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MODELS OF EXTRAGALACTIC RADIO SOURCES WITH A CONTINUOUS ENERGY SUPPLY FROM A CENTRAL OBJECT

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SUMMARY

This paper explores the dynamics of radio source models in which energy is carried from a nucleus to the radio components by a relativistic beam. Only a small fraction of the energy supply can be radiated away from the tip of the beam, and the rest lingers on in a cavity surrounding the beam. For the dynamical development of the source it is not of great importance whether the 'beam' consists of strong electromagnetic waves or of fast particles with or without magnetic fields, or even if the energy is supplied from a moving massive object instead of the tip of a beam. If the energy in the cavity contains both fast electrons and magnetic fields, then the cavity rather than the tip of the beam is likely to be the site for the greater part of the radio emission. When the lateral expansion of the cavity becomes sub-sonic, the outer parts of the cavity swell at the expense of the parts nearest the massive nucleus, where the thermal gas pressure is higher.

Not long after the discovery of pulsars it began to be suggested that pulsars, or larger objects in some way analogous to them, might provide the mechanism by which gravitational energy could be harnessed to provide the energy supply for radio galaxies and QSOs. In particular, rotating magnetic objects could provide beams of low-frequency electromagnetic radiation, which in turn could accelerate beams of relativistic charged particles and thus provide energy in a form very suitable for radio sources (Morrison 1969; Rees 1971). In an earlier paper Longair, Ryle and Scheuer (1973) have endeavoured to show that to avoid losses by adiabatic expansion any adequate energy supply from the central object of a radio source to the distant radio components must indeed consist of bulk kinetic energy, so that, if the energy is in the form of relativistic particles and of photons it has to be a beam moving at a speed close to the speed of light.

In this paper I shall investigate the consequences of postulating that a relativistic beam of things—for many (though not all) purposes, it does not matter too much whether they are fast particles or low-frequency electromagnetic waves—is thrown out into a supposed intergalactic gas. The approach adopted here differs from Rees' (1971) approach in that I shall be concerned more with the dynamics of the models, and less with the radiation mechanism.

I propose to begin by analysing a model subject to the very simplest assumptions (and some fairly rough, but not really crucial approximations to simplify the algebra); this is Model A. Model A will then be subjected to various criticisms on the grounds of (a) conflict with observations, and (b) inconsistency with simple physical principles, and some alternative possibilities which emerge will then be discussed.

MODEL A

Suppose that a central energy source ejects a beam of low-frequency electromagnetic waves (hereinafter referred to as LFEMW) in a cone of solid angle Ω. (There should be two such beams, ejected in opposite directions, to produce a double radio source; this and the following discussion refers to just one side of such a source.) The beam pushes the intergalactic gas out of its way, excavating a tunnel through which it can propagate, up to a distance D where it encounters intergalactic gas of density ρ, which in this model is assumed to be uniform until it is disturbed. I follow Rees (1971) in assuming that the LFEMW are totally absorbed at their first encounter with matter, and the energy, Q per second, is converted into relativistic particles. The pressure which balances the ram pressure ρV^2 of the gas retreating at speed V is the radiation pressure plus the reaction of the relativistic particles, i.e. it is

$$K_1 Q/\Omega D^2 c \tag{1}$$

where K_1 is a number depending somewhat on the range of directions over which the relativistic particles are thrown out. K_1 is certainly less than 2, and probably greater than 1, since $K_1 = 1$ corresponds to radiation pressure alone.

It will be assumed throughout that $V \ll c$. This assumption is easily justified in the conventional plasma cloud models by the predominance of more or less symmetrical double sources (Scheuer 1967; Mackay 1973), since the velocity V at which the radio components move away from the centre is then the velocity of the radiating material. In the canonical Rees model this is not necessarily true; indeed, the fast particles might travel predominantly backwards through the source, in which case the retreating component of a double source should appear the brighter. However, it seems highly improbable that the relativistic particles should be produced over a range of angles that makes their mean velocity relative to the absorbing front precisely $-V$, so that it still seems reasonable to assume that V is not relativistic.

As the fast electrons pass through the front end of the beam of LFEMW they emit synchro-Compton radiation, which in the canonical Rees model accounts for the radio emission. Where do they go then? This is the problem of providing a waste energy basket, already posed in Longair *et al.* (1973) and set out in more detail in the Appendix. If the intergalactic gas were strictly free of magnetic fields,

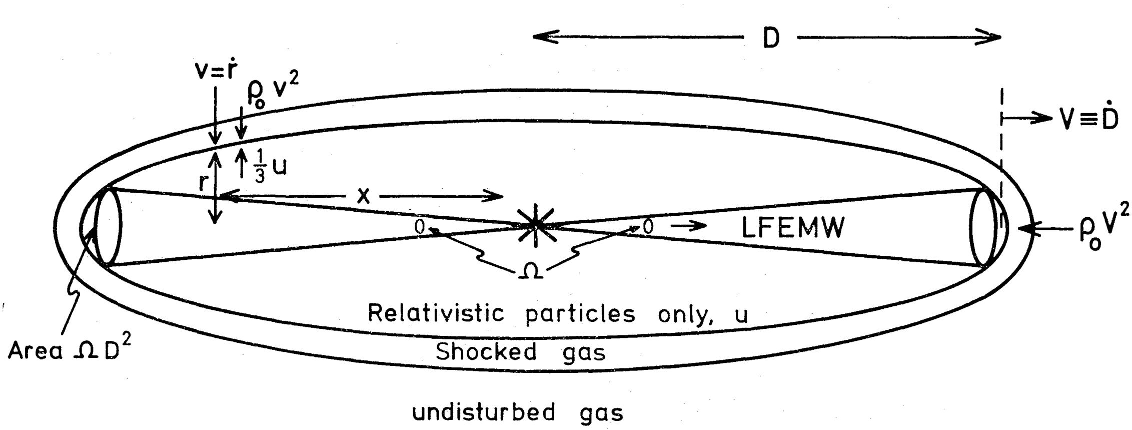

FIG. 1. *Model A.*

they might travel outwards freely (though even then they might be hindered by the development of plasma instabilities of the two-stream type). But even a field of 10^{-16} T would give a 1 BeV electron a Larmor radius of only 1 pc, so that it does not seem probable that electrons could stream through the intergalactic medium; they must rather push it back, and in so doing they will face the ram pressure of the receding gas, just as the LFEMW do at the front of the radio component. The electrons may of course mix with the gas eventually, by convection or diffusion or both, but this does not essentially alter the dynamical situation, which is that the electrons must share their momentum with the gas that stands in the way of their escape. Thus we arrive at the picture illustrated in Fig. 1. The fast particles discarded by the radio components fill a spindle-shaped cavity, in which they have randomly directed velocities; as the cavity expands, they lose energy in proportion to the cube root of the volume. This is the picture; to simplify the calculations, it is useful to add the following approximations:

(i) $\Omega \ll 1$;

(ii) Q is constant in time (after $t = 0$), and ρ is uniform in space;

(iii) The energy density u of the accumulated fast particles is uniform through the cavity. Since $V \ll c$, the speed of sound within the cavity is much greater than V, even if the fast particles are mixed with some very small amount of gas. Thus this assumption is reasonable over most of the cavity, though it must break down within $D\Omega^{1/2}$ of the ends, where the energy density must be comparable with that within the ' radio components ';

(iv) The expansion of the cavity is at right angles to the source axis, and its speed of expansion v is given simply by balancing the ram pressure ρv^2 with the fast particle pressure $u/3$. That is, we ignore the numerical factor introduced because there is a layer of shocked gas. It will be verified *a posteriori* that the cavity does remain rather narrow over the relevant part of the source's life.

Subject to these simplifications, the development of the model can be calculated as follows.

From the equation of pressure balance at the end of the beam,

$$\rho V^2 = K_1 Q/\Omega D^2 c \tag{2}$$

$$D\frac{dD}{dt} = DV = (K_1 Q/\Omega c\rho)^{1/2} \tag{3}$$

and therefore

$$D = (K_1 Q/\Omega \rho c)^{1/4}\,(2t)^{1/2}. \tag{4}$$

Hence

$$D = 2Vt.$$

We now use the approximations (iii) above to find the form of the cavity and the energy density in it. If the radius at distance x from the centre is r, then $dr/dt = v$ where v is determined by

$$\rho v^2 = u/3. \tag{5}$$

The total energy U in the cavity is increased in time dt by the input $Q\,dt$ from the LFEMW and decreased by the work $(u/3)$ d(volume) done in expanding the cavity:

$$dU = Q\,dt - (u/3)\,d(\text{volume}). \tag{6}$$

Since the parameters Q, ρ, c determine a characteristic length we do not have a self-similar solution, and we cannot expect that volume $\propto D^3$. However, the volume determines the energy density u through the above differential equation (6), and the energy density as a function of time determines the volume through equation (5), since

$$\text{volume} = \int_0^D \pi r^2 \, dx$$

where

$$r = \int v \, dt,$$

integrated from the time when D was equal to x until the present time. A self-consistent solution can be obtained with the volume proportional to a constant power of D, and application of the calculations just indicated shows that the power must be the (7/2)th power, i.e.

$$\text{volume} = CD^{7/2} \tag{7}$$

(I assume that this solution is unique, though I have not proved that it is.) Substituting (7) into (6), and using (3),

$$dU = Q \frac{D \, dD}{(K_1 Q/\Omega \rho c)^{1/2}} - \frac{7}{6} U \, dD/D$$

which is easily integrated to yield

$$U = (6/19)(Q\Omega\rho c/K_1)^{1/2} D^2 = (12/19) Qt. \tag{8}$$

Using (8), (7) and (5) we find v, and on integrating obtain

$$r = (2/19C)^{1/2} (\rho\Omega^3 c^3/K_1{}^3 Q)^{1/4} (4/5)(D^{5/4} - x^{5/4}) \tag{9}$$

and hence the volume. The volume is consistent with the formula (7) provided that

$$\begin{aligned} C &= (\pi/38)^{1/2} (64/63)^{1/2} (\rho\Omega^3 c^3/K_1{}^3 Q)^{1/4} \\ &= (\pi/38)^{1/2} (64/63)^{1/2} (\Omega c/K_1 DV)^{1/2}. \end{aligned} \tag{10}$$

Formula (10) gives C in terms of the current values of D and V, as well as in terms of the constants of the model, since the former is more illuminating in comparisons with observation. Thus, (10) verifies immediately that the cavity is a thin spindle-shaped cavity, since its volume $CD^{7/2}$ is much less than D^3, so long as $\Omega < V/c$, and that condition must hold at least in the stronger radio sources. On the other hand, the volume of the cavity exceeds the volumes $(1/3)\,\Omega D^3$ inside the conical beam of LFEMW by a factor $(32\pi/133)^{1/2} (c/K_1 V\Omega)^{1/2}$, so that it was justifiable to neglect the volume in the beam when solving equations (5) and (6). The cavity comes to a sharp point at each end, its outline making an angle

$$v/V = (63/152\pi)^{1/4} (\Omega c/K_1 V)^{1/4} \tag{11}$$

with the source axis. It also follows that the energy density in the cavity is much less than the energy density in the ends of the beam, for their ratio is

$$u/u_{\text{beam}} = 3\rho v^2/\rho V^2. \tag{12}$$

On the other hand, the total energy U in the cavity is much greater than the energy in the radio component at the end of the beam; their ratio is

$$U/U_{\text{beam}} = (6/19)(c/K_1V)(D/h) \tag{13}$$

where h is the length of the radio component along the source axis. Thus the approximations made in the model appear reasonable. We now proceed to question the physical assumptions on which it is based.

Critique of Model A

(1) If the cavity is a good vacuum, the relativistic particles stored there will permeate the beam of LFEMW, and therefore give rise to a bridge of synchro-Compton emission at radio frequencies, between the two radio components at the ends of the beam. The fraction of the radio power P coming from the bridge may be estimated as follows:

The emissivity (power P per unit volume) is, as in synchrotron radiation, given by an expression of the form

$$\text{constant} \times KB^{1+\alpha} \tag{14}$$

where K is the constant in the particle energy distribution

$$dn = K\gamma^{-(2\alpha+1)}\, d\gamma \tag{15}$$

that specifies the number dn of electrons per unit volume with energies between $\gamma m_e c^2$ and $(\gamma + d\gamma)\, m_e c^2$. Now,

$$K = K'Q/D^2\Omega c \tag{16}$$

inside the component and

$$K = K_2K'u \tag{17}$$

in the cavity, where K' is a factor, common to (16) and (17), depending on the low-energy cut-off on the distribution (15), while K_2 is a numerical factor somewhat less than 1 which is introduced to represent the change in K' in the cavity due to adiabatic expansion. Also, the field strength B in the LFEMW falls as $1/x$ with distance from the central object, so that at general x, $B = B_{\text{end}}(D/x)$ where B_{end} is the field strength in the radio component at the end of the beam. Hence, from (14),

$$\begin{aligned}\frac{P_{\text{comp}}}{P_{\text{bridge}}} &= \frac{D^2\Omega h(K'Q/D^2\Omega c)\, B_{\text{end}}{}^{1+\alpha}}{\int_0^D K_2K'u\,.\,x^2\Omega\, dx\,.\,(B_{\text{end}}D/x)^{1+\alpha}} \\ &= (2-\alpha)\, hQCD^{1/2}/K_2Uc\Omega \\ &= \frac{2-\alpha}{K_2}\left(\frac{152\pi}{567}\right)^{1/2}\frac{h}{a}\left(\frac{V}{K_1\pi c}\right)^{1/2}\end{aligned} \tag{18}$$

using (10), and (8) in the form $U = (12/19)\, QD/2V$. Here h stands for the effective length of the radio component generated by the fast particles as they escape for the first time from the beam of LFEMW, and a is its radius defined by $\pi a^2 = \Omega D^2$. The numerical factor in equation (18) is about 2, and h will not be very much greater than a, while V is considerably less than c; thus according to Model A all radio sources should have a bridge contributing at least as much to the flux density as the components one first thought of. While there are such sources, most sources do not have prominent bridges between the components, so that Model A clearly conflicts with the observations.

(2) To avoid the difficulty raised in (1) above, suppose that the fast electrons in the cavity are somehow prevented from re-entering the beam of LFEMW. (The mechanism might well be that the particles in the cavity are tied to a small magnetic field, and a density gradient is set up in the outer parts of the beam which refracts the LFEMW towards the source axis. However, the nature of the mechanism is irrelevant to the present argument.) Then the relativistic ' gas ' exerts a pressure $u/3$ which tends to compress the beam, and which must be balanced by an outward pressure from the beam. The simple radial beam envisaged in Model A provides no outward pressure; such a pressure requires that the trajectories of the relativistic things in the beam should be continually bent inward towards its axis, so that the external pressure is balanced by the centrifugal force of the relativistic material (whether LFEMW, fast electrons, particles plus magnetic field, or tomatoes). Such considerations lead naturally to Model B, worked out in the next section of this paper, in which the things in the relativistic beam travel at a small angle to the source axis. In the context of LFEMW the beam is self-focusing, and goes down a waveguide of its own making, as described by Rees (1971). It will appear that Model B also has some undesirable features.

(3) So far, it has been assumed that the cavity contains only fast particles, with no magnetic field (or an extremely small one, sufficient only to bind them into a gas). If an appreciable part of the energy in the cavity were magnetic energy, ordinary synchrotron radiation would be radiated in the radio band. The maximum radio power obtainable from the cavity is given by the usual minimum energy formula; since approximate equipartition also occurs in the radio components in the beam (between fast electrons and LFEMW) the ratio of volume emissivities of ' radio component ' and cavity is simply

$$(u_{\text{comp}}/u_{\text{cavity}})^{7/4}$$

and therefore

$$\frac{P_{\text{comp}}}{P_{\text{cavity}}} = \left(\frac{u_{\text{comp}}}{u_{\text{cavity}}}\right)^{3/4} \frac{U_{\text{comp}}}{U_{\text{cavity}}}$$

$$= \frac{19}{3\pi^{1/2}} \left(\frac{152\pi}{567}\right)^{3/8} \frac{h}{2a} \left(\frac{K_1 V}{c}\right)^{11/8} \Omega^{1/8} \tag{19}$$

using equations (12), (11) and (8). The numerical factor in (19) is about 4, so that the radio power from the ' radio component ' is considerably less than the maximum radio power from the cavity. While some radio sources have bridges of radio emission between the components, few if any have the greater part of their radio power in a spindle-shaped region such as that shown in Fig. 1, broadest in the middle; the bridges that are found tend to be narrow.

Of course the argument leading to the ratio (19) only applies if an appreciable fraction of the energy in the cavity is in the form of magnetic fields. Magnetic fields are likely to appear in the cavity in several ways, notably (i) by amplification of intergalactic fields through turbulent motion (ii) from the ' pulsars ' in the central object; the energy emitted by these objects is likely to be intermediate between the pure LFEMW radiated by a magnet rotating *in vacuo* and the relativistic stellar wind envisaged by Goldreich & Julian (1969) and by Michel (1969) in which ordinary magnetic fields with a non-reversing component are wound up and supported by net currents in streams of fast particles.

Further discussion of these possibilities will now be postponed until Model B has been described.

MODEL B

This model was set up in order to explore the consequences of excluding the fast particles of the cavity from the beam. Consider first whether the beam of Model A, consisting of radially streaming relativistic material, would be focused

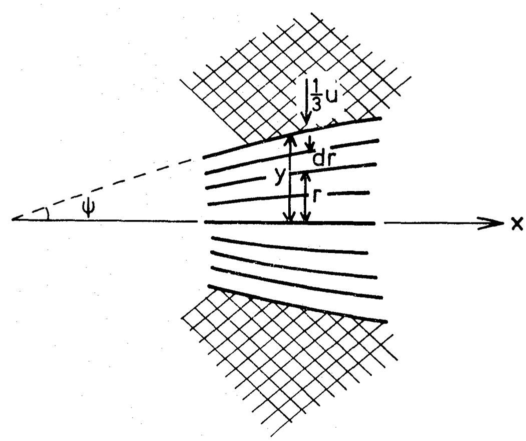

FIG. 2. *The compression of a beam by external pressure.*

appreciably by the external pressure $u/3$ (Fig. 2). There will be a transverse pressure gradient $\simeq u/3y$, which changes the transverse momentum of the stream at radius r:

$$(u/3y)\, dr \simeq \text{net transverse force per unit area}$$

$$\simeq (Qc^{-2}/\pi y^2)\, dr(D\psi/Dt).$$

It follows that the beam will be focused again and again, at intervals along the axis of the order of

$$(Q/uc)^{1/2} \simeq (\text{volume of cavity}/ct)^{1/2} \ll D. \tag{20}$$

Having established that the external pressure has a strong effect on the beam, one has to ask next what the beam might look like. Suppose we imagine the central object as a rapidly rotating disc of matter, in which upheavals (e.g. new pulsars) occur from time to time, and puffs of energy appear out of both sides of the disc, in a fairly broad cone of directions. To investigate what happens to the energy as the cone widens, the energy will be treated at first as a collection of free relativistic particles (or rays of LFEMW) bouncing specularly off the sides of the crater which they have made (Fig. 3). The direction of motion of a particle will be closer to the axis by $2\, dy/dx$ after a reflection, so that the beam is directed more and more accurately forwards. In the limit of a long narrow crater, simple quantitative relations can be found which may be a fair guide to the behaviour of such systems. In a narrow crater, the distance between reflections is $2y \cot \theta$ for rays inclined at an angle θ to the x axis; thus

$$d\theta/dx = -(2\, dy/dx)/(2y \cot \theta)$$

which, integrated, gives

$$y \sin \theta = \text{constant} = Y \tag{21}$$

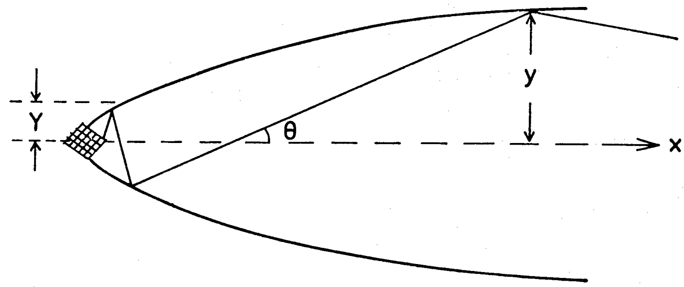

FIG. 3. *Collimation of relativistic material bouncing off the walls of a crater.*

where the constant Y is evidently the half-width of the beam near its source, where it had a large opening angle. The shape of the crater is evidently controlled by the available external pressure, which presumably goes down with increasing height x above the disc and at each x the external pressure is balanced by the outward pressure from the beam, $u_{\text{beam}}\langle\sin^2\theta\rangle$. (In general, there will be a range of angles θ, and in equation (21) $\sin\theta$ should be interpreted as the root mean square value of $\sin\theta$.) The relation (21) between the width of the channel and the rms direction of motion of the particles is not peculiar to the case of free particles. The same relation holds for particles streaming along in a divergent bundle of magnetic field lines: there, the adiabatic invariant $\sin^2\theta B^{-1}$ is conserved for each particle, and so is the magnetic flux $\pi y^2 B$. The relation is still true if the particles collide with each other, as may be shown by applying the relativistic form of Bernoulli's law (e.g. Landau & Lifshitz 1959, p. 503, problem 2) to a gas of highly relativistic particles.

In Model B it is still assumed that the far end of the beam advances into the intergalactic gas at a speed $V \ll c$, and that the energy density u in the cavity surrounding the beam is independent of position as soon as the beam is far away from the central object. The values of y and $\sin\theta$ then become constant, being determined by the condition of pressure balance between beam and cavity:

$$u_{\text{beam}}\langle\sin^2\theta\rangle = u/3. \tag{22}$$

The general picture of Model B is now becoming clear (Fig. 4). Qualitatively the geometry is similar to Model A; the difference is that the width $2y$ of the beam is now constant over most of its length at any one time, but varies with time according to the pressure in the cavity.

The total energy supply to the beam is

$$Q = u_{\text{beam}}\pi y^2 c. \tag{23}$$

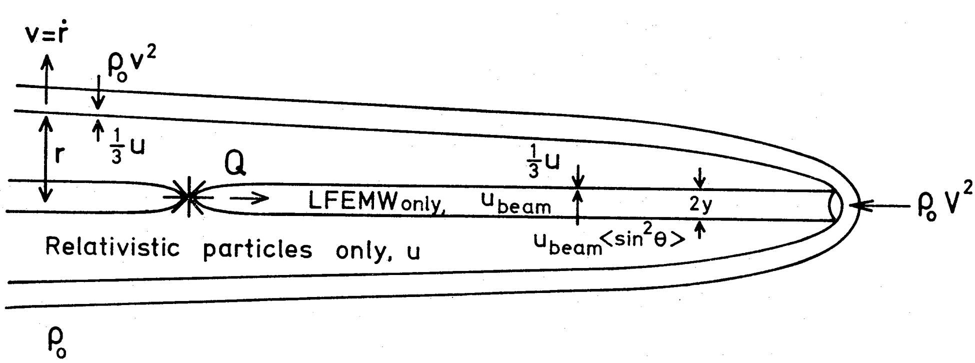

FIG. 4. *Model B.*

Balancing ram pressure and internal pressure at the end of the beam and at the sides of the cavity gives

$$K_1 u_{\text{beam}} = \rho V^2 \tag{24}$$

and

$$u/3 = \rho v^2. \tag{25}$$

The remaining relations are (like equation (25)) identical with the corresponding relations in Model A; the energy U in the cavity increases according to

$$dU = Q dt - (U/3)\, d(\text{volume})/\text{volume} \tag{26}$$

$$\text{volume} = \int_0^D \pi \left\{ \int_{t_{D=x}}^t v\, dt \right\}^2 dx \tag{27}$$

$$D = \int_0^t V\, dt. \tag{28}$$

The eight equations (21)–(28) can be solved for the eight variables V, v, y, $\sin\theta$, volume, D, u and u_{beam} by a process similar to that used in Model A, but more complicated because the calculation of V and D as functions of time cannot now be separated from the rest of the algebra. The principal results (from which others are easily deduced) are:

$$U = (27/44)\, Qt \tag{29}$$

$$D = (9/7)\, Vt \tag{30}$$

$$= (9/7)(119/594)^{1/9}\, K_1^{4/9} Y^{-4/9} c^{-2/9} t^{7/9} \tag{31}$$

$$\text{volume} = (49\pi/102 K_1)\, D^3 \sin^2\theta \tag{32}$$

$$= (343\pi^2/4480 K_1)^{1/2} (c/V)^{1/2}\, D^2 y \tag{33}$$

$$y/D = (154/51 K_1)^{1/6} (V/c)^{1/6} (Y/D)^{2/3}. \tag{34}$$

Since the beam is narrower than in Model A the beam pushes its way through the intergalactic gas faster, and $D \propto t^{7/9}$ (equation (31)) instead of $D \propto t^{1/2}$ (equation (4)). Equations (32) and (33) show that the cavity is much longer than wide, yet much wider than the beam. Indeed the beam is extremely narrow; according to equation (34) the ratio of beam width to beam length (i.e. of component size to component separation) is of the order of $(Y/D)^{2/3}$. The evidence of source variability and of VLB interferometry indicates that Y is of the order of 0·1 pc, while D is typically of the order of 100 kpc, and with these values we should have $Y/D \approx 10^{-4}$! It is time to begin criticizing Model B.

Critique of Model B

(1) As noted above, the predicted size of the radio component at the end of the beam is much smaller, in relation to its distance from the central object, than observations seem to indicate.

(2) If the beam consists only of fast particles, there is no reason to predict a bridge of radio emission along the beam (but conversely there is also a difficulty in inventing a suitable mechanism for radio emission at the ends of the beams). If the beam consists of LFEMW, some (necessarily fast) particles must penetrate into the beam in order to refract it towards its axis, and synchro-Compton emission

must take place in the beam; similarly, if the beam consists of fast particles and fields, synchrotron emission, or synchro-Compton emission, or both, must take place in the beam. In Model B, however, this emission will be beamed strongly in a forward direction, because the emitting particles are swept along with the beam at relativistic speeds, and no contradiction with observation need arise. In this respect, Model B is better than Model A.

(3) So far as radio emission from the cavity is concerned Model B is no better than Model A. From the equations given it is easy to deduce that the maximum radio emission from the cavity is related to that in the radio components by

$$\frac{P_{\text{comp}}}{P_{\text{cavity}}} = \frac{88}{21}\left(\frac{616}{459K_1}\right)^{3/8}\left(\frac{V}{c}\right)^{11/8}\left(\frac{h}{D}\right)^{1/4}\left(\frac{h}{y}\right)^{3/4} \tag{35}$$

from which it is clear that the radio emission from the cavity is likely to exceed that from the components at the ends of the beams if any appreciable fraction of the energy in the cavity is in the form of magnetic fields.

(4) In Model B we also have to give serious consideration to the stability of the beam. A small-perturbation analysis of a plane boundary between a gas and a relativistic stream has been carried out (unpublished work with B. Turland) for the case where the stream may be treated as a fluid (the relativistic Kelvin–Helmholtz instability) and also for the case where the objects in the stream reflect from the boundary but not from each other (such as LFEMW). In both cases, growing waves are found which propagate almost at right angles to the velocity of the stream. Much more work is needed before the fate of a stream is properly understood but it seems likely that, so long as it is not stopped entirely, it will broaden out more than in Model B, and perhaps as much as in Model A, with the entrainment of some of the surrounding medium. The diameter of the stream is important because it controls the rate of advance of the front end of the beam; the sharp beam of Model B exerts a greater pressure than the blunt beam of Model A, and therefore advances faster. Instabilities other than fluid dynamic instabilities at the surface may also occur, depending on the nature of the beam (e.g. Max & Perkins 1972).

(5) The observed systematic downward trend of linear polarization with decreasing frequency indicates that there is some internal Faraday rotation, and therefore some matter in the radio components.

HOW CAN ONE BUILD AN ACCEPTABLE MODEL?

If we regard the relativistic beams in Models such as A and B as energy sources which cause fast electrons and magnetic fields to appear at their ends, the radio picture would look like Fig. 5, with a small intense component at each end and a broader bridge, with a larger total flux density, stretched between them. This picture would fit the observations of typical powerful radio sources rather well if only there were:

(i) a mechanism for removing the middle of the bridge; and
(ii) a mechanism for introducing some matter into the cavity.

The modification of Models A and B that I find least implausible at present stems from the realization that quite large thermal pressures may exist in clusters of galaxies, pressures sufficient to balance the pressures required in some radio sources. If the X-ray emission observed from the Coma cluster is interpreted as thermal bremsstrahlung, the thermal pressure could contain an energy density of 2 to 5×10^{-13} J m^{-3} (Gursky *et al.* 1971). Gull & Northover (1973) have taken this point of view much further, and show that even a powerful radio source like Cygnus A could be contained by external thermal pressure without contradiction with the X-ray data so far published. We shall not require such large pressures here.

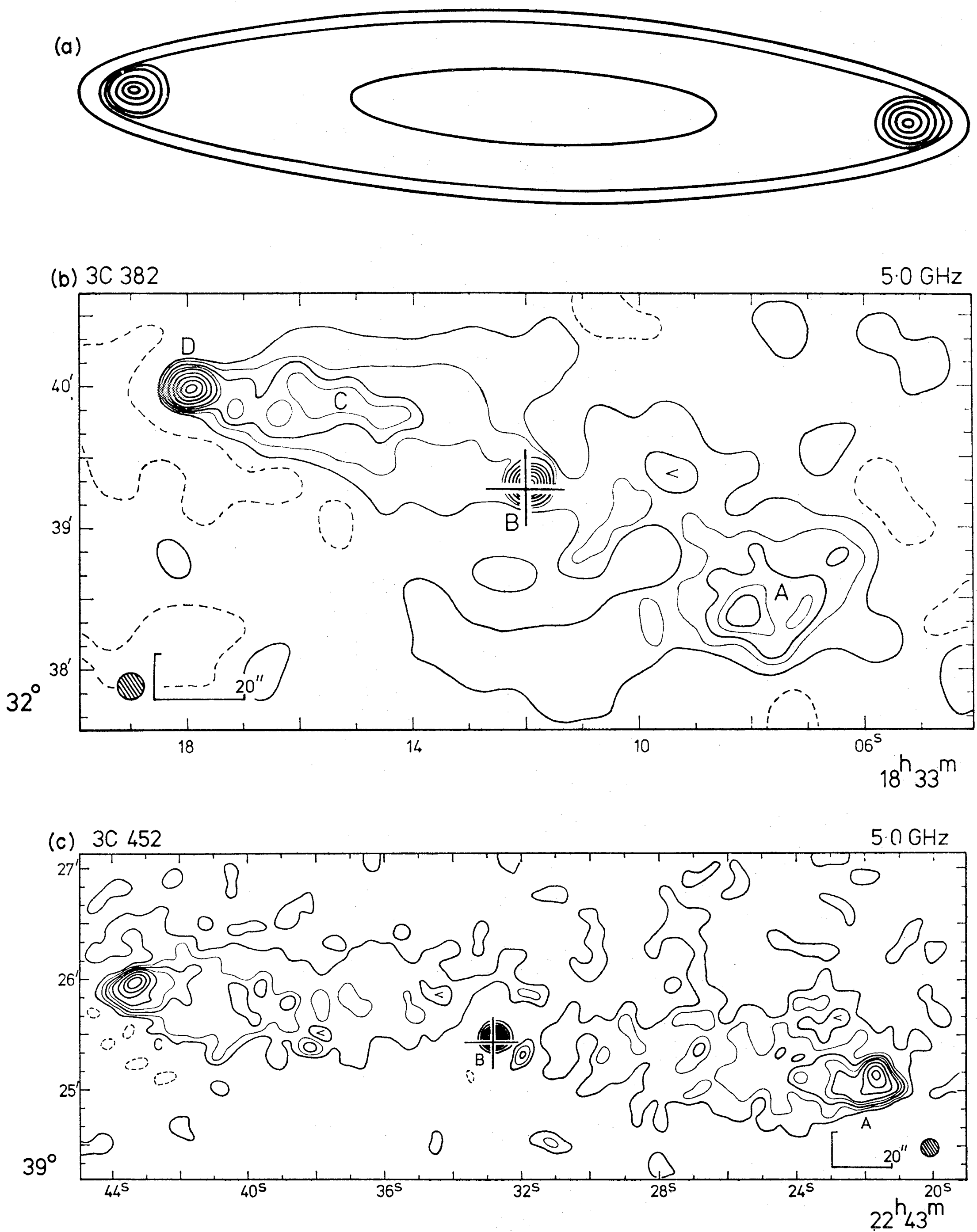

FIG. 5. (*a*) *Sketch map of radio emission from Model B, with the assumption that the cavity contains magnetic field as well as relativistic electrons. For comparison, maps of two radio sources;* (*b*) *3C 382; and* (*c*) *3C 452, after Riley & Branson* (1973).

Hot gas may well appear in a cluster of galaxies because it falls into the cluster in the aftermath of cluster formation, as described by Gunn & Gott (1972); further contributions to heating may come from the motion of galaxies through the gas (Ruderman & Spiegel 1971) and from earlier violent events in member galaxies.

MODEL C

The rms proton speeds corresponding to 10^7, 10^8, 10^9 K are 0·002, 0·005, 0·016*c* respectively. Therefore we should contemplate the possibility that the lateral expansion of the cavity becomes subsonic. It will do so when the energy density u in the cavity falls to a value comparable with the thermal energy density in the ambient gas. The speed V at which the end of the beam advances is in general much greater, and will remain supersonic.

As a first attempt to understand this new situation we consider the extreme subsonic case, in which the cavity remains in pressure balance with the ambient gas. The cavity expands at constant pressure as it is pumped up by the newly supplied energy, but its shape is quite indeterminate in this lowest approximation.

In the next approximation, we take note of the pressure difference between the cavity and the ambient gas. Where the cavity is newly formed near the end of the beam, we have a long thin almost cylindrical region at pressure $p+\Delta p$ (and zero density) surrounded by a cylindrical shell of expanding gas, bounded by a *weak* shock moving out at the sound speed c_s, in the ambient gas at pressure p and density ρ. Now, over times small compared with the age of the source, p is constant, so we may get some insight by investigating the idealized cylindrically symmetrical situation in which the cavity is kept pumped up to a constant excess pressure Δp. Then the quantities determining the radius r of the cavity after time t are (taking $t = 0$ when $r = 0$)

$$(\Delta p/p),\ c_s,\ \rho,\ t$$

and the only dimensionless quantities we can form are $\Delta p/p$ itself and $r/c_s t$. Therefore

$$v = r/t = c_s \times (\text{function of } \Delta p/p) = \text{constant}.$$

Thus the problem is equivalent to determining the excess pressure on a cylinder expanding at constant speed, or (equally well) on a very long cone of small semi-vertical angle χ moving at very high speed V' into the gas (without friction) along its own axis. The last problem is solved in Landau & Lifshitz (1959, Section 105, p. 418)*, and the result (their equation 105.8) is

$$\Delta p = \rho V'^2 \chi^2 \{\ln [2\chi^{-1}(V'^2/c_s^2-1)^{-1/2}]-\tfrac{1}{2}\}.$$

If we let $\chi \to 0$ and $V' \to \infty$ while the radial expansion velocity at a fixed place $v = \chi V'$, remains finite, the above equation reduces to

$$\Delta p = \rho v^2 (\ln (2c_s/v)-\tfrac{1}{2}). \tag{36}$$

This relation governing the expansion is not too different from the relation $p_{\text{cavity}} \approx \rho v^2$ which we adopted for the supersonic case, the main change being the

* The more general problem of sound waves generated by long thin supersonic projectiles with more complicated profiles has also been studied extensively; *cf.* Landau & Lifshitz (1959) Section 115.

replacement of the pressure in the cavity by the pressure difference between the cavity and the external gas.

More interesting things happen if the external pressure is not uniform, because of the gravitational forces of a massive central galaxy. If the galaxy is surrounded by a gaseous atmosphere in approximate hydrostatic equilibrium, the pressure is greater near the galaxy than at great distances; in the cavity, however, the pressure $u/3$ is uniform (since the mass density in the cavity is negligible). If the cavity is in pressure balance with its surroundings near its ends, it will be squeezed ($\Delta p < 0$) in the middle; thus the middle of the cavity is constricted, while the tips of the cavity are forced to expand. This is another way of saying that buoyancy forces the relativistic material in the cavity upwards from the galaxy. However, the rise time for the cavity is much shorter than the rise time for a bubble in Gull & Northover's model, since the relativistic material can pass along the length of the cavity at the speed of light; the rate of transfer is restricted only by the lateral speed v. A bubble of radius r takes a time of the order of $(D/r)^{1/2}(D/g)^{1/2}$ to rise distance D in gravitational field g, while the relativistic material in a cavity of radius r and length D can migrate to the top in a time of the order of

$$r(\gamma\Delta p/\rho)^{-1/2} \approx r(gD)^{-1/2} \approx (r/D)(D/g)^{1/2}.$$

It appears from equation (36) that the time required to close the central part of the cavity is of the order of $(p/\Delta p)^{1/2}$ times the time it took to expand to its maximum radius, so that radio emission will be concentrated into the tips of the cavity when the source is several times older than it was when the motion first became subsonic. This may in fact be an overestimate, since the gas immediately around the central portion of the cavity is the gas that was shocked first, while u was very great, and which consequently has a much higher sound speed than (but now of course the same pressure as) the ambient gas.

Having shown that Model C gives a good account of the structure of radio components, we also have to ask whether it can account for the presence of matter in radio components.

MATTER IN RADIO COMPONENTS

Let us estimate the density of matter required to account for the observed depolarization. Like the energy density, it is much greater in Cygnus A than in almost any other source, since Cygnus A is already noticeably depolarized at about 4 GHz. A region 5″ arc in diameter at the distance of Cygnus A, with a uniform field of 10^{-8} T will introduce a Faraday rotation of π radians at 4 GHz if the electron density is 800 m^{-3}. However, the polarization found at high frequencies is not much more than 10 per cent, indicating that the field is far from uniform. There is still not enough information to make a good estimate of the number of field reversals along a line of sight, but a reasonable guess would be that the mean field component along the line of sight is one-quarter of the total field; we should then raise the estimate of electron density to 3000 m^{-3}. That is still a reasonable gas density in a cluster of galaxies. Indeed, to provide the ram pressure ρV^2 required to balance the minimum energy density in the most compact components of Cygnus A we need a density > 1000 m^{-3} if $V = 0{\cdot}1c$. Thus a density comparable with the cluster gas density is sufficient to account for depolarization.

Several ways of introducing matter into the radio components can be imagined, including the following.

(i) Matter may be thrown out of the central object, but it was shown in Longair *et al.* (1973) that such ejected material is unlikely to contribute to the mass in the radio components.

(ii) Some cluster gas may be entrained in the relativistic beam. Such a process is also subject to the objections raised in Longair *et al.* (1973), though the difficulties become less severe if the matter is entrained far from the central object, where the beam has already spread out to a large fraction of its final width.

(iii) It is also possible that some cluster gas falls into the cavity. So long as the energy is supplied at a constant or steadily diminishing rate Q, the lateral expansion velocity v decreases, and the boundary between the cavity and the surrounding shocked gas is stable. However, if $Q(t)$ varies there may be times when $v(t)$ as well as $Q(t)$ increases; the cavity boundary then becomes Rayleigh–Taylor unstable, and some gas 'falls' into the cavity. The greatest extent to which Rayleigh–Taylor instabilities could mix cluster gas into the cavity may be estimated as follows. The e-folding time for a perturbation of linear scale λ is $\sim(\lambda/g)^{1/2}$, so that the largest perturbations that can develop a large amplitude in time t have $\lambda \sim \frac{1}{2}gt^2$, where g is determined by $\rho v^2 \simeq \frac{1}{3}u$ (equation (5); see also equation (36)) and hence $2g/v = (2/v)(dv/dt) = (1/u)(du/dt) \sim 1/\tau$ where τ is the time scale of the rise in the input power Q. Hence we may expect penetration to a depth

$$\lambda_{\text{max}} \sim \tfrac{1}{2}(v/2\tau)\,\tau^2 \sim \tfrac{1}{4}v\tau$$

during the rise in Q; that is, the cluster gas never penetrates an appreciable fraction of the radius $r \sim vt$ of the cavity unless τ is comparable with the whole age of the source.

CONCLUSIONS

In attempting to follow the development of a model of a radio source whose energy is supplied from the centre by a pair of relativistic beams, we have discarded two models (A and B) which can be formulated quantitatively (albeit with some rough approximations) but which are found unsatisfactory, and found one model (C) which is full of uncertainties but holds some promise. The picture of a radio source represented by Model C differs from Rees' (1971) model a little in that the nature of the beam is not specified, but more importantly in that most of the radio emission does not come from the tip of the beam at all, but from the energy accumulated in the cavity. The beam is chiefly an energy source in the models discussed here, and in many respects it could be replaced by some compact but invisible energy source flung out from the central object (e.g. Saslaw, Valtonen & Aarseth 1974). (Note, however, that cluster gas is still necessary to produce the observed asymmetry between heads and tails of radio components.) Model C also requires the tails of the radio components to be contained by pressure balance with the surrounding gas, but differs from the bubble model of Gull & Northover in that Model C radio components have heads whose energy density is limited by ram pressure, which could be much larger than thermal pressure.

It is not possible to make explicit calculations on the dynamics of Model C as for Models A and B, even roughly, because we do not yet know how to predict the way in which a beam will spread. If the beam spreads out at a constant angle, then the algebra of Model A applies; if the beam spreads more slowly, the model will behave dynamically more like Model B, so that these models give some indication of the range of behaviour to be considered, and that is my excuse for letting these physically wrong models remain in this paper.

Clearly Model C gives a good account of the observed structure of radio components; there is less difficulty in accounting for the energy containment than in the bubble model and one requires a more modest energy supply than in Rees' model. Intuitively it seems natural that the radio component should also incorporate some of the surrounding gas, and the mixing process wind up an appreciable magnetic field, sufficient to account for the observed depolarization by Faraday rotation. However, I have evidently not been able to find a satisfactory mechanism for doing so.

The most serious problem in models involving beams of energy is, in my opinion, the stability of the beam. That is largely a theoretical problem. An important observational problem is to estimate the pressure in clusters of galaxies, either from their X-ray emission, or perhaps by the method suggested by Sunyaev & Zeldovich (1972) which in principle determines just the same combination nT of electron density and temperature that also gives the pressure.

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APPENDIX

THE NECESSITY FOR A WASTE ENERGY BASKET

Consider the radio emission P W Hz^{-1} ster^{-1} at frequency ν; a well-known argument shows that the minimum energy required to produce P by synchrotron or synchro-Compton radiation in a volume r^3 is

$$U_{\min} \simeq 3{\cdot}25 \times 10^{34} \nu_8{}^{2/7} P^{4/7} r_{\rm kpc}{}^{9/7} \text{ Joules}$$

$U_{\min}$ must be present in the source at any one time; since the beam enters the radio component at speed c, and constitutes at least half of the energy density, the power supply Q must exceed $\frac{1}{2}U_{\min}(c/r)$.

The above formula for $U_{\min}$ includes the energy required for all the radiation at frequencies above ν, assumed to have a power law distribution with spectral index $\sim 0{\cdot}75$, but $U_{\min}$ is not sensitive to the behaviour of the spectrum at high frequencies; most of $U_{\min}$ is needed even if the spectrum does not extend very far above ν.

The power radiated by the radio component,

$$4\pi \int_{\nu_{\min}}^{\nu_{\max}} P\, d\nu = 4\pi \frac{\nu P(\nu)}{1-\alpha} \left\{ \left(\frac{\nu_{\max}}{\nu}\right)^{1-\alpha} - \left(\frac{\nu_{\min}}{\nu}\right)^{1-\alpha} \right\}$$

does depend on some upper limit $\nu_{\max}$, since generally $\alpha < 1$ in the observed region of the spectrum and the expression then diverges as $\nu_{\max} \to \infty$. However, if we ask whether the electrons chiefly responsible for radiation at frequency ν can radiate away most of their energy while in the radio component, we can restrict attention to radiation in the part of the spectrum near ν; $\nu_{\min} = 0{\cdot}1\nu$ to $\nu_{\max} = 10\nu$ is a generous estimate. Then

$$\text{Power radiated} = 4\pi f \nu P(\nu)$$

where the factor f is $5{\cdot}7$ for $\alpha = 0{\cdot}5$ and $4{\cdot}9$ for $\alpha = 0{\cdot}8$; we therefore adopt $f = 5$.

Then

$$\frac{\text{Power supplied}}{\text{Power radiated}} \simeq \frac{3{\cdot}25 \times 10^{34} \nu_8{}^{2/7} P^{4/7} r_{\rm kpc}{}^{9/7} (\frac{1}{2}c/r)}{4\pi f \nu P}$$

$$= 25 \nu_8{}^{-5/7} P_{28}{}^{-3/7} r_{\rm kpc}{}^{2/7}$$

Here P_{28} is P in units of 10^{28} W Hz^{-1} ster^{-1}; since even the components of Cygnus A have $P_{28} < 1$ and $r_{\rm kpc} > 1$, it is clear that only a small fraction of the power supplied can be radiated away from a component of an extended radio source, interpreted as the tip of the beam, and that most of the energy must be dumped elsewhere.

Note that the result is rather more sensitive to ν than to r or P; thus components radiating principally at infra-red or optical wavelengths could well radiate energy as fast as it is supplied by a beam.