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Models of the water retention curve for soils with a fractal pore size distribution

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Abstract. The relationship between water content and water potential for a soil is termed its water retention curve. This basic hydraulic property is closely related to the soil pore size distribution, for which it serves as a conventional method of measurement. In this paper a general model of the water retention curve is derived for soils whose pore size distribution is fractal in the sense of the Mandelbrot number-size distribution. This model, which contains two adjustable parameters (the fractal dimension and the upper limiting value of the fractal porosity) is shown to include other fractal approaches to the water retention curve as special cases. Application of the general model to a number of published data sets covering a broad range of soil texture indicated that unique, independent values of the two adjustable parameters may be difficult to obtain by statistical analysis of water retention data for a given soil. Discrimination among different fractal approaches thus will require water retention data of high density and precision.

Introduction

It has long been recognized that the behavior of water in soils depends on pore space geometry. Quantification of this geometry by means of fractal concepts offers an opportunity to relate soil water properties to soil structural properties. Fractal objects exhibit three defining attributes: similar structure over a range of length scales, intricate structure that is scale-independent, and irregular structure that cannot be captured entirely by classical (i.e., Euclidean) geometrical concepts, necessitating, for example, the use of a spatial dimension that is not an integer [*Mandelbrot*, 1983; *Falconer*, 1990]. Like the mathematical objects in Euclidean geometry, circles, spheres, squares, or cubes, the objects in fractal geometry are idealizations that can only approximate the pore scale structures encountered in natural soils, but nonetheless they are useful to represent some of the inherent complexity in these porous media.

Recent efforts to apply fractal concepts to hydraulic phenomena in soils have sought new organizing principles for understanding soil structure (see the review by *van Damme* [1995]). Structural properties (particle number-size distributions [*Tyler and Wheatcraft*, 1989, 1992b; *Wu et al.*, 1993], aggregate number-size distributions [*Perfect and Kay*, 1991; *Rieu and Sposito*, 1991a, b], pore size distributions [*Friesen and Mikula*, 1987; *Bartoli et al.*, 1991], aggregate density–sample size relationships [*Rieu and Sposito*, 1991a, b; *Young and Crawford*, 1991], porosity [*Katz and Thompson*, 1985; *Ghilardi et al.*, 1993] or pore-solid interfacial areas [*Pfeifer and Avnir*, 1983; *van Damme and Ben Ohoud*, 1990]), when measured at different scales of resolution, often appear to be power law functions of a relevant length scale. The exponent in these power laws can be interpreted in terms of a fractal dimension which may be related to fundamental soil structural characteristics.

A basic soil hydraulic property is the relationship between water content and water potential, termed the water retention curve (or soil water characteristic). The water retention curve is intimately related to the soil pore size distribution through a standard method of measurement [*Danielson and Sutherland*, 1986]. Since soil water content often is found to be expressed empirically as a power law function of the water potential, there is at hand an experimental exponent that may be modeled or even predicted by a fractal dimension. *Tyler and Wheatcraft* [1989, 1990, 1992a] pioneered the search for this type of interpretation, but subsequent theoretical work by *Rieu and Sposito* [1991a, b, c] has produced an apparently different concept of the relationship between fractal dimension and the water retention curve. These two model approaches do not differ in the way they relate pore size distribution to the water retention curve, since that is conventional [*Danielson and Sutherland*, 1986]. Both of the resulting expressions for the water retention curve may be used to estimate a fractal dimension, and the question arises as to which approach is the more appropriate for a given soil. In this paper an attempt is made to resolve the conundrum by deriving a general fractal model of the pore size distribution that leads to an equation for the water retention curve that includes as special cases the results of *Tyler and Wheatcraft* [1990] and *Rieu and Sposito* [1991c]. It is shown that the difference between the two models lies in the way fractal concepts are applied to represent real porous media. The new general equation is applied systematically to a variety of data sets in order to shed light on current ambiguities surrounding the application of fractal models to interpret soil water behavio

Derivation of the Model Water Retention Curve

Fractal Pore Size Distribution

Pfeifer and Avnir [1983; 1984] have suggested that any porous medium containing solid particles with surfaces rough enough

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to be described by a fractal dimension D will exhibit a power law pore size distribution:

$$-\frac{d[V > \ell]}{d\ell} \propto \ell^{2-D} \tag{1}$$

where ℓ is a measure of pore size (e.g., pore radius as determined by conventional intrusion porosimetry or water desorption methods [*Danielson and Sutherland*, 1986]) and $[V > \ell]$ is our notation for the volume of pores whose size exceeds ℓ. *Pfeifer and Avnir* [1983, 1984] derived (1) explicitly for a model system of cylindrical pores but hypothesized that it should be applicable to arbitrary pore shapes and extendable to Euclidean spatial dimensions other than three [*Pfeifer*, 1984; *Pfeifer et al.*, 1984]. The essential physical concept behind (1) is that a fractal surface structure entails a power law pore size distribution featuring a surface fractal dimension D.

Equation (1) is the definition of a fractal pore-size distribution adopted by *Ahl and Niemeyer* [1989a, b] and that chosen by *Friesen and Mikula* [1987] and *Bartoli et al.* [1991] as their starting point to estimate the fractal character of pore size distributions based on experimental studies using intrusion porosimetry. Equation (1) also can be derived from the fractal solid-pore interface model described by *de Gennes* [1985], as well as from generic "lacunarity" models of porous media, in which gaps or holes are created recursively within an initially solid geometrical object under a constraint of self-similarity [*Rieu and Perrier*, 1996]. In this latter class of models, the fractal dimension D does not necessarily connote surface roughness.

Whether D in (1) in fact represents a surface fractal dimension or a volume fractal dimension thus remains a debated question. *Ghilardi et al.* [1993] have denoted by FSV (fractal surface and volume) geometrical objects that may be interpreted both as surface and volume fractals having the same dimension D. The Menger sponge has been shown to be an FSV [*Toledo et al.*, 1990; *Ghilardi et al.*, 1993], but some fractal surfaces can exhibit a divergent total area while enclosing a nonfractal pore space or solid [*Crawford et al.*, 1993]. Using similar examples, *Friesen and Mikula* [1987] showed the theoretical independence of the fractal behavior of the bulk from that of the boundary of any geometrical object. They concluded that measurement of a pore size distribution alone cannot distinguish between a fractal surface and a fractal volume if the fractal dimension lies in the commonly observed range $2 < D < 3$. In the present study, (1) will be taken simply as the defining equation for a fractal pore size distribution in three-dimensional space (in that it depicts quite generally a basic scaling feature common to previous models) without assuming any particular geometrical structure for the porous medium as a whole.

This heuristic perspective can be reinforced by a simple derivation of (1) that generalizes those sketched by *Pfeifer and Avnir* [1983] and *Jullien and Botet* [1987, section III-4-3]. *Mandelbrot* [1983, chapter 13] has stated that the cumulative size distribution,

$$N\,(\text{volume}^{1/E} > \lambda) = F\lambda^{-D} \qquad 0 < D < E \tag{2}$$

should be of broad applicability to fractal objects of dimension D embedded in a Euclidean space of dimension E. *Falconer* [1990, chapter 3] has discussed the mathematical requirements attendant to the Mandelbrot conjecture. Equation (2) is interpreted as giving the number of fractal objects whose size, measured by the Eth root of their Euclidean volume (strictly, their E-dimensional measure) exceeds the value $\lambda > 0$. The differential size distribution corresponding to (2) is the product of $dN/d\lambda$ times the Euclidean volume (or measure), the latter simply being proportional to λ^E. If λ is interpreted physically as a pore radius ℓ, the differential pore size distribution that follows from (2) is, then, in the notation of (1),

$$-\frac{d[V > \ell]}{d\ell} = \beta(E - D)\ell^{E-D-1} \qquad 0 < D < E \tag{3}$$

where β is a positive constant related to F in (2) and to the geometrical factor that connects volume (or measure) to ℓ^E. Equation (3) depends mathematically only on the Mandelbrot conjecture. It is well known to be applicable to a wide variety of "lacunar" fractal objects, including the Cantor dust ($E = 1$, $D = 0.6309$), the Sierpinski carpet ($E = 2$, $D = 1.8928$), and the Menger sponge ($E = 3$, $D = 2.7268$) [*Mandelbrot*, 1983, pp. 80 and 144; *Perrier*, 1994].

Two-Parameter Model

Integration of (3) with respect to ℓ leads to an equation for $[V > \ell]$:

$$[V > \ell] = -\beta\ell^{E-D} + V_0 \qquad 0 < D < E \tag{4}$$

The constant of integration V_0 (which was omitted in a similar integration presented by *Ahl and Niemeyer* [1989a]) can be evaluated by imposing physical conditions on (4). Let $\ell_{\min}$ be the smallest pore size in the medium. Then $[V > \ell_{\min}] \equiv V_P$ is the total pore volume, by definition. When $\ell = \ell_{\min}$, (4) takes the form

$$V_P = -\beta\ell_{\min}^{E-D} + V_0 \tag{5}$$

and V_0 is seen to be equal to V_P as $\ell_{\min} \downarrow 0$. The definition,

$$[V > \ell] + [V \le \ell] \equiv V_P \tag{6}$$

can be applied to transform (4) into an alternative expression for V_0 in terms of physical quantities:

$$V_0 = V_P + \beta\ell^{E-D} - [V \le \ell] \qquad \ell_{\min} \le \ell \le \ell_{\max} \tag{7}$$

where $\ell_{\max}$ is the largest pore size in the medium; i.e., $[V \le \ell_{\max}] \equiv V_P$ according to (6). Thus

$$V_0 = \beta\ell_{\max}^{E-D} \tag{8}$$

follows from setting $\ell = \ell_{\max}$ in (7).

In the conventional measurement of the soil pore size distribution by water desorption experiments [*Danielson and Sutherland*, 1986], the water potential (or soil-water suction) is assumed to be inversely proportional to an equivalent pore radius ℓ. The largest pore radius, $\ell_{\max}$, corresponds to the very small water potential $h_{\min}$, defined operationally by *Danielson and Sutherland* [1986] as that under which the soil remains visibly saturated with water at equilibrium in a desorption apparatus. The volumetric water content $\theta_{\max}$ that is observed when this saturation equilibrium condition is established is identified as the soil porosity. As the water potential is increased, a smaller volumetric water content follows. In this way, a pore radius–cumulative pore volume data set is created [*Danielson and Sutherland*, 1986, Table 18-1]. This experimental methodology permits (6) to be rewritten in the alternate form

Table 1. Fractal Dimension D Obtained From Fitting Either (11) or (12) to Log-Transformed Water Retention Curves

Porous Medium	Reference	Data Points	Equation (11)		Equation (12)	
			D	r^2	D	r^2
Ariana silty clay loam	*Rieu and Sposito* [1991b]	27	2.90	0.99	2.71	0.98
Berea sandstone	*Davis* [1989]	9	2.99	0.98	2.55	0.99
Delhi sand	*Toledo et al.* [1990]	4	2.97	0.98	2.59	0.99
Panoche loam	*Rieu and Sposito* [1991c]	9	2.97	0.95	2.92	0.95
Yolo clay loam	*Rieu and Sposito* [1991c]	9	2.95	0.999	2.87	0.997

measured water retention curves. We tested these two one-parameter models on several data sets (Table 1) by classical methods. Equation (13) was used [e.g., *Rieu and Sposito*, 1991b] to determine the fractal dimension D from the slope of a regression line fitted to (θ, h) retention data by a log-log plot of $(h_{\min}/h)$ versus $(\theta(h) + 1 - \theta_{\max})$, whereas (12) was used to determine the fractal dimension D from the slope of a regression line fitted to a plot of $\log (h_{\min}/h)$ versus $\log (\theta(h)/\theta_{\max})$ [e.g., *Toledo et al.*, 1990]. Expressions (12) and (13) both were fit very well by linear regression ($r^2 \geq 0.95$), but they led to quite different estimates of the fractal dimension D. This discrepancy is especially striking, given the very small range of fractal dimension [2.4, 3] calculated typically for soils in three-dimensional Euclidean space.

Application to Experimental Water Retention Data

Several theoretical studies [*Tyler and Wheatcraft*, 1990, *Toledo et al.*, 1990] and numerical applications [*Davis*, 1989, *Toledo et al.*, 1990; *Brakensiek and Rawls*, 1992; *Rawls and Brakensiek*, 1995] have made fractal interpretations of (12). Indeed, a good fit of this equation to experimental water retention data should determine a fractal dimension $D = 3 - \lambda_B$ in three-dimensional space. We reanalyzed the original data of *Brooks and Corey* [1964] in which six experimental water retention curves were studied. Similar results were obtained with each data set. One example, Touchet silt loam, is presented in Figure 1 based on Brooks and Corey's values of θ_r and $h_{\min}$, which they selected by trial and error. (Incorporation of a nonzero residual water content θ_r into (11) is straightforward, incidentally, since θ_r is simply subtracted from both sides of the equation.) In the original data fitting, $\theta_r = 0.131$, and the water content corresponding to $h_{\min}$ was $\theta_{\max} = 0.485$, leading to $A = \theta_{\max} - \theta_r = 0.354$ in order to achieve consistency with (12). The straight line through the data points in Figure 1a has a slope equal to 1.83, implying $D = 1.17$. Figure 1b shows a data fitting based on $A = \theta_{\max} = 0.485$, yielding $D = 2.06$, and Figure 1c shows a fitting for $A = 1.0$, yielding $D = 2.66$. It is evident that the estimated fractal dimension depends strongly on the choice of A in the model water retention curve.

Data-fitting results for Ariana silty clay loam [*Rieu and Sposito*, 1991c] are given in Figure 2. Figure 2a shows the original least squares adjustment of (13) and the resulting fractal dimension $D = 2.90$ estimated by *Rieu and Sposito* [1991b], where $h_{\min} = 0.22$ m, measured at $\theta_{\max} = 0.46$ m^3 m^{-3}, was assumed to be the smallest value of the water potential. Figure 2b shows what would have been obtained if (12) had been used instead, with $A = \theta_{\max} = 0.46$. The estimated value of D is 2.71. We then applied nonlinear regression analysis (SAS software, Newton or Marquardt optimization) to the data using (11) instead of (13) to search for optimal estimates of A and D according to a least-squares criterion. For the Ariana soil, with a numerical constraint imposed on A ($A \leq 1$), several equally acceptable pairs of estimates were obtained: ($A = 0.57$, $D = 2.80$), ($A = 0.65$, $D = 2.84$), or ($A = 1$, $D = 2.90$), the last of which of course corresponds to the direct use of (13). No convergence to the pair ($A = 0.46 = \theta_{\max}$, $D = 2.71$), which corresponds to (12), was found. Without a constraint on the optimization process, convergence was met using (11) for the pair ($A = 3.25$, $D = 2.97$; Figure

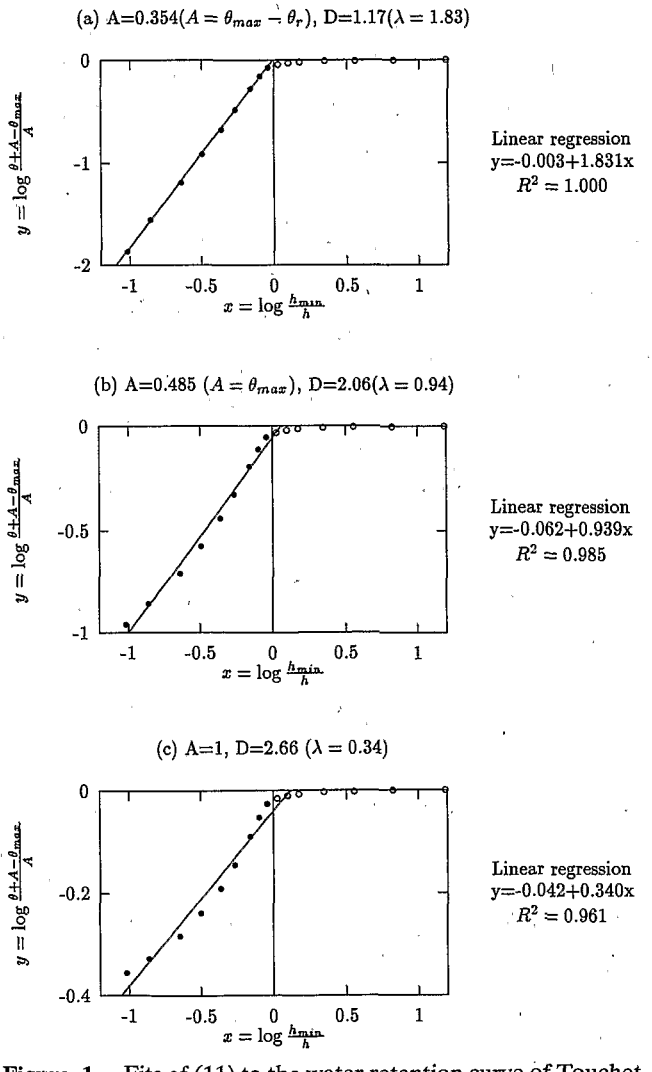

Figure 1. Fits of (11) to the water retention curve of Touchet silt loam [*Brooks and Corey*, 1964]: (a) $A = \theta_{\max} - \theta_r = 0.345$, (b) $A = \theta_{\max} = 0.485$, (c) $A = 1.0$.

$$\frac{[V > \ell(h)]}{V_T} + \theta(h) = \theta_{\max} \tag{9}$$

where V_T is the total volume of a soil sample, $\theta_{\max} \equiv V_P/V_T$ is the soil porosity, and the dependence of pore radius ℓ and water content θ on the water potential h has been noted explicitly. Equation (9) is a mathematical relationship between the water retention curve $\theta(h)$ and the soil pore size distribution $[V > \ell(h)]/V_T$, based only on (6) and a standard method of determining pore size distributions.

If the soil pore size distribution is modeled as fractal, then (3) applies and its integrated form (4) can be introduced into (9):

$$\frac{-\beta[\ell(h)]^{E-D}}{V_T} + \frac{V_0}{V_T} + \theta(h) = \theta_{\max} \qquad 0 < D < E \tag{10a}$$

Equation (8) can be applied to put the first two terms on the left side of (10a) into the form:

$$\frac{V_0}{V_T}\left(1 - \left[\frac{\ell(h)}{\ell_{\max}}\right]^{E-D}\right) + \theta(h) = \theta_{\max} \qquad 0 < D < E \tag{10b}$$

Finally, given the inverse proportionality between $\ell(h)$ and h, and between $\ell_{\max}$ and $h_{\min}$, the ratio of pore sizes in (10b) can be expressed as a ratio of water potentials instead:

$$\frac{V_0}{V_T}\left(1 - \left[\frac{h_{\min}}{h}\right]^{E-D}\right) + \theta(h) = \theta_{\max} \qquad 0 < D < E \tag{11}$$

Equation (11) describes a water retention curve for any soil with the fractal pore size distribution defined by (3). Given that $\theta_{\max}$ and $h_{\min}$ are always measured in a water desorption experiment [*Danielson and Sutherland*, 1986], the only adjustable parameters in (11) are the fractal dimension D and the ratio, V_0/V_T, hereinafter denoted by A for convenience in applications. The parameter A is expected to lie in the range, $\theta_{\max} \leq A \leq 1$, since V_0 is an upper bound on V_P as $\ell_{\min} \downarrow 0$.

Relation to Other Fractal Models

Equation (11) can be transformed to the Brooks-Corey form [*Brooks and Corey*, 1964],

$$\theta(h) = \theta_{\max}\left(\frac{h_e}{h}\right)^{\lambda_B} \tag{12}$$

after equating A with $\theta_{\max}$ and interpreting the parameters $h_{\min}$ and 3-D (i.e., $E = 3$) in (11) to be respective correspondents of the original Brooks-Corey parameters, h_e ("air-entry suction") and λ_B ("pore size distribution index"), with neglect of any residual water content. A physical consequence of setting $\theta_{\max} = A = V_0/V_T$ is that $\ell_{\min} = 0$ in (5), which is consistent with ignoring the residual water content. The Brooks-Corey model has been shown in many studies to provide a reasonably accurate representation of the water retention curve for soil water contents that are not close to that at saturation or that at oven dryness (for reviews, see *Milly* [1987] and *Rossi and Nimmo* [1994]). *Brakensiek and Rawls* [1992] have tabulated the Brooks-Corey parameter λ_B for 11 U.S. Department of Agriculture textural classes based on measurements made on 1323 soils in the United States. Geometric-mean values of λ_B for the textural classes ranged from 0.127 (silty clay) to 0.592 (sand). This range of λ_B corresponds to a fractal dimension D, defined in three-dimensional space by $D \equiv 3 - \lambda_B$, that ranges from 2.873 for silty clay to 2.408 for sand.

Tyler and Wheatcraft [1990, 1992a] used the Sierpinski carpet as a model for a fractal soil pore space, in effect thereby mapping the three-dimensional soil pore network onto a plane. Their model for the water retention curve is obtained from (11) by setting $E = 2$ and $\theta_{\max} = A = V_0/V_T$; i.e., it is the same as a Brooks-Corey equation applicable to two-dimensional space. Thus *Brakensiek and Rawls* [1992], who applied the two-dimensional model of *Tyler and Wheatcraft* [1990, 1992a], reported fractal dimensions that are smaller by 1.0 than those given above for the case $E = 3$.

Rieu and Sposito [1991a, b, c] developed a "lacunarity" model of an aggregated soil based on a space partition of the solid initiator into N parts, which then are reduced by a factor r to define N replicas of the initiator surrounded by gaps (or holes). This process is repeated recursively, replacing at each iteration the N replicas by copies of the generator reduced by the factor r. The resulting model equation for the water retention curve can be derived from (11) by setting $A = 1$ with $E = 3$:

$$\theta(h) = \theta_{\max} - 1 + (h_{\min}/h)^{3-D} \qquad 0 < D < 3 \tag{13}$$

Rieu and Sposito [1991b, c] tested (13) with experimental water retention curve data for six soils whose texture varied from silty clay to sand. They found excellent fits of (13) to the data, with D values ranging from 2.758 (sand) to 2.986 (silty clay).

Equation (5) shows that the parameter A represents the largest value possible for the fractal porosity, which is achieved as $\ell_{\min} \downarrow 0$. Equation (13) implies further that this upper limit of porosity is 1.0, which corresponds to an infinite number of recursive steps in a lacunarity model [*Perrier*, 1994]. It is this case that appears in the model of *Rieu and Sposito* [1991a], but the upper limit of porosity, 1.0, was not achieved because they set $\ell_{\min} \neq 0$ in order to have their model represent the geometry of both the pore space and the solid particles [see *Rieu and Sposito*, 1991a]. Equation (12), on the other hand, implies that the upper porosity limit is $\theta_{\max}$, not 1.0. This case appears in the model of *Tyler and Wheatcraft* [1990], in which $\ell_{\min} \downarrow 0$, allowing the upper limit of porosity to be achieved, so $\theta_{\max} = A$. Because infinitely small pores (i.e., $\ell_{\min} = 0$) are assumed to occur, the solid phase can vanish, leaving the fractal structure to represent solely the pore space. Thus the fractal model of *Tyler and Wheatcraft* [1990] describes only a pore size distribution, not the geometry of soil structure. In this paper we also do not assume a particular geometrical soil structure but consider only the implications of any fractal pore size distribution consistent with (3)—and any such distribution may be represented "geometrically" by the holes of a generalized Sierpinski carpet, as done in the approach of *Tyler and Wheatcraft* [1990]. However, if we consider that this carpet can represent a real pore size distribution only over a limited range of length scales, (i.e., $\ell_{\min} \neq 0$), the porosity $\theta_{\max}$ does not reach the upper limit A that would be achieved if $\ell_{\min} = 0$. In this case, $\theta_{\max} < A$, and (12) no longer holds. Irrespective of the choice of $\ell_{\min}$, the general equation that follows from (3) and (9) is (11).

The models proposed by *Rieu and Sposito* [1991a] and *Tyler and Wheatcraft* [1990] thus do not differ as to the mathematical object used, since the lacunarity model and the Sierpinski carpet have identical fractal properties, but the two models do not portray soil water properties in the same way. This important conceptual difference between (12) and (13) can be illustrated quantitatively by using each equation to infer D from

2c); but this result is quite unrealistic, given the physical interpretation of A as a porosity!

The conclusion to be drawn from non-linear optimization using (11) with all the data sets given in Table 1 (plus other sets for the same soil series), is that A and D are not independent parameters. Given (h_{min}, θ_{max}) as the bounding point of the fitting domain investigated, several pairs of attractors (A, D) were found to be equivalent, and increasing both A and D generally led to very good fits in terms of a least squares criterion. The optimization actually converged uniquely only for the data set used by *Davis* [1989], with a value of A slightly larger than θ_{max} ($A = 1.032\ \theta_{max}$) and a value of D ($D = 2.72$) larger than the value $D = 2.55$ estimated by *Davis* [1987], who used (12). But, as in all other examples, the confidence intervals for the estimated parameters were untenably large (e.g., with the data of *Davis* [1987] we found [0.99 θ_{max}, 1.07 θ_{max}] for A and [2.62, 2.82] for D). We tried also to optimize (11) in a derivative form ($d\theta/dh \propto h^{D-4}$), which is analogous to an expression applied by *Friesen and Mikula* [1987], but this also led to unacceptably large confidence intervals for the estimates of D (e.g., $D = 2.98 \pm 0.26$ for the Ariana soil), possibly because of a relative lack of precision in water retention data by comparison to the mercury intrusion data analyzed by *Friesen and Mikula* [1987].

Discussion

Estimation of the Fractal Dimension From Water Retention Data

When either (12) or (13) is postulated as the appropriate equation with which to model a soil water retention curve, there is no difficulty in calculating fractal dimensions for many soils [cf. *Brakensiek and Rawls*, 1992], even considering the vicissitudes of log-log transforms and the sizes of the confidence intervals for the estimated fractal dimension. However, if no particular simplified form of (11) is assumed, it was found to be far more difficult to estimate a unique value of D. Nonetheless, (11), whether used in integral form or in a derivative form, in principle should provide the means to discriminate between (12) and (13), or any other particular model. Using simulated water retention data, we have found that when good log-log transform linear fits are obtained using (12), those obtained using (13) are very poor [*Perrier et al.*, 1995], and vice versa [*Perrier*, 1994]. This last result suggests that more precise and abundant experimental water retention data should help to determine which fractal model is the more appropriate.

Young and Crawford [1991] published a critical review of determinations of the fractal dimension based on (13) using water retention data and noted that systematically lower estimates of the fractal dimension resulted as compared to other methods, such as aggregate bulk density-size measurements. They suggested that this discrepancy may be the effect of an incomplete representation of the pore space when pore network connectivity is neglected. It must be stressed, however, that the only evaluation method they used was the fitting of water retention data to (13). Comparing the order of magnitude of published fractal dimensions, we can propose another explanation: that use of (13) may produce an underestimation of D, while using (12) may overestimate D. Intermediate values might result from a very careful use of (11).

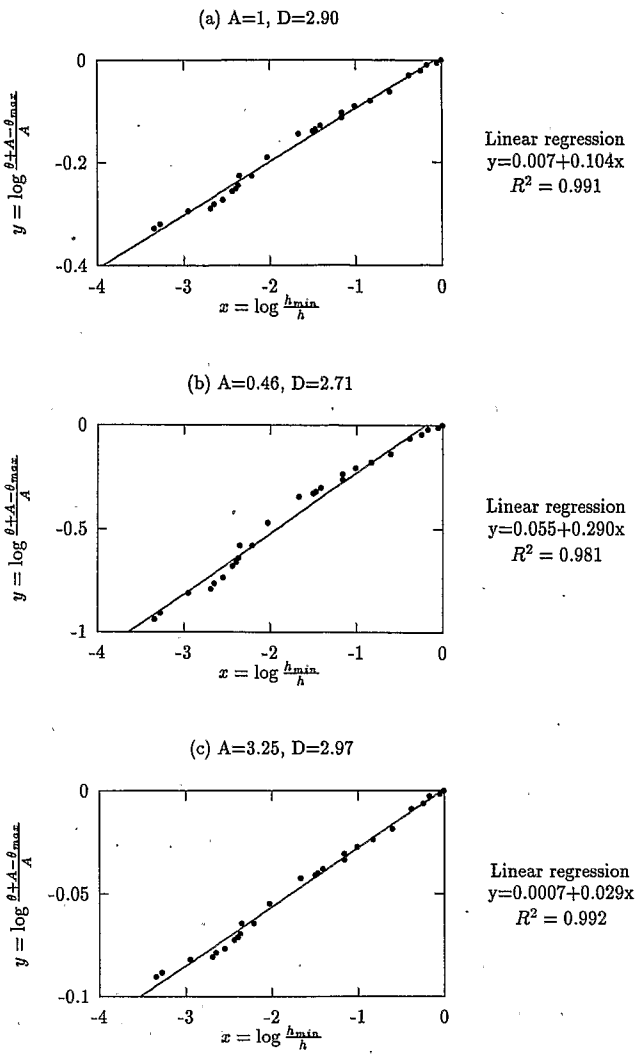

Figure 2. Fits of (12) to the water retention curve of Ariana silty clay loam [*Rieu and Sposito*, 1991b]: (a) $A = 1.0$, (b) $A = \theta_{max} = 0.46$, (c) $A = 3.25$.

Prediction of the Water Retention Curve From the Fractal Dimension

Fractal scaling of soil structural properties can help to interpret the shape of the water retention curve in soils, in that the fractal dimension D leads to an estimate of the exponent in a power law function describing $\theta(h)$. A powerful application would be the prediction of this basic hydraulic property from soil structural properties that are more easily measured and, although it goes beyond the scope of this paper, the prediction of the hydraulic conductivity [*Rieu and Sposito*, 1991b, c; *Shepard*, 1993; *Rawls and Brakensiek*, 1995]. A fundamental question still remains: Which fractal dimension best describes the water retention curve? The fractal dimension D is related to the pore size distribution, whatever may be its geometric origin, assuming the conventional capillary model of the water potential [*Danielson and Sutherland*, 1986], such that an equivalent pore-size distribution can be obtained from mercury intrusion or water retention data that are fit to power-law equations. Thus fractal geometry is, in effect, used only to rename an empirical exponent without specifying any physical concept of soil structure. Another way to proceed would to be to make

independent measurements of the pore size distribution from an image analysis of soil thin sections, if the limitations of this two-dimensional approach can be overcome.

What may be even more useful is to determine a relation between D and fractal dimensions that pertain to the solid phase. These latter fractal dimensions are readily estimated from scaling laws observed for the particle size or aggregate size distribution as obtained by mechanical analysis [*Tyler and Wheatcraft*, 1989, 1992a, b; *Perfect and Kay*, 1991; *Rieu and Sposito*, 1991b, c; *Wu et al.*, 1993]. In most cases, fractal structure models do not make a theoretical connection between the scaling behavior of soil pores and that of solid grains. *Tyler and Wheatcraft* [1992b, p. 368] postulated an "intuitive" relationship between fractal particle size distributions and fractal pore size distributions but acknowledged that "a theoretical development is not yet available". *Rieu and Sposito* [1991b] showed that their soil structure model can lead to a volume fractal dimension, determined from aggregate density or mass measurements, which is the same as that characterizing the pore size distribution and the water retention curve as modeled by (13). *Agnese et al.* [1994] successfully predicted the water retention curve in this way based on a fractal analysis of the aggregate density-size distribution of clayey aggregated soils. *Rieu and Sposito* [1991b] discussed why partial destruction of the soil structural organization as customarily performed prior to mechanical analysis might lead to fractal aggregate number-size distributions characterized by a somewhat smaller fractal dimension than that for the undisturbed soil structure.

Concluding Remarks

A fractal analysis of the water retention curve cannot be done without also analyzing the underlying fractal object in respect to its geometrical interpretation. Thus only experiments carried out to measure, on the same soil, both water retention data and structural properties will enable progress to occur in understanding the fractal nature of soils. Although fractal objects provide idealized and simplified models of real porous media, they do give valuable insight as to the geometrical coherence that must underlie any attempt to relate fractal dimensions corresponding to different physical definitions with that describing water retention curves. For example, soils have been found to be pore fractals [e.g., *Katz and Thompson*, 1985; *Ghilardi et al.*, 1993] on the basis of a fractal model that represents porous media conceptually in exactly the opposite way to that presented in this paper; namely, the mathematical model is a "lacunar" fractal object, but the gaps or holes represent the solid grains instead of the pores. In this case, the pore volume is fractal, but the pore size distribution is not [*Rieu and Perrier*, 1996]. Since no geometrical model offering a realistic partition of a soil into pores and solid grains is yet available to relate a fractal pore volume to a fractal pore size distribution, any attempt to use a pore fractal dimension to predict a water retention curve in the form of (11) can succeed only fortuitously.

Scale-invariant processes apply to a whole porous medium structure, and the same scaling exponents may characterize different parts of this structure [*Hillel and Elrick*, 1990]. Our view is that these exponents are fundamental physical indicators of soil water behavior, but that a fractal approach must rely on geometrically consistent models. Geometrical models whose scaling properties mimic those encountered in natural soils also may provide useful representations of soil structure organization. Hence it is possible to go beyond the mere illustration of a fractal pore size distribution to take into account the detailed connectivity of the soil pore network. Simulations of random fractal soil structures [*Perrier et al.*, 1995] have shown that connectivity conditions have strong influence on hydraulic properties, especially in respect to the well-known hysteresis behavior of water retention curves. Further investigation is necessary to evaluate the effect of hysteresis on fractal analyses, which currently are based on the convenient assumption of a one-to-one correspondence between the pore size distribution and the water retention curve.

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