# Modes of clustered star formation 

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#### Abstract

Context. The recent realization that most stars form in clusters, immediately raises the question of whether star and planet formation are influenced by the cluster environment. The stellar density in the most prevalent clusters is the key factor here. Whether dominant modes of clustered star formation exist is a fundamental question. Using near-neighbour searches in young clusters, Bressert and collaborators claim this not to be the case. They conclude that - at least in the solar neighbourhood - star formation is continuous from isolated to densely clustered environments and that the environment plays a minor role in star and planet formation. Aims. We investigate under which conditions near-neighbour searches in young clusters can distinguish between different modes of clustered star formation. Methods. Model star clusters with different memberships and density distributions are set up and near-neighbour searches are performed. We investigate the influence of the combination of different cluster modes, observational biases, and types of diagnostic on the results. Results. We find that the specific cluster density profile, the relative sample sizes, the limitations of the observation, and the choice of diagnostic method decide, whether modelled modes of clustered star formation are detected by near-neighbour searches. For density distributions that are centrally concentrated but span a wide density range (for example, King profiles), separate cluster modes are only detectable under ideal conditions (sample selection, completeness) if the mean density of the individual clusters differs by at least a factor of $\sim 65$. Introducing a central cut-off can lead to an underestimate of the mean density by more than a factor of ten especially in high density regions. The environmental effect on star and planet formation is similarly underestimated for half of the population in dense systems. Conclusions. Local surface-density distributions are a very useful tool for single cluster analyses, but only for high-resolution data. However, in a simultaneous analysis of a sample of cluster environments, it is found that effects of superposition suppress characteristic features very efficiently and thus promote erroneous conclusions. While multiple peaks in the distribution of the local surface density in star forming regions imply the existence of different modes of star formation, the converse conclusion is impossible. Equally, a smooth distribution is no proof of continuous star formation, because such a shape can easily hide modes of clustered star formation.


Key words. open clusters and associations: general - methods: statistical - stars: formation

## 1. Introduction

Most stars form in proximity to other stars within embedded clusters rather than being uniformly distributed throughout molecular clouds (Testi et al. 1999; Carpenter 2000; Lada \& Lada 2003; Porras et al. 2003; Allen et al. 2007). The density in young clusters in the Milky Way varies over many orders of magnitude from $<1 \mathrm{stars} / \mathrm{pc}^{3}$ in relatively sparse clusters to $>10^{5}$ stars $/ \mathrm{pc}^{3}$ in the central areas of dense clusters. The key factor in determining the relative importance of the environment for star and planet formation is the stellar density in the young clusters. Stars forming in the sparse cluster environments are largely unaffected by the presence of their fellow cluster members. In contrast, one can expect a strong influence on star and planet formation by the environment in the densest of these young clusters. Theoretical investigations predict that this environmental influence on star formation might manifest itself in a different initial mass function (Freitag et al. 2006; Pfalzner 2006; Marks et al. 2012), the binary fraction (Marks et al. 2011; Kaczmarek et al. 2011), and the disc frequency in high
stellar-density environments (Scally \& Clarke 2001; Pfalzner et al. 2005, 2006; Olczak et al. 2006, 2010). Observations have found indications of a dependence of these properties on the stellar density in young clusters (Hillenbrand \& Hartmann 1998; Harayama et al. 2008; Stolte et al. 2010). In dense clusters, interactions might lead to a lower disc frequency resulting in a lower planetary-system incidence rate and different properties of the planetary system.

For the stellar population as a whole, the question is whether the properties of prestellar cores largely determine the stellar properties, like in the case of isolated star formation (Shu et al. 2004; Larson 2005; Tan et al. 2006), or whether most stars form in a more dynamic way, where external forces and interactions dominate over initial conditions (e.g. Bonnell et al. 2001; Bonnell \& Bate 2006).

Hence, a fundamental question of current star formation research is whether there exists a type of cluster with a particular number of members or density that is the dominant environment for star formation? At first sight, it would seem easy enough to
answer this by simply collecting cluster data and determining the distribution of the mean density in young clusters, but this approach has a number of obstacles. Most star formation occurs inside the spiral arms and close to the centre of our Galaxy where it is difficult to identify clusters owing to our position within the plane of the Galactic disc. This means we have nothing like a complete census of the young clusters in the Milky Way. In principle, looking at nearby galaxies should help, but the larger distance means that the detection of low-mass clusters is impeded by their low luminosity.

There are different strategies for tackling the issue indirectly. One method is to look at either the initial mass function (e.g. Bonnell et al. 2007; McKee \& Tan 2008; Bate 2009; Da Rio et al. 2012) or the binary development (e.g. Durisen \& Sterzik 1994; Brandner \& Koehler 1998; Duchêne 1999; Connelley et al. 2008; Fregeau et al. 2009; Kaczmarek et al. 2011; Marks et al. 2011) in different types of young clusters and compare them to those in the field. Any similiarities are then interpreted as signs of a dominant cluster mode. However, since many cluster modes contribute simultaneously, a one-to-one relation is difficult to establish.

Another method is to measure the local surface-density distribution in different cluster environments. Several observational studies (e.g. Gutermuth et al. 2009; Bressert et al. 2010; Kirk \& Myers 2012) have tried to answer the above questions by analysing large samples of young stellar objects in terms of their local surface density, $\Sigma$, predominantly in the solar neighbourhood. Here, it is argued that if different discrete modes existed they should manifest themselves as peaks in a surface density distribution (e.g. Strom et al. 1993; Carpenter 2000; Weidner et al. 2004; Wang et al. 2009; Bressert et al. 2010).

This latter simple approach has the advantage that it does not rely on the definition of stellar groups, but uses the local separation of the star from its nearest neighbours. The local surface density is simply defined as $\Sigma=(n-1) /\left(\pi r_{n}^{2}\right)$, where $n$ is the considered number of nearest neighbours including the star itself and $r_{n}$ is the distance to the $n$th neighbour. Higher values of $n$ correspond to a lower spatial resolution, but smaller fractional uncertainty (Casertano \& Hut 1985; Gutermuth et al. 2009).

Using this method, Bressert et al. (2010) found no peaks in the combined surface density distribution of several clusters in the solar neighbourhood (see their Fig. 1). They concluded from the absence of such peaks that star formation is continuous from isolated to densely clustered environments. In addition, they deduced a mean stellar surface density of $20 \mathrm{stars} / \mathrm{pc}^{2}$ for the star forming regions in the solar neighbourhood and concluded that the environment plays a minor role in star and planet formation because only a small fraction of stars is found in high-density regions.

In the present study, we discuss the effect of different clusterdensity profiles, the dependence on the sample selection and the influence of observational constraints on the obtained results. We demonstrate that local surface-density measurements are rather limited in their ability to determine different starformation modes owing to superposition effects. Therefore, the question of whether dominant modes of clustered star formation exist in the solar neighbourhood remains unclear.

## 2. Method

### 2.1. Cluster types

The determination of the general shape of the stellar density distribution of young clusters can be observationally challenging.

Owing to the significant amount of dust in young embedded clusters, not all stars are visible and even in young exposed clusters, crowding in the central high-density regions poses problems even for high resolution instruments such as the Hubble Space Telescope (HST) (e.g. McCaughrean \& Stauffer 1994).

A century ago, Plummer (1911) found that
$\rho_{\mathrm{P}}(r)=\left(\frac{3 M}{4 \pi a^{3}}\right)\left(1+\frac{r^{2}}{a^{2}}\right)^{-5 / 2}$,
provides a good fit to the density distribution of globular clusters. Here $M$ is the total cluster mass and $a$ is the Plummer radius, a scale parameter for the cluster core-size $r_{\mathrm{c}}$. This model is widely used for all types of star clusters, largely thanks to its success in fitting globular cluster profiles, but also because of its convienient analytical form.

King (1966) developed an improved empirical law, leading to the so-called family of King models. These consist of an energy distribution function of the form
$f_{\mathrm{K}}(\mathcal{E})= \begin{cases}\rho_{1}\left(2 \pi \sigma_{\mathrm{K}}^{2}\right)^{-3 / 2}\left(\mathrm{e}^{\mathcal{E} / \sigma_{\mathrm{K}}^{2}}-1\right): & \mathcal{E}>0 \\ 0 & : \\ \mathcal{E} \leq 0,\end{cases}$
with $\mathcal{E}=\Psi-\frac{1}{2} v^{2}$ and $\Psi=-\Phi+\Phi_{0}$ being the relative energy and relative potential of a particle, respectively. Here $f(\mathcal{E})>0$ for $\mathcal{E}>0$ and $\sigma_{\mathrm{K}}$ is the King velocity dispersion. The stellar density distribution can only be obtained by numerical integration. The King paramter $W_{0}=\Psi / \sigma_{\mathrm{K}}^{2}$ characterizes the sequence of King profiles with decreasing relative size of the cluster core $r_{\mathrm{c}} / r_{\mathrm{hm}}$ for increasing $W_{0}$, where $r_{\mathrm{hm}}$ is the half-mass radius.

In the following, we describe our investigation of two types of model clusters - those based on Plummer and King distributions. While the Plummer distribution is well-approximated by a King model with $W_{0} \approx 4$, young clusters are best represented by King models with $W_{0} \geq 7$ (e.g. Hillenbrand \& Hartmann 1998; Sung \& Bessell 2004; Harfst et al. 2010). Thus, the term "King model" is used here as an equivalent to King distributions with high $W_{0}$.

### 2.2. Diagnostics

To determine the conditions for which local surface density allow us to distinguish different spatial modes of star formation, we constructed a representative set of numerical cluster models that span the expected parameter space. We generated model clusters with Plummer and King stellar density profiles containing 100, 1000, and 10000 stars. Each cluster has a half-mass radius of $r_{\mathrm{hm}}=1 \mathrm{pc}$. Hence, configurations with different numbers of stars imply different volume and surface densities. In our model clusters, the mean surface densities are $12.6,126$, and 1260 stars $/ \mathrm{pc}^{2}$, respectively. The distributions have been set up as only single stars, so without a primordial binary population.

To ensure equally statistically significant results, each cluster population was generated repeatedly with different random seeds for a total of $10^{5}$ stars for each of the considered cases. We used a tree-based algorithm to reduce the computational effort for the near neighbour search (Kennel 2004).

As pointed out by Casertano \& Hut (1985), an intermediate number of neighbours has the advantage of neither missing small dense structures nor introducing artificial overdensities produced by strongly bound multiple systems. We tested the influence of the number of nearest neighbours (3-27) on the resulting surface density diagnosed for our King models. For the clusters with 1000 and 10000 stars, no obvious difference was visible in the


Fig. 1. Comparison of model clusters with 10000 stars and a half-mass radius of 1 pc obeying a Plummer distribution (open circles) with those of a King $\left(W_{0}=12\right)$ distribution. Here $\left.\mathbf{a}\right)$ shows the number of stars as a function of the radial distance to the cluster centre, whereas $\left.\mathbf{b}\right)$ shows the number of stars of a given surface density and $\mathbf{c}$ ) the same in cumulative normalized form.
results averaged over the set of simulations. Only the results for the cluster consisting of 100 stars depends slightly on the number of neighbours considered. However, even these differences are within the error bars. Hence, we included the contribution of the 8 nearest neighbours throughout our investigation.

### 2.3. Model vs. observed clusters

In some respect, our model clusters represent the ideal of what one would like to observe. In observations of even the closest star-forming regions, it is, however, nearly impossible to detect each and every star of the cluster. One reason is that owing to the limited spatial resolutions of telescopes, crowding becomes a severe problem in the central regions of dense clusters. For example, the Spitzer Space Telescope as used in the study of Bressert et al. (2010) can only marginally resolve the inner 0.3 pc of the Orion Nebula Cluster. To avoid observational biases caused by crowding, they excluded this inner cluster area from their analysis. This means that the average stellar densities obtained in this way can only be regarded as lower limits. For high-resolution telescopes such as the HST, this is less of a problem.

Another limitation is the maximum contrast that an instrument can image. This means that low-mass stars are less likely to be detected close to massive stars and therefore the surface density around massive stars, which are mostly located in the central dense cluster area, is underestimated.

Finally, the magnitude limits of a given survey impose a limit on the faintest observable isolated object. With decreasing mass the number of stars in a star cluster grows rapidly, so the estimated density is a strong function of the magnitude limit. Field contamination usually imposes another serious observational bias. However, the young members of star forming regions can usually be distinguished rather well from the much older population of field stars.

The observational limitations outlined above basically affect studies of any star-forming region. The effect of all these limitations is to lower estimates of the cluster density. This is particularly true for the maximum local density that typically reaches its highest value where crowding and massive stars impose the most severe observational biases.

## 3. Single clusters

### 3.1. Cluster density profile

We first compare single model clusters with a Plummer profile to those with a King $\left(W_{0}=12\right)$ profile. We note that all models have normalized half-mass radii ( $r_{\mathrm{hm}}=1 \mathrm{pc}$ ). Plummer models are commonly used as initial models for numerical simulations
of young star clusters. However, observations of very young star clusters ( $<3 \mathrm{Myr}$ ) typically have a more concentrated distribution, close to that of an isothermal sphere. From a numerical point of view, a King model with $W_{0}=12$ is a rather good representation of such an isothermal sphere. The basic difference between these two models is that the stellar density in a King model of $W_{0}=12$ increases much more towards the cluster centre than in a Plummer model.

This is clearly visible in Fig. 1a, which shows the projected radial number profile of both models. Their maxima roughly correspond to the half-mass radius $r_{\mathrm{hm}}$. While the Plummer-model clusters (grey lines) contain only a small fraction of stars at small (projected) distances to the cluster centre, this fraction in King-model clusters (black lines) is considerably larger. In the local surface-density plot (Fig. 1b), this translates into a sharply peaked asymmetric distribution for the Plummer model and a Gaussian-shaped distribution for the King model. Most stars in the Plummer-shaped cluster share the same local density that marks roughly the maximum local density of the entire cluster. In contrast, the King-shaped cluster has a long high-density tail that extends well beyond the maximum local density of the Plummer model. In the cumulative local surface-density distribution (Fig. 1c), this difference is encoded in the steeper slope at the end of the distribution for Plummer-type clusters.

### 3.2. Incompleteness

As described in Sect. 2.3, observations of real star clusters always suffer from observational limitations that potentially influence the resulting surface-density distribution. Here we mimic these observational limitations by applying "filters" to the data in our diagnostics. We first emulate the observational resolution of the Spitzer's IRAC camera of $2.5^{\prime \prime}$ for a cluster at the same distance as the ONC corresponding to a resolution of $1035 \mathrm{AU} \approx$ 0.005 pc . In our diagnostics, we scan all particles and mark those that lie in projection within 1000 AU from the current star as not being observable.

Figure 2a demonstrates the effects of this observational limitation for the King model cluster with $N=10^{3}$ and $N=$ $10^{4}$ stars, where grey indicates the case without any filter and black the filtered case. Observational limitations lead to the neglect of any stars with local surface densities exceeding roughly ten times the median density of stellar systems. In the intermediate-density regime, a slight increase in the counted number of stars is seen, because in high density areas the local surface density is lower around the stars that remain observable. Here and in the following, the number of stars in the distributions have been normalized to the total number of stars in the sample.


Fig. 2. Comparison of the effects of different observational biases (black) on the unbiased local two-dimensional (2D) density distribution (grey) for star clusters consisting of 1 k (dashed) and 10 k (solid) particles with a King parameter of $W_{0}=12$. a) Resolution limit of the IRAC camera of the Spitzer Space Telescope. b) Cutout of the densest inner 0.5 pc . c) Combined effects $\mathbf{a}$ ) and $\mathbf{b}$ ).

Hence, adopting the Spitzer-like resolution significantly reduces the number of stars at the high-density end. The average number of observed stars in the filtered case reduces for the cluster containing 100 stars to 99 , that with 1000 stars to 864 , and the one with 10000 to about 6460 . In addition, for the dense clusters the limited resolution ensures that the existing Gaussianlike shape has a much more peaked curve with a steeper decline at high densities - very similar to a Plummer distribution (cf. Fig. 1). As a consequence of the observational limitation, the median observed density of the densest of our model cluster would be reduced to less than half its real value (see Table 1).

The problem of crowding is often circumvented by excluding stars in central high stellar-density regions from the sample. Here we mimic this by excluding all stars closer than 0.5 pc from the cluster centre (Fig. 2b). As in the case of the Spitzer resolution limitation, the relative number of stars with low local two-dimensional (2D) densities ( $\lesssim 10^{2} \mathrm{pc}^{-2}$ ) remains nearly unchanged, the number of stars with high local 2D densities (higher than ten times the median density of the cluster) being entirely removed, but the number of stars with intermediate local 2D densities rising owing to the adoption of this filter. However, this time the increase in the number of stars with intermediate densities is much more pronounced than in the Spitzer resolution case because of boundary effects at the filter cutoff radius. The result is an apparent increase in the number of stars with intermediate densities.

Combining both (Fig. 2c), observational limitations lead to a significant underrepresentation of the number of stars in highdensity regions of dense clusters. As a result, an observer would underestimate the median local density by more than a factor of two and the average local density by more than an order of magnitude (see Table 1). The quoted values can only be regarded as lower limits.

## 4. Multiple modes

In the following, we analyse idealized samples of stars constructed from different cluster modes. Technically, this is achieved by scaling accordingly the data sets from Sect. 3. The aim is to determine under which circumstances one would be able to detect different cluster modes from the (cumulative) surface density distributions.

### 4.1. Relative sample size

In reality, sample sizes from different clusters often differ considerably. In many cases, only a few tens of data points are



Fig. 3. a) Differential and b) cumulative local surface-density distribution for stars from two Plummer-shaped model clusters of approximate mean densities of $12.6 \mathrm{pc}^{-3}$ and $1260 \mathrm{pc}^{-3}$, where the first one contains 100 stars and the latter 1000 stars.
available for low-mass clusters but several hundreds to thousand for high-mass clusters such as the ONC. Therefore, high-mass clusters may dominate the results. To test the limitations, we start with a model consisting of two modes of clustered star formation - one corresponding to a relatively dense and the other a less dense environment.

Combining two Plummer-type clusters, where one has a ten times higher median density than the other, Fig. 3a shows the


Fig. 4. Differential and cumulative local surface-density distribution for stars from two Plummer-shaped (a) and b)) and two King-shaped $\mathbf{c}$ ) model clusters. The approximate mean densities of each individual cluster is $10 \operatorname{stars~}_{\mathrm{pc}^{-3}}$ and 1000 stars $\mathrm{pc}^{-3}$ in $\mathbf{a}$ ) and $100 \mathrm{stars}^{\mathrm{pc}}{ }^{-3}$, and $1000 \mathrm{stars}^{\mathrm{pc}}{ }^{-3}$ in b) and $\mathbf{c}$ ). For each cluster mode, 10000 stars were considered.
differential local surface-density distribution and Fig. 3b its cumulative form for the case where one cluster corresponds to the 100 and the other to the 1000 star models described in Sect. 3. This illustrates a situation where 10 times as many stars formed in the denser environment than in the less dense one. It can be seen that one would not detect two peaks. Similarly the cumulative surface-density distribution increases steadily and does not show any "bumps", even though two different cluster modes are present.

This demonstrates that the limited sample-size can prevent the detection of an existing bi-modal clustered star-formation process. We tested the maximum possible difference in sample size that allows the identification of existing cluster modes and found that generally the sample sizes must not differ by less than a factor of five for existing cluster modes to be identifiable.

In reality, one either considers a smooth distribution of young stars throughout a single cloud or one combines the results from multiple distinct clusters. It is obvious that the above reservations apply in the first case. In the second case, one could argue that there will be many more stars in high-mass clusters than in low-mass clusters, but many more low-mass clusters than high-mass clusters. So in principle one can construct equal-sized samples.

### 4.2. Two equal modes of clustered star formation

We now analyse the case of an idealized sample by combining two identical sample sizes of two different cluster modes, that is, the case where stars form with equal likelihood in one of two clustered modes.

We start with two Plummer-type clusters, where one has a hundred times higher median density than the other. Figure 4a shows the surface density on top and its cumulative form underneath for the case where one cluster corresponds to the 100 and the other to the 10000 star models. Scaling is applied to avoid the non-detection of cluster modes owing to sample size effects. The combined surface density shows a strong double peak and
the cumulative distribution a saddle point. These features are still visible if the density in one cluster is only ten times that of the other cluster (see Fig. 4b).

These multiple peaks in the surface density distribution and the "bumpy" nature of the cumulative distribution is what is expected for multi-modal clustered star formation. Conversely, the absence of these features is often taken as proof of continuous star formation ranging from low to high density regions (see, for example, Bressert et al. 2010). It is argued that the peaks are so densely packed that the result is a continuous function. We show that this argument is only valid under very specific conditions that are usually not fulfilled in young cluster environments.

As mentioned above, Plummer profiles are widely used in theoretical investigations owing to the existence of an analytical solution. However, they seem less suitable for modelling young clusters. King profiles with high $W_{0}$ are regarded as a better choice. By performing the same investigation as above but now for two of the King-type clusters deviating by a factor of ten in density, we obtained results quite different from the Plummer case. Instead of two peaks, a single one appears in the surface density distribution (Fig. 4c), which is no longer " M "-shaped but nearly Gaussian and wider than in the case of a single Kingshaped cluster.

As a result, the cumulative surface distribution (Fig. 4c) looks very much like that of a single cluster with only a slightly different slope. Hence, despite being the result of two distinct modes of clustered star formation with a factor of ten difference in mean cluster density, this would be inferred from neither the differential nor the cumulative local surface-density distribution in this case.

The reason that the two different modes of clustered star formation are detectable for Plummer-type but not King-type clusters is the different shape of the surface distribution of each individual cluster at the high-density end. For King-type clusters, the high-density tail of the lower-density cluster overlaps with the low-density end of the high-density cluster, creating a peak in the middle between the two mean cluster densities. As there
is no high-density tail in Plummer-type clusters, the steep drop leads to two clearly distinct peaks. Consequently, the threshold for identifying distinct peaks in the superpositions of King-type cluster modes is much higher and requires a ratio of the median densities of $\sim 65$.

The result that the shape of the distribution is relevant for the detectability of different cluster modes does not only hold for the cases of Plummer and King models, but applies to other distributions as well: distinct cluster modes are easily detectable for narrow distributions whereas for concentrated but broader distributions can these modes be obscured. In the following, we continue to speak of King-type clusters, but the reader should keep in mind that this is valid for any type of broader distribution.

For concentrated King-type clusters, the absence of peaks in the surface density distribution therefore does not allow the conclusion that there are no multiple modes of star formation present nor that star formation proceeds continuously over all cluster densities. At the same time, a smooth surface distribution does not allow one to draw the conclusion that no distinct scale for stars clustering within nearby star-forming regions exists as, for example, stated by Bressert et al. (2010).

In view of the above findings, local surface-density distributions contain limited information about existing modes of clustered star formation.

### 4.3. Observational biases

We have demonstrated above that for two King-type clusters differing in density by a factor of ten only a single peak appears in the surface density profile. How far do observational limitations affect the above result? Figure 5 shows the surface density distribution that results from the two observationally limited King-model clusters shown in Fig. 2. It can be seen that observational limitations lead to a non-physical cut-off at the high-density end of the surface density distribution. Although the observational limitations lead to an under-representation of high-density areas, the two underlying cluster modes are still not detected. Different cluster modes are only revealed if the peak densities of the two modes differ by more than a factor of $\sim 65$.

### 4.4. Multiple modes of clustered star formation

If there are more than two modes of clustered star formation, the surface density distribution becomes an increasingly unreliable diagnostic of multiple modes. Figure 6 shows the combination of three King-type cluster models (non-detection limited cluster A, B, and C in Table 1) of different average density but with an equal number of stars in each mode. As in the case of two cluster modes, here again the underlying three cluster modes would not show up as separate peaks but one obtains a more or less Gaussian-shaped smooth distribution with a single (although this time broader) peak. In the cumulative surface plot, this is represented by a smooth but somewhat flatter curve than the ones for the single clusters. This might possibly provide a way of detecting the underlying cluster modes.

We emphasize that we do not advocate that all star formation happens in two, three, or more modes but that surface density distributions are of limited use in inferring underlying modes of clustered star formation. In the solar neighbourhood in particular, there have so far been no indications of different modes of star formation. However, on Galactic scales that might, at least for massive clusters, be different (Hunter 1998; Maíz-Apellániz 2001; Pfalzner 2009).


Fig. 5. Same as Fig. 4c but this time observational limitations are modelled for two clusters with a King profile (see Fig. 2).

## 5. Influence on star and planetary system formation

Observations often apply the technique of surface density plots to identify the degree to which the cluster environment influences planet and star formation. These studies presume a density limit above which they assume that the interactions between the stars become important. Determining the relative proportion of stars that reside in areas with stellar densities above and below that limit, this is then used as an argument for or against the importance of the environment for star and planet formation.

The value of $10^{4}$ stars $\mathrm{pc}^{-3}$ (see Gutermuth et al. 2005) is often quoted as the threshold for determining whether the cluster environment plays a role. Gutermuth et al. (2009) translated this into a local surface density exceeding $200{\text { star } \mathrm{pc}^{-2} \text { (see }}^{\text {s }}$ Bressert et al. 2010). These values are only rough estimates and this value of the local surface-density limit at which environmental effects play a role depends strongly on the actual aspect of star and planet formation that one considers. Stellar mergers, disc destruction, or modifications of the disc structure will correspond to very different local surface-density limits.

For the moment, we take the estimated local surface density threshold - 200 stars $\mathrm{pc}^{2}$ - at face value to investigate how the cluster profile, sample, and incompleteness effects influence the estimate of the relative importance of the cluster environment on star and planet formation. The effects of observational limitations lead to underestimating high local surface densities for the limits applied in Bressert et al. (2010). For the three cluster

Table 1. Properties of the King-type cluster models used in Sect. 4.

| Property | Cluster A | Cluster B | Cluster C |
| :--- | :---: | :---: | :---: |
| Model clusters |  |  |  |
| No. stars | 10000 | 10000 | 10000 |
| Median density | 12.6 | 126 | 1260 |
| Average density | 21.2 | 530 | 11000 |
| Above threshold | 0 | 3300 | 8000 |
| Spitzer resolution sample |  |  |  |
| No. stars | 9923 | 8638 | 6458 |
| Median density | 12.6 | 79.4 | 501 |
| Average density | 21.2 | 151 | 668 |
| Above threshold | 0 | 1813 | 4327 |
| Radial cut-off and Spitzer resolution sample |  |  |  |
| No. stars | 8476 | 7246 | 5990 |
| Median density | 7.9 | 50.1 | 501 |
| Average density | 11.7 | 77.1 | 518 |
| Above threshold | 0 | 290 | 3893 |

Notes. The "above the threshold" lines denote the "detectable" number of stars in an environment where the stellar density exceeds 200 stars $\mathrm{pc}^{-2}$. The densities are median and average surface densities and are given in units of $\mathrm{pc}^{-2}$.
modes of different densities, Table 1 provides - in terms of observational limitations - the number of observed stars, the resulting change in average and median surface density, and the number of stars detectable above the local surface density threshold of $200 \mathrm{stars}_{\mathrm{pc}}{ }^{-2}$.

The values in our model clusters are scaled in such a way that all three clusters contain 10000 stars but their densities differ by a factor of 10 and 100 , respectively. In the least dense cluster, no stars are located in regions above the local surface-density threshold, whereas $80 \%$ of stars in the densest cluster encounter higher local densities and are thus potentially affected by the cluster environment.

For a Spitzer resolution-limited sample, the densest cluster obviously has the largest number of undetected stars in highdensity regions. However, in relative terms it is the same in intermediate- and high-density clusters - in both cases observational limitations result in our missing $\sim 45 \%$ of the stars potentially affected by the environment.

Excluding the central area of the cluster from the study (see Bressert et al. 2010) again lowers the number of detected stars in high-density regions. If such a cut-off is applied to our highdensity and even intermediate-density clusters, the mean and average surface density are underestimated. However, whereas in high-density clusters resolution limitations prevent most of the stars being detected owing to the high stellar density, in intermediate-density clusters stars that would normally be resolved are heavily affected by the exclusion of the central area. Bressert et al. state that excluding the central areas would at most change the number of affected stars by a factor of two. For our model cluster B, the cut-off procedure and the Spitzer limitations would reduce the number of stars above the threshold to less than a tenth of the real value.

## 6. Summary and conclusions

We have investigated the circumstances under which categorical distributions of local surface densities of young stellar objects


Fig. 6. Local surface density distribution of the superposition of three King-type cluster modes with $10^{2}, 10^{3}$, and $10^{4}$ stars, each with the same total stellar population size of $10^{5}$.

- which we refer to as surface density plots - are suitable tools for investigating modes of clustered star formation and the dynamical influence of the star cluster environment on star and planet formation. Using different types of model star clusters, we demonstrate how sensitively the results depend on the actual cluster density profile. While for narrow (for example Plummer-shaped) density distributions discrete cluster modes are easily identified as multiple peaks in the surface density plot, this is often not the case for distributions that span a wider density range - for example, concentrated King-type density distributions. Our findings imply that the surface density plots of star forming regions do not reveal multiple peaks unless the median density of the individual cluster modes differs by more than a factor of $\sim 65$.

The relative population size also plays a role. The detection of discrete modes in the surface density plot is only possible if the population sizes do not differ by more than a factor of five. Even, if one constructs equal-sized samples, difficulties might arise. If one combines different low-mass clusters to form a single sample, it is very difficult to guarantee that they are in the same evolutionary stage. The cluster age is, at least for embedded clusters, not a reliable indicator of their dynamical state. The reason is that if star formation is ongoing and accelerated then averaging will always lead to approximately the same mean cluster age of $\sim 1-2$ Myr. So a cluster just starting to form stars and one that has nearly finished the star formation process will both be attributed the same age. However, during which phase
the cluster size, profile, and surface density evolves considerably (Pfalzner 2009, 2011; Parmentier \& Pfalzner 2012). Including these different clusters in the same sample would lead to erroneous results. This means that the right sample choice is vital to determine whether a dominant mode of clustered star formation exists.

This means that although one can determine from multiple peaks in the surface density plot whether discrete modes of clustered star formation exist, the reverse is not true. We point out that unlike other publications (e.g. Bressert et al. 2010) a smooth surface density plot does not rule out the existence of dominant modes of clustered star formation. We thus caution against the use of surface density plots to determine whether dominant modes of clustered star formation exist.

However, surface density plots are potentially very useful in determining the dynamical influence of the cluster environment on star and planet formation. A robust estimate requires high-resolution observations of rich star clusters to map the entire stellar population. Here we have demonstrated that excluding regions of high local surface-density in rich star clusters (as in Bressert et al. 2010) leads to an underestimation of the average local surface density not as estimated in their study by at most a factor of two, but by up to more than an order of magnitude. Observations with instruments other than Spitzer (such as HST) are important for determining high surface-density regions in such clusters.

Another limitation that biases our understanding of star formation modes arises from restrictions of observational samples to the solar neighbourhood. Although there are good reasons for this approach such as sample completeness, one has to be aware that these results cannot be generalized to the Galaxy, because, for example, starburst clusters with their mostly much higher local surface-densities are excluded.

Similarly, the age of the clusters included in the sample is an important factor. Dynamical interactions and stellar evolution in star clusters induce cluster expansion and hence act to lower their median local surface-density with time. This effect becomes even more pronounced if gas expulsion is taken into account (see e.g. the review of Vesperini 2010). Hence, using a sample with a spread of cluster ages leads to an underestimate of the median local surface density of a given mode. This is of particular relevance to low-mass clusters that are expected to dissolve faster owing to their short relaxation time.

In summary, a consistent analysis of the modes of clustered star formation requires a sample of isochronal clusters that is unlimited in mass and the development of a tool suitable for revealing potential discrete modes.

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