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MODFLOW's River Package: Part 2: Correction, Combining Analytical and Numerical Approaches

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

Most widely used integrated hydrologic models still describe the flow interaction between streams and aquifers using primitive early concepts. In the previous article the shortcomings of the methodology were shown in great details. In this second part means are presented by which improvements can be introduced into the procedures. Accuracy and numerical efficiency will be improved. The article describes in details the proposed alternatives for both the saturated and the unsaturated connections. In the article reference is made specifically to the code MODFLOW. Most of the other integrated hydrologic models used for large-scale regional studies apply essentially the same methodology to estimate seepage.

Keywords: Seepage; saturated/unsaturated connection; combined analytical-numerical techniques; leakance coefficient.

1. INTRODUCTION

Large-scale hydrologic models such as MODFLOW [1] try to be as physically based as possible. Out of necessity these mathematical models must greatly simplify a complex reality and as a result they become highly conceptual. However a proper conceptualization process should be done without violating basic physical processes. Part 1 has shown that, in

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MODFLOW's conceptualization for the estimation of seepage, some well-known physical principles were ignored. In this article, Part 2, a methodology is introduced to improve the estimation of seepage under conditions of saturated or unsaturated hydraulic connection. Naturally still some simplifications were necessary, and unavoidable, but at least in the author's opinion, no basic physical laws were violated.

2. PROPOSED COMBINED ANALYTICAL-NUMERICAL ESTIMATION OF SEEPAGE UNDER A SATURATED CONNECTION

Fig. 1 displays the flow pattern of saturated seepage from a rectangular cross-section of a river toward some distance away in the surrounding aquifer.

It is clear in Fig. 1 that the average head in the aquifer cell is less than the river head, which in this case is 104 m. The boundary condition at both ends of the region was a uniform head of 103 m. As the flow approaches the right and left sides of the system it tends to become horizontal. The question is: how to combine such analytical solution with an overall numerical code such as e.g. MODFLOW? In the large-scale regional studies the water-table aquifer is treated as a single calculation layer, which means that the model is using the Dupuit-Forchheimer assumption that in the aquifer the head distribution in the vertical direction is hydrostatic. In other words the flow in that water-table aquifer is considered horizontal. Yet it is clear from Fig. 1 that the flow pattern in the vicinity of the river is not horizontal.

The proposed solution is to treat the flow for what it is locally that is 2-dimensional in the vertical plane and reattach it at some distance away from the river bank to a 2-dimensional numerical solution in the horizontal plane. To achieve that result one distinguishes the aquifer cell that contains the river, the "river cell", from an adjacent neighboring cell as shown in Fig. 2. (There may or may not exist a clogging layer). The lateral grid size, G, is chosen, at a minimum, such that by the time the seepage flow from the river has reached the center of the right (or left) half of the river cell it has become horizontal. That way the Dupuit-Forchheimer assumption to calculate the flow between the river cell and the adjacent cell is legitimate. The analytical solution for the flow [2,3,4] as shown in Fig. 1, has demonstrated that horizontal flow will hold conservatively, in case of isotropy, at a minimum distance from the bank of the river equal to twice the aquifer thickness, D_{aq}.

(This distance of twice the aquifer thickness is quite excessive as a look at Fig. 1 shows quite clearly. In practice one can use shorter grid sizes than the one conservatively needed to determine the minimum grid size). The seepage discharge from the river on one side, $Q_S^{one-sided}$ (say the left side) is given by the relation:

$$Q_S^{one-sided} = K_H L_R \Gamma_{one-sided}(h_S - h_f) \quad (1)$$

where K_H is the aquifer hydraulic horizontal conductivity, L_R is the river reach length, h_S is the head in the river and h_f is the average head in the aquifer river cell (i.e. the cell that contains the river). $\Gamma_{one-sided}$ is the SAFE (Stream-Aquifer Flow Exchange) dimensionless conductance. That $\Gamma_{one-sided}$ or simply Γ has been estimated exactly analytically. It is a function of the normalized wetted perimeter of the river,

$$W_p^N = \frac{W_p}{D_{aq}} \quad (2)$$

of the degree of penetration, $\frac{H}{D_{aq}}$ (3)

where H is the river stage, of the degree of anisotropy,

$$\rho_{anis} = \frac{K_V}{K_H} \quad (4)$$

of the excess distance from the minimum standard far distance,

$$\Delta = \frac{G}{4} - (2\frac{D_{aq}}{\sqrt{\rho_{anis}}} + B) \quad (5)$$

which means that the minimum grid size must be

$$G_{\min} = 8\frac{D_{aq}}{\sqrt{\rho_{anis}}} + 4B \qquad (6)$$

and of the presence of a real clogging layer defined by its leakance coefficient,

$$\Lambda_{rcl} = \frac{K_{rcl}}{e_{rcl}} \qquad (7)$$

The symbol for Γ when all the effects of anisotropy, excess distance over the minimum standard far distance and presence of a real clogging layer are explicitly accounted is $\Gamma_{anis-\Delta-rcl}$ if necessary, though otherwise for brevity still labeled as Γ. The total seepage discharge is thus:

$$Q_S^{safe} = 2L_R K_H \Gamma(h_S - h_f) \qquad (8)$$

On the other hand the MODFLOW equation is:

$$Q_S^{\mathrm{mod}} = L_R W_p \Lambda_{\mathrm{mod}}(h_S - h_f) \qquad (9)$$

If there is a tight streambed (clogging layer) MODFLOW proposes for the leakance coefficient the expression:

$$\frac{K}{M} = \frac{K_{cl}}{e_{cl}} = \Lambda_{\mathrm{mod}} \qquad (10)$$

However MODFLOW does not provide a procedure to estimate these clogging layer parameters except possibly through calibration.

If there is no tight streambed within some limited conditions MODFLOW proposes:

$$\Lambda_{\mathrm{mod}} = \frac{K_{aq}}{1} = \frac{K_V}{1} \qquad (11)$$

Identification of Eq. (8) and (9) shows that as long as there is saturated connection, whether there is a tight streambed or not, the choice for MOFLOW should be:

$$\Lambda_{\mathrm{mod}} = \Lambda_{safe} = 2K_H \Gamma / W_p = \frac{K_H \Gamma}{B + H} \qquad (12)$$

References [3,4] have provided all the information necessary to calculate Γ in terms of the local conditions and the values of the parameters defining the system. It requires only a few algebraic calculations [5,4].

When using the leakance coefficient of Eq. (10) in the MODFLOW Eq. (9) for seepage discharge the river cell head used is h_{ijk}, that is the finite difference average value of head in the full river cell, which is precisely the average value of head in the half river cell and a very close approximation for the head at the center of the half river cell, which is the head needed for the validity of Eq. (9).

3. PROPOSED COMBINED ANALYTICAL-NUMERICAL ESTIMATION OF SEEPAGE UNDER AN UNSATURATED CONNECTION

This is a more complicated situation. The complete physical system consists of a river, a clogging layer (riverbed), an unsaturated zone below, a capillary fringe, a water table mound, a river cell and an adjacent cell (Figs. 3 and 4).

3.1 The Simplified Description of the Unsaturated Zone

The goal is to describe approximately, simply but with sufficient accuracy, the transient flow exchange between surface water (river, canal or pond) and the underlying aquifer under an unsaturated connection. The riverbed acts as a clogging layer. In the aquifer just below the clogging layer, the flow may be saturated or unsaturated. The word interface refers to the boundary between the bottom of the clogging layer and the top of the underlying aquifer, while we use the term capillary zone for the combination of both the unsaturated zone and the capillary fringe.

The approach is to simplify the analysis of the unsaturated situation by approximating the shape of the water content profile in the unsaturated zone.

The selected profile for the water content is the one that would convey the current seepage steadily and uniformly through the unsaturated zone.

In this document the unsaturated relative conductivity and the capillary pressure functions are characterized by the Brooks-Corey formulation as described in Appendix 1.

For illustration, the parameter M=2.5 (power in the capillary pressure curve expressed as a function of normalized water content) and p=5 (power in the relative permeability curve expressed as a function of normalized water content) are chosen.

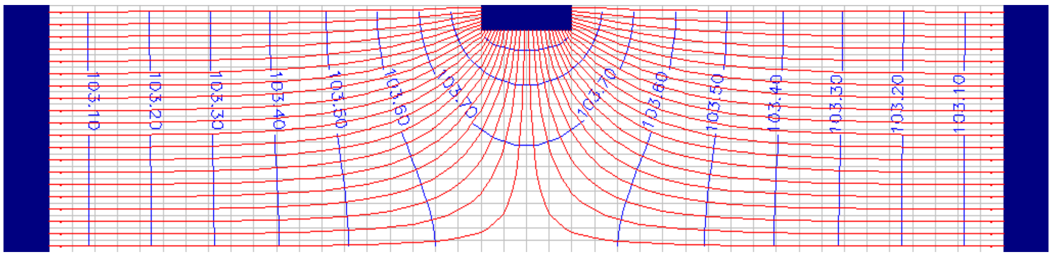

Fig. 1. Exact analytical flow pattern from a rectangular cross-section with a moderate degree of penetration. After Miracapillo and Morel-Seytoux, 2014

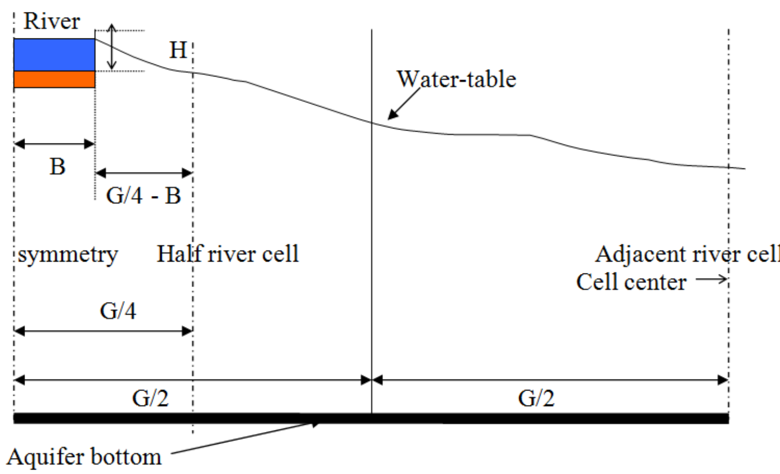

Fig. 2. Cross-section view showing the different components of the stream-aquifer system, applicable in the case of saturated connection

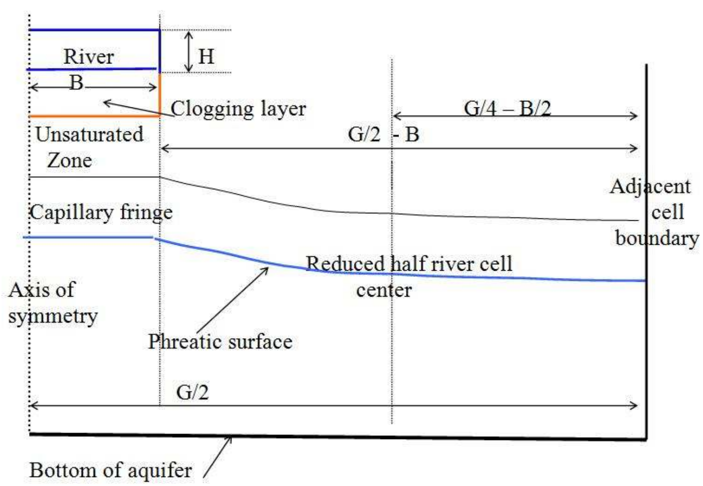

Fig. 3. Cross-section view showing the different components of the stream-aquifer system, applicable in the case of unsaturated connection

In this case the normalized capillary pressure head profile (details in Appendix 2) is:

$$h_c^* = \frac{1 + h_{cI}^* \sqrt{i_S^*} - e^{D_z z^*}(1 - h_{cI}^* \sqrt{i_S^*})}{\sqrt{i_S^*}[1 + h_{cI}^* \sqrt{i_S^*} + e^{D_z z^*}(1 - h_{cI}^* \sqrt{i_S^*})]} \quad (13)$$

where $z^*=z/z_f$ is the normalized coordinate, z is the vertical coordinate with origin at the interface oriented positive downward, and z_f denotes the position (depth) of the bottom of the unsaturated zone from the bottom of the clogging layer.

At the interface between the clogging layer and the aquifer on the aquifer side there is a water content, θ_I, distinct from the average one within the unsaturated zone, θ. Furthermore, θ_s is the saturated water content in the aquifer, h_{cI} is the capillary pressure at the interface, and its normalized value is $h_{cI}^* = h_{cI} / h_{ce}$ where h_{ce} is the drainage entry pressure. The seepage rate at the interface is i_s. Dividing it by the vertical hydraulic conductivity of the aquifer, K_V, its normalized value is $i_s^* = i_s / K_V$. The coefficient D_z is:

$$D_z = \ln\left\{\frac{(1 + h_{cI}^* \sqrt{i_S^*})((1 - \sqrt{i_S^*})}{(1 + \sqrt{i_S^*})(1 - h_{cI}^* \sqrt{i_S^*})}\right\} \quad (14)$$

One can see from Eq. (13) that the capillary pressure takes the proper values at the water table and the interface (details in Appendix 2). The normalized water content is obtained as $\theta^* = (h_c^*)^{-1/M}$, while z_{rf} denotes the position (height) of the current water table (mound) as shown in Fig. 4.

While the choice of the water content profile in the unsaturated zone is approximate, the process maintains mass balance and the essential dynamics of the process. D is the maximum thickness of the water table aquifer *including the clogging layer* below the river, that is:

$$D = e_{rcl} + z_f + h_{ce} + z_{rf} \quad (15)$$

In other words, D is the sum of the streambed thickness, e_{crl}, the unsaturated zone thickness, the capillary fringe thickness, h_{ce}, and the water table height.

Fig. 5 displays the shape of the unsaturated zone water content profile for a given set of parameters.

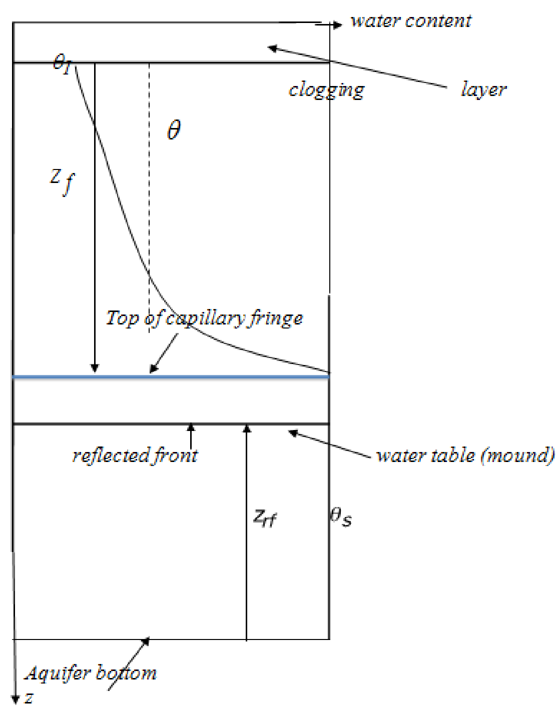

Fig. 4. Water content profile below the riverbed and above the water table mound

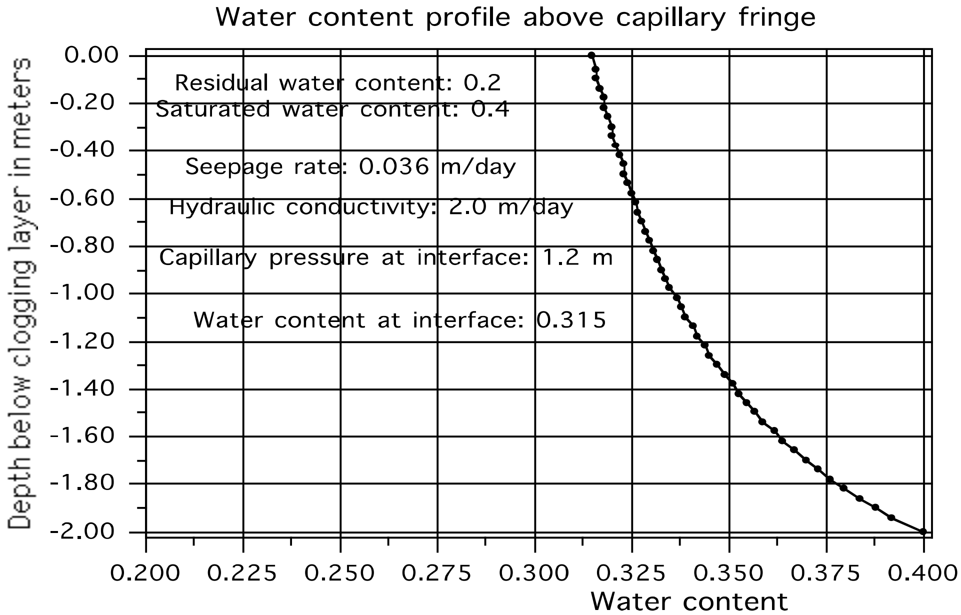

Fig. 5. Water content profile in the unsaturated zone above the capillary fringe

(Had there been no flow the capillary pressure at 2 m above the capillary fringe would have been 2.3 m but it is only 1.2 m because there is downward flow).

Several different initial conditions are defined. (Some are applicable for the case of saturated connection). It could be (1) incipient desaturation or hydrostatic condition or (3) general saturated condition. These conditions are described in Appendix 3.

3.2 Estimation of Recharge Rate to the Water Table under Unsaturated Connection

In that case $h_{cI} \geq h_{ce}$ (16a)

and

$$\theta_I^* \leq \theta^* \leq 1 \quad (16b)$$

Dynamic estimation of the water velocity from the bottom of the streambed to the top of the capillary fringe will provide the average flow rate in the unsaturated zone. The expression for that average (in space) dynamic water velocity is):

$$\frac{v}{K_V} = v^* = \frac{-H_{cS}[1-(\theta_I^*)^{p-M}] + k_{rw}(\theta) z_f}{z_f} \quad (17a)$$

or

$$\frac{v}{K_V} = v^* = \frac{-H_{cS}[1-(h_{cI}^*)^{-\frac{p-M}{M}}] + k_{rw}(\theta) z_f}{z_f} \quad (17b)$$

This is an instantaneous value of a space average over the unsaturated zone. Note that the first term on the right hand side of Eq. (17) expresses the capillary resistance to flow on the part of the water table. That capillary resistance being a potential is known exactly. It only depends on the end boundary conditions and is independent of the actual profile shape. On the other hand the second term that represents the always down force of gravity is approximate because it depends upon the choice of the water content profile.

For simplicity in writing let:

$$-H_{cS}[1-(h_{cI}^*)^{-\frac{p-M}{M}}] = C_{ap} R_{es} \quad (18)$$

where $C_{ap} R_{es}$ is the capillary resistance, a negative value. Then Eq. (24) has a simpler expression:

$$v = K_V [\frac{C_{ap} R_{es}}{z_f} + k_{rw}(\theta)] \quad (19)$$

From a mass balance point of view the recharge rate to the top of the capillary fringe is the sum of the seepage rate through the clogging layer and of the amount of drainage from the unsaturated zone, symbolically:

$$v_{rech}^{mass} = i_S + [\frac{(\theta^o - \theta_r)z_f^o}{\Delta t} + \frac{(\theta_S - \theta_r)(z_f - z_f^o)}{\Delta t}] - \frac{(\theta - \theta_r)z_f}{\Delta t} \tag{20}$$

(Even though the numerical value of *Δt* is 1 (day), as a check on proper dimensionality of the derived expressions it is better to keep it explicitly. The superscript "mass" is not generally shown when mass estimate is meant). The superscript "o" refers to *old* values, at the beginning of a period (time step). The superscript "ν" (or no superscript) refers to *new* values, at the end of the period.

The space average instantaneous water flow rate in the unsaturated zone is:

$$v = K_V[\frac{C_{ap}R_{es}}{z_f} + k_{rw}(\theta)] = (i_S + v_{rech}^{dyn})/2 \tag{21}$$

from which one deduces:

$$v_{rech}^{dyn} = 2K_V[\frac{C_{ap}R_{es}}{z_f} + k_{rw}(\theta)] - i_S \tag{22}$$

The two Eqs. (20) and (22) for the recharge rate must give the same result. By equating the two expressions one obtains an expression for the depth of the unsaturated zone as a function of the capillary pressure at the interface:

$$i_S + [\frac{(\theta^o - \theta_r)z_f^o}{\Delta t} + \frac{(\theta_S - \theta_r)(z_f - z_f^o)}{\Delta t}] - \frac{(\theta - \theta_r)z_f}{\Delta t} = 2K_V[\frac{C_{ap}R_{es}}{z_f} + k_{rw}(\theta)] - i_S \tag{23a}$$

Multiplying by z_f and dividing by $2K_V$ one obtains:

$$\frac{(\theta_S - \theta)(z_f)^2}{2K_V\Delta t} - \{k_{rw} - i_S^* + \frac{(\theta_S - \theta^o)z_f^o}{2K_V\Delta t}\}z_f - C_{ap}R_{es} = 0 \tag{23b}$$

Setting $a = \dfrac{(\theta_S - \theta)}{2K_V\Delta t}$ (24a)

$$b = -\{k_{rw} - i_S^* + \frac{(\theta_S - \theta^o)z_f^o}{2K_V\Delta t}\} \tag{24b}$$

and $c = -C_{ap}R_{es}$ (24c)

the solution is:

$$z_f = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{25}$$

Note that, since this value of z_f is obtained by requiring that the recharge rate v_{rech} be the same whether evaluated by mass balance or dynamically, in the later sections the stipulation that v_{rech} is the mass balance or the dynamic estimate is superfluous since they have the same value.

3.3 Evolution of (Water Table) Mound below the River Bed

Because the driving force behind the transient evolution of the unsaturated seepage is the head in the river cell (the aquifer cell that contains the river reach cross-section), we look at how the aquifer zones react to that head and to the head in the river. Because of the complex interaction between these different zones (river, mound, river cell away from river banks, adjacent cells) to keep derivations (and illustrations) simple we simply look at how the mound reacts to the head in the half aquifer river cell not under the clogging layer (see Fig. 3), *hf*. (This is a reduced half river cell as it excludes the water-table mound below the river bottom). Naturally in practice the river head is affected by the river flow and its interaction with the aquifer below. Similarly the head in the river cell is affected by the heads in adjacent cells, conditioned by what happens in the full river-aquifer system, as a result of pumping, artificial recharge, etc. These heads are not realistic boundary conditions. Here we want to focus on the procedures to estimate seepage and therefore eliminate all complexities resulting from a full system that would obscure the manner in which seepage is estimated.

The water table mound is excited by the recharge rate from the river and the lateral outflow to (or inflow from) the part of the river cell, which is not below the river. Mass balance for the position of the mound is:

$$\phi_{erf}(B+H)\frac{dz_{rf}}{dt} = (B+H)v_{rech} - \Gamma K_H(z_{rf}-h_f)$$
$$= (B+H)\{2K_V[\frac{C_{ap}R_{es}}{z_f}+k_{rw}]-i_S\} - \Gamma K_H(z_{rf}-h_f) \quad (26a)$$

In this expression ϕ_{erf} is the specific yield (effective porosity) in the mound region.

The position of the center of the part of the half river cell on the right (or left) away from the river bank, which is *G*/4 – *B*/2, must exceed the standard far distance [4]. This requirement is necessary to guarantee: (1) the applicability of the SAFE Γ as the proper dimensionless conductance and (2) that the flow between the river cell and the adjacent cell will be horizontal, i.e. meets the Dupuit-Forhheimer criterion. This puts a limit on the minimum lateral size of the river cell. Let Δ be that excess distance. Also the SAFE dimensionless conductance appearing in Eq. (26) must be $\Gamma_{flat-anis-\Delta}$ accounting for the fact that there is no longer river penetration, but the possibility of anisotropy in the aquifer and for an excess distance over the standard far distance. Eq. (26a) slightly rewritten is:

$$\phi_{erf}(B+H)\frac{dz_{rf}}{dt} + \Gamma K_H z_{rf} = \Gamma K_H h_f + (B+H)v_{rech} \quad (26b)$$

Dividing throughout by *ΓKH*, setting $\sigma_{rf} = \frac{B+H}{K_H\Gamma}$ (27a)

$$C_{rf} = \phi_{erf}\sigma_{rf} \quad (27b)$$

one obtains:

$$C_{rf}\frac{dz_{rf}}{dt} + z_{rf} = h_f + \sigma_{rf}v_{rech} \quad (28)$$

or more simply defining the excitation as:

$$E_{rf} = h_f + \sigma_{rf}K_V\{2[k_{rw}(\theta) + \frac{C_{ap}R_{es}}{z_f}] - i_S^*\} \quad (29a)$$

or

$$E_{rf} = h_f + \sigma_{rf}v_{rech} \quad (29b)$$

thus $C_{rf}\frac{dz_{rf}}{dt} + z_{rf} = E_{rf}^o + (E_{rf}^v - E_{rf}^o)t$ (29c) with structure of a Linear Reservoir hydrologic routing model with constant "time constant" with a linear variation of the excitation with time.

The expression (see Appendix 4) applied for $z_{rf}(n)$ (where n is the period (usually day) number for time) is:

$$z_{rf}^{dyn}(n) = \rho_{rf} z_{rf}(n-1) + \alpha_{rf}[h_f(n-1) + \sigma_{rf} v_{rech}(n-1)]$$

$$+\beta_{rf}[h_f(n) + \sigma_{rf} v_{rech}(n)] \quad (30)$$

with

$$\rho_{rf} = e^{-\frac{1}{C_{rf}}} \quad (31a)$$

$$\alpha_{rf} = [C_{rf}(1-\rho_{rf}) - \rho_{rf}] \quad (31b)$$

$$\beta_{rf} = [1 - C_{rf}(1-\rho_{rf})] \quad (31c)$$

3.4 Procedural Steps

The external excitations to the system are the stage (maximum water depth) in the river, *H*, and the head in the part of the half river cell away from the banks, h_f. The first step is to estimate (guess) the value of the interface capillary pressure, h_{cl}, and thus determine θ_l, θ and i_S as well. Then one estimates a value for z_f by requiring that the recharge rates estimated by mass balance and dynamically be the same, using Eq. (25). That defines a value of z_f. Next the value of z_{rf} is obtained by mass balance and dynamically.

One estimates the value of z_{rf} by mass balance:

$$z_{rf}^{mass} = z_{rf} = D - e_{cl} - z_f - h_{ce} \quad (32)$$

and dynamically, z_{rf}^{dyn}, using Eq. (30).

Had one chosen the right value for h_{cl} the two estimated values for z_{rf} would be the same. If they are not the same then iteratively one chooses other values of h_{cl} so that ultimately the two values match within a given tolerance. Once that tolerance is met the right values of h_{cl} and of all the other variables are obtained.

4. NUMERICAL EXAMPLE

The purpose of that example is to show the difference in results using the strict MODFLOW approach and the one proposed in this article. The results provide an idea of how large the differences can be, though of course the magnitude of the differences will depend strongly on the values of the parameters. The example also illustrates the fact that the differences not only depend on the parameters but also exist due to the structural differences in the conceptualization of the processes.

Parameters of the system are provided in Table 1.

The minimum grid size must be $8\bar{D}_{aq} + 4B$. In this case the cell size should equal or exceed 160 + 20 = 180 m. Nevertheless the grid size is chosen conservatively to be 200 m. The excess far distance is $\frac{G}{4} - B - 2\frac{D_{aq}}{1}$ =200/4 – 5 – 2(20) = 5.

Fig. 6 displays the evolution of the head in the river, the mound and the river cell. To facilitate the interpretation of the results the river stage is maintained constant at a value of 0.1 m. Thus affecting the evolution of seepage and recharge is the variation of the head in the river cell. It varies in such a way that at times the hydraulic connection between the river and the aquifer is saturated and at other times it is unsaturated. As long as the connection is saturated the head in the river cell and in the mound below the river bottom are the same.

At first the river is gaining from the aquifer as the head in the aquifer exceeds the river stage. The seepage is algebraically negative in that case as Fig. 7 shows. At time 20 the head which had been declining starts to rise.

Table 1. Parameters of the system

Parameter	Definition	Unit	Value
D	Aquifer thickness below river bottom	m	20
B	Half-width of the river	m	5
G	Lateral grid	m	200
Δ	Excess far distance	m	5.0
K_H	Aquifer hydraulic conductivity (horizontal)	m/day	2.5
K_V	Aquifer hydraulic conductivity (vertical)	m/day	2.5
K_{rcl}	Hydraulic conductivity of clogging layer	m/day	0.01
e_{rcl}	Thicknesss of clogging layer	m	0.4
h_{ce}	BC air entry value, aquifer	m	0.30
	BC air entry value, clogging layer	m	2.00
M	BC exponent, aquifer	-	2.5
	BC exponent, clogging layer	-	2.5
p	BC Exponent conductivity aquifer	-	5
	BC exponent conductivity clogging layer	-	5
H_{ini}	Water level in river	m	0.1
h_f^{ini}	Initial head in the aquifer river cell	m	20.7

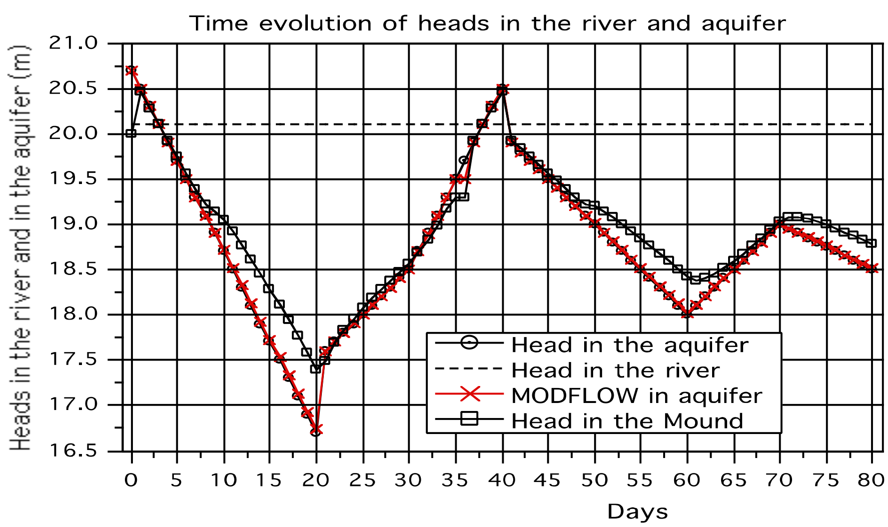

Fig. 6. Heads in the river, the mound and the aquifer cell

It rises so much that by time 35 resaturation is taking place and by time 38 the river is gaining from the aquifer. Then it declines again and by time 49 desaturation occurs and it remains the condition till the end of the simulation.

In the case of MODFLOW there is no distinction between seepage and recharge. It is assumed that the seepage rate instantly recharges the aquifer cell below the river bottom as shown in Fig. 7. Also the Figure shows quite clearly that the differences are not the results of different values of the parameters but due to structural differences in the compared approaches. Under MODFLOW the seepage remains constant for long period of times while in reality it changes very significantly.

Fig. 8 displays the evolution of capillary pressure at the interface. Whenever that value exceeds the entry pressure (0.30 m) seepage is occurring under an unsaturated connection, At time 8 the capillary pressure exceeds the entry pressur (0.3 m), the connection becomes unsaturated and recharge now exceeds the seepage as a result of drainage of moisture below the riverbed.

Fig. 9 shows the water content distribution within the unsaturated zone with time.

(Appendix 5 summarizes the results and provides a glossary of terms).

5. DISCUSSION BASED ON THE RESULTS OF THE NUMERICAL EXAMPLE

For this particular set of parameters, given that a very tight clogging layer exists, under a saturated condition the predictions between MODFLOW and the proposed method are very close. In this case MODFLOW's assumption that all the resistance is taking place vertically through the clogging layer (and none to allow for the flow to turn from a vertical to a horizontal direction) is practically correct. However when the connection becomes unsaturated then the difference is major. The assumption that the head drop driving the flow is the head difference between the river and the elevation of the bottom of the riverbed is not valid. The location of the water-table mound below the riverbed does have an impact on the seepage rate.

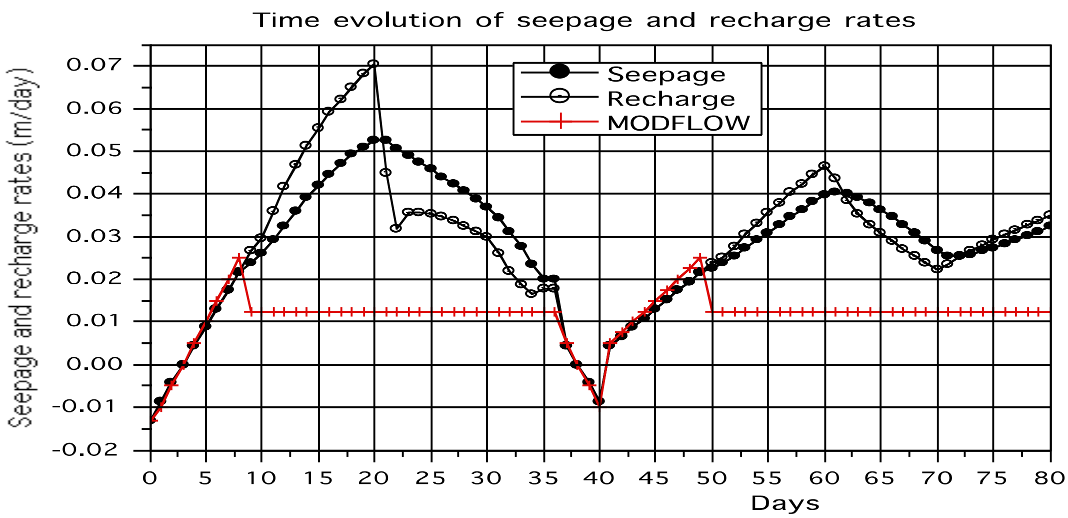

Fig. 7. Seepage from the river and recharge rate to the aquifer

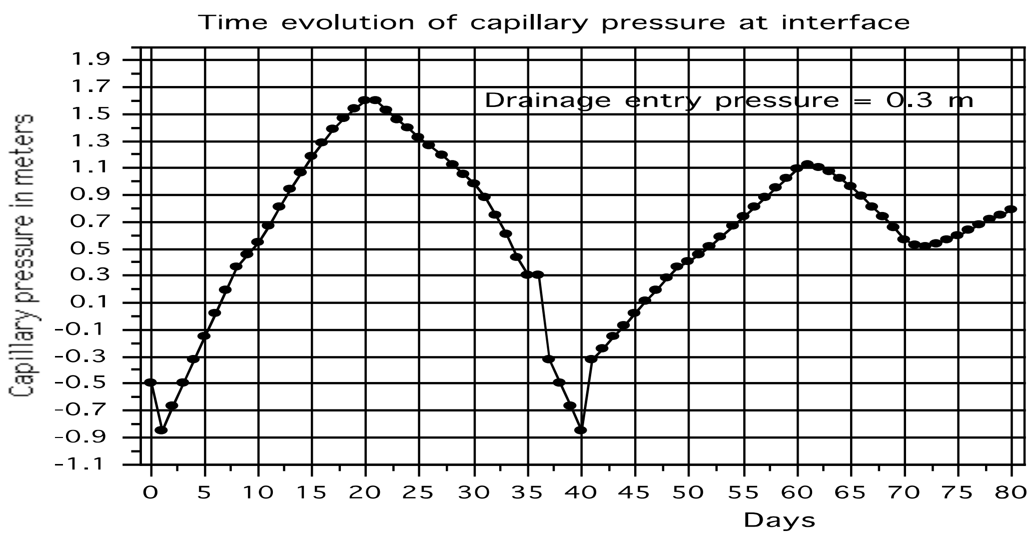

Fig. 8. Capillary pressure at interface

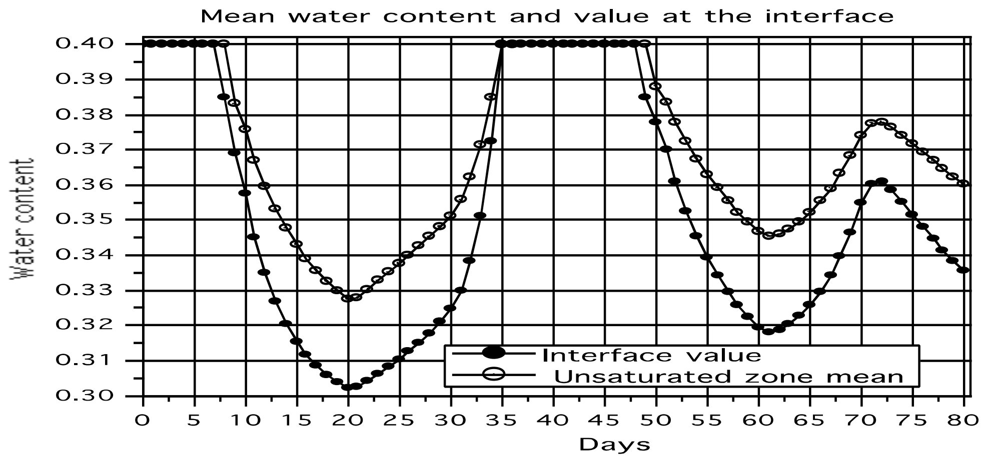

Fig. 9. Average water content in the unsaturated zone and value at interface

A major problem with these old methods is the basic assumption that the flow is driven by a difference of head between the river and an average head in a river cell whose dimensions are large compared to the width of the river. Clearly the head difference should be with a head in the aquifer that is close to the bottom of the river. Under an unsaturated connection it should be quite clear that the relevant head is not the average head in a huge river cell but the head of the water-table mound present below the riverbed.

If a single leakance coefficient is used under saturated or unsaturated connection as is done here for the estimation by MODFLOW's approach then clearly it cannot be accurate under all circumstances. With the new approach presented here that leakance coefficient is constantly changing based on the physical situation and the prevailing circumstances, as demonstrated in the numerical example.

6. POTENTIAL QUESTIONS OF THE PRACTICAL MERIT OF THE PROPOSED APPROACH

Fig. 1 shows an analytical solution for saturated seepage in the case of a homogenous saturated flow condition. Such analytical solution is also available in the case of anisotropy [4]. Is this approach possible in a heterogeneous case? Probably not or with great difficulty and only under very specific mathematical assumptions on the type of heterogeneity. However the purpose of these recent investigations that span a decade, e.g. [2,6] is to improve the accuracy of MODFLOW and similar models for large-scale regional studies where Finite Difference or Element numerical tools are used. In such studies the river cell width is much larger than the width of the river and in that cell the hydraulic conductivity is uniform. In that cell there is perfect homogeneity. For the compatibility and combination of the analytical approach with the numerical techniques some of the approximations of the numerical techniques have be accepted.

Whether in MODFLOW or in this article there is a constant mention of a clogging layer, characterized by its thickness and hydraulic conductivity. They always appear together in the form of a ratio, $\Lambda_{rcl} = \dfrac{K_{rcl}}{e_{rcl}}$, called the leakance coefficient. How could that leakance coefficient be determined in practice? With MODFLOW such leakance coefficient is obtained by calibration on historical records of groundwater levels and streamflows. However what is calibrated is a composite value that includes the resistance of a clogging layer if such really exists and other factors such as resistance due to the curving of the flow lines (see Fig. 1) and the size of the river cell. What the new approach provides is an ability given the calibrated value of the leakance coefficient to extract from it the leakance coefficient that is actually due to a clogging layer if it does exist. That procedure is discussed in a separate forthcoming article.

Accuracy of approximate analytical unsaturated zone solution: One might legitimately question the accuracy of the proposed analytical solution. However a simple look at Fig. 7 shows that the approach would provide a better estimation of seepage with the ability to separate what is actually seepage from what is actually recharge of the aquifer. Also it is clear that MODFOW's approach with a constant value of the recharge rate is not realistic and must be in significant error. Still without questioning the improvement provided by this new approach one may still wonder how accurate the results may be. A comparison with a fine-grid numerical solution based on unsaturated flow equations (such as Richards eq.) would be valuable from a theoretical point of view. Such comparison should be pursued and it would be best if it was pursued by others than the author as it would be unbiased. It might provide ideas on how to improve the approximate analytical solution.

Testing of approach on a real case: As mentioned in the previous section and for the same reason it would be best done by others than the author. The problem with real systems is that the representation of a real system (itself never perfectly known) by MODFLOW is itself a theoretical concept. In addition to the misrepresentation of some of the physical principles in the model the knowledge of the parameters obtained primarily by calibration is always uncertain. Thus results of tests on a real case are always themselves subject to great deal of uncertainty. It would be best to create a theoretical but realistic system for which there is no uncertainty in the parameters and where to represent the unsaturated zone a fine-grid numerical solution based on unsaturated flow equations is used to test any new approach.

7. CONCLUSION

Many previous studies have shown that the early methodology to estimate the flow interaction between a river and a connected aquifer, as described in a number of manuals, was not very physically based. Yet that methodology is still much in used today, particularly in large-scale regional studies. That situation is especially critical when the connection becomes unsaturated and the situation alternates between the two conditions. An alternative approach is presented which has a sound physical basis and allows the situation to alternate between a saturated and unsaturated connection. This is done with recourse to simple analytical procedures and avoids reliance on complex and time-consuming numerical solutions of the two-dimensional unsaturated flow equations.

Because in this article the emphasis is on the estimation of seepage and recharge the river stage and the aquifer river cell are treated as the decision variables. That way comparison with MODFLOW is not obscured by the influence of many other factors. In actual studies they are not decision variables but rather state variables depending on routing of flow in the river and the influence of adjacent cells in a large system. Other articles have already suggested more efficient analytical routing procedures and how to treat the river cell head as a state variable depending on the recharge from the river and the influence of the heads in the adjacent cells and more articles will explore these aspects and publish them in greater details in the future.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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APPENDICES

Appendix 1. Using the Brooks-Corey formulation

Normalized water content is defined as:

$$\theta^* = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{1}$$

Capillary pressure (head), h_c, expressed as an equivalent water height is represented in the unsaturated zone by a power law (Brooks-Corey, BC) as:

$$h_c = h_{ce}(\theta^*)^{-M} \tag{2}$$

where h_{ce} is the entry pressure. In the BC original notations a parameter λ is used and M is simply the inverse of λ) That capillary pressure is positive in the unsaturated zones and in the capillary fringes. h_{cI} denotes the capillary pressure at the interface between the bottom of the clogging layer and the aquifer below. This pressure is continuous across the interface.

Relative permeability, k_{rw}, in the unsaturated zone is defined by a power law as:

$$k_{rw} = (\theta^*)^p \tag{3}$$

In the BC original notations a power $\varepsilon = \frac{\eta}{\lambda}$ is used which is simply p. Note that the power p is always much greater than M. BC suggested a relation between p and M, $p = 3 + 2M$. Actually p can be less than 3 so this is a rough approximation. Then

$$k_{rw} = (\theta^*)^p \tag{4a}$$

for $h_c \geq h_{ce}$

$$k_{rw} = 1 \tag{4b}$$

for $h_c \leq h_{ce}$

$$(\frac{h_c}{h_{ce}}) = (\theta^*)^{-M} \tag{5a}$$

for $h_c \geq h_{ce}$

or vice versa $\theta^* = (\frac{h_c}{h_{ce}})^{-\frac{1}{M}}$ (5b)

Also $k_{rw} = (\frac{h_c}{h_{ce}})^{-\frac{p}{M}}$ (6a)

for $h_c \geq h_{ce}$

and

$$k_{rw} = 1 \tag{6b}$$

otherwise.

Appendix 2. Steady-state unsaturated seepage water content profile

Darcy's equation:

$$v^* = \frac{v}{K_V} = k_{rw}[\frac{h_{ce}dh_c^*}{dz} + 1] \tag{1}$$

Expressing k_{rw} as a function of h_c:

$$k_{rw} = (h_c^*)^{-\frac{p}{M}} = (h_c^*)^{-\alpha} \tag{2}$$

Substitution in Eq. (1) yields:

$$v^* = (h_c^*)^{-\alpha}[\frac{h_{ce}dh_c^*}{dz} + 1] \quad \text{or} \quad \frac{v^* - (h_c^*)^{-\alpha}}{(h_c^*)^{-\alpha}} = \frac{h_{ce}dh_c^*}{dz} \tag{3}$$

Separation of variables yields:

$$\frac{(h_c^*)^{-\alpha} dh_c^*}{v^* - (h_c^*)^{-\alpha}} = \frac{dz}{h_{ce}} \tag{4}$$

Let $x = h_c^* \sqrt{v^*}$ then $h_c^* = x / \sqrt{v^*}$ and $dh_c^* = \frac{dx}{\sqrt{v^*}}$

Substitution in Eq. (4) yields:

$$\frac{(\frac{x}{\sqrt{v^*}})^{-\alpha}\frac{dx}{\sqrt{v^*}}}{v^*-(\frac{x}{\sqrt{v^*}})^{-\alpha}}=\frac{dz}{h_{ce}} \tag{5}$$

In case $\alpha = 2$

$$\frac{(\frac{v^*}{x^2})\frac{dx}{\sqrt{v^*}}}{v^*-\frac{v^*}{x^2}}=\frac{dz}{h_{ce}} \quad \text{or} \quad \frac{\frac{dx}{x^2\sqrt{v^*}}}{1-\frac{1}{x^2}}=\frac{dz}{h_{ce}} \quad \text{or} \quad \frac{dx}{1-x^2}=-\sqrt{v^*}\frac{dz}{h_{ce}} \tag{6}$$

Note that Eq. (4) is also integrable exactly for values of α equal to 3 and 4. Integration of Eq. (5b) between the limits $h_c^*\sqrt{v^*}$ and $h_{cI}^*\sqrt{v^*}$ yields:

$$\frac{1}{2}\ln(\frac{1+x}{1-x})\Big|_{h_c^*\sqrt{v^*}}^{h_{cI}^*\sqrt{v^*}}=\frac{\sqrt{v^*}z}{h_{ce}} \tag{7}$$

or ultimately:

$$\frac{1}{2}\ln\{(\frac{1+h_{cI}^*\sqrt{v^*}}{1+h_c^*\sqrt{v^*}})(\frac{1-h_c^*\sqrt{v^*}}{1-h_{cI}^*\sqrt{v^*}})\}=\frac{\sqrt{v^*}}{h_{ce}}z \tag{8}$$

When $h_c^* = 1$, one is at the top of the capillary fringe and then:

$$\frac{1}{2}\ln\{(\frac{1+h_{cI}^*\sqrt{v^*}}{1+\sqrt{v^*}})(\frac{1-\sqrt{v^*}}{1-h_{cI}^*\sqrt{v^*}})\}=\frac{\sqrt{v^*}}{h_{ce}}z_f \tag{9}$$

Defining: $$D_z=\ln\{(\frac{1+h_{cI}^*\sqrt{v^*}}{1+\sqrt{v^*}})(\frac{1-\sqrt{v^*}}{1-h_{cI}^*\sqrt{v^*}})\} \tag{10}$$

and dividing Eq. (8) by Eq. (9) one obtains:

$$z^*=\frac{z}{z_f}=\ln\{(\frac{1+h_{cI}^*\sqrt{v^*}}{1+h_c^*\sqrt{v^*}})(\frac{1-h_c^*\sqrt{v^*}}{1-h_{cI}^*\sqrt{v^*}})\}/D_z \tag{11}$$

which solved for the normalized capillary pressure yields:

$$h_c^* = \frac{1 + h_{cI}^* \sqrt{v^*} - e^{D_z z^*}(1 - h_{cI}^* \sqrt{v^*})}{\sqrt{v^*}[1 + h_{cI}^* \sqrt{v^*} + e^{D_z z^*}(1 - h_{cI}^* \sqrt{v^*})]} \tag{12}$$

One can verify that for $z^* = 0$ one obtains correctly $h_c^* = h_{cI}^*$. For $z^* = 1$ one obtains also correctly $h_c^* = 1$. That follows from the very definition of the parameter D_z.

If $v^* < 0$ let $\tan(h_{cI}^* \sqrt{-v^*}) = A$, $\tan(\sqrt{-v^*}) = P$ and $D_z = A - P$

The relation between normalized capillary pressure and normalized unsaturated zone coordinate $z^* = \dfrac{z}{z_f}$ is:

$$z^* = \frac{A - \tan(h_c^* \sqrt{-v^*})}{D_z} \quad \text{or} \quad h_c^* = \frac{\tan^{-1}[A - D_z z^*]}{\sqrt{-v^*}}$$

while the full thickness of the unsaturated zone is: $z_f = \dfrac{h_{ce} D_z}{\sqrt{-v^*}}$

Appendix 3. Initial Conditions

Hydrostatic condition:

At initial incipient desaturation the seepage (infiltration) flux (area per time) through the interface on one side is:

$$(B + H) i_S^{ini} = (B + H) K_{rcl} [\frac{h_{ce} + H + e_{rcl}}{e_{rcl}}] \tag{1}$$

where *B* is half the width of the river bottom, K_{rcl} is the conductivity of a (real) clogging layer, e_{rcl} its thickness, and *H* the water depth in the river. The flux transmitted out of the mound into the river cell (e.g. Morel-Seytoux et al., 2014; Morel-Seytoux, 2009) is:

$$K_H \Gamma (z_{rf} - h_f) = (B + H) i_S^{ini} \tag{2}$$

where Γ is the SAFE dimensionless conductance (Morel-Seytoux et al., 2016), K_H is the aquifer horizontal conductivity and h_f is the head in the part of the half river cell away from the river bank.

At incipient desaturation, since z_f=0,

$$z_{rf}^{ini} = D - e_{rcl} - h_{ce} \quad (3)$$

and it follows from Eq. (2) that

$$h_f^{ini} = z_{rf}^{ini} - \frac{(B+H)i_S^{ini}}{K_H \Gamma} \quad (4)$$

These are possible chosen initial conditions in the river and the aquifer so that simulation starts at incipient desaturation time and continues unsaturated.

Hydrostatic condition:

$$i_S^{ini} = \frac{K_H \Gamma (h_S^{ini} - h_f^{ini})}{B + H^{ini}} z_f^{ini} = 0 z_{rf}^{ini} = h_f^{ini} - e_{rcl} \quad (1)$$

$$z_f^{ini} = 0 \quad (2)$$

$$z_{rf}^{ini} = D - e_{rcl} - h_{ce} \quad (3)$$

$$i_S^{ini} = 0 = v_{rech}^{ini} \quad (4)$$

$$h_{cI}^{ini} = -(H_{ini} + e_{rcl}) \quad (5)$$

General saturated condition:

In this case $h_S^{ini} = D + H_{ini}$ (1) $h_f^{ini} = D + l$ (2) where l is an arbitrary number but greater than the negative of the entry pressure $-h_{ce} \le l$ so that no unsaturated zone exists below the riverbed at initial time.

$$i_S^{ini} = \frac{K_H \Gamma (h_S^{ini} - h_f^{ini})}{B + H^{ini}} \quad (3) \quad z_f^{ini} = 0 \quad (4) \quad z_{rf}^{ini} = h_f^{ini} - e_{rcl} - h_{ce} \quad (5)$$

Appendix 4. Constant C Linear Reservoir type equation with a right hand-side excitation varying linearly in time

The excitation varies linearly in time and thus the basic governing equation is:

$$C\frac{dU}{dt} + U = E_o + (E_v - E_o)t \quad (1)$$

We look for a solution of the form:

$$U(t) = A + Mt + De^{-\frac{t}{C}} \tag{2}$$

$$\frac{dU(t)}{dt} = M - \frac{D}{C} e^{-\frac{t}{C}} \tag{3}$$

Substitution in Eq. (1) yields:

$$C(M - \frac{D}{C} e^{-\frac{t}{C}}) + [A + Mt + De^{-\frac{t}{C}}] = E_o + (E_v - E_o)t \tag{4}$$

Satisfaction of the equation requires that:

$$M = (E_v - E_o) \tag{5}$$

and

$$A = E_o - C(E_v - E_o) \tag{6}$$

Substitution in Eq. (2) yields:

$$U(t) = E_O - C(E_V - E_O) + (E_V - E_O)t + De^{-\frac{t}{C}} \tag{7}$$

At time zero then: $U(0) = E_o - C(E_v - E_o) + D$ (8)

which yields D.

Substitution in Eq. (7) yields:

$$U(t) = U(0)e^{-\frac{t}{C}} + [E_o - C(E_v - E_o)](1 - e^{-\frac{t}{C}}] + (E_v - E_o)t \tag{9}$$

Application for end of period n making $t = 1$ and setting $\rho_U = e^{-\frac{1}{C}}$ (10)

yields:

$$U(n) = \rho_U U(n-1) + (1 - \rho_U)\{E(n-1) - C[E(n) - E(n-1)]\} + [E(n) - E(n-1)] \tag{11}$$

Grouping terms:

$$U(n) = \rho_U U(n-1) + \{(1 - \rho_U)(1 + C) - 1\} E(n-1) + \{1 - C(1 - \rho_U)\} E(n) \tag{12}$$

or

$$U(n) = \rho_U U(n-1) + [C(1-\rho_U) - \rho_U]E(n-1) + [1 - C(1-\rho_U)]E(n) \quad (13)$$

with $\alpha_U = [C(1-\rho_U) - \rho_U]$ (14a)

$$\beta_U = [1 - C_U(1-\rho_U)] \quad (14b)$$

then Eq. (13) becomes:

$$U(n) = \rho_U U(n-1) + \alpha_U E(n-1) + \beta_U E(n) \quad (15)$$

Appendix 5. Tabulated results for the numerical example

```
Name of the file unsatseep#22_results_2018.mpw.  It was run on December
13, 2018

JUNSAT is index to indicate if condition is currently unsaturated or
saturated seepage (abbreviated in Table as simply JU)
JUNSAT = 1 means unsaturated case
JUNSAT = - 1 means saturated case

All lengths are in units of meters.
Distance from bottom of river (top of clogging layer) to aquifer
impervious bottom D = 20.0 m
Grid size is GRID = 200.0 m
Initial condition for this run is one of general saturated condition.
Initial head in
the aquifer is 20.7 m. Initially the river is gaining from the aquifer.
So one can see that at time (day) = 8  the condition starts again as
desaturated case
ZF is the thickness of the unsaturated zone (that does not include the
capillary fringe)
HSTAGE is the head in the river.  The datum for all heads are the
bottom of the aquifer.
River stage in the river is maintained constant at a value of 0.1 m.
Thus head in the river
is maintained constant at a value of 20.1 m
ZRF is the elevation of the top of the water-table mound
HF is the head in the part of the river cell that excludes the river
AIS is the seepage rate (velocity) at the bottom of the river bed
AISRP is meant to be the seepage rate using MODFLOW's approach
(simplest River Package).
AISRP is the MODFLOW seepage rate (velocity) at the bottom of the river
bed.  It is also the recharge rate.
VRECH is the recharge rate into the mound.

Parameters for the run are KH (horizontal conductivity) = 2.5 m/day,
AKRCL (real clogging layer conductivity = 0.01 m/day
ERCL (real clogging layer thickness) = 0.4 m

 Drainage entry pressure HCE = 0.30 m
 Capillary pressure power exponent M = 2.5
 Relative conductivity power exponent p = 5.0
 Saturated water content WCSAT = 0.4; Residual water content WCRES = 0.2
 Effective porosity of aquifer PHIEFFEC = 0.2
```

unsatseep#22_results_2018.mp

DAY	JU	HCI	ZF	HSTAGE	ZRF	HF	HFRP	AISRP	AIS	VRECH	WCI	WC
0	-1	-0.5000	0.0000	20.1000	20.0000	20.7000	20.7000	-.0130	-.0130	-.0130	0.4000	0.4000
1	-1	-0.8477	0.0000	20.1000	20.4584	20.5000	20.5000	-.0100	-.0087	-.0087	0.4000	0.4000
2	-1	-0.6737	0.0000	20.1000	20.2790	20.3000	20.3000	-.0050	-.0043	-.0043	0.4000	0.4000
3	-1	-0.5000	0.0000	20.1000	20.1000	20.1000	20.1000	0.0000	0.0000	0.0000	0.4000	0.4000
4	-1	-0.3265	0.0000	20.1000	19.9214	19.9000	19.9000	0.0050	0.0043	0.0043	0.4000	0.4000
5	-1	-0.1533	0.0000	20.1000	19.7431	19.7000	19.7000	0.0100	0.0087	0.0087	0.400	0.4000
6	-1	0.0197	0.0000	20.1000	19.5653	19.5000	19.5000	0.0150	0.0130	0.0130	0.400	0.4000
7	-1	0.1923	0.0000	20.1000	19.3878	19.3000	19.3000	0.0200	0.0173	0.0173	0.400	0.4000
8	1	0.3647	0.0892	20.1000	19.2108	19.1000	19.1000	0.0250	0.0216	0.0216	0.385	0.4000
9	1	0.4559	0.1585	20.1000	19.1415	18.9000	18.9123	0.0125	0.0239	0.0266	0.369	0.3831
10	1	0.5444	0.2501	20.1000	19.0499	18.7000	18.7178	0.0125	0.0261	0.0295	0.357	0.3758
11	1	0.6705	0.3843	20.1000	18.9157	18.5000	18.5212	0.0125	0.0293	0.0358	0.345	0.3672
12	1	0.8047	0.5333	20.1000	18.7667	18.3000	18.3238	0.0125	0.0326	0.0416	0.334	0.3596
13	1	0.9379	0.6906	20.1000	18.6094	18.1000	18.1260	0.0125	0.0359	0.0468	0.326	0.3531
14	1	1.0649	0.8531	20.1000	18.4469	17.9000	17.9279	0.0125	0.0391	0.0514	0.320	0.3477
15	1	1.1825	1.0191	20.1000	18.2809	17.7000	17.7296	0.0125	0.0421	0.0555	0.315	0.3430
16	1	1.2888	1.1879	20.1000	18.1121	17.5000	17.5312	0.0125	0.0447	0.0591	0.311	0.3390
17	1	1.3834	1.3592	20.1000	17.9408	17.3000	17.3327	0.0125	0.0471	0.0622	0.308	0.3356
18	1	1.4667	1.5343	20.1000	17.7657	17.1000	17.1339	0.0125	0.0492	0.0651	0.306	0.3326
19	1	1.5404	1.7172	20.1000	17.5828	16.9000	16.9348	0.0125	0.0510	0.0680	0.303	0.3299
20	1	1.6046	1.9072	20.1000	17.3928	16.7000	16.7353	0.0125	0.0526	0.0705	0.302	0.3275
21	1	1.5971	1.8126	20.1000	17.4874	17.6000	17.5943	0.0125	0.0524	0.0450	0.302	0.3278
22	1	1.5293	1.6074	20.1000	17.6926	17.7000	17.6996	0.0125	0.0507	0.0318	0.304	0.3303
23	1	1.4609	1.4664	20.1000	17.8336	17.8000	17.8017	0.0125	0.0490	0.0355	0.306	0.3328
24	1	1.3941	1.3379	20.1000	17.9621	17.9000	17.9032	0.0125	0.0474	0.0355	0.308	0.3352
25	1	1.3280	1.2229	20.1000	18.0771	18.0000	18.0039	0.0125	0.0457	0.0353	0.310	0.3376
26	1	1.2616	1.1164	20.1000	18.1836	18.1000	18.1043	0.0125	0.0440	0.0347	0.312	0.3400
27	1	1.1940	1.0156	20.1000	18.2844	18.2000	18.2043	0.0125	0.0423	0.0337	0.315	0.3426
28	1	1.1244	0.9185	20.1000	18.3815	18.3000	18.3042	0.0125	0.0406	0.0326	0.317	0.3453
29	1	1.0525	0.8239	20.1000	18.4761	18.4000	18.4039	0.0125	0.0388	0.0312	0.321	0.3482
30	1	0.9782	0.7310	20.1000	18.5690	18.5000	18.5035	0.0125	0.0370	0.0298	0.324	0.3513
31	1	0.8812	0.6151	20.1000	18.6849	18.7000	18.6992	0.0125	0.0345	0.0262	0.330	0.3558
32	1	0.7530	0.4706	20.1000	18.8294	18.9000	18.8964	0.0125	0.0313	0.0218	0.338	0.3624
33	1	0.6028	0.3098	20.1000	18.9902	19.1000	19.0944	0.0125	0.0276	0.0187	0.351	0.3716

34	1	0.4353	0.1370	20.1000	19.1630	19.3000	19.2930	0.0125	0.0234	0.0166	0.372	0.3849
35	1	0.3002	0.0013	20.1000	19.2987	19.5000	19.4897	0.0125	0.0200	0.0179	0.399	0.4000
36	-1	0.3002	0.0013	20.1000	19.2987	19.7000	19.4897	0.0125	0.0200	0.0179	0.399	0.4000
37	-1	-0.3268	0.0000	20.1000	19.9218	19.9000	19.9000	0.0050	0.0043	0.0043	0.400	0.4000
38	-1	-0.5000	0.0000	20.1000	20.1000	20.1000	20.1000	0.0000	0.0000	0.0000	0.400	0.4000
39	-1	-0.6735	0.0000	20.1000	20.2786	20.3000	20.3000	-.0050	-.0043	-.0043	0.400	0.4000
40	-1	-0.8472	0.0000	20.1000	20.4576	20.5000	20.5000	-.0100	-.0087	-.0087	0.400	0.4000
41	-1	-0.3263	0.0000	20.1000	19.9210	19.9000	19.9000	0.0050	0.0043	0.0043	0.400	0.4000
42	-1	-0.2400	0.0000	20.1000	19.8323	19.8000	19.8000	0.0075	0.0065	0.0065	0.400	0.4000
43	-1	-0.1534	0.0000	20.1000	19.7433	19.7000	19.7000	0.0100	0.0087	0.0087	0.400	0.4000
44	-1	-0.0669	0.0000	20.1000	19.6544	19.6000	19.6000	0.0125	0.0108	0.0108	0.400	0.4000
45	-1	0.0195	0.0000	20.1000	19.5656	19.5000	19.5000	0.0150	0.0130	0.0130	0.400	0.4000
46	-1	0.1058	0.0000	20.1000	19.4769	19.4000	19.4000	0.0175	0.0151	0.0151	0.400	0.4000
47	-1	0.1921	0.0000	20.1000	19.3882	19.3000	19.3000	0.0200	0.0173	0.0173	0.400	0.4000
48	-1	0.2783	0.0000	20.1000	19.2997	19.2000	19.2000	0.0225	0.0195	0.0195	0.400	0.4000
49	1	0.3644	0.0887	20.1000	19.2113	19.1000	19.1000	0.0250	0.0216	0.0216	0.385	0.4000
50	1	0.4028	0.1041	20.1000	19.1959	19.0000	19.0100	0.0125	0.0226	0.0238	0.377	0.3881
51	1	0.4507	0.1531	20.1000	19.1469	18.9000	18.9126	0.0125	0.0238	0.0250	0.370	0.3835
52	1	0.5175	0.2221	20.1000	19.0779	18.8000	18.8142	0.0125	0.0254	0.0278	0.360	0.3779
53	1	0.5901	0.2980	20.1000	19.0020	18.7000	18.7154	0.0125	0.0273	0.0305	0.352	0.3725
54	1	0.6646	0.3775	20.1000	18.9225	18.6000	18.6164	0.0125	0.0291	0.0332	0.345	0.3675
55	1	0.7393	0.4589	20.1000	18.8411	18.5000	18.5174	0.0125	0.0310	0.0357	0.339	0.3631
56	1	0.8130	0.5416	20.1000	18.7584	18.4000	18.4183	0.0125	0.0328	0.0380	0.334	0.3592
57	1	0.8852	0.6253	20.1000	18.6747	18.3000	18.3191	0.0125	0.0346	0.0403	0.329	0.3556
58	1	0.9555	0.7098	20.1000	18.5902	18.2000	18.2199	0.0125	0.0364	0.0424	0.325	0.3523
59	1	1.0234	0.7951	20.1000	18.5049	18.1000	18.1207	0.0125	0.0381	0.0445	0.322	0.3494
60	1	1.0887	0.8812	20.1000	18.4188	18.0000	18.0214	0.0125	0.0397	0.0465	0.319	0.3467
61	1	1.1191	0.9199	20.1000	18.3801	18.1000	18.1143	0.0125	0.0405	0.0437	0.318	0.3455
62	1	1.1050	0.8970	20.1000	18.4030	18.2000	18.2104	0.0125	0.0401	0.0384	0.318	0.3460
63	1	1.0689	0.8475	20.1000	18.4525	18.3000	18.3078	0.0125	0.0392	0.0353	0.320	0.3475
64	1	1.0179	0.7813	20.1000	18.5187	18.4000	18.4061	0.0125	0.0379	0.0328	0.3227	0.3496
65	1	0.9567	0.7055	20.1000	18.5945	18.5000	18.5048	0.0125	0.0364	0.0307	0.325	0.3523
66	1	0.8881	0.6240	20.1000	18.6760	18.6000	18.6039	0.0125	0.0347	0.0288	0.329	0.3554
67	1	0.8138	0.5391	20.1000	18.7609	18.7000	18.7031	0.0125	0.0328	0.0271	0.334	0.3591
68	1	0.7353	0.4521	20.1000	18.8479	18.8000	18.8024	0.0125	0.0309	0.0254	0.339	0.3634
69	1	0.6533	0.3638	20.1000	18.9362	18.9000	18.9018	0.0125	0.0288	0.0238	0.346	0.3683

70	1	0.5686	0.2745	20.1000	19.0255	19.0000	19.0013	0.0125	0.0267	0.0223	0.354	0.3740
71	1	0.5211	0.2254	20.1000	19.0746	18.9500	18.9564	0.0125	0.0255	0.0234	0.360	0.3776
72	1	0.5174	0.2218	20.1000	19.0782	18.9000	18.9091	0.0125	0.0254	0.0253	0.360	0.3779
73	1	0.5357	0.2408	20.1000	19.0592	18.8500	18.8607	0.0125	0.0259	0.0267	0.358	0.3764
74	1	0.5652	0.2717	20.1000	19.0283	18.8000	18.8116	0.0125	0.0266	0.0280	0.355	0.3742
75	1	0.6000	0.3083	20.1000	18.9917	18.7500	18.7623	0.0125	0.0275	0.0292	0.351	0.3718
76	1	0.6372	0.3477	20.1000	18.9523	18.7000	18.7129	0.0125	0.0284	0.0304	0.348	0.3693
77	1	0.6753	0.3887	20.1000	18.9113	18.6500	18.6633	0.0125	0.0294	0.0316	0.344	0.3669
78	1	0.7138	0.4304	20.1000	18.8696	18.6000	18.6137	0.0125	0.0303	0.0327	0.341	0.3646
79	1	0.7522	0.4726	20.1000	18.8274	18.5500	18.5641	0.0125	0.0313	0.0338	0.338	0.3624
80	1	0.7903	0.5152	20.1000	18.7848	18.5000	18.5145	0.0125	0.0323	0.0349	0.335	0.3603

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