in terms of modelling the ET process.

METHODOLOGY

ANN model development for ET_0 Estimation

In general, an ANN tries to fit a functional relationship between the input and output variables. In the case of estimation of ET_0, the input (independent) variables are generally temperature, dew point, sunshine hours, radiation, wind speed, humidity, etc. The functional form

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of this type of model is:

$$\mathbf{y} = f(\mathbf{X^n}) \qquad (2)$$

where f is the unknown function mapped by the model and $\mathbf{X^n}$ is an n-dimensional input vector consisting of the variables described above. The main task in developing any ANN model is to identify the input vector (independent variables) to the network and to identify the optimal network architecture so as to produce the desired output accurately. As stated earlier, the current investigation is focused mainly on identifying the minimum combination of variables to estimate ET_0. To achieve this, many combinations of input variables were presented to the ANN.

Description of data and availability

In the present investigation, hourly data (temperature, dew point, sunshine radiation, wind speed, humidity) for a period of two years (1990 and 1991) were used for estimation of ET_0. These data pertain to a few stations in the Reynolds Creek Experimental Watershed in South-western Idaho, USA. A description of the watershed and the data set is available in Slaughter *et al.* (2001). The data were downloaded from the web site of the Agricultural Research Service of US Department of Agriculture (ftp.nwrc.ars.usda.gov). Periods of missing data records were discarded in the current analysis. Using these observed climatic data, hourly values of ET_0 were initially computed by using Equation (1) and the steps recommended by Allen *et al.* (1998). These computed ET_0 values were used to train the ANN models. All the ANN models were trained using data for the period 11 February 1990 to 31 December 1990, and validated for the rest of the data (1 February 1991 to 31 December 1991). The rationale behind this division into training and validation is that that one full seasonal cycle each was used for training and testing. This also ensures the statistical properties of the training and testing data to be of similar order. As the climatic characteristics of the watershed are important in assessing the applicability of the models in general, the variation of different climatic parameters in the study area are presented in Figure 1. It is noted that the variation of the climatic parameters in the study area is similar to that experienced in a typical catchment. For example, for the data of the study period, the daily values of temperature ranged from 34 °C to −28 °C, relative humidity varied from 0·98 to 0·50, radiation ranged between 0·0 and 3·6 W m^{-2}, and the range of wind speed was from 0·0 to 0·16 m s^{-1}. Hence any model developed on this data set should have wide application.

Selection of input variables

The correlation (Box and Jenkins, 1976) matrix between all the input variables is presented in Table I. It can be observed from Table I that the linear correlation between radiation and ET_0 is 0·72, implying that any model built using radiation will certainly be able to compute the ET_0 satisfactorily. The model's accuracy can be improved by incorporating variables that account for aerodynamic effects on ET, such as humidity and dew point temperature in addition to radiation. The second highest correlation exists between temperature and

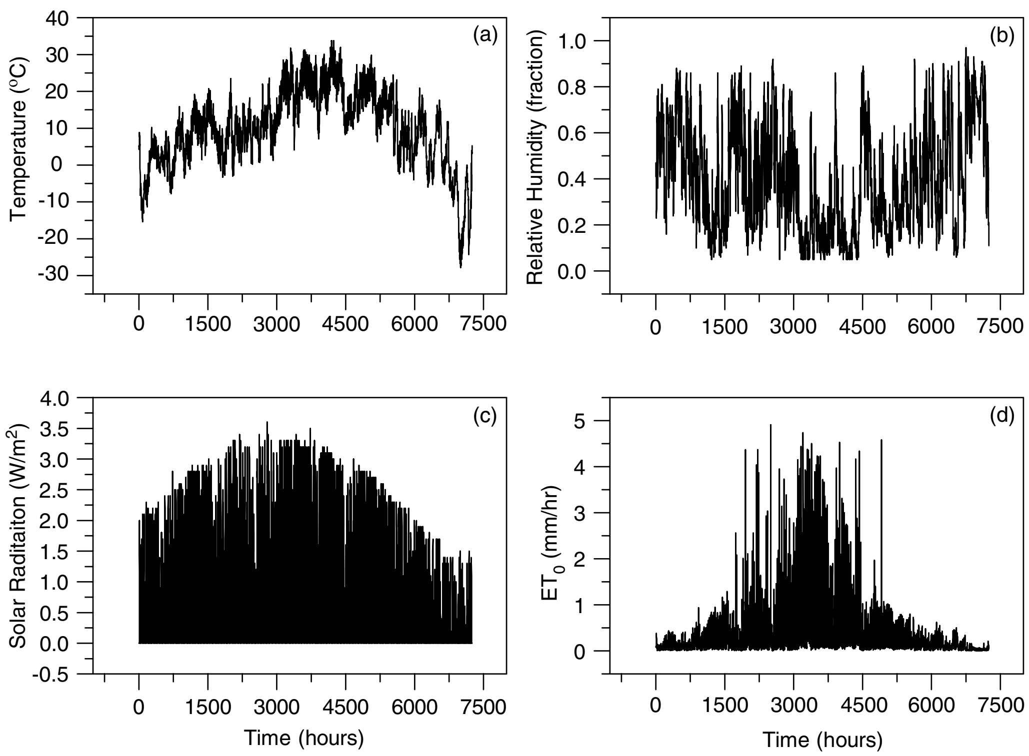

Figure 1. Variation of climatic parameters and evapotranspiration with time during the training data period: (a) temperature; (b) relative humidity; (c) solar radiation; and (d) ET_0 (time index '0' refers to the beginning of the training period)

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Table I. Correlation matrix between input and output variables

	Temperature	Humidity	Dew point temperature	Radiation	Wind speed	ET_0
Temperature	1·00	−0·74	0·39	0·40	0·00	0·46
Humidity	−0·74	1·00	0·26	−0·34	0·05	−0·37
Dew point temperature	0·39	0·26	1·00	0·08	0·03	0·10
Radiation	0·40	−0·34	0·08	1·00	0·17	0·72
Wind speed	0·00	0·05	0·03	0·17	1·00	0·13
ET_0	0·46	−0·37	0·10	0·72	0·13	1·00

ET_0. Note that humidity and temperature are also highly correlated. So, a combination of radiation and temperature may also provide a good estimate. As dew point temperature is not well correlated with ET_0, inclusion of this variable in the input vector may give little improvement in estimates. However, it is to be noted that these inferences are based on the linear correlation between the variables, while the evapotranspiration process is considered to be highly nonlinear. Hence, these inferences can be considered only as guidelines for deciding on the input variable combination. In addition, radiation data may not be available in many cases and we wanted to examine the predictive ability of ANN when such data are not available. Consequently the current study investigated the six combinations listed in Table II.

ANN Model Construction and Evaluation

A standard back propagation algorithm was employed to estimate the network parameters (weights and biases). The data were scaled to fall between 0 and 1, as the activation function used in the hidden and output node is a sigmoid function. This scaling was achieved by using the maximum and minimum of each variable of interest. Adaptive learning and momentum rates (Nayak *et al*., 2005) were employed for model training. Determination of an appropriate architecture for a neural network (number of hidden nodes) for a particular problem is an important issue as the network topology directly affects its computational complexity and its generalization capability. Many studies have revealed that larger-than-necessary networks tend to over-fit the training samples and thus have poor generalization performance, while too-small networks (that is, with very few hidden neurons) will have difficulty in learning the training data. Currently there is no established methodology for selecting the appropriate network architecture before training (Coulibaly *et al*., 2001).

Table II. Combinations of input variables considered in developing ANN models

Model	Input vectors
Model 1	Temperature, Humidity, Dew point, Radiation, Wind speed
Model 2	Temperature, Humidity, Radiation, Wind speed
Model 3	Temperature, Humidity, Wind speed
Model 4	Temperature, Humidity, Radiation
Model 5	Temperature, Radiation
Model 6	Temperature, Humidity

In the current study, the number of hidden neurons in the network was identified by various trials. The trial and error procedure started with two hidden neurons initially, and the number of hidden neurons was increased to 12 with a step size of 1 at each trial. For each set of hidden neurons, the network was trained in batch mode to minimize the mean square error at the output layer. In order to check any over-fitting during training, cross-validation was performed by keeping track of the efficiency of the fitted model. Though the available data was divided into only two sets (training and validation), 10% of the training examples were set aside for cross-validation, and this set was not employed in training. The training was stopped when there was no significant improvement in the efficiency. The model was then tested for its generalization properties by examining the computational accuracy of the trained model on the validation data set. The parsimonious structure that resulted in minimum error and maximum efficiency during training as well as testing was selected as the final form of the ANN model.

The values of ET_0 computed by all the models were analysed statistically using various indices employed for the performance analysis of models. The goodness-of-fit statistics considered are the coefficient of correlation (CORR) between ANN computed and targeted ET and the model efficiency (Nash and Sutcliffe, 1970). Based on this analysis, the best architecture for each ANN model was identified. These selected models were subjected to further evaluation for their effectiveness in estimating the ET_0, and a comparative analysis was carried out.

RESULTS AND DISCUSSION

Model Architecture Selection

The results, in terms of the performance indices for the trial and error procedure employed to identify the appropriate network architecture for each input combination, are depicted in Figure 2. It is evident from Figure 2 that as the number of hidden neurons increases, the model performance also increases, up to a certain level. Thereafter, any additional neuron in the hidden layer dampens model performance. It is observed that for Model 1, the correlation and efficiency statistics during training increase along with the number of hidden neurons. However, the validation efficiency is found to deteriorate for more than eight neurons in the hidden layer, suggesting that the

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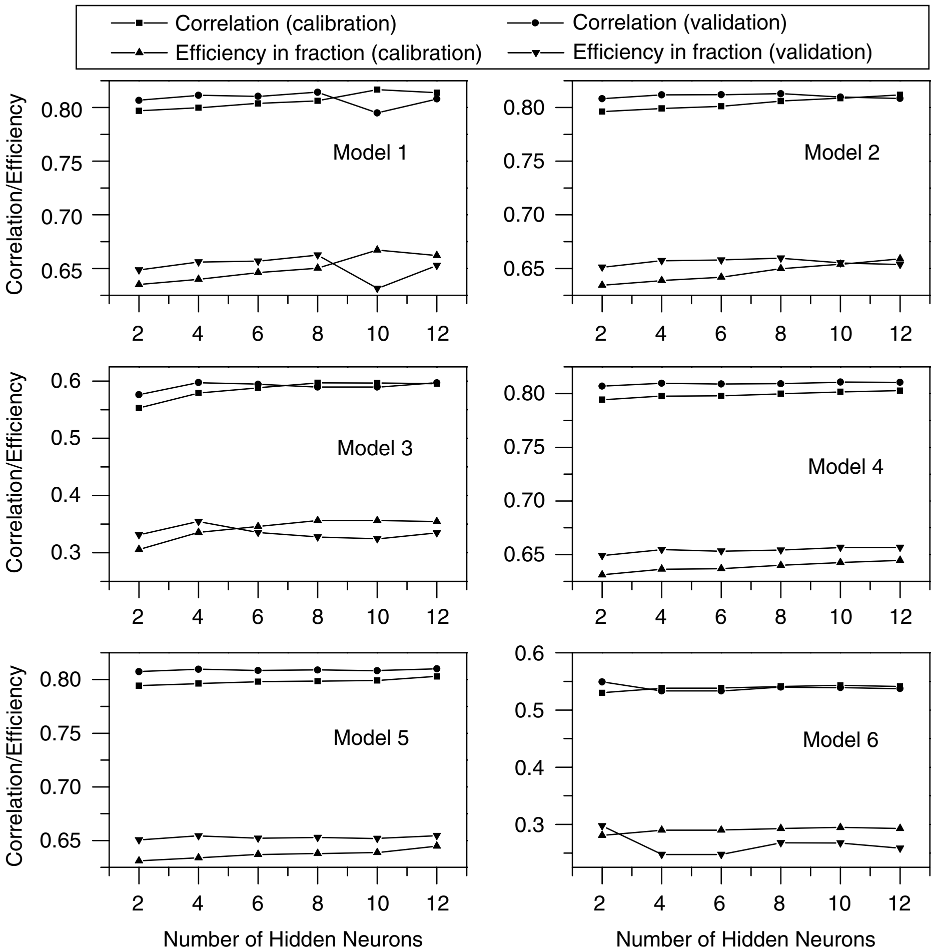

Figure 2. Variation in performance of different models with number of hidden neurons in the ANN architecture in terms of correlation and efficiency statistics

model is trying to over-fit the data. Therefore, eight hidden neurons were considered to be appropriate for Model 1. In a similar way, the optimal number of hidden neurons was determined for the other models. The number of hidden neurons identified for the other models was eight, eight, ten, eight and ten, respectively, for Model 2, Model 3, Model 4, Model 5, and Model 6.

It is noted from Figure 2 that the performance of Model 3 and Model 6 was not satisfactory in estimating the PM ET_0 (plausible reasons for this underperformance are discussed in later sections). Consequently, Model 3 and Model 6 were not considered for further evaluation.

Performance Evaluation of Models

Figure 3 shows scatter plots of PM-estimated and ANN-computed ET_0, from which it can be observed that the ANN models compute the PM ET_0 well at low values, but fails to preserve its accuracy at higher values of ET_0. It is worth mentioning that the greatest deviations in ANN-computed ET_0 from PM-ET_0 occur for the hours around 1 pm to 3 pm in the months June to August, when temperatures are high (usually above 24 °C) and the relative humidity is low (generally below 0·15). Note that Irmak *et al.* (2003) suggest that the weather measurements need to be taken at a properly watered and maintained agricultural setting, otherwise adjustments to air temperature, humidity, and wind speed measurements may be necessary when applying the PM method for ET_0 estimation. No such adjustment was made in this study in the absence of requisite information. Hence the higher value of PM-ET_0 during this period (where ANN was not able to map it effectively) requires further examination, but is beyond the scope of this paper. However, it may be noted that all the models result in a coefficient of determination (r^2) value of 0·81, despite different input combinations. It is worth mentioning that Model 5, which considers temperature and radiation values as the only input variables, computes a PM-ET_0 comparable to that of Model 2, which considers most of the influencing variables. The scatter plot also shows that the data mostly follow the 45° line but many points are located above/below the line. The points above the line are scattered and those below the line seem to follow another line. Such behaviour could be due to the large range of the controlling variables (temperature, radiation).

Note that Figure 3 gives only a visual presentation of the model performance, and the robustness of the models cannot be clearly assessed. Hence the models are evaluated using additional evaluation measures, i.e. sum of squared error (SSE), standard error of estimates (SEE), average absolute relative error (AARE), noise to

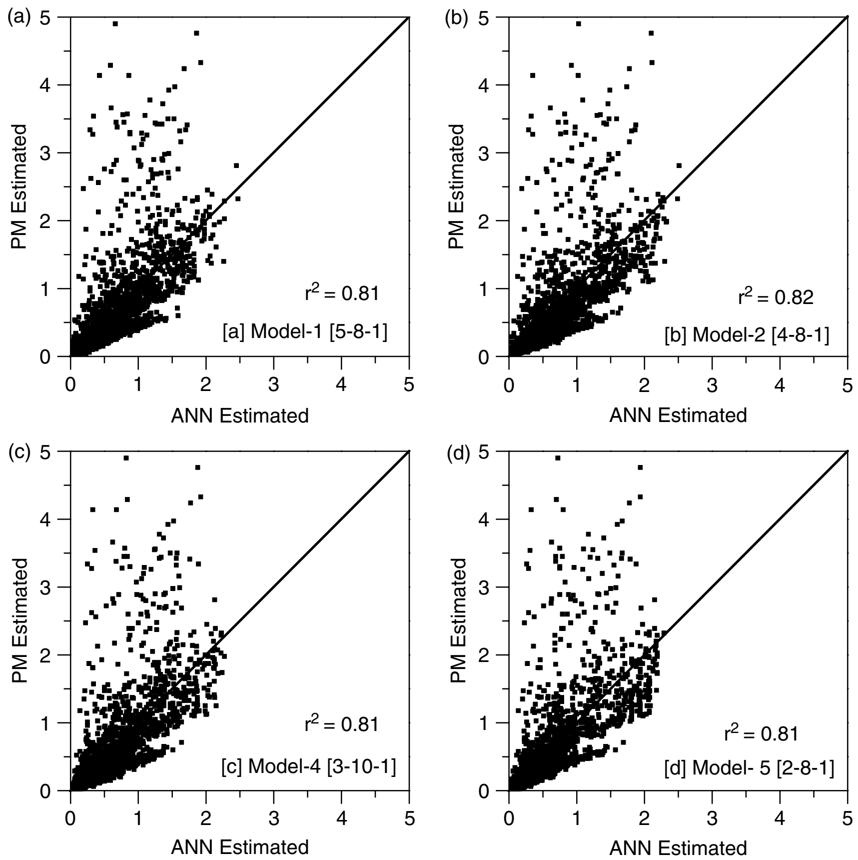

Figure 3. Scatter plots showing evapotranspiration estimated by Penman–Monteith (PM) method and selected ANN models: (a) Model 1; (b) Model 2; (c) Model 4; and (d) Model 5

signal ratio (NS) and mean absolute error (MAE). The definitions of these performance measures are given in Table III.

The values of performance indices for all the models are presented in Table IV. The SSE value is a measure of the unexplained variance and is found to be the least for Model 2. A significant observation is that the unbiased standard error of estimates (SEE) is comparable for all the models. The results presented in Table IV suggest that the ANN models developed can be ranked in terms of performance as Model 2, Model 1, Model 4, and Model 5 in decreasing order of performance. While Model 2 shows a consistent performance in terms of all indices, other models are not significantly inferior in performance. The results suggest that the ANN approach can compute PM-ET_0 efficiently from limited climatic data and this observation is in agreement with Sudheer *et al.* (2003). It may be noted that Sudheer *et al.* (2003) used an ANN to estimate ET for a particular crop, while here the focus is on estimation of ET_0. This observation is significant since the data requirement to obtain an accurate estimation of ET_0 can be considerably reduced.

Since the climatic parameters are highly correlated with ET_0, one obvious question may be: how best is the linear model compared to the ANN models developed in this study? Hence, in addition to ANN, a multi-linear regression (MLR) model was also established to estimate ET_0 from the independent variables. Note that a single

Table III. Definitions of performance criteria

Evaluation criteria	Definition
Sum of Squared Errors (SSE)	$SSE = \sum_{i=1}^{n} \left(y_i^o - y_i^c\right)^2$
Standard Error of Estimate (SEE)	$SEE = \sqrt{\dfrac{\sum_{i=1}^{n} \left(y_i^o - y_i^c\right)}{\nu}}$
Average Absolute Relative Error (AARE)	$AARE = \frac{1}{n} \sum_{i=1}^{n} \lvert RE_t \rvert$ in which, $RE_t = \dfrac{y_t^o - y_t^c}{y_t^o} \times 100$
Noise to Signal Ratio	$NS = \dfrac{SEE}{\sigma_y}$
Mean Absolute Error (MAE)	$MAE = \frac{1}{n} \sum_{i=1}^{n} \left\lvert \left(y_i^c - y_i^0\right) \right\rvert$
Coefficient of correlation	$r = \dfrac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sigma_x \sigma_y}$
Nash and Sutcliffe efficiency	$\eta = 1 - \dfrac{\frac{1}{n}\sum_{i=1}^{n}(y_i^c - y_i)^2}{\frac{1}{n}\sum_{i=1}^{n}(y_i^o - y_i)^2}$

y_i^o and y_i^c, respectively are the PM-estimated and ANN-computed ET_0 values at time t, ν is the number of degrees of freedom, σ_y is the standard deviation of the PM-estimated and n is the total number of data points. A bar over the variable denotes its mean value.

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Table IV. Statistical performance indices of selected models during calibration and validation

Evaluation criteria	Model	Calibration	Validation
Sum of Squared Errors (SSE)	Model 1	636·79	533·60
	Model 2	629·68	524·66
	Model 4	642·17	535·42
	Model 5	650·12	541·81
	MLR*	831·84	672·80
Standard Error of Estimate (SEE)	Model 1	0·30	0·26
	Model 2	0·30	0·26
	Model 4	0·30	0·26
	Model 5	0·30	0·26
	MLR	0·34	0·29
Average Absolute Relative Error (AARE)	Model 1	0·35	0·39
	Model 2	0·34	0·34
	Model 4	0·48	0·56
	Model 5	0·51	0·62
	MLR	2·12	1·56
Noise to Signal Ratio (NS)	Model 1	0·60	0·59
	Model 2	0·59	0·58
	Model 4	0·60	0·59
	Model 5	0·60	0·59
	MLR	0·68	0·66
Mean Absolute Error (MAE)	Model 1	0·10	0·09
	Model 2	0·10	0·09
	Model 4	0·11	0·10
	Model 5	0·11	0·10
	MLR	0·16	0·14

* MLR—Multiple linear regression model

MLR was developed using all the potential influencing variables. The model had the following form:

$$ET_0 = a \times \text{Temperature} + b \times \text{Humidity} + c \times \text{Radiation} + d \times \text{Wind speed} + \text{Constant}$$

The best values of the parameters were estimated by using the least squares error algorithm and these turned out to be: $a = -0{\cdot}0596$, $b = 0{\cdot}1151$, $c = -0{\cdot}0076$, $d = 0{\cdot}2469$, Constant $= 0{\cdot}0248$. The performance of the MLR model in terms of various statistical indices are also presented in Table IV. It is evident from Table IV that the ANN models outperform the MLR and this is an indication of their ability to capture the nonlinear input–output relationship.

Physical Interpretation of the ANN ET_0 Models

It is a common belief that the ANN models of any physical process are purely black box models as they do not explain the process being modelled. However, it must be realized that the data employed for developing ANN models do contain important information about the physical process being modelled. Hence, the ANN models can be interpreted as representing the physical process by performing analyses such as input variable sensitivity with respect to the output variable. There is no widely accepted method for extracting knowledge from the weights of a trained ANN. However, a number of methods have been proposed for interpreting the weights of a trained ANN so as to assess the relative importance of each of the input variables (Garson, 1991; Olden and Jackson, 2002, Olden *et al*., 2004). The relative contributions of ANN inputs in calculating the output are dependent on the magnitude and direction of the connection weights. When the sign of the weights connecting the input-hidden and hidden-output layers are the same (i.e. either both positive or both negative), the input will have a positive impact on the output. On the other hand, if the signs of these connection weights are opposite, the specific input will have an inhibiting effect on the output. Further, the overall contribution of the input on the output will depend on the positive and inhibiting effect of it through different hidden nodes. In this study, a connection weight procedure suggested by Garson (1991) is employed to evaluate the sensitivity of each climatic variable on PM-ET_0 with the help of the parameters of the ANN models. While the physical process of evapotranspiration is well understood, this analysis helps explain why an ANN model is able to accurately compute ET_0 with limited climatic data.

Garson (1991) advocated the following procedure to physically interpret the working of ANNs. Consider an ANN having l, m, n neurons in input, hidden and output layers, respectively. Let i, j, k represent the index of neurons in these respective layers. Let W_{ij} and W_{jk} represent the weight parameter between the input-hidden and hidden-output connections. The algorithm starts by computing the positive or inhibitive effect (P_{ij}) of each input variable through each of the hidden nodes on the kth output:

$$P_{ij} = W_{ij} \times W_{jk} \; for \; \forall i \; and \; for \; \forall j \tag{3}$$

Then the combined effect of P_{ij} for all input nodes on each hidden node $j(S_j)$ is estimated by arithmetic summation of P_{ij} for all input nodes connecting to node j. Thus,

$$S_j = \sum_{i=1}^{l} P_{ij} \tag{4}$$

Subsequently, the P_{ij} for each input variable is normalized using the combined effect on every hidden node (S_j) such that:

$$P'_{ij} = \frac{P_{ij}}{S_j} \tag{5}$$

The individual contribution by each of the input (S_i) is then calculated by summation of the normalized contribution of each input to all hidden nodes. That is,

$$S_i = \sum_{l=1}^{m} P'_{ij} \tag{6}$$

Table V. Relative importance (%) of input variable for ET_0 evaluated from ANN weights

	Temperature	Humidity	Radiation	Wind speed	Dew point temperature
Model 1	17	25	30	7	21
Model 2	28	20	34	17	
Model 3	52	21		26	
Model 4	36	17	47		
Model 5	58		45		
Model 6	55	42			

Finally the relative importance of each input variable in computing the kth output is estimated by:

$$RI_i = \frac{S_i}{\sum_{i=1}^{l} S_i} \times 100 \tag{7}$$

where RI_i is the relative importance (expressed in per cent) of variable at neuron i in the input layer on the variable at neuron k in the output layer. The whole computation is repeated for each output neuron (i.e. for $k = 1, \ldots, n$).

The relative importance of each of the input variables in all the models computed according to Garson (1991) is presented in Table V. It can be observed from Table V that in Model 1, radiation has the maximum influence (30·10%) on ET_0 estimation as discussed earlier. Note that when radiation is not present in the ANN model as one of the inputs (e.g. Model 3 and Model 6), temperature data has the greatest influence, which indirectly takes care of the effect of radiation (temperature is very well correlated with radiation; Table I). It may also be noted that in the current data set, temperature is more strongly correlated with humidity than with radiation (Table I), implying that temperature may also account for the vapour pressure deficit. As a result, Model 3 (and Model 6) appears not to represent the effects of radiation, and fails to compute ET_0 effectively. As temperature can be considered to be an indicator of the vapour pressure deficit, when it is combined with radiation data in Model 5, a better estimate of ET_0 is obtained. Note that while investigating the relative importance of input variables in computing ET_0, ANN models were developed with a single input that considered only temperature and only radiation as input variables. However, the results were not promising and hence, these models were not investigated further and their results are not presented here.

SUMMARY AND CONCLUSIONS

This paper discusses a research study conducted to develop ANN based models to estimate ET_0 from limited climatic data. The motivation for the study was the cumbersome procedure and large data requirement (not easily available in many situations) for estimating ET_0 using the FAO recommended Penman–Monteith method. The results of the study show that an ANN technique can be used successfully to estimate ET_0 from climate data. It is observed that for accurate estimation of ET_0 using an ANN, temperature and radiation data are the most crucial inputs. A sensitivity analysis of the input variables on ET_0 performed using the connection weights of the ANN models confirmed this. The results of the study indicate that the ANN can estimate ET_0 accurately even if data for only these two variables are available. An ANN model whose input consists of temperature and humidity or temperature, humidity and wind speed cannot provide a good estimate of ET_0 because the major predictor variable (radiation) is not present in the input vector. Among the input combinations that were examined in this study, an ANN with inputs of temperature, humidity, dew point, radiation, and wind speed provides the best estimate of reference evapotranspiration.

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