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MODELS OF EXTENDED RADIO SOURCES

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SUMMARY

Some of the basic requirements of viable models of double radio sources are examined. Some models require large mass densities in the most intense radio components and can now be excluded because they predict observable optical emission from the radio components which is not observed. The process of transporting energy from the central object to the radio components is studied, with a view to defining the conditions in which energy can be conveyed without prohibitive losses by adiabatic expansion. It appears likely that energy, and also mass, is brought into the radio components over a time scale comparable with the age of the source. A basic difficulty in accumulating mass in a radio component is outlined.

1. INTRODUCTION

In this paper we shall try to define the features required of a model if it is to represent a real radio galaxy or quasi-stellar source with extended components. In the first part we summarize the most relevant observational evidence and the basic problems which this evidence poses. In the second part we examine various models, particularly with a view to finding out which are now untenable.

We do not discuss the nature of the redshifts of quasi-stellar objects. This paper will be written as if they were cosmological, but almost all the arguments apply to radio galaxies in any case, even if the radio sources around quasi-stellar radio sources turn out to be unrelated phenomena. Many of the arguments we use are well known to specialists, but so far as we are aware they have not hitherto been set down systematically.

2. THE BASIC FACTS AND PROBLEMS

2.1 *Observational evidence*

Much of the information we use comes from maps of a complete sample of 200 bright extragalactic radio sources (all those having $|b| > 10^\circ$, $\delta > 10^\circ$, $S_{178} \geqslant 9 \times 10^{-26}$ W m^{-2} Hz^{-1}; Macdonald, Kenderdine & Neville 1968; Mackay 1969, 1971a) and a subset of 45 of these mapped with angular resolution $6''\cdot5 \times 6''\cdot5$ cosec δ (Branson *et al.* 1972, and references (4)–(9) therein). Hubble's constant will be taken to be 50 km s^{-1} Mpc^{-1} throughout.

The most striking general features shown by extragalactic sources are:

(a) The very wide dispersion in intrinsic radio luminosity, ranging from sources such as 3C 272.1 (M84) ($P_{178} \sim 4 \times 10^{21}$ W Hz^{-1} sr^{-1}) which is only an order of magnitude more powerful than normal galaxies to those with luminosities 10^7 times greater, such as the QSO 3C 9. It is useful to remember that for normal galaxies there is no difficulty in accounting for their energy requirements in terms of supernova or pulsar activity at the observed rate of formation of such objects in the galaxy.

(b) A characteristic double structure, the components being more or less symmetrically disposed on either side of the nucleus of the galaxy or QSO at distances typically 100–200 kpc (Longair & Macdonald 1969; Mackay 1971a). Among sources which fall in the category of normal galaxy there are also examples of double structure but on a much smaller scale (e.g. NGC 3726 and 4736, van der Kruit 1971; 3C 272.1, Riley 1972). The most powerful radio sources ($P_{178} \geqslant 10^{26}$ W Hz^{-1} sr^{-1}) which have overall sizes greater than 1″ arc appear to be exclusively double sources with occasionally a compact radio source associated with the nucleus (see (c) below). The components of these sources are small in comparison with the distance to the optical object and contain a very large amount of energy. The ratio of the distance D of a component from the optical nucleus to the component radius r takes a value greater than 40 in the case of Cygnus A (Mitton & Ryle 1969) and greater than 60 in that of 3C 390.3 (Harris 1972). Miley & Wade (1971) have shown that there is structure having scale less than 1″ arc in the components of Cygnus A which suggests even larger values of D/r. Values of $D/r \gtrsim 100$ are also found in the knots in the optical jet associated with the massive galaxy M87 (Virgo A). De Young (1972) has claimed that double sources in clusters are of smaller physical extent than those in the general field. Recent high resolution data on the 200-source sample do not support this result (Hooley, private communication).

Miley's data (1971) on the total linear size of double QSOs suggest that at large redshifts these sources are of smaller physical extent than those with small redshifts.

(c) Within the nuclei of radio galaxies and QSOs there is frequently evidence of violent activity. More than one in ten of all powerful double sources have compact radio components associated with the nucleus of the radio galaxy or QSO (' triple sources '). Particularly striking examples are 3C 109 (Branson *et al.* 1972) and 3C 390.3 (Harris 1972). Interferometry with very large baselines provides evidence of complex radio structures on scales less than 20 pc in compact sources such as 3C 273B, 279, 120, 84A (e.g. Cohen *et al.* 1971). These results have been interpreted by some workers (e.g. Whitney *et al.* 1971) as indicating the presence of double structure on a very small scale while others infer instead the presence of independent events (Dent 1972a, b). Several of the most compact radio sources are known to be variables at centimetre wavelengths (e.g. Kellermann & Pauliny-Toth 1968). Optical evidence of variability (e.g. 3C 120), broad emission lines, and expanding systems of gas filaments suggesting past explosions (NGC 1275, M82) all support the view that the nuclei of radio galaxies are the seats of continuing violent activity.

(d) More complex structures are found among the 200 sources (Macdonald *et al.* 1968; Mackay 1969) but these seem to be associated with radio sources of intermediate power, typically having luminosities $P_{178} \sim 10^{24} - 3 \times 10^{25}$ W Hz^{-1} sr^{-1}, i.e. only about 1000 times greater than those of normal galaxies. This association is not a selection effect arising from the greater spatial resolution with which weaker nearby sources can be observed. Complex sources are often associated with clusters of galaxies but the compactness of the outer components is not so extreme as in the powerful double sources.

(e) There is definite evidence that in some double sources (e.g. 3C 390.3, 3C 98, 3C 47) the substructure of the components is not aligned with the axis of the double radio source (Harris 1972, 1973, in preparation).

(f) There is also some evidence indicating that the radio source axis is aligned with the major axis of those radio galaxies which have an elliptical appearance (Mackay 1971b).

(g) The polarization of the radio emission from radio sources is always much less than 100 per cent and there does not seem to be a universally preferred orientation of the intrinsic polarization vectors with respect to the axis of the double source (Mitton 1972). In most cases the percentage polarization decreases with decreasing frequency.

2.2 *The basic problems*

We can summarize the basic problems under four headings.

(*a*) *The source of energy*. The evidence of variable sources, triple sources, and the optical activity strongly suggests that the energy originates in the nucleus of the radio galaxy either continuously or in a large number of outbursts. It should eventually be possible to decide whether the observed events in the nuclei of radio galaxies and QSOs can be attributed to ' known ' processes such as supernova or pulsar activity or whether it is necessary to invoke more exotic mechanisms involving black holes, supermassive discs, etc., by which much more than one solar mass of energy can be liberated in a single event. We shall not devote much attention to this aspect of the problem although the discussion below indicates which forms of energy supply are most suitable for producing double-radio sources.

There are obvious attractions in supposing that the energy supply is in the form of *relativistic particles* since these are known to be present in the nuclei of some radio galaxies. The acceleration of the particles is attributed to the violent events in the nucleus. Alternatively the nucleus may be a source of intense *low-frequency electromagnetic radiation* as has been advocated by Rees (1971). Provided that these waves can escape from the nucleus they are an efficient means of transferring energy over large distances and of accelerating particles which are far from the point of origin of the waves. A third possibility is that the supply is in the form of *kinetic energy* of relatively cold matter. All explosions liberate a great deal of energy in this form, and there are mechanisms by which it can be converted into relativistic particle energy. In practice it may be that energy is released in all three forms. Many of the problems which arise are the same in each case.

(b) *The directive ejection of the energy*. The evidence of very long baseline interferometry suggests that the double structure may already be present in the compact nuclei of radio galaxies and that the structure of the nuclear regions is therefore the key to the origin of the directivity.

(c) *Energy transport without loss*. In many models in which fields and relativistic particles are supplied from the nucleus, most of the energy is lost by adiabatic expansion before it can reach the radio components. In Section 3 we discuss how energy may be conveyed without excessive adiabatic losses.

(d) *Containment of radio components*. In view of the large energy requirements of radio sources it may be desirable not to waste relativistic particle energy, and several models have been proposed in which the fields and relativistic electrons are held within the radio components. The principal containment mechanisms are (i) gravity, (ii) external thermal or magnetic pressure, (iii) the inertia of the matter in the radio component, and (iv) ram pressure. Alternatively, if the energy problem is neglected, models can be constructed in which the particles escape freely from the radio components and the energy is continually replenished; these will be called ' continuous flow models '. The problems of containment and of continuous flow models will be described in Section 4.

3. THE ADIABATIC LOSS PROBLEM

3.1 *The expansion of a radio source component*

The simplest conception of the evolution of a radio source is that clouds of plasma containing fast particles and magnetic fields are ejected from the nucleus of the radio galaxy and expand as they travel outwards to become the ' extended components '. The evidence of variable sources and of very long baseline interferometry indicates that the clouds originate in a region smaller than 1 pc, but grow to components 1 kpc, 10 kpc, or even more in diameter. If they merely expand, magnetic flux freezing ensures that the magnetic field $B \propto r^{-2}$, while each relativistic particle's energy E varies as $E \propto r^{-1}$; hence the distribution of particle energies

$$N(E)\,dE = K(r)E^{-(2\alpha+1)}\,dE$$

has $K \propto r^{-2\alpha}$, and the radio luminosity due to synchrotron radiation decreases with radius as

$$P(\nu) \propto K(r)B^{\alpha+1} \propto r^{-(4\alpha+2)}.$$

This classic result due to Shklovskii (1960) has been modified by Kellermann and by van der Laan to take account of self-absorption at low frequencies, and then gives a good account of the behaviour of about half of the known variable radio sources (Epstein 1972). However, expansion from 1 pc to 1 kpc would reduce the radio luminosity by a factor

$$(1000)^{-(4\alpha+2)} = 10^{-15} \quad (\text{for } \alpha = 0{\cdot}75)$$

so this simple picture is totally inadequate to account for the extended double-radio sources. Clearly

(a) The particle and field energy present in a radio component at any one stage of its evolution will have become seriously devalued when it has expanded by a further factor of two; these forms of energy must be replenished at every stage. We must therefore suppose either that fast particles and fields are generated within the radio components or that they are supplied from the central object.

(b) In the latter case, we must investigate how energy may be transferred from the compact central object to the extended components without excessive adiabatic losses.

We now deal with the latter.

3.2 *Who profits from adiabatic loss?*

In the classical textbook situation of a perfect gas in a cylinder from which a piston is slowly withdrawn, the internal energy U diminishes because work is done on the piston: $dU = -p\,dV$. For a relativistic gas (or a tangled magnetic field) the pressure $p = U/3V$, hence $U \propto V^{-1/3}$ as in the Shklovskii expansion. Microscopically what happens is that particles colliding with the receding piston rebound with lower energy—their de Broglie wavelength is redshifted owing to the Doppler effect.

In the derivation of the Shklovskii expansion, however, no mention is made of pistons; the result does not depend on the existence of external constraints. What happens in a free expansion? Consider a cloud of relativistic particles released in a vacuum. The cloud expands with the speed of light, and no particle loses energy. Yet, because of the sorting of momenta, each bit of the resulting spherical shell of particles is relatively cold: the ' internal energy ' of the cloud has been largely ' converted ' into bulk kinetic energy of expansion. If the relativistic particles had been tied to cooler plasma by a magnetic field, the kinetic energy of expansion would be shared with the slow particles, and because of its greater mass the cool plasma would get most of the energy. In thermodynamic terms, we should say that the inner parts of the cloud had done work in pushing against the outer parts and accelerating them.

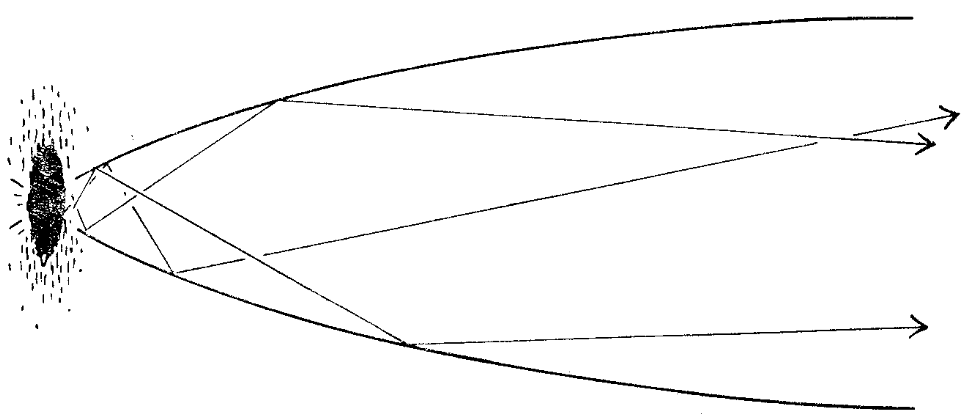

FIG. 1.

A similar ' conversion ' of energy into bulk kinetic energy occurs when fast particles flow freely from the narrower to the wider end of a smooth rigid tunnel (Fig. 1), the walls of the tunnel being rigid so that no work can be done on them. Although each particle retains its whole energy, after each reflection from a wall the particle's velocity makes a smaller angle with the axis of the tunnel; if we start with a group of particles near the central source, moving in random directions, the particles move more and more nearly in a parallel beam as they go down the tunnel. In accordance with Liouville's theorem, such a group occupies a fixed volume of phase space; to make up for its larger ' real ' volume after expansion down the tunnel, it occupies a smaller volume of momentum space. The group has turned most of its ' heat ' energy into bulk kinetic energy. As before, that kinetic energy is still relativistic particle energy only because the particles are free; if they had been wrapped up in a plasma cloud, the kinetic energy would be shared with the plasma. Two points should be noted here:

(a) The smooth rigid tunnel of Fig. 1 is chosen for simplicity only and the same considerations apply to more plausible situations, such as: (i) particles streaming down a diverging magnetic field, conserving their adiabatic invariants and therefore moving in helices with decreasing pitch angles; and (ii) low-frequency electromagnetic radiation which expels some particles and accelerates those left in the tunnel, to yield a situation in which the refractive index is close to unity in the tunnel and decreases outwards. Then photons are totally internally reflected in just the way illustrated in Fig. 1.

(b) The 'rigid' walls of the tunnel have to withstand a pressure which is large close to the source, but very small (energy density $\times \langle \sin^2 \theta \rangle$) further out, where the particle paths make small angles θ with the sides of the tunnel. This is clearly a desirable feature, since it is difficult to think of ways of providing strong walls far from a galaxy or QSO.

3.3 *Continuous energy supply*

There are several arguments in favour of models in which energy is supplied continuously to the source components. First, there is the evidence of violent activity in the nuclei of radio galaxies and in QSOs. At least 10 per cent of double radio sources have compact radio components associated with their nuclei. There would be little likelihood of observing events each lasting only 0·1–10 yr if they did not occur more or less continuously over much of the lifetime of the radio source. Second, the energies associated with events in these central components do not greatly exceed those which might be obtained from supernovae. A single radio outburst does not require more than 10^{47} J (Kellerman 1972). The optical variations of 3C 273 (assumed to be at the distance corresponding to its redshift) over a timescale of 10 yr require energy to be produced in rather larger units, 10^{47} to 10^{48} J. Third, a crude comparison of the rate of energy supply from the compact components with the rate necessary to produce double sources suggests that their orders of magnitude are the same.

Energy in the form of relativistic particles and/or fields could be supplied either 'wrapped', i.e. trapped in gas clouds which are ejected in the right direction by the central object, or 'loose', meaning that the particles and fields do not have any cool plasma associated with them.

If the energy is 'wrapped', adiabatic losses occur, but need not be as severe as in the case of a single expanding cloud described in Section 3.1. Suppose that each year for 10^6 yr a source of radio power P_{compact} and diameter 1 pc is formed in the central object, and ejected, and that these contributions accumulate to form an extended radio component 1 kpc in diameter. Then each little component has to expand linearly by a factor $\{(1\text{ kpc})^3/(10^6 \times 1\text{ pc}^3)\}^{1/3} = 10$, so its radio power diminishes by a factor $10^{-(4\alpha+2)}$; the radio power of the extended component would then be

$$P_{\text{extended}} = 10^{4-4\alpha} P_{\text{compact}} = 10\, P_{\text{compact}} \quad \text{if } \alpha = 0{\cdot}75.$$

More generally, with N contributions and a ratio R of linear sizes,

$$P_{\text{extended}} = R^{-(4\alpha+2)}\, N^{(4\alpha+5)/3}\, P_{\text{compact}}.$$

There cannot be more than 10^{10} contributions at the rate of 1 per year, and probably $N < 10^9$, so this process cannot yield extended components stronger than the central source if $R \gtrsim 10^5$ (e.g. 10 kpc/0·1 pc). A serious difficulty with this model (and many others) is indicated in Section 4.6.

If the energy is supplied 'loose', adiabatic losses can be avoided, provided that the energy passes through a tunnel whose walls are sufficiently resistant to motion so that no significant work is done on them. Then, as described in Section 3.3, the energy is converted into a relativistic beam, and the energy 'lost' in adiabatic expansion is converted into the kinetic energy associated with the systematic motion of the particles or fields. It seems necessary that the beam should be well collimated, rather than bouncing back and forth across the tunnel at large angles. Apart from the self-collimating tendency of a beam in a diverging tunnel, noted above, we can give two reasons. First, if the forward velocity of the beam were not much greater than the speed V of the radio component, the containment problem would be more severe in the tunnel than in the radio component itself. Second, more energy would then be in the tunnel than in the radio component; the tunnel might therefore be expected to produce more radio emission than the radio component. It also appears likely that the beam is collimated in or close to the central object. The interaction of a gas with beams of relativistic particles or strong low-frequency waves is still a matter for speculation, but it is unlikely to be described by analogy with wave optics. The impinging energy can deform the walls of the tunnel, and instabilities are likely to develop which distort the interface, scatter the beam, and thus destroy the collimation. Such disturbances are least likely to be troublesome if there is as little interaction as possible with the tunnel walls, that is, if the particles or waves are injected in the correct direction. The concept of a tunnel then becomes less important or even irrelevant. Quite apart from these theoretical considerations, the discovery of very compact double sources (by very long baseline interferometry) suggests that we should look to the nuclei of radio galaxies for the origin of double structure.

So far we have assumed that the energy for the extended components is supplied from the centre in the form of fast particles or fields or both. The alternative is that fast particles and fields are produced continuously within the extended component, from some other form of energy that does not exert pressure and thus does not suffer adiabatic losses in transit. Two such possibilities are:

(1) The central object ejects compact bound systems, perhaps pulsar-like objects, which are efficient particle accelerators (Burbidge 1967; Saslaw *et al.* 1973); and (2) the kinetic energy of plasma associated with the extended radio components is partially converted into fast particle and field energy.

In both of these cases, it seems likely that most of the energy must be supplied from the central object in one relatively brief event, for reasons discussed in Section 4.6.

4. CONTAINMENT OF RADIO COMPONENTS

4.1 *Introduction*

So far we have considered the transfer of energy from the nucleus of a galaxy to the two distant radio components. Next we ask how the energy is kept there—if indeed it is. We shall use the phrase ‘ useful energy ’ for the energy of the fields and fast particles responsible for radio emission.

Gravitational binding has been considered (Burbidge 1967) but in most models the fast electrons and fields are prevented from dispersing at the speed of light by some pressure ρv^2 where ρ is the density and v is a characteristic velocity of the resisting material. The pressure is required to balance that due to the ‘ useful energy ’ and is one-third of the energy density u, the latter having at least the value

$$\text{constant } \nu^{2/7}(P/\text{volume})^{4/7}$$

given by the minimum energy argument for synchrotron radiation. (P/volume) is the radio power per unit volume at frequency ν. The minimum energy density

TABLE I

Radio source	Type of source	Components	U_{min}	Reference
Cygnus A	Powerful radio galaxy	Compact components having $\theta \leqslant 3''$	$1 \cdot 0 \times 10^{-9}$ J m^{-3}	Mitton & Ryle (1969)
		Extended tails	$2 \cdot 2 \times 10^{-12}$ J m^{-3}	Mitton & Ryle (1969)
		Very compact components $\theta \lesssim 0''{\cdot}75$	3×10^{-9} J m^{-3}	Miley & Wade (1971)
3C 465	Complex weak radio galaxy	Extended resolved components		
		3	$8 \cdot 7 \times 10^{-13}$ J m^{-3}	Macdonald, Kenderdine & Neville (1968)
		7	$4 \cdot 9 \times 10^{-13}$ J m^{-3}	
		Partially resolved components		
		1	$1 \cdot 5 \times 10^{-12}$ J m^{-3}	
		2	$6 \cdot 3 \times 10^{-13}$ J m^{-3}	
		5	$4 \cdot 9 \times 10^{-13}$ J m^{-3}	
		6	$5 \cdot 6 \times 10^{-13}$ J m^{-3}	
M87	Weak radio galaxy	Radio double	$4 \cdot 6 \times 10^{-10}$ J m^{-3}	Graham (1970)
		Radio halo	$1 \cdot 0 \times 10^{-13}$ J m^{-3}	Graham (1970)
		Optical blobs	$5 \cdot 6 \times 10^{-10}$ J m^{-3}	Felten (1968) (refers only to optical waveband)
3C 66	Weak complex radio galaxy	Extended components	$7 \cdot 4 \times 10^{-13}$ J m^{-3}	Macdonald, Kenderdine & Neville (1968)
3C 98	Weak radio galaxy with bent components	Bent tail double	$6 \cdot 3 \times 10^{-13}$ J m^{-3}	Branson *et al.* (1972)
3C 83.1B	Extended radio tail	Tail	$1 \cdot 4 \times 10^{-13}$ J m^{-3}	Ryle & Windram (1968)
3C 129	Extended radio tail	Tail	$6 \cdot 8 \times 10^{-14}$ J m^{-3}	Hill & Longair (1971)

is the same for synchro-Compton radiation as for synchrotron radiation, since the power emitted and the characteristic frequency of emission depend in the same way on the electron energy and the field strength. Typical energy densities encountered in different types of radio sources are indicated in Table I.

The restraining pressure may be due to:

(a) *External gas pressure*, in which case ρ is the density and v the r.m.s. thermal velocity in the gas.

(b) *The inertia of relatively cold gas inside the radio component.* Since the ' useful energy ' has to be replaced each time the component doubles its volume, its kinetic energy of expansion must be at least the ' useful energy ' at half its present volume and the pressure is roughly equal to ρv^2 where ρ is the density of the cold gas and v the expansion velocity. This type of model has been discussed by Scheuer (1967).

(c) *Ram pressure exerted by the ambient intergalactic gas* which is pushed out of the way as the radio component moves outward from the parent galaxy or QSO. In this case ρ is the density of the ambient gas and v is the speed of the radio component. A model of this type was proposed by de Young & Axford (1967), and variations of it by Christiansen (1969), Mills & Sturrock (1970) and Mills (1971) In the original de Young & Axford model, the radio emitting material, consisting of relativistic particles, magnetic field and plasma, forms an atmosphere in hydrostatic equilibrium in the cavity formed by the bow wave and is held down against the apex of the cavity by the gravitational field g equivalent to the deceleration of the component.

It is characteristic of both (b) and (c) that the ' useful energy ' can only be a small fraction of the bulk kinetic energy of the plasma clouds, these fractions being of the order of $(r/D)^2$ in (b) and (r/D) in (c), where r is the component size and D its distance from the central object. A modest conversion of kinetic energy into ' useful energy ' could therefore keep the components radiating. Such a conversion is not possible until the component meets resistance since it has no kinetic energy in its own rest-frame; it only becomes available through interaction with the ambient gas which takes up some momentum. The latter statement is also true if the energy is supplied in relativistic beams from the central object (Section 3.3), although the ratio of kinetic energy to momentum is then much higher, and most of the energy is likely to appear as ' useful energy ' in the radio component.

We now consider these types of containment.

4.2 *Gravitational binding*

A radio component is gravitationally bound if its mass M is so great that $GM^2/r \gtrsim U$, where U is the total internal energy and r is the radius of the component. Gravity is important if (a) discrete gravitationally bound objects are ejected from the nucleus, or (b) another containment mechanism requires so much cold matter to be channelled into the component that the above inequality is satisfied. Burbidge (1970) has discussed the first possibility as an explanation for the jet in M87 and similar phenomena. While structures on the scale of 1″ arc are found in the radio components outside galaxies, the really compact radio sources on the scale of 0·001″ arc detected by VLBI have hitherto always been found to coincide with the nucleus of the central object, never with an external radio component. We do not discuss this possibility further in this paper.

4.3 *External pressure*

Until quite recently it was thought that bremsstrahlung from an intergalactic gas with density corresponding to $\Omega = 1$ ($q_0 = 0.5$) would exceed the observed X-ray background unless its temperature were below $\sim 10^6$ K. More recent measurements of the X-ray background, together with a lower adopted value of Hubble's constant, now allow temperatures of 2×10^8 K with $\Omega = 1$ (e.g. Field 1972). Such a gas would exert a pressure of 1.3×10^{-14} N m^{-2} and could thus contain fast particles and fields with an energy density 4×10^{-14} J m^{-3}. Table I shows that this is comparable with the minimum energy densities in the most diffuse sources.

Reference to Fig. 1 of Field (1972) indicates that a lower density ($\Omega = 0.1$) would be compatible with much larger temperatures ($\gtrsim 10^{10}$ K) and thus with energy densities $\gtrsim 2 \times 10^{-13}$ J m^{-3}. However, still larger pressures are excluded by the consideration that the universal pressure must not exceed the total pressure in the disc of the Galaxy.

In the Coma cluster the observed X-ray emission is consistent with a central density $\sim 10^2$ m^{-3} and temperature 7×10^7 K (Gursky *et al.* 1971), and comparable X-ray emission has been observed from a number of other clusters (Gursky *et al.* 1972). The thermal pressure of the gas in the Coma cluster could balance the pressure of 3×10^{-13} J m^{-3} of useful energy. This is of the order of the minimum energy densities in the extended components of weak radio galaxies (see Table I). It is therefore quite likely that thermal pressure plays a large part in containing extended radio components of low surface brightness.

Gull & Northover (1973) have discussed the possibility that the external pressure of very hot, dense gas ($T \sim 10^9$ K, $n \sim 10^4$ m^{-3}) could contain the energy densities of 10^{-9} J m^{-3} found in the components of Cygnus A. They show that such a possibility is not excluded by existing observations of the X-ray source provisionally associated with the Cygnus A cluster. We will not pursue this line of argument in the present paper but refer the reader to their paper.

4.4 *Inertia*

The simplest view, that the radio component of mass M and internal energy U always expands at a speed close to the generalized sound speed $(U/M)^{1/2}$, immediately indicates that the radius of the cloud after time t is $r \simeq (U/M)^{1/2}\, t$. Its distance D from the central object is Vt, so that the kinetic energy is

$$\frac{1}{2} MV^2 \simeq \frac{1}{2}\left(\frac{D}{r}\right)^2 U$$

as stated earlier.

However, $(U/M)^{1/2}$ is not in general a constant and we should consider the time dependence of the energy supply. If the energy is supplied by a relativistic beam from the central object, it is most natural to assume a uniform rate of energy input W. If, on the other hand, a large mass is ejected at an early stage and its kinetic energy is gradually made available by interaction with intergalactic gas, it is more natural to assume that some fraction ϵ of the kinetic energy of the incident intergalactic gas is converted into internal energy. We should distinguish between 'internal energy' U which exerts pressure and 'useful energy' responsible for radio emission; the latter cannot exceed U but it may be much less. We make the most favourable assumption, that all the internal energy is in the form of relativistic particles or magnetic fields, so that $p = u/3$. Hence the power input is

$$\epsilon \pi r^2 \rho_e V^3$$

where ϵ is an efficiency factor which we can only guess but which must be less than 1, and ρ_e is the ambient density. The rate of change of internal energy, which would be given by $(d/dt)(Ur) = 0$ for adiabatic expansion of a relativistic gas, becomes

$$\frac{d}{dt}(Ur) = \begin{cases} Wr & \text{uniform energy supply} \\ \epsilon \pi r^3 \rho_e V^3 & \text{` kinetic ' energy supply} \end{cases}$$

The development of such source models is worked out in Appendix A. Note that these models assume that all the mass of the source component is supplied at $t = 0$ and that the energy supplied is deposited uniformly throughout the component with its present radius r.

In many sources there is no compelling reason for rejecting this type of model. The total energy requirements (equation (A7)) are large but, as we know so little about the energy source, that cannot be regarded as evidence against the model unless the energy requirements approach $c^2 \times$ (a galactic mass). The mass needed in a compact component now depends on the assumed value of V. The frequency with which approximately symmetrical double sources are observed suggests that V is generally not more than 0·1 or 0·2c (e.g. Ryle & Longair 1967; Mackay 1973).

Both the mass and the total energy required are greatest for powerful sources with large ratios D/r ($=Vt/r$ of Appendix A); thus we consider Cygnus A.* We use the data of Mitton & Ryle (1969), scaled to $H = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$; projected distances from the galaxy are then ~ 120 kpc, the compact regions have radii ~ 3 kpc and the minimum energy in each is $U_{\min} \simeq 2 \times 10^{51}$ J. According to equation (A7) the kinetic energy is $2 \times 10^{51} (120/3)^2 = 3 \cdot 2 \times 10^{54}$ J, and if $V = 0 \cdot 2$ to 0·1c the mass required is 2 to 7×10^{39} kg $=$ 1 to $4 \times 10^9 \, M_\odot$. The corresponding energy and matter densities are $u_{\min} \simeq 6 \times 10^{-10} \text{ J m}^{-3}$ and $n \simeq (0 \cdot 3 \text{ to } 1 \cdot 4) \times 10^6$ hydrogen atoms m^{-3}. These values are disturbingly large, and we now consider whether they conflict with observation. The observational limits on n are:

(a) *Free-free absorption*. The optical depth exceeds unity at frequencies below

$$\nu_{FF} = 3 \times 10^{-5} \, n_e r_{kpc}{}^{1/2} T_4{}^{-3/4} \text{ MHz}$$

so that even if the clouds were as cool as 10^4 K, absorption would be small above 100 MHz and negligible at 5 GHz, the frequency of Mitton & Ryle's observations.

(b) *X-ray bremsstrahlung* may be important if $T > 10^6$ K. However, it can readily be shown that the X-ray luminosity in the 1–10 keV energy band reaches a maximum if $T = 10^8$ K, at which temperature the X-ray luminosity of a cloud 3 kpc in radius and with $n_e = 10^6 \text{ m}^{-3}$ would be $\sim 10^{36}$ W, more than an order of magnitude less than the reported luminosity of Cygnus A (Giacconi *et al.* 1972).

(c) *Optical emission*. The emission measure for $n_e = (0 \cdot 3 \text{ to } 1 \cdot 4) \times 10^6 \text{ m}^{-3}$ and $r = 3$ kpc would be 600 to 12 000 cm^{-6} pc, and galactic H II regions with such an emission measure are readily observable. It is not so clear that such a region only 3 to 4 arc sec in diameter would have been noticed.

* As is well known Cygnus A is anomalously close to the Galaxy (by a factor of about 10) by comparison with typical radio sources of the same luminosity class. As a result it has been studied with the highest spatial resolution of all the luminous sources. Other bright radio sources either have much smaller radio luminosities or have been studied with ten times poorer spatial resolution.

More serious problems arise when we consider the small bright spots inferred by Miley & Wade (1970). The brightest of these contains about 10 per cent of the flux of one component and has a diameter $\lesssim 1$ arc sec. The result of diminishing P by a factor of 10 while diminishing r by a factor of 4 is to change the parameters in the following way:

$$u_{\min} \propto P^{4/7} r^{-12/7} \propto 10^{-4/7} 4^{12/7} = 10^{0.46}$$

$$n \propto Mr^{-3} \propto u_{\min}(D/r)^2 \propto P^{4/7} r^{-12/7-2} \propto 10^{1.66}$$

$$\nu_{FF} \propto n_e r^{1/2} \propto 10^{1.36}.$$

For a given temperature the optical and X-ray emission increase proportionally to

$$n^2 r^3 \propto P^{8/7} r^{-43/7} \propto 10^{1.51} = 4 \text{ mag}.$$

To estimate the optical emission we need to know the gas temperature T; if $T > 10^4$ K most of the emission is in the UV and beyond. As the thermal pressure alone must not expand the components beyond their present size in a time D/V, the thermal velocity must never have exceeded $(r/D)V$, and hence $T < 8 \times 10^7$ K, 5×10^6 K for the 3-kpc and 0·75-kpc regions, respectively. At such high temperatures bremsstrahlung is the main radiative heat loss, and it has little effect in time D/V. However, adiabatic cooling is likely to have lowered the temperature far below this initial upper limit, and radiative cooling due to collisional excitation of ions becomes far more powerful below 10^6 K. Therefore the gas is likely to be not far from a balance between heating by the relativistic electrons and radiative cooling. The energy lost by the relativistic electrons can be calculated fairly reliably from standard formulae (e.g. Ginzburg & Syrovatskii 1964), and so long as the gas is fully ionized a large part of the energy becomes heat. (If there is a large departure from equipartition, there are fewer fast electrons, but the total energy and hence n_e rises, and the rate of energy loss rises too.) The results are shown in Table II. The radiative losses are much more sensitive to the condition of the gas: Spitzer (1968, Fig. 4.1) gives loss rates for H II regions ionized by ultra-violet radiation, Cox & Daltabuit (1971) give considerably larger loss rates for gas whose ionization temperature equals its kinetic temperature. Both the above estimates assume normal 'metal' abundances; Gould & Ramsay (1966) give loss rates appropriate for intergalactic gas containing only hydrogen and helium. With our estimates of heating by relativistic electrons, all the temperature estimates lie between $\sim$15 000 K and $\sim$50 000 K for the 3-kpc components; for the 0·75 kpc bright spots the heating may just exceed the peak of the cooling rate at 10^5 K on Gould & Ramsay's curve, but with quite a modest 'metal' abundance the temperature would again fall below 10^5 K. Table II lists two V magnitudes for the components. One if for free-free emission only, and is clearly an underestimate. The total radiative cooling (taken to be equal to the heating rate) would be an overestimate, since most of the energy is radiated in ultra-violet lines. We therefore make a fairly conservative estimate of the optical emission by assuming that energy is radiated uniformly over a frequency interval corresponding to 10^5 K. This second estimate of V is somewhat brighter than the observed upper limit (kindly given to us by Dr C. D. Mackay), allowing for absorption.

(d) *Rotation measure*. With a uniform magnetic field close to the equipartition value of 30 nT, and $n_e = 3 \times 10^5$ m^{-3}, there would be an internal rotation measure over 3 kpc of $2{\cdot}5 \times 10^6$ rad m^{-2} (and about twice as much in the 0·75 kpc bright

TABLE II

	Radio components (Mitton & Ryle 1969)	Small bright spots
Velocity V (assumed)	0·2 c	0·2 c
Radius r	3 kpc	0·75 kpc
Energy density (minimum)	1×10^{-9} J m^{-3}	3×10^{-9} J m^{-3}
Electron density n_e	$0{\cdot}3 \times 10^{6}$ m^{-3}	$1{\cdot}4 \times 10^{6}$ m^{-3}
Maximum temperature if thermal energy < particle and field energy	8×10^{7} K	5×10^{6} K
Cooling time from that temperature	5×10^{8} yr	3×10^{7} yr
Heat input by fast electrons	$5{\cdot}7 \times 10^{35}$ W $1{\cdot}7 \times 10^{-25}$ W m^{-3}	$1{\cdot}0 \times 10^{36}$ W $3{\cdot}0 \times 10^{-23}$ W m^{-3}
Visual magnitude:		
Free-free only	$V = 23{\cdot}5 + 1{\cdot}25 \log T_4$	$22{\cdot}3 + 1{\cdot}25 \log T_4$
If heat input radiated uniformly in frequency interval $k(10^5 \text{ K})/h$	$V = 20{\cdot}5$	19·4
Absorption	$0^{m}{\cdot}9$ (Sandage 1972)	
Observational limit	$V \geqslant 22$ for a 2″ diameter object (Mackay, private communication)	

spots), which would result in zero net polarization at centimetre wavelengths. The observed distribution of polarization over sources, and the low percentage polarization at high frequencies, show that the field is far from uniform. However, in an irregular field with N cells along each line of sight the net statistical fractional polarization on each line of sight through the component is $N^{-1/2}$ of the value for a uniform field [$N^{-3/2}$ for the whole source] and equally the net rotation is $N^{-1/2}$ of that for a uniform field (*cf.* Burn 1966). To obtain any net polarization at centimetre wavelengths, the rotation measure must be less than 10^3 rad m^{-2} and hence $N \geqslant (2{\cdot}5 \times 10^6/10^3)^2 \simeq 6 \times 10^6$. Thus the net fractional polarization along each line of sight should only be 4×10^{-4} and the net total from the whole component only 6×10^{-11}, both in contradiction with observation. There is a way out of this difficulty (Scheuer 1967); if an irregular field is sheared, the field configuration becomes anisotropic and the synchrotron radio emission is partially linearly polarized, but the Faraday rotation is still reduced by a factor of $\sim N^{-1/2}$ below the uniform field value, where N is the number of field reversals on a line of sight. In the case of the Cygnus A components even the latter model becomes implausible, on physical grounds, for to avoid depolarization at 10 GHz we require a rotation measure $\lesssim 10^3$, hence $N^{1/2} \gtrsim 10^7/10^3$, implying turbulence on a scale $\lesssim 10^{12}$ m.

4.5 *Ram pressure*

The basic de Young & Axford model (1967) is governed by the following relations:

Mass conservation $$M = C_M \rho_a h^3, \quad (1)$$

Pressure balance $$U = C_U \rho_e V^2 h^3, \quad (2)$$

Hydrostatic equilibrium $$\rho_a g h = \rho_e V^2. \quad (3)$$

Here ρ_e is the external density, ρ_a is the internal density at the apex of the radio component, h is a scale height defined by equation (3), $g = -dV/dt$ is the effective acceleration due to gravity in the component and C_M, C_U (and C_P—see Appendix B) are dimensionless constants whose values depend only on the form of the equation of state, i.e. the relation between pressure and density. To provide a basis for discussion, their values are calculated in Appendix B both for an 'isothermal' atmosphere (de Young & Axford's original model) in which $p \propto \rho$, and for a well-stirred atmosphere with $\gamma = 4/3$ (i.e. the pressure due to relativistic particles $\gg$ thermal pressure). We may use such a model to represent the source at each stage provided that changes in the model, e.g. the doubling of h, are slow compared to the sound travel time $h/(U/M)^{1/2}$ across the component. The symbol h is used here instead of the radius r since h has a well-defined meaning in this non-spherical model.

There is nothing in the 'instantaneous' model to determine the mass M in terms of observable quantities, but if we suppose that the component decelerated from velocity V_1 a distance D back along its track, then to reach its present position it must have had

$$V_1^2 > 2gD \approx \frac{2\rho_e V^2}{\rho_a h} D$$

$$\tfrac{1}{2} M V_1^2 > \left(\frac{C_M}{C_U}\right)\left(\frac{D}{h}\right) U, \tag{4}$$

i.e. the initial kinetic energy is at least (D/h) times the present 'useful energy'. Our lower limit for M then depends, as in the case of containment by inertia, upon how high a value of the initial velocity we are prepared to accept, but the limit is lower by a factor D/h (and is therefore lower by a factor 120 for the bright spots in Cygnus A).

A test of the model which is independent of guesses about the past history comes from equation (B13). This expresses the condition that one-third of the minimum energy density required to account for the radio emission does not exceed the ram pressure $\rho_e V^2$; it comes directly from the assumption of ram pressure confinement and is insensitive to details of the model. If the ambient density is the 'cosmological' density 10 atoms m^{-3}, we require $V = 0{\cdot}4$ c for the components of Cygnus A described by Mitton & Ryle (1969) and rather larger values $V \approx 0{\cdot}6$ c for the bright spots of Miley & Wade (1971). If we suppose, following Mitton & Ryle, that Cygnus A is in a cluster where the particle density may be 100 m^{-3}, V returns to the more acceptable range 0·1–0·2 c. Comparable values of velocity and density are necessary if the optical components of the M87 jet are confined by ram pressure. These lie within the galaxy M87 where still larger densities may exist.

Thus the model is acceptable on the grounds of present energy and mass requirements. Objections to the model may be raised on the following grounds:

(a) A firmly held belief that no intergalactic matter exists. At present there is no decisive observational evidence either way.

(b) The radio components predicted by the models (*cf.* Christiansen 1969, and Appendix B) are wider than they are long whereas observed radio components are generally said to be longer than they are wide. We are not inclined to worry about this apparent inconsistency because (i) Mills & Sturrock (1970) have proposed a regular magnetic field structure which enables particles to diffuse backwards; (ii) the theoretical model must in any case be drastically modified (see (c) and (d) below); and (iii) the supposed elongated radio components are probably the result of inadequate resolution. Where observations with improved resolution have been made, they either split into a chain of two or more separate components or consist of a still unresolved head containing part of the flux density and an irregular tail.

(c) The models are hydrodynamically unstable (Blake 1972). Rayleigh–Taylor instabilities near the apex cause shocked intergalactic gas to bubble into the components. Kelvin–Helmholtz instabilities round the sides lead to the entrainment of component material into the bow shock. The smallest scale instabilities grow fastest. Blake found that perturbations on a scale which would cause the whole component to break up, e-fold in a time comparable with that required for the deceleration of the component to the stage where the expansion velocity approaches V, i.e. the lifetime of the model. Though these effects necessarily spoil the simplicity of the model, this does not mean that some turbulent mess will not travel outwards, held together essentially by ram pressure.

At the same time, the instabilities supply a natural input of turbulent energy with which the frozen-in magnetic field is likely to reach equipartition. Whether relativistic particle acceleration can take place in such conditions is less clear. If particle and field energy are created from the turbulence in comparable amounts with constant efficiency, we have a ‘ kinetic ’ energy supply and the history of a model based on this scheme is worked out in Appendix C.

(d) The model components presented in the literature succeed in preserving themselves while they travel from the galaxy to their present positions but only at the expense of starting virtually ready-made at the galaxy with diameters not much less than their present diameters of many kiloparsecs. It appears likely that radio sources have their origin in regions ~ 1 pc in size. The authors of the papers are certainly aware of this large gap in the theory but it has not received much attention. We have argued in Section 3 that the answer lies in the continuous injection of energy; initially, while the component has little internal energy U, the component is small (*cf.* equation (2)) and the energy density is always approximately $\rho_e V^2$. A higher density ρ_e close to the central object also helps to make the component small there.

4.6 *The supply of mass to radio components*

In both the inertial and the ram pressure models for containing radio components, the components are required to have large masses originating in the central object, and most of the energy of the source must be in the kinetic energy of these masses. While we have given several reasons for believing that the ‘ useful ’ energy of the radio source is supplied continuously (Section 3), by far the largest part of the total energy would have then to be supplied in some initial explosion. That such a dual energy supply is aesthetically objectionable is a feeble argument, the more so since we know that in supernovae the initial explosion is (sometimes, if not always) followed by a continuous energy supply from a pulsar. However, in the case of extragalactic sources we have the further problem that the hypothetical ejection of matter must be highly directional, so that we need a very strong nozzle to control the flow.

There is a third restriction on the sudden ejection of large masses from compact regions of radius $r_1 \lesssim 1$ pc. The initial internal energy must be sufficient to prevent gravitational collapse, but it must not be so great that it will expand the mass beyond its present size in a time D/V_1. In the case of components contained by their own inertia (Section 4.4), this restriction may be significant. Unless we can contrive an ejection mechanism in which the mass has exactly enough energy to escape gravitational binding and no more, this means that

$$(GM/r_1)^{1/2} < V_1 r/D.$$

The lower limit to the mass M is given by $\frac{1}{2}MV_1{}^2 \gtrsim \frac{1}{2}(D/r)^2\, U$, hence

$$r_1 > G(D/r)\, D^3 u V_1{}^{-4}.$$

For the bright spots in Cygnus A we find, using u from Table I and $V_1 = 0{\cdot}2$ c, that $r_1 > 3$ pc. If the mass had originated in a region smaller than 3 pc, it would now have grown larger than its observed size. This figure is already larger than our provisional estimate of 1 pc, though we have used the most favourable parameters for u and V_1.

As an alternative to an initial explosion one may suppose that the mass is ejected continuously, or in small fragments, over some long time t with a range of velocities V to $V + \Delta V$. If for the moment we ignore collisions between the fragments, the size of the component that is formed after the travel time D/V is at least

$$Vt + \Delta V(D/V),$$

the first term being the length of the cloud or string of clouds as ejected, and the second term the subsequent spread. We now consider what would happen if the gas ejected later consistently had higher velocity than the gas ejected earlier, or if the front end of the component were decelerated by ram pressure so that the gas ejected later can catch up with it. There is inevitably a kinetic energy of the order of $\frac{1}{2}M\Delta V^2$ in the centre of mass frame of the radio component, and this will either cause the clouds to bounce apart again elastically, or will become internal energy, increase the pressure, and soon become kinetic energy of expansion. In old supernova remnants heat can be radiated away effectively, and consequently a very narrow shell can be formed, but in the conditions of extragalactic radio sources both theoretical expectations and the lack of observable optical emission from the radio components argue against an analogous process. There may be a way of converting most of the energy in a cloud–cloud collision into ultra-relativistic particles which can radiate effectively, or there may be a hydrodynamic process in which most of the energy is carried away in a splash involving only a small proportion of the total mass, but we do not know of any plausible way to dispose of the energy in such collisions. Thus we think it is most probable that the effective expansion velocity of the radio component is ΔV, whether the ejected gases collide with each other or not. The observations of component size r then require that

$$\Delta V/V < r/D.$$

Furthermore, since the gas ejected later must catch up with the gas ejected first, within the time of travel D/V, we have:

$$\Delta V(D/V) > Vt$$

and hence

$$t < (\Delta V/V)(D/V) < r/V$$

using the condition on $\Delta V/V$ above. Thus, without regard to the details of the process, 'continuous' mass ejection must take place within a time such that

$$Vt < h$$

and within a velocity range

$$V(1 \pm r/D).$$

Since $r/D < 1/30$ in many sources, and in the case of the bright spots in Cygnus A $r/D \approx 1/120$, it appears that ' continuous ' ejection of matter can only take a small fraction of the lifetime of the source, and the velocity of ejection must be controlled with uncanny precision. Therefore we regard the whole process with a good deal of incredulity, and would prefer models which do not require large masses to be ejected.

4.7 *Continuous flow models*

Since the particles and fields in the radio component must be replaced at frequent intervals, it is no longer clear that attempts should be made to construct models in which they are held together rather than passing through the radio component at the speed of light and escaping freely. The only model of this type which has been discussed in the literature is that of Rees (1971) in which beams of low-frequency electromagnetic radiation are emitted by pulsars or pulsar-like objects in the nucleus. These beams push back the intergalactic gas in their path and accelerate particles at the leading edge of the beam to high energies. These particles then radiate synchro-Compton radiation in the field of the low-frequency waves before escaping. This model has the great attraction that it links the radio components directly to a plausible energy source and that there appears to be no need for any ejection of matter.

Though at first sight this model is radically different from those described earlier, it has many features in common with them:

(a) The minimum ' useful energy ' that must be in the radio component at any one time is the same as if the radiation were synchrotron radiation; (b) we still need an intergalactic gas to turn the bulk kinetic energy of the low-frequency radiation into a form useful for radio emission; and (c) the energy density u in the radio component exerts a pressure $\approx u$ (between $\frac{1}{2}u$ and u in this case, rather than $\frac{1}{3}u$) which must be balanced by the ram pressure $\rho_e V^2$ of the intergalactic gas as the beam sweeps it back at speed V.

A complication arises from the apparently innocuous statement that the useful energy is allowed to escape freely. Since neither low-frequency radiation (if there is any which is not absorbed at the front of the component) nor relativistic particles (if there is a magnetic field, even an extremely weak one, e.g. 10^{-14} T) can penetrate the surrounding gas, they must blow a cavity around the radio component in which the energy density balances the ram pressure of the retreating intergalactic gas which now has to be pushed sideways. Since the energy in the component is completely renewed in a time r/c, the total energy supplied is at least

$$\frac{U_{\min}}{(r/c)}\frac{D}{V} = \left(\frac{c}{V}\frac{D}{r}\right) U_{\min},$$

i.e. 10^2 to 10^3 $U_{\min}$. Even though considerable adiabatic losses occur, the accumulated energy in the cavity is large.

This is an important general consideration for continuous flow models and can be viewed from a slightly different point of view. To supply sufficient momentum to sweep back the intergalactic gas by a relativistic beam, more energy is needed than merely to provide $U_{\rm min}$. The momentum which must be supplied by the beam is of order $Dr^2\rho_e V$. If energy is supplied at rate ϵ, the total momentum supplied by the beam is $(\epsilon/c)(D/V)$. But $\rho_e V^2 r^3 \approx U_{\rm min}$ and therefore the total energy supplied is $(D/r)(c/v)\, U_{\rm min}$. It is an essential requirement of these models that this extra energy be removed from the source component.

There is no cold matter within the cavity. Therefore there is no effective gravitational force to confine the relativistic matter to the leading end of the cavity as occurs in the de Young & Axford model. The fast particles in the cavity will not by themselves produce radio emission though they will produce X-rays by inverse Compton scattering of the microwave background. If, however, a little matter is entrained with the streams of particles, the irregular motions may well generate a magnetic field by dynamo action and then we should again have observable radio emission. In that case we should again be close to the situation envisaged earlier in which a beam of low-frequency radiation acts as the energy supply to a radio component which radiates chiefly conventional synchrotron radiation.

The dynamics of such models will be considered in greater detail in a separate paper.

5. CONCLUSIONS

We have not produced an answer to the radio source problem. What we have attempted to do is to point to certain features demanded of models which are to be consistent with observation and with basic physical principles. The principal points we have raised are:

(a) The severity of adiabatic losses, and the consequent need for a supply of 'useful energy' that continues through much of the source's lifetime.

(b) The restrictions on the form of the energy supply. The observations of compact central components suggests a long-lived energy source at the centre; we argue that if the central source continuously supplies the extended radio components without great adiabatic losses it must do so via relativistic beams.

(c) The thermal pressure of hot intergalactic gas of the densities and temperatures inferred from X-ray observations of clusters of galaxies can confine the extended components of weak complex sources. Gas with much higher density and temperature may surround powerful sources (Gull & Northover 1973).

(d) Containment of radio components by their own inertia is a model which fails in the case of the bright spots in Cygnus A, because (i) it predicts observable optical emission, and (ii) it predicts complete depolarization even at centimetre wavelengths. While the evidence is not yet as simple and clear-cut as one would like it to be, there is scope for improved observational limits on optical emission and also for better theoretical estimates.

(e) The details of the ram pressure containment models of de Young & Axford are probably not applicable to real sources, because of instabilities and also because a continuous supply of energy and momentum would destroy the simple internal structure of the radio components.

(f) There are basic physical difficulties in getting large masses of gas from a central object into small radio components. These difficulties do not seem to have been mentioned before, and they lead us to doubt both the inertia and the ram pressure models for radio source containment. It still appears to be true that in general radio sources are less polarized at low frequencies than at high frequencies, and the only convincing known explanation for such a systematic trend is Faraday rotation within the radio components. Thus some gas is required in the components, but the densities needed are low, and might even be supplied from ambient gas in clusters of galaxies, rather than ejected from the central object.

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REFERENCES

Blake, G. M., 1972. *Mon. Not. R. astr. Soc.*, **156**, 67.
Branson, N. J. B. A., Elsmore, B., Pooley, G. G. & Ryle, M., 1972. *Mon. Not. R. astr. Soc.*, **156**, 377.
Burbidge, G. R., 1967. *Nature*, **216**, 1287.
Burbidge, G. R., 1970. *Astrophys. J.*, **159**, L105.
Burn, B. J., 1966. *Mon. Not. R. astr. Soc.*, **133**, 67.
Christiansen, W., 1969. *Mon. Not. R. astr. Soc.*, **145**, 327.
Cohen, M. H., Cannon, W., Purcell, G. H., Shaffer, D. B., Broderick, J. J., Kellermann, K. I. & Jauncey, D. L., 1971. *Astrophys. J.*, **170**, 207.
Cox, D. P. & Daltabuit, E., 1971. *Astrophys. J.*, **167**, 113.
Dent, W. A., 1972a. *Science*, **175**, 1105.
Dent, W. A., 1972b. *Astrophys. J.*, **175**, L55.
De Young, D. S., 1972. *Astrophys. J.*, **173**, L7.
De Young, D. S. & Axford, W. I., 1967. *Nature*, **216**, 129.
Epstein, E. E., 1972. *IAU Symposium No.* 44, *External galaxies and quasi-stellar objects*, p. 227, ed. D. S. Evans, Reidel, Dordrecht.
Felten, J. E., 1968. *Astrophys. J.*, **151**, 861.
Field, G. B., 1972. *A. Rev. Astr. Astrophys.*, **10**, 227.
Giacconi, R., Murray, S., Gursky, H., Kellogg, E., Schreier, E. & Tananbaum, H., 1972. *Astrophys. J.*, **178**, 281.
Ginzburg, V. L. & Syrovatskii, S. I., 1964. *The origin of cosmic rays*, pp. 141–3, Pergamon Press Ltd, Oxford.
Gould, R. J. & Ramsay, W., 1966. *Astrophys. J.*, **144**, 587.
Graham, I., 1970. *Mon. Not. R. astr. Soc.*, **149**, 319.
Gull, S. F. & Northover, K. E., 1973. *Nature*, **244**, 80.
Gursky, H., Kellogg, E., Murray, S., Leong, C., Tananbaum, H. & Giacconi, R., 1971. *Astrophys. J.*, **167**, L81.
Gursky, H., Solinger, A., Kellogg, E. M., Murray, S., Tananbaum, H., Giacconi, R. & Cavaliere, A., 1972. *Astrophys. J.*, **173**, L99.
Harris, A., 1972. *Mon. Not. R. astr. Soc.*, **158**, 1.
Hill, J. M. & Longair, M. S., 1971. *Mon. Not. R. astr. Soc.*, **154**, 125.
Kellermann, K. I., 1972. *IAU Symposium No.* 44, *External galaxies and quasi-stellar objects*, p. 190, ed. D. S. Evans, Reidel, Dordrecht.
Kellermann, K. I. & Pauliny-Toth, I. I. K., 1968. *A. Rev. Astr. Astrophys.*, **6**, 417.
Longair, M. S. & Macdonald, G. H., 1969. *Mon. Not. R. astr. Soc.*, **145**, 309.
Macdonald, G. H., Kenderdine, S. & Neville, A. C., 1968. *Mon. Not. R. astr. Soc.*, **138**, 259.
Mackay, C. D., 1969. *Mon. Not. R. astr. Soc.*, **145**, 31.
Mackay, C. D., 1971a. *Mon. Not. R. astr. Soc.*, **154**, 209.
Mackay, C. D., 1971b. *Mon. Not. R. astr. Soc.*, **151**, 421.
Mackay, C. D., 1973. *Mon. Not. R. astr. Soc.*, **162**, 1.
Miley, G., 1971. *Mon. Not. R. astr. Soc.*, **152**, 477.
Miley, G. & Wade, C. M., 1971. *Astrophys. Letts.*, **8**, 11.
Mills, D. M., 1971. *Stanford University Institute of Plasma Research Report*, No. 408.
Mills, D. M. & Sturrock, P. A., 1970. *Astrophys. Letts.*, **5**, 105.
Mitton, S., 1972. *Mon. Not. R. astr. Soc.*, **155**, 373.
Mitton, S. & Ryle, M., 1969. *Mon. Not. R. astr. Soc.*, **146**, 221.
Rees, M. J., 1971. *Nature*, **229**, 312.
Riley, J. M., 1972. *Mon. Not. R. astr. Soc.*, **157**, 349.

Ryle, M. & Longair, M. S., 1967. *Mon. Not. R. astr. Soc.*, **136,** 123.
Ryle, M. & Windram, M. D., 1968. *Mon. Not. R. astr. Soc.*, **138,** 1.
Saslaw, W. C., Valtonen, M. J. & Aarseth, S. J., 1973. (preprint Dec., 1972).
Scheuer, P. A. G., 1967. *Plasma astrophysics*, p. 262, Enrico Fermi Summer School Course 39, ed. P. A. Sturrock.
Shklovskii, I. S., 1960. *Astr. J. USSR*, **37,** 256. *Sov. Astr. A.J.*, **4,** 243.
Spitzer, L., 1968. *Diffuse matter in space*, Interscience, New York.
Van der Kruit, P., 1971. *Astr. Astrophys.*, **15,** 110.
Whitney, A. R., Shapiro, I. I., Rogers, A. E. E., Robertson, D. S., Knight, C. A., Clark, T. A., Goldstein, R. M., Marandino, G. E. & Vandenberg, N. R., 1971. *Science*, **173,** 225.

APPENDIX A

Since any conceivable method of supplying energy will disturb the radio source component, it is not worth attempting more than a rough calculation. Accordingly, we say that a force of the order of surface area × pressure $= 4\pi r^2 . \frac{1}{3}u = U/r$ is tending to expand the radio component of mass M and radius r; hence

$$\ddot{r} = CU/Mr. \tag{A1}$$

Here u is the energy density, U the internal energy, and C is a numerical constant of the order of unity.

From the text,

$$(d/dt)(Ur) = Qr^{\beta} \tag{A2}$$

where $Q = W$, $\beta = 1$ for a 'beam' energy supply and $Q = \epsilon\pi\rho_e V^3$, $\beta = 3$ for a 'kinetic' energy supply. The general solution of (A1) and (A2) is frightful, but we can easily find solutions for which $U = Ar^{\alpha}$, which are the solutions corresponding to the desirable boundary conditions that U, r, and $\dot{r}$ are all small at the same time. Equations (A1) and (A2) require that the constants A and α must be

$$A^3 = 3MQ^2C^{-1}\beta(2\beta+3)^{-2} \tag{A3}$$

and

$$\alpha = 2\beta/3. \tag{A4}$$

Hence

$$U = (3MQ^2C^{-1}\beta(2\beta+3)^{-2})^{1/3}\, r^{2\beta/3}. \tag{A5}$$

Substituting for U in (A2) we find

$$\dot{r} = (Q/A(\alpha+1))\, r^{\beta/3} = (3C/\beta)^{1/2}(U/M)^{1/2} \tag{A6}$$

confirming that the model expands at roughly the sound speed. Integrating (A6) yields r as a function of time t, and from this we obtain the ratio of internal to kinetic energy

$$U/\tfrac{1}{2}MV^2 = (2\beta/3C)(1-\tfrac{1}{3}\beta)^{-2}(r/Vt)^2. \tag{A7}$$

For a 'beam' supplying energy at a uniform rate W we find

$$r = (8CW/15M)^{1/2}\, t^{3/2} \tag{A8}$$

and

$$U = \tfrac{2}{5}Wt \tag{A9}$$

so that the maximum possible radio power (i.e. assuming that all of U is in fast electrons and magnetic field energy, in optimum proportions) is

$$P_{\max} \propto U^{7/4} r^{-9/4} \propto t^{-13/8}. \tag{A10}$$

For a ' kinetic ' energy supply, we have the rather special case $\beta = 3$; integrating (A6) yields

$$r = \text{constant} \times \exp(t/\tau),$$

where

$$\tau = 3A/Q = (M/AC)^{1/2} \quad \text{(A11)}$$

and

$$U = Ar^2 = \tfrac{1}{3}Q\tau r^2 = \epsilon\pi r^2 \rho_e V^3(\tfrac{1}{3}\tau) \quad \text{(A12)}$$

the ' energy input in time $\frac{1}{3}\tau$ ',
and

$$P_{\max} \propto r^{5/4} \quad \text{(A13)}$$

Equation (A7), which fails at $\beta = 3$, is replaced by

$$U/\tfrac{1}{2}MV^2 = 2Ar^2/MV^2 = (r/V\tau)^2(2A\tau^2/M) = (2/C)(r/V\tau)^2 \quad \text{(A14)}$$

so that the proportion of ' useful energy ' may be considerably greater than $(r/Vt)^2$ if τ is several times smaller than the age of the source.

In the above we have assumed, wherever relevant, that the component has not been decelerated significantly, i.e. V is constant.

APPENDIX B

In this appendix we derive quantitative relations between the parameters of a classical de Young–Axford radio component and its maximum radio emission, its observed ' diameter ', etc. We work in the rest-frame of the radio component; we measure height z from the apex of the component and the radius r to the surface of the component at height z, from the source axis. The angle χ between the surface of the component and the z axis is given by $\tan\chi = dr/dz$. Like de Young & Axford (1967) we assume that the pressure at the surface of the component due to the incident intergalactic gas (transmitted by the layer of shocked gas) is adequately approximated by $\rho_0 V^2 \sin^2\chi$; also that the deceleration g of the component is so small that the interior may be regarded as a stratified atmosphere in hydrostatic equilibrium under a gravitational field g. De Young & Axford assumed that the energy density u and mass density ρ were distributed in the same way with height (an isothermal atmosphere); here we shall take the slightly more general case of an arbitrary ' equation of state ' $p = p(\rho)$. In particular, we shall also consider the case of a well-stirred atmosphere with an adiabatic lapse rate. So long as the pressure in the cloud is due almost entirely to things with negligible rest-mass (ultra-relativistic particles and magnetic fields),

$$p = \tfrac{1}{3}u$$

and

$$u = \text{constant}.\ \rho \text{ for the ' isothermal ' model}$$

$$u = \text{constant}.\ \rho^{4/3} \text{ for the ' adiabatic ' model.}$$

In the model (Fig. B1), the pressure p is a function of z only, and is determined by the following conditions:

Hydrostatic equilibrium $$dp = -\rho g dz \quad \text{(B1)}$$

Pressure continuity at boundary $$p = \rho_e v^2 \sin^2\chi \quad \text{(B2)}$$

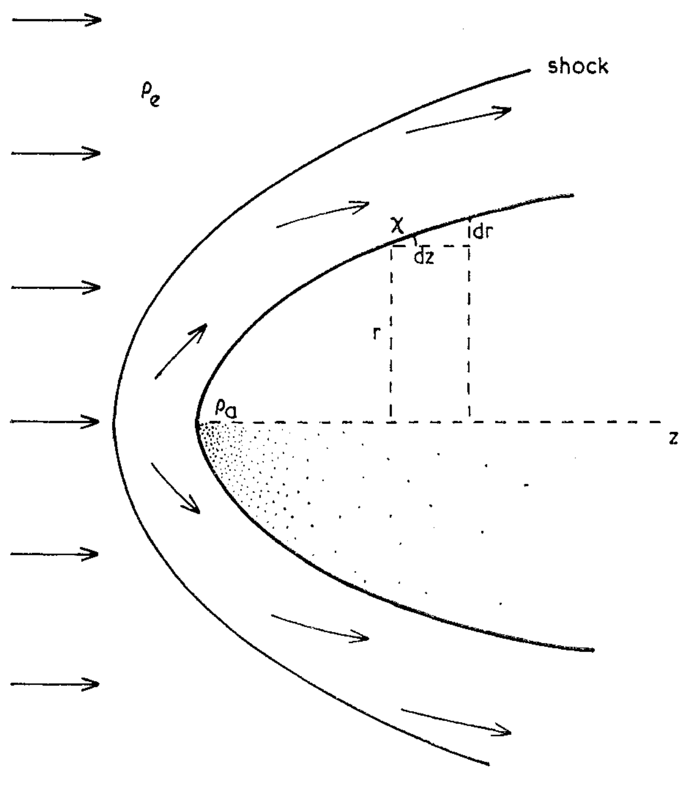

Fig. B1.

It will be convenient to express the density ρ, pressure p, energy density u, etc. in the cloud as fractions of their values ρ_a, p_a, u_a etc. at the apex of the cloud. We define ξ by

$$\rho = \rho_a \xi;$$

the equation of state will be written as

$$p/p_a = f(\rho/\rho_a) = f(\xi),$$

i.e.

$$p = \rho_e v^2 f(\xi),$$

by (2), since $\chi = \pi/2$ at the apex.

Equation (1) now becomes

$$\rho_e v^2 f'(\xi)\, d\xi = -\rho_a g \xi\, dz$$

or

$$dz = -hf'(\xi)/\xi\, d\xi \tag{B3}$$

where the 'scale height'

$$h = \rho_e v^2/\rho_a g \tag{B4}$$

is introduced as a convenient length scale.

Hence

$$z(\xi) = hZ(\xi) = h\int_\xi^1 \frac{f'(\xi')}{\xi'}\, d\xi'. \tag{B5}$$

Equation (2) becomes

$$f(\xi) = \sin^2 \chi \tag{B6}$$

and may be applied to find the shape of the cloud, for

$$dr = \tan\chi\, dz$$
$$= -\{f(\xi)/(1-f(\xi))\}^{1/2}\, h(f'(\xi)/\xi)\, d\xi$$

using (B3) and (B6), so that

$$r(\xi) = hR(\xi) \equiv h\int_{\xi}^{1} f(\xi')/(1-f(\xi'))\}^{1/2}(f'(\xi')/\xi')\, d\xi'. \quad \text{(B7)}$$

The dimensionless functions $Z(\xi)$ and $R(\xi)$ are determined solely by the form of the 'equation of state' $p/p_a = f(\xi)$. The total mass of the cloud is

$$M = \int_0^{z_{max}} \rho\pi r^2\, dz$$
$$= \int_1^0 \rho_a\xi\pi h^2 R^2(\xi)(-h)(f'(\xi)/\xi)\, d\xi$$
$$= C_M\rho_a h^3 \quad \text{(B8)}$$

where

$$C_M = \int_0^1 \pi R^2(\xi) f'(\xi)\, d\xi$$

is a pure number determined solely by the form of the 'equation of state'.

Similarly, the total internal energy is

$$U = C_U \tfrac{1}{3} u_a h^3 = C_U \rho_e v^2 h^3$$

where the number

$$C_U = \int_0^1 3\pi R^2(\xi) f(\xi)(f'(\xi)/\xi)\, d\xi \quad \text{(B9)}$$

and the total retarding force is

$$F = \int 2\pi r\, ds . p \sin\chi = \int_0^{r_{max}} p . 2\pi r\, dr$$
$$= \int \pi r^2\, dp \quad \text{integrating by parts}$$
$$= \int \pi r^2 \rho_g\, dz = gM.$$

The most interesting parameter to be calculated, because unlike M, U, and F it is observable, is the radio power P emitted per Hz and per steradian. The value of P depends on the division of the energy U between fields, electrons, protons, and other forms, and on the energy spectrum of the relativistic particles. The purpose of the calculation is to test the theory; therefore those assumptions will be used that lead to the largest possible power P so that, if greater values of P are observed, a clear contradiction with the theory can be established.

For a homogeneous source it is well known that, given a total energy U, there is a maximum radio power P given by

$$U = AP^{4/7}(\text{Volume})^{3/7} \quad \text{(B10)}$$

which occurs close to equipartition of energy between relativistic electrons and magnetic fields. In deriving this formula, it is assumed that the electrons have an energy distribution

$$dN \propto E^{-(2\alpha+1)}\, dE \tag{B11}$$

with a cut-off below some limit $E_{\min}$; the higher $E_{\min}$ the lower U, or the higher the power P for given energy U. Thus the highest possible power P is obtained by taking for $E_{\min}$ the largest value consistent with observation. Now, if the spectrum of the source is observed to have spectral index α (defined by flux density $S \propto \nu^{-\alpha}$) down to a frequency ν, then $E_{\min}/m_0c^2 < (\nu/\nu_g)^{1/2}$ (ν_g = gyro-frequency of electrons, 28 Hz per nT); if this upper limit is used as the value for $E_{\min}$, and P represents the power at the lowest frequency of observation, then A is proportional to $\nu^{-2/7}$. For our source model we obtain the highest possible P by using the highest allowed $E_{\min}$ for each small volume ΔV in the model, and then

$$\Delta U = A\, \Delta P^{4/7}\, \Delta V^{3/7}$$

or

$$\Delta P = A^{-7/4}(\Delta U/\Delta V)^{7/4}\, \Delta V = A^{-7/4}(u_a f(\xi))^{7/4}\, \Delta V.$$

Hence the maximum power from the whole source is

$$P_{\max} = A^{-7/4} C_P (\rho_e v^2)^{7/4}\, h^3 \tag{B12}$$

where

$$C_P = \int_0^1 \{3f(\xi)\}^{7/4}\, R^2(\xi) f'(\xi)\, d\xi/\xi.$$

Numerically, taking $\alpha = 0{\cdot}75$,

$$P_{\max} = 3\cdot10^{26}\, C_P\{(\rho_e/10 \text{ hydrogen atoms m}^{-3})(v/c)^2\}^{7/4}\, h_{\text{kpc}}^3\, \nu_8^{-1/2} \text{ W Hz}^{-1} \text{ ster}^{-1}. \tag{B13}$$

In the 'isothermal' model, defined by $f(\xi) = \xi$, the cloud extends backwards indefinitely, with a radius asymptotic to πh; ρ, p, and u and radio emissivity $(\Delta P/\Delta V)$ all decrease exponentially with z.

The 'adiabatic' model, defined by $f(\xi) = \xi^{4/3}$, has $\rho/\rho_a = \xi = (1 - z/4h)^3$, so that the model ends at the plane $z = 4h$.

The values of the pure numbers C_R (where $C_R h$ is the maximum radius), C_M, C_U and C_P have been computed for these two models and are shown in Table BI. Some of them can be calculated analytically:

Isothermal model

$$C_R = \pi;\; C_M = 2\pi(\tfrac{1}{4}\pi^2 - 1);\; C_U = 3C_M$$

Adiabatic model

$$C_R = (8/\pi)^{1/2}\{(-\tfrac{1}{4})!\}^2;\; C_U = (32\pi/5)(C_R - 34/21) \tag{B14}$$

and these values have been used to check the machine computations.

For comparison with the observed source diameters, Table I also shows the square roots of the second moments about the centroid of the brightness distribution seen by a remote observer on a line perpendicular to v. The square root of the second moment about the z axis of the projected brightness distribution is hR_{rms}, and that about a perpendicular axis, hZ_{rms}. If the axis of the source model makes an angle θ with the line of sight, the corresponding quantities for the observed brightness distribution are hR_{rms} and $h(R_{\text{rms}}^2 \cos^2\theta + Z_{\text{rms}}^2 \sin^2\theta)^{1/2}$.

TABLE BI

	C_R	C_M	C_U	C_P,	$\bar{Z}$ i.e. $\frac{\int z\,dP}{h\int dP}$,	Z_{rms} i.e. $\left(\frac{\int (z-\bar{z})^2\,dP}{h^2\int dP}\right)^{1/2}$,	R_{rms} i.e. $\left(\frac{\int r^2\,dP}{h^2\int dP}\right)^{1/2}$
'Isothermal' model	3·14159	9·220	27·66	23·35	1·042	0·737	0·899
'Adiabatic' model	2·39628	7·470	15·627	16·054	0·747	0·458	0·772

The reason for tabulating these quantities is that the ' diameters ' found when a source is barely resolved by the largest interferometer spacing depend only on the second moments. Upper limits are often quoted as upper limits to the diameter between half-power points if the distribution were Gaussian. The second moments implied by these statements give $hZ_{\rm rms}$ (or $hR_{\rm rms}$) $= (8 \ln 2)^{-1/2}$ (Diameter between half-power points).

APPENDIX C

In this appendix we follow the development of a de Young–Axford radio component starting with mass M_1 and velocity V_1, but with negligibly small size and internal energy, whose internal energy is gradually increased by a fraction of the kinetic energy it loses.

The rate of loss of kinetic energy is

$$-(d/dt)(\tfrac{1}{2}MV^2) = MVg - \tfrac{1}{2}V^2\, dM/dt. \qquad \text{(C1)}$$

If the amount of matter swept up is proportional to the area of the radio component, say

$$dM/dt = \eta C_M \rho_e V h^2 \qquad \text{(C2)}$$

where η is some numerical constant, then the rate of loss of kinetic energy becomes

$$-(d/dt)(\tfrac{1}{2}MV^2) = (1 - \tfrac{1}{2}\eta)\, C_M \rho_e V^3 h^2 \qquad \text{(C3)}$$

using equations (1) and (3) to rewrite the first term of (C1). The input of internal energy cannot exceed this amount; on the other hand, we expect the energy input into the radio component to be comparable to the kinetic energy of the intergalactic matter with which it mixes,

$$\eta C_M \rho_e h^2 V \,.\, \tfrac{1}{2}V^2.$$

We therefore define an ' efficiency ' ϵ, which is less than $(1 - \frac{1}{2}\eta)$ and probably less than $\frac{1}{2}\eta$, such that the input of internal energy is

$$\epsilon C_M \rho_e h^2 V^3. \qquad \text{(C4)}$$

We first relate mass and velocity changes:

$$dV/dt = -g = -\rho_e V^2/\rho_a h = -(V/\eta M)\, dM/dt \qquad \text{(C5)}$$

using (1), (3), and (C2), and therefore

$$(M/M_1) = (V/V_1)^{-\eta}. \qquad \text{(C6)}$$

To allow for the adiabatic expansion losses in energy correctly, we need to know the index γ; for the moment we assume that most of the energy is in relativistic particles and in magnetic fields, so that $\gamma = 4/3$. Then

$$(d/dt)(Uh) = h \times \text{energy input} = \epsilon C_M \rho_e V^3 h^3$$
$$= -\epsilon C_M V h^3 \,.\, \rho_a h\, dV/dt = -\epsilon V h M_1 (V/V_1)^{-\eta}\, dV/dt.$$

We now use equation (2) to express h in terms of Uh, and integrate the above equation, setting $u = 0$ when $V = V_1$. Using equation (2) again, one finds

$$h^3 = \frac{3\epsilon M_1}{4 C_U \rho_e (\frac{3}{2} - \eta)} \{(V_1/V)^{3/2} - (V_1/V)^{\eta}\}. \qquad \text{(C7)}$$

Similarly for other values of γ one finds

$$h^3 = (2-\gamma\eta)^{-1}(\epsilon M_1/C_U\rho_e)\{(V_1/V)^{2/\gamma}-(V_1/V)^\eta\}. \tag{C8}$$

Having found its mass and size in terms of V_1/V, we can also find the component's distance D from its starting point and the time t it took to get there in terms of V_1/V. Using (C6) and (C2) in (C5)

$$dV/dt = -(V/\eta M)\, dM/dt = -C_M\rho_e V^2 M_1^{-1}(V/V_1)^\eta h^2$$

and substituting for h^2 from (C8) we find

$$dt/\tau = X^\eta(X^{2/\gamma}-X^\eta)^{-2/3}\, dX \tag{C9}$$

where $X = V_1/V$ and

$$\tau = (C_M V_1)^{-1}(M_1/\rho_e)^{1/3}((2-\gamma\eta)\, C_U/\epsilon)^{2/3}. \tag{C10}$$

Similarly, we can express $dD = V\, dV(dV/dt)^{-1}$ in terms of V_1/V. Equation (C9) and the corresponding expression for D can be integrated in terms of incomplete beta functions, or numerically. If the amount of mixing is small, so that we can make the approximation $\eta = 0$, and if $\gamma = 4/3$, the integration is elementary, and gives

$$t/\tau = -\ln\{X^{1/2}-(X^{3/2}-1)^{1/3}\}+\tfrac{2}{3}\sqrt{3}\tan^{-1}\{\sqrt{3}(1+2(1-X^{-3/2})^{-1/3})^{-1}\} \tag{C11}$$

and

$$D/V_1\tau = \ln\{1+(X^{3/2}-1)^{1/3}\}-\tfrac{1}{2}\ln X+\tfrac{2}{3}\sqrt{3}\tan^{-1}\{\sqrt{3}(2(X^{3/2}-1)^{-1/3}-1)^{-1}\}. \tag{C12}$$

In (C12) the argument of the arctangent starts at 0 (when $X = 1$), rises to $+\infty$ (i.e. $\tan^{-1}(\ \ ) = \pi/2$) and finishes ($X \to +\infty$) at $-\sqrt{3}$, the value of $\tan^{-1}(-\sqrt{3})$ between $\pi/2$ and π being taken. Thus the separation of the component from the central object tends toward the finite limit

$$D_{max} = 4\pi 3^{-3/2} V_1\tau. \tag{C13}$$

In the general case, the qualitative behaviour is the same; as V becomes small (C8) shows that the component expands to a great size, and its distance from its initial position tends toward D_{max} given by

$$\frac{D_{max}}{V_1\tau} = \frac{\gamma(-2/(6-3\gamma\eta))!(-2/3)!}{(2-\gamma\eta)(-\gamma\eta/(6-3\gamma\eta))!} \tag{C14}$$

$$\simeq 3^{-1/2}\pi\gamma \text{ if } \eta \ll 1.$$

So long as the radio component has not been decelerated much, i.e. $(V_1-V) \ll V_1$, it follows from (C9) that $(V_1-V) \propto t^3$ and substituting in (C8) one finds that

$$h \simeq (\epsilon C_M/3\gamma C_U)\, D \tag{C15}$$

so that, in the early stages, there is a constant ratio of component size to component separation. As we have seen, $\epsilon \lesssim 0.5$ and $C_U \simeq 3C_M$, so that $D/h \gtrsim 25$.

The maximum radio power P_{max} allowed by the minimum energy rule is proportional to $u^{7/4}h^3$, and therefore to

$$V^{7/2}\{(V_1/V)^{2/\gamma}-(V_1/V)^\eta\};$$

it rises at first because of the increasing internal energy and falls later because the kinetic energy supply is almost exhausted and the adiabatic losses continue having a maximum at $V/V_1 = 0{\cdot}69, 0{\cdot}64$, if $\gamma = 4/3$, $\eta = 0, 1$, respectively.

The initial condition, that the component starts with negligible size and internal energy, is not essential; clearly, starting with a radio component of finite size just corresponds to starting at some time $t > 0$ in the sequence calculated above.

We have assumed that the structure of the component is at each stage given by calculations such as those in Appendix B, which ignore the mixing with incident intergalactic matter. However, the changes in structure affect the development described here only through changes in the constants C_M, C_U etc., not in any essential way. More important is the assumption that the ambient density ρ_e is uniform, and that the fractions ϵ and η are constant; it is likely that ρ_e decreases with distance from a galaxy, and ϵ and η are likely to increase as the density of the radio component falls to values closer to the intergalactic density. However, the calculations illustrate the general features of models of this type; the real difficulties concern the supply of matter to the radio components, as discussed in the text.