Modified Compact Genetic Algorithm for Thinned Array Synthesis

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***Abstract*—In this letter, a new optimization algorithm, the Modified compact Genetic Algorithm (M-cGA) is introduced and applied to the synthesis of thinned arrays. The M-cGA has been derived from the compact Genetic Algorithm (cGA), properly modified and improved by implementing more than one probability vector (PV) and adding suitable learning scheme between these PVs. The so-obtained algorithm has been applied to the optimized synthesis of different-size linear and planar thinned arrays: In all the considered cases, it outperforms not only the cGA, but also the other optimization schemes previously applied to this kind of problem, both in terms of goodness of the solution (minimization of the peak sidelobe level) and of computational cost.**

***Index Terms*—Antenna, compact genetic algorithm, optimization algorithm, thinned array.**

I. Introduction

In RECENT years, thinned arrays have attracted significant attention from researchers because of their advantages such as the reduction of the array weight and of the complexity of the feeding network, thus resulting in an overall cost reduction. However, array thinning has also some disadvantages, the main of which is the decreasing of the maximum gain value, which corresponds to an increase of the sidelobe level (SLL) with respect to a fully populated array with the same equivalent size [1].

To circumvent this drawback, several techniques have been proposed, aimed to find the best location of the active elements inside the array grid [2]–[12]. Deterministic approaches have been first adopted, but they did not show significant improvements with respect to the random element placement [2], [3]. Recently, dynamic program [4] and stochastic optimization techniques, including Genetic Algorithm (GA) [5], simulated annealing (SA) [6], [7], and Ant Colony Optimization (ACO) [8], [9], have been applied to the optimization of thinned array. The obtained results are remarkable, even if they could be further improved.

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The combination of deterministic approaches and stochastic optimization have been proposed exploiting the available knowledge of Different Sets (DS) or Almost Different Sets using GA [10]–[12]. These combinations proved that this procedure is very effective most of the times. However, the operations performed by the optimizer still presents the inherent disadvantage of stochastic based optimization, i.e., the process convergence can be really slow, with a resulting increase of its computational cost.

In [13], the previous hybrid approach has been extended to planar thinned arrays, while in [14] the synthesis of these last have been carried out by the combination of others optimization algorithm (PSO) and combinatorial method. Also, [15] deals with the synthesis of planar thinned arrays, proposing two techniques that are the hybridization of a deterministic approach (the density tapering), in one case with the random location of the elements, in the other with the iterative Fourier Transform. This last is instead used alone for the design of large planar arrays in [16].

In this framework, the compact Genetic Algorithm (cGA) [17] seemed to be a good candidate for the optimized synthesis of thinned arrays. The authors have recently introduced an improved version, named Modified cGA (M-cGA), with the aim of overcoming the limitation of the former one [18]. Some preliminary results have been presented in [19] and [20], showing that M-cGA provided good solutions with a reduced computational cost, i.e., it converged faster. In view of these encouraging results on its application to different test functions and simple electromagnetic problems, the use of M-cGA was further investigated: In this letter, the results of its application to the optimization of several, different-size linear and planar thinned arrays are reported and compared to the results obtained with other approaches. The letter is structured as follows. In Section II, the compact genetic algorithm (cGA) is briefly introduced, followed by the description of the M-cGA; in Section III, the results of the optimization of planar and linear thinned arrays are shown, while in Section IV, some conclusions are drawn.

II. Modified Compact Genetic Algorithm

Despite its name, the compact Genetic Algorithm, first presented in [17], belongs to the Estimation Distribution Algorithms since, in order to get the distribution of good solutions, it uses a probability vector (PV) to represent a possible solution; this PV is managed in place of the population of entities typical of Evolutionary Algorithms. The length of the PV corresponds to the number N of variables of the problem, and the value of

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Step 1: PV initialization:
                PV(i) = 0.5      ∀ i = 1...N
Step 2: Generation of two individuals: a, b
Step 3: Deterministic competition
                winner, loser:= compete(a,b)
Step 4: PV update (∀ i = 1...N):
                IF {winner(i) ≠ loser(i)}
                    IF {winner(i) = 1}
                        PV(i) = PV(i) + 1/n;
                    ELSE
                        PV(i) = PV(i) - 1/n;
                    ENDIF
                ENDIF
Step 5: Convergence check
Parameters:
n: population size,          N: chromosome length.
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Fig. 1. Pseudocode of the cGA.

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Step 4: PV update:
        Local update (∀ i = 1...P, ∀ j = 1...N)
            IF {winner(i,j) ≠ loser(i,j)}
                IF {winner(i,j) = 1}
                    PV(i,j) = PV(i,j) + 1/n;
                ELSE
                    PV(i,j) = PV(i,j) - 1/n;
                ENDIF
            ENDIF
        Global update (∀ i = 1...P)
            PVbest = best(PV)
            PV(i)  = PV(i) + c (PVbest - PV(i))
Parameters:
P: number of Pvs,          c: learning factor,
N: length of PV,           n: population size.
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Fig. 2. New updating rules for the M-cGA.

the PV elements represents the probability of a variable to get a particular value. A full treatment of the method can be found in [17] and [21], but for the sake of clarity and uniformity of notation, it is briefly summarized in Section II-A.

A. Compact Genetic Algorithm

The pseudocode of the cGA is shown in Fig. 1. Initially, each element of the PV is set equal to 0.5, assuming a uniform distribution for each one. At the following step, two individuals are generated from each element of the current PV. They compete with each other, and the winner is responsible for updating the corresponding PV's element: Its value is increased or decreased by a factor $1/n$ (where n is the population size) according to the value of the winner. In the standard implementation of cGA, this competition is of deterministic type, i.e., the winner is the individual with the lower cost, when facing a minimization problem, and vice versa. The cGA will stop when all the PV's elements are equal to 0 or 1, i.e., the optimal solution is found.

A first variation of the standard cGA has already been introduced in [17] by increasing the number of generated offspring and applying a different kind of competition, as the tournament one, i.e., simulating higher selection pressure. This modification, however, has a high computational cost since it needs to store and evaluate a considerable number of individuals.

In [21], Ahn proposed new versions of cGA introducing elitism. He created two different approaches, i.e., the persistent elitism cGA (pe-cGA) and the nonpersistent elitism cGA (ne-cGA). The elitism-based cGAs outperform the original cGA in term of function evaluations, but they do not perform better in term of solution quality.

B. Modified Compact Genetic Algorithm

The idea behind the M-cGA is to enhance the exploration capability of the cGA, which tends to stagnate, by adding one of the operators typical of the stochastic algorithms. Therefore, starting from the ne-cGA, the Modified cGA was implemented by introducing more PVs and integrating a learning scheme in the update procedure. In Fig. 2, the new "step 4" of the M-cGA is reported, which describes the updating procedure, different from that of the standard cGA. In fact, in M-cGA, each element of each PV is updated according to the rule used in the standard cGA, which represents its self-knowledge, but it is also influenced by the elements of the other PVs, i.e., by a global knowledge, and in particular by the best element among those of all the PVs. This concept is taken from the well-known Particle Swarm Optimization (PSO) [22]. The new global update, in particular, depends on the learning factor c, introduced in [18], which is taken from the velocity update rule of PSO. In preliminary simulations, we found that the value typically used in PSO ($c = 2$) is suitable also in the M-cGA case.

In this way, it is possible to enhance the exploration properties of the algorithm and increase the ability to avoid local optimum, with a reduced increase of the computational cost: In fact, the number of operations performed by the M-cGA is equal to that carried on by the cGA, just multiplied by the number of PVs, which is generally very small (2–6).

III. Thinned Array Synthesis

In view of the preliminary results reported in [18]–[20], the application of the M-cGA to thinned array has been further investigated. Several configurations of both linear and planar thinned arrays have been considered. The performance of the M-cGA has been compared to results available in literature, obtained by other approaches on the same configurations. They have been compared both in terms of their capability to obtain a good solution, i.e., a configuration that minimizes the peak sidelobe level (PSL), and of their computational cost. In all the considered situations, the M-cGA uses 4 PVs, and the reported results are the average values over 50 independent trials.

A. Synthesis of Linear Thinned Array

For what concerns linear arrays, five different configurations have been considered: arrays with 96, 198, and 502 elements,

TABLE I
AVERAGE PSL [dB] OBTAINED WITH DIFFERENT METHODS FOR ARRAYS WITH THE 50% OF THE ELEMENTS SWITCHED OFF

Array	GA[12]	ADS-GA [12]	cGA	M-cGA
98/49	−19.82	−20.4	−19.8	−20.45
198/99	−18.20	−19.24	−19.9	−21.9
502/251	−20.83	−21.31	−20.4	−23.53

TABLE II
COMPARISON OF THE M-cGA AND THE ADS-GA [12] IN TERMS OF MINIMUM PSL AND COMPUTATIONAL COST

Array	PSL [dB]		No. of cost function eval.	
	ADS-GA [12]	M-cGA	ADS-GA [12]	M-cGA
198/79	−20.25	−21.10	126,126	60,000
200/46	−23.05	−23.75	305,600	100,000

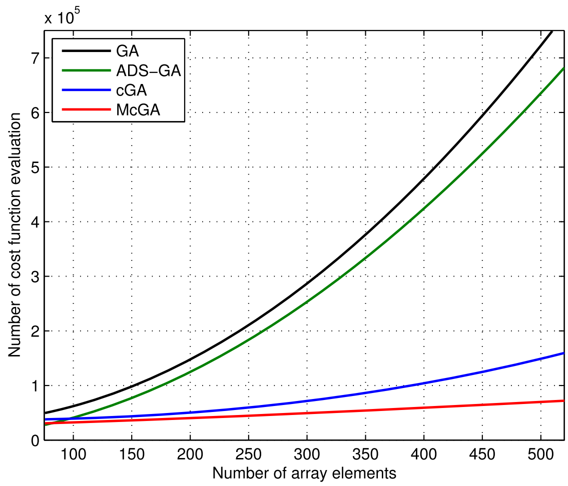

Fig. 3. Function evaluation of different thinned arrays.

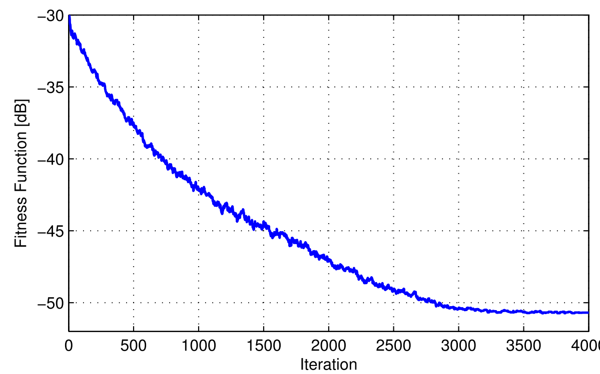

Fig. 4. Average curve of convergence of the M-cGA applied to the optimization of the 20×10 thinned array.

50% of which is turned on; a 198-element array, with 79 elements switched off; and an array with 200 elements, 46 of which are off. These configurations were chosen due to the availability of previous results in literature, therefore it was possible to compare the performance of the M-cGA not only to the standard cGA but also to other established approaches [12].

Table I reports the PSL of the first three configurations, i.e., the arrays with the 50% of elements switched off, obtained with the M-cGA, the cGA, the GA, and the hybrid ADS-GA [12], respectively. These results show that the cGA works almost always as the GA, while the M-cGA outperforms both the GA and the cGA in all cases, most significantly when the size of the array increases; its performance is comparable to those of the ADS-GA for the smallest array, but it becomes better than the latter when increasing the problem size. Moreover, after running the M-cGA simulation over 50 independent trials, the resulting standard deviation is 0.4009 for the first array (98/49), 0.5953 for the second (198/99), 0.261 for the last (502/251).

Fig. 3 gives information about the computational cost of the four considered methods applied to arrays with the 50% of elements switched off since it shows the variation of the number of cost function evaluations versus the total number of array elements. This plot highlights the advantage of using the probability vector instead of the population since it allows a drastic reduction of the workload. Moreover, it proves that M-cGA outperforms cGA since the use of more PVs speeds up the convergence.

Finally, Table II summarizes the results for the last two considered arrays, for what concerns both the minimum PSL and the number of cost function evaluations, relative to the M-cGA and compared to those for the ADS-GA [12]; in fact, from the above analysis, the latter appears to provide better results than the cGA and the GA. Also in these two cases, the PSL values obtained with the M-cGA are slightly better than those given by the ADS-GA, but the M-cGA outperforms the ADS-GA for what concerns the computational cost, which is reduced to one half in the first case and even to one third for the second array.

B. *Synthesis of Planar Thinned Array*

In this section, results of the synthesis of planar, i.e., square and rectangular, thinned arrays, are shown. Similarly to the linear case, different configurations have been considered based on previous literature availability. In all the cases, the fitness function optimized by the M-cGA is the sum of PSLs in two main planes, i.e., $\phi = 0°$, and $\phi = 90°$ and the probability vectors are one-dimensional vectors as for the linear array.

The first configuration considered is a 20×10-element planar array, in which 108 elements are turned on. In Fig. 4, the M-cGA average curve of convergence is plotted: The value of the fitness function after 3000 iterations corresponds to an array configuration whose radiation pattern is shown in Fig. 5. The PSL is equal to −26.6 dB in the $\phi = 0°$ plane, and to −23.5 dB in the $\phi = 90°$ plane. These achieved values are lower than those obtained with the GA in [5], and with the modified real genetic algorithm (MGA) that optimized also the position of the elements switched on [23]. Moreover, the number of fitness function evaluations required to converge is around 12 000 for the M-cGA, i.e., less than half of those needed by the MGA [23].

As a last example of application of the M-cGA to the optimized synthesis of planar thinned arrays, different square arrays have been considered, with different size and percentage of switched off elements. The obtained PSLs, which in these cases is equal in the two planes, are reported in the third column of Table III. In the columns 4–7, the results obtained with the

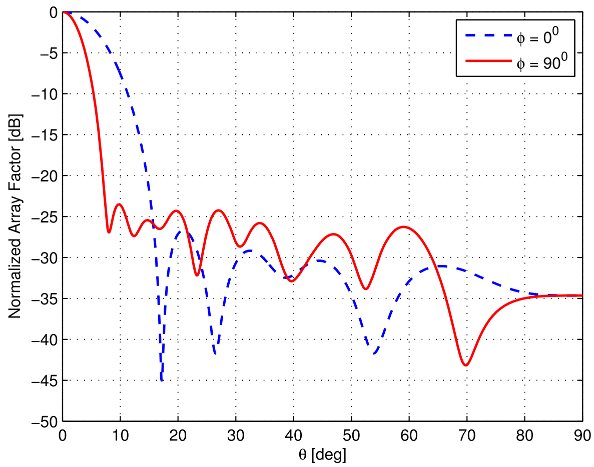

Fig. 5. Far-field patterns in the two main planes for the 20×10 optimized thinned array.

TABLE III
MINIMUM PSL [dB] OBTAINED WITH DIFFERENT METHODS APPLIED TO DIFFERENT-SIZE PLANAR THINNED ARRAYS

array size	**% of ON elements**	**M-cGA**	**cGA**	**HSPSO [14]**	**ACO [9]**	**IFTDT [15]**
12×12	48	-19.4	-17.9	-16.7	-	-17.6
24×24	44	-23.3	-22.0	-19.0	-	-22.8
30×30	60	-24.6	-23.9	-	-23.5	-24.3

cGA, the HSPSO [14], the ACO [9], and the IFTDT [15] are also shown.

IV. CONCLUSION

In this letter, the M-cGA, an enhanced version of the cGA recently introduced integrating learning mechanism in cGA, is applied to the synthesis of thinned arrays. The results here presented reveal that the M-cGA is able to well control the PSL of both linear and planar thinned array, with a reduced computational cost.

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