Modified cosmology through Barrow entropy

Ahmad Sheykhi*

Department of Physics, College of Sciences, Shiraz University, Shiraz 71454, Iran
Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran

We investigate the cosmological consequences of the modified Friedmann equations when the entropy associated with the apparent horizon, given by Barrow entropy, $S \sim A^{1+\delta/2}$, where $0 \leq \delta \leq 1$, represents the amount of the quantum-gravitational deformation of the horizon. We study implications of this model in a flat Friedmann-Robertson-Walker (FRW) universe with/without cosmological constant. Taking the cosmological constant into account, this model can describe the current accelerated expansion, although the transition from deceleration phase to the acceleration phase takes place in the lower redshifts. We investigate the evolution of the scale factor and show that with increasing δ, the value of the scale factor increases as well. We also estimate the age of the universe in Barrow cosmology which is smaller than the age of the universe in standard cosmology.

I. INTRODUCTION

Inspired by the Covid-19 virus structure, recently Barrow discussed [1] that quantum-gravitational effects may deform the geometry of the black hole horizon leading to an intricate, fractal features. He argued that the area law of the black hole entropy get modified and is given by

$$S_h = \left(\frac{A}{A_0} \right)^{1+\delta/2}, \qquad (1)$$

where A is the black hole horizon area and A_0 is the Planck area. The exponent δ ranges as $0 \leq \delta \leq 1$ and represents the amount of the quantum-gravitational deformation effects. The area law is reproduced in case of $\delta = 0$ and $A_0 \to 4G$. On the other hand, $\delta = 1$ corresponds to the most intricate and fractal structure of the horizon. In the cosmological setup, the effects of Barrow entropy on the cosmic evolution have been investigated from different viewpoints. For example, modification of the area law leads to a new holographic dark energy model based on Barrow entropy [2, 3]. A cosmological scenario based on Barrow entropy was proposed in [4], where it was shown that new extra terms that constitute an effective dark energy sector appear in the Friedmann equations. Although, it was argued in [4] that the modified Friedmann equations based on Barrow entropy (1) can describe the thermal history of the universe from early deceleration to the late time acceleration, with the dark-energy epoch following the matter one, regardless of the presence of cosmological constant Λ, nevertheless, it seems this conclusion is only correct in the presence of cosmological constant [4]. In other words, when $\Lambda = 0$ the equation of state (EoS) parameter of the dark sector is always positive ($w_{DE} > 0$) [4] (see Appendix). This implies that the exponent δ in Barrow entropy cannot reproduce any term which may play the role of dark energy

[4]. On the other hand, it was recently proven that Barrow entropy as well as any other known entropy (Tsalis, Renyi, Kaniadakis, etc) is just sub-case of generalized entropy expression introduced in [5, 6]. Other studies on the cosmological consequences of the Barrow entropy can be carried out in [7–17].

In the present work, we are going to investigate cosmological implications of the modified Friedmann equations when the entropy associated with the apparent horizon is given by the Barrow entropy (1). Our work differs from [4] in that the author of [4] modifies the cosmological field equations in such a way that leads to an extra component of energy in the Friedmann equations. In this approach the gravity side of the Friedmann equation is not modified and the Barrow entropy acts as an effective dark energy in the right hand side of the field equations. However, our studies in the present work is based on the modification of the geometry part (left hand side) of the cosmological filed equations [18] and keeping the energy content of the universe in the form of ordinary matter and radiation. This approach is well motivated and more physically reasonable, since basically the entropy depends on the geometry of spacetime (gravity part of the action). Any modification to the entropy expression should affect directly the gravity side of the field equations. In the Appendix of the present work, we compare the results of [4] with the present work and clarify the difference of our work with [4]. Throughout this paper we set $\kappa_B = 1 = c = \hbar$, for simplicity.

This paper is structured as follows. In the next section, for completeness, we briefly review the procedure of deriving the modified Friedmann equations describing the evolution of the universe, when the entropy associated with the apparent horizon is in the form of Barrow entropy (1). We shall see that the cosmological constant can appear as a constant of integration in the Friedmann equations. In section III, we investigate the cosmological consequences of the modified Friedmann equations in the presence/absence of cosmological constant. We also estimate the age of the universe in this section. We finish with closing remarks in the last section.

*asheykhi@shirazu.ac.ir

II. MODIFIED FRIEDMANN EQUATIONS BASED ON BARROW ENTROPY

In this section, we briefly review the approach of constructing the modified Friedmann equations based on Barrow entropy by using the gravity-thermodynamics conjecture. We refer to [18] for details of calculations.

In the background of FRW universe, the line elements of the metric is given by

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right),\tag{2}$$

where $a(t)$ is scale factor of the universe, $k = 0, \pm 1$ stand for the curvature parameter, and (t, r, θ, ϕ) are the comoving coordinates. We further assume $a_0 = a(t = t_0) = 1$, at the present time. Assuming the apparent horizon as boundary of the universe, the temperature associated with the horizon is given by [19]

$$T_h = -\frac{1}{2\pi\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right),\tag{3}$$

where $\tilde{r}_A = 1/\sqrt{H^2 + k/a^2}$ is the apparent horizon radius [20]. From the thermodynamical viewpoint the apparent horizon is a suitable horizon consistent with first and second law of thermodynamics [21–26]. We further assume the energy-momentum tensor of the universe is $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$, where ρ and p are the energy density and pressure, respectively. The energy-momentum tensor is conserved, $\nabla_\mu T^{\mu\nu} = 0$, which results the continuity equation, $\dot{\rho} + 3H(\rho + p) = 0$ where $H = \dot{a}/a$ is the Hubble parameter. The work density associated with the volume change of the expanding universe, is also given by $W = (\rho - p)/2$ [27]. To employ the gravity-thermodynamics conjecture, we propose the first law of thermodynamics on the apparent horizon satisfies as

$$dE = T_h dS_h + W dV,\tag{4}$$

where $E = \rho V$ is the total energy of the universe enclosed by the apparent horizon, and T_h and S_h are, respectively, the temperature and entropy associated with the apparent horizon. Here $V = \frac{4\pi}{3}\tilde{r}_A^3$ is the volume enveloped by a 3-dimensional sphere with the area of apparent horizon $A = 4\pi\tilde{r}_A^2$. Taking differential form of the total matter and energy, we find $dE = 4\pi\tilde{r}_A^2\rho d\tilde{r}_A + \frac{4\pi}{3}\tilde{r}_A^3\dot{\rho}dt$, which after combining with conservation equation, we arrive at

$$dE = 4\pi\tilde{r}_A^2\rho d\tilde{r}_A - 4\pi H\tilde{r}_A^3(\rho + p)dt.\tag{5}$$

Differentiating the Barrow entropy (1), yields

$$dS_h = (2+\delta)\left(\frac{4\pi}{A_0}\right)^{1+\delta/2}\tilde{r}_A^{1+\delta}\dot{\tilde{r}}_A dt.\tag{6}$$

Finally, combining Eqs. (3), (5) and (6) with the first law of thermodynamics (4), after some algebraic calculations

and using continuity relation, we arrive at

$$-\frac{2+\delta}{2\pi A_0}\left(\frac{4\pi}{A_0}\right)^{\delta/2}\frac{d\tilde{r}_A}{\tilde{r}_A^{3-\delta}} = \frac{d\rho}{3}.\tag{7}$$

After integration, we find the first modified Friedmann equation in Barrow cosmology,

$$\left(H^2 + \frac{k}{a^2}\right)^{1-\delta/2} = \frac{8\pi G_{\rm eff}}{3}\rho + \frac{\Lambda}{3},\tag{8}$$

where Λ is a constant of integration which can be interpreted as the cosmological constant, and $G_{\rm eff}$ stands for the effective Newtonian gravitational constant,

$$G_{\rm eff} \equiv \frac{A_0}{4}\left(\frac{2-\delta}{2+\delta}\right)\left(\frac{A_0}{4\pi}\right)^{\delta/2}.\tag{9}$$

If we define $\rho_\Lambda = \Lambda/(8\pi G_{\rm eff})$, Eq. (8), can be rewritten as

$$\left(H^2 + \frac{k}{a^2}\right)^{1-\delta/2} = \frac{8\pi G_{\rm eff}}{3}(\rho + \rho_\Lambda).\tag{10}$$

In this way, we derive the modified Friedmann equation by starting from the first law of thermodynamics, and assuming the entropy associated with the apparent horizon has the form (1). When $\delta = 0$, the area law of entropy is restored and $A_0 \to 4G$. In this case, $G_{\rm eff} \to G$, and Eq. (8) reduces to the standard Friedmann equation in General Relativity.

To get the second Friedmann equation, we can combine the continuity equation with the first Friedmann equation (8). It is a matter of calculations to show that [18]

$$(2-\delta)\frac{\ddot{a}}{a}\left(H^2 + \frac{k}{a^2}\right)^{-\delta/2} + (1+\delta)\left(H^2 + \frac{k}{a^2}\right)^{1-\delta/2}$$
$$= -8\pi G_{\rm eff}(p + p_\Lambda),\tag{11}$$

where $p_\Lambda = -\Lambda/(8\pi G_{\rm eff})$. This is the second modified Friedmann equation governing the evolution of the universe based on Barrow entropy. In the limiting case where $\delta = 0$ ($G_{\rm eff} \to G$), Eq. (11) reduces to the second Friedmann equation in standard cosmology. Combining Eqs. (8) and (11), yields

$$(2-\delta)\frac{\ddot{a}}{a}\left(H^2 + \frac{k}{a^2}\right)^{-\delta/2}$$
$$= -\frac{8\pi G_{\rm eff}}{3}\left[3(p + p_\Lambda) + (\delta + 1)(\rho + \rho_\Lambda)\right]$$
$$= -\frac{8\pi G_{\rm eff}}{3}(\rho + \rho_\Lambda)(3w_{\rm tot} + \delta + 1),\tag{12}$$

where $w_{\rm tot}$ is the EoS parameter of the total energy and matter, defined as

$$w_{\rm tot} = \frac{p + p_\Lambda}{\rho + \rho_\Lambda}.\tag{13}$$

From Eq. (12), one may notice that the condition for the acceleration of the cosmic expansion ($\ddot{a} > 0$), yields

$$1 + \delta + 3w_{\text{tot}} < 0 \quad \longrightarrow \quad w_{\text{tot}} < -\frac{1+\delta}{3}. \tag{14}$$

For $\delta = 0$, we have $w_{\text{tot}} < -1/3$, while for $\delta = 1$ we find $w_{\text{tot}} < -2/3$. The former is the standard cosmology, while the later represents the most intricate and fractal structure of the horizon. As we shall see in the next section, our model can reproduce the accelerated expansion provided we take the cosmological constant into account. This is, perhaps, the main difference between Barrow and Tsallis cosmology [28], despite the origin of the corrections in entropy which are completely different in these two cases [29]. It was argued that choosing the non-extensive parameter in Tsallis cosmology as $\beta < 1/2$, leads to an accelerated universe, without invoking any kind of dark energy (cosmological constant) [28] (see also [30, 31] for dark energy models based on Tsallis entropy). However, in Barrow cosmology, we observe that in order to have $\ddot{a} > 0$, the EoS parameter should be always negative in the allowed range of δ, requiring an additional component of dark energy/cosmological constant to reproduce an accelerated universe [4].

Given the modified Friedmann equations (10) and (11) at hand, in the next section, we investigate the cosmological implications of this model.

III. MODIFIED COSMOLOGY

In this section we are going to investigate cosmological consequences of the modified Friedmann equations given in Eqs. (10) and (11). For simplicity, we focus on the flat universe ($k = 0$), although the study can be carried out for $k = \pm 1$.

A. The case $\Lambda = 0$

Let us first consider the case where the cosmological constant is zero ($\Lambda = 0$) and the universe is dominated with pressureless matter. Integrating, the continuity equation $\dot{\rho}_m(t) + 3H\rho_m(t) = 0$, immediately yields

$$\rho_m(t) \propto a^{-3} \Rightarrow \rho_m(t) = C_1 a^{-3}, \tag{15}$$

where C_1 is a constant of proportionality. In order to derive the evolution of the scale factor, we insert ρ_m from (15) in the modified Friedmann equation in Barrow cosmology (10). We find

$$\left(\frac{\dot{a}}{a}\right)^{2-\delta} = \frac{8\pi G_{\text{eff}}}{3} C_1 a^{-3}, \tag{16}$$

which can be rewritten as

$$\left(\frac{da}{dt}\right)^{2-\delta} = C_2 a^{-1-\delta}, \tag{17}$$

where the constant C_2 is defined

$$C_2 \equiv \frac{8\pi G_{\text{eff}} C_1}{3}. \tag{18}$$

One can easily integrate Eq. (17), which has the solution

$$a(t) = C_3 t^{(2-\delta)/3}, \tag{19}$$

where

$$C_3 \equiv \left[\frac{3}{2-\delta} C_2^{1/(2-\delta)}\right]^{(2-\delta)/3}. \tag{20}$$

When $\delta = 0$, we have

$$a(t) = \left[\frac{3}{2}\sqrt{C_2}\right]^{2/3} t^{2/3}, \tag{21}$$

which is the well-known result of standard cosmology. The second time derivative of the scale factor is given by

$$\ddot{a}(t) = -\frac{C_3}{9}(1+\delta)(2-\delta)\, t^{-(4+\delta)/3}, \tag{22}$$

which is always negative ($\ddot{a} < 0$) during the evolution of the universe in the allowed range of δ. Thus, in Barrow cosmology, one should take into account a dark energy/cosmological constant component to explain the acceleration of the cosmic expansion. This is consistent with the argument given in [4].

The evolution of the energy density, the Hubble and the deceleration parameters are calculated as

$$\rho_m(t) \propto \frac{1}{t^{2-\delta}}, \tag{23}$$

$$H(t) = \frac{\dot{a}}{a} = \frac{2-\delta}{3t}, \tag{24}$$

$$q(t) = -1 - \frac{\dot{H}}{H^2} = \frac{1+\delta}{2-\delta}. \tag{25}$$

Again, all above parameters reduce to those of standard cosmology for $\delta = 0$. Looking at the deceleration parameter indicates $q > 0$, implying a decelerated universe in Barrow cosmology filled with pressureless matter.

Now we consider a universe filled with radiation. This case makes only sense at the early stage of the universe where the radiation was dominated. Since our Universe is expanding, the proper momenta of freely moving particles decreases as $P(t) \sim 1/a(t)$. This implies that the random velocities of particles seen today should have been large in the past when the scale factor was much smaller than its present value [32]. As a result, the pressureless approximation is break down in the early universe. Our aim here is to obtain the evolution of the Universe in the framework of Barrow cosmology, when the energy content of the universe is composed of highly relativistic gas (radiation) with EoS $p_r = \rho_r/3$. In this case from the continuity equation, $\dot{\rho}_r(t) + 4H\rho_r(t) = 0$, we can get

$$\rho_r(t) \propto a^{-4} \Rightarrow \rho_r(t) = B_1 a^{-4}, \tag{26}$$

where B_1 is an integration constant. Combining $\rho_r(t)$ given in (26) with the first Friedmann equation (10) for $k = 0 = \Lambda$, one gets

$$\left(\frac{\dot{a}}{a}\right)^{2-\delta} = \frac{8\pi G_{\text{eff}}}{3} B_1 a^{-4}, \tag{27}$$

which can be rewritten as

$$\left(\frac{da}{dt}\right)^{2-\delta} = B_2 a^{-2-\delta}, \tag{28}$$

where we have defined

$$B_2 \equiv \frac{8\pi G_{\text{eff}}}{3} B_1. \tag{29}$$

Solving Eq. (28) for scale factor, we arrive at

$$a(t) = B_3 \, t^{(2-\delta)/4}. \tag{30}$$

where

$$B_3 = \left[\frac{4}{2-\delta} B_2^{1/(2-\delta)}\right]^{(2-\delta)/4}. \tag{31}$$

The standard cosmology is deduced by setting $\delta = 0$, yielding

$$a(t) = \sqrt{2} B_2^{1/4} \, t^{1/2}. \tag{32}$$

Taking the second time derivative of the scale factor (30) leads to

$$\ddot{a}(t) = -B_3 \left(\frac{4-\delta^2}{16}\right) t^{-(6+\delta)/4} < 0. \tag{33}$$

which is an expected result for radiation dominated era, where the universe was undergoing a decelerated phase ($\ddot{a}(t) < 0$). Now, we calculate the energy density, the Hubble and the deceleration parameters in the radiation dominated era. We find

$$\rho_r(t) \propto \frac{1}{t^{2-\delta}}, \tag{34}$$

$$H(t) = \frac{\dot{a}}{a} = \frac{2-\delta}{4t}, \tag{35}$$

$$q(t) = -1 - \frac{\dot{H}}{H^2} = \frac{2+\delta}{2-\delta} > 0, \tag{36}$$

which again confirm that $q > 0$ ($\ddot{a}(t) < 0$). This implies that in the early stage of the universe where the relativistic particles have been dominated, our Universe has been in a decelerated phase. In summary, in the presence of radiation and pressureless matter, Barrow cosmology cannot explain the cosmic phase transition from a deceleration phase to an acceleration phase during the history of the universe, unless the dark energy/cosmological constant is taken into account [4]. This is in contrast to the Tsallis cosmology [28] where by suitably choice of the nonextensive parameter, it is quite possible to explain the history of the universe from a deceleration to acceleration phase without invoking any kind of dark energy/cosmological constant.

B. The case with $\Lambda \neq 0$

In this case, we define the density parameters as

$$\Omega_m = \frac{\rho_m}{\rho_c}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c}, \quad \rho_c = \frac{3H^{2-\delta}}{8\pi G_{\text{eff}}}. \tag{37}$$

Therefore, in terms of the density parameters, the first Friedmann equation (10) can be written as

$$\Omega_m + \Omega_\Lambda = (1 + \Omega_k)^{1-\delta/2}, \tag{38}$$

where, as usual, the curvature density parameter is given by $\Omega_k = k/(a^2 H^2)$. We consider a flat universe filled with pressureless matter ($p = p_m = 0$) and cosmological constant, and hence

$$\Omega_m + \Omega_\Lambda = 1. \tag{39}$$
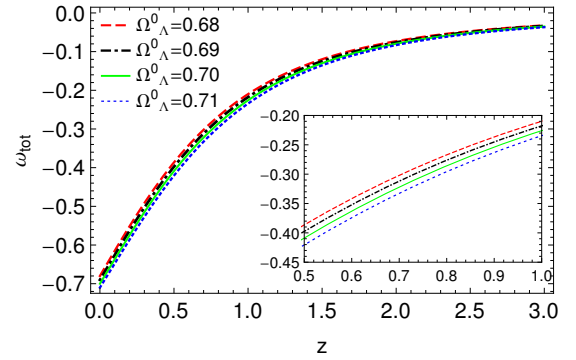
The total EoS parameter can be written

FIG. 1: Evolution of w_{tot} as a function of redshift parameter z in modified Barrow cosmology for different values of Ω_Λ^0.

$$w_{\text{tot}} = \frac{p_m + p_\Lambda}{\rho_m + \rho_\Lambda} = \frac{p_\Lambda}{\rho_m + \rho_\Lambda} = \frac{p_\Lambda/\rho_\Lambda}{\rho_m/\rho_\Lambda + 1},$$
$$= -\frac{1}{\rho_m/\rho_\Lambda + 1}, \tag{40}$$

where $p_\Lambda/\rho_\Lambda = -1$, according to the definition of p_Λ and ρ_Λ. Taking into account the fact that $\rho_\Lambda = \rho_\Lambda^0$, and $\rho_m = \rho_m^0(1+z)^3$, and using the definition of density parameters in Eq. (37), we obtain

$$w_{\text{tot}}(z) = -\frac{\Omega_\Lambda^0}{\Omega_m^0(1+z)^3 + \Omega_\Lambda^0}. \tag{41}$$

If we take $\Omega_\Lambda^0 \simeq 0.7$ and $\Omega_m^0 \simeq 0.3$, we have

$$w_{\text{tot}}(z) = -\frac{0.7}{0.7 + 0.3(1+z)^3}. \tag{42}$$

At the present time where $z \to 0$, we have $w_{\text{tot}} = -0.7$, while at the early universe where $z \to \infty$, we get $w_{\text{tot}} = 0$. This implies that at the early stages, the universe undergoes a decelerated phase while at the late time it

experiences an accelerated phase. The behaviour of the total EoS parameter in term of the redshift is plotted in Fig. 1. This figure shows that w_{tot} decreases with increasing Ω_Λ^0, which is an expected result.

We can also obtain the deceleration parameter defined as $q = -1 - \dot{H}/H^2$. It is a matter of calculations to show that

$$q(z) = -1 + \frac{3}{2-\delta}(1-\Omega_\Lambda^0)(1+z)^3$$
$$\times \left[\Omega_\Lambda^0 + (1-\Omega_\Lambda^0)(1+z)^3\right]^{-1}. \quad (43)$$

The behaviour of q versus z are plotted in Figs. 2 and 3. In Fig. 2, we keep $\delta = 0.4$ and investigate the effects of Ω_Λ^0 on $q(z)$. We observe that q is not sensitive to the present values of Ω_Λ^0, while in Fig. 3, we see that q is very sensitive to the Barrow exponent δ. From Fig. 3, we see that with increasing δ, the transition from deceleration phase ($q > 0$) to the acceleration phase ($q < 0$) takes place at lower redshifts. Indeed, the best consistency with observation for the phase transition happens for $\delta = 0$ at $z \approx 0.63$.
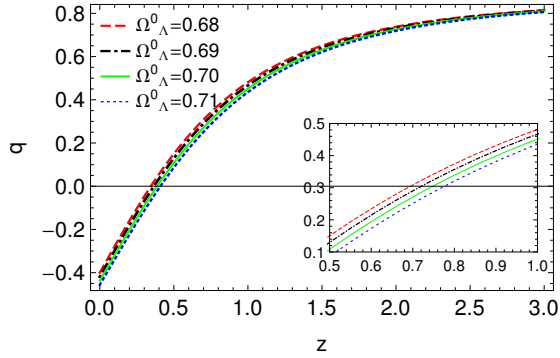

FIG. 2: Evolution of q as a function of redshift parameter z in modified Barrow cosmology for $\delta = 0.4$ and different values of Ω_Λ^0.
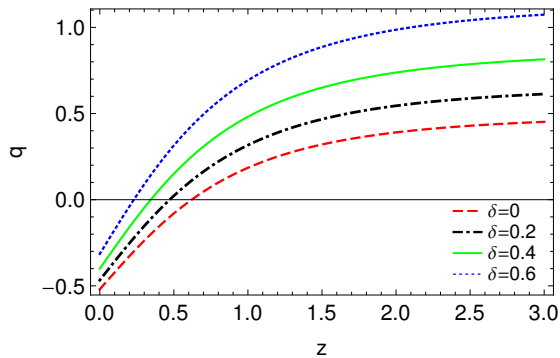

FIG. 3: Evolution of q as a function of redshift parameter z in modified Barrow cosmology for $\Omega_\Lambda^0 = 0.68$ and different values of δ.

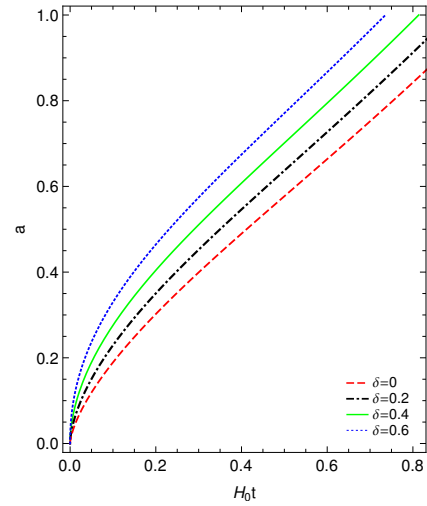Next, we investigate the scale factor of the universe. The first Friedmann equation (10) for flat universe ($k =$

FIG. 4: Evolution of scale factor a versus $H_0 t$ in modified Barrow cosmology for $\Omega_\Lambda^0 = 0.70$ and different values of δ.
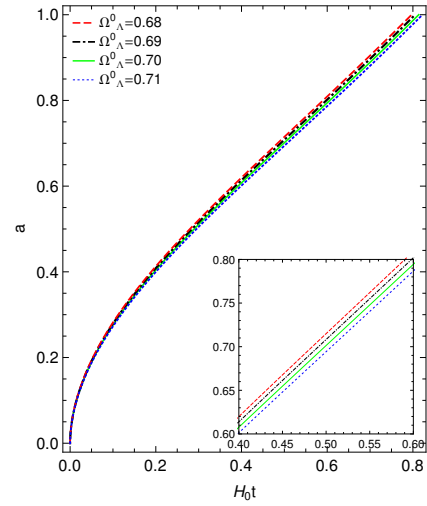

FIG. 5: Evolution of scale factor a versus $H_0 t$ in modified Barrow cosmology for $\delta = 0.4$ and different values of Ω_Λ^0.

0) can be rewritten as

$$H^{2-\delta} = H_0^{2-\delta}\left[\Omega_m^0 a^{-3} + \Omega_\Lambda^0\right],$$
$$\Rightarrow \quad \frac{da}{dt} = H_0 a \left[\Omega_m^0 a^{-3} + \Omega_\Lambda^0\right]^{1/(2-\delta)}, \quad (44)$$

where $H_0 = H(t = t_0)$ is the Hubble parameter at the present time t_0. Integrating (44), yields

$$H_0 t = \int a^{-1} \left[(1-\Omega_\Lambda^0)a^{-3} + \Omega_\Lambda^0\right]^{1/(\delta-2)} da. \quad (45)$$

Let us look at the above relation for the special case where $\Omega_\Lambda^0 \approx 1$ and $\Omega_m^0 \approx 0$. In this case, we have $H_0 t \sim \ln a$, and hence $a(t) \sim \exp(H_0 t)$, which describes a de-Sitter universe, independent of the value of δ. For $\Omega_\Lambda^0 \simeq 0.7$ and $\Omega_m^0 \simeq 0.3$, we have

$$H_0 t = \int a^{-1} \left[0.3a^{-3} + 0.7\right]^{1/(\delta-2)} da. \quad (46)$$

In principle, one can solve the Eq. (46) to obtain the scale factor of the universe for different values of δ. Indeed, for a given value of δ, it is better to plot the scale factor a versus $H_0 t$ which are shown in Figs. 4-5. From Fig. 4, we see that at each time, the scale factor increases with increasing δ. Therefore, in modified Barrow cosmology the radius of the universe increases comparing to the standard cosmology.

C. Age of the universe

Given the Hubble parameter at hand, we can estimate the age of the universe at the present time ($t = t_0$). In the absence of cosmological constant ($\Lambda = 0$) and for the matter dominated universe, from (24) we can estimate the age of the universe in modified Barrow cosmology as

$$t_0|_B = \frac{2-\delta}{3H_0} = \frac{2-\delta}{2}t_0|_S, \tag{47}$$

where $H_0 = H(t_0)$ is the Hubble constant and $t_0|_S = 2/(3H_0)$ is the age of the universe in standard cosmology with $\Lambda = 0$. Thus, compared to the standard cosmology, the age of the universe decreases by factor $(2-\delta)/2 < 1$. This implies that in modified cosmology based on Barrow entropy, the age problem cannot be alleviated. This is in contrast to the Tsallis cosmology [28], where the age problem can be alleviated provided one take the non-extensive parameter $\beta < 1/2$ [28]. When we take the cosmological constant into account, the age of the universe can be obtained through relation

$$t_0 - t = \int_0^z \frac{dz}{H(z)(1+z)}, \tag{48}$$

where $a = (1+z)^{-1}$. Substituting $H(z)$ from Eq. (44), we arrive at

$$t_0 - t = \frac{1}{H_0} \int_0^z \frac{dz}{(1+z)\left[\Omega_m^0(1+z)^3 + \Omega_\Lambda^0\right]^{1/(2-\delta)}}. \tag{49}$$

In order to obtain the age of the universe, we must take the limit $t \to 0$ when $z \to \infty$. Rewriting the above expression in terms of $x = (1+z)^{-1}$, we have

$$t_0 = \frac{1}{H_0} \int_0^1 \frac{dx}{x\left[\Omega_m^0 x^{-3} + \Omega_\Lambda^0\right]^{1/(2-\delta)}}. \tag{50}$$

We observe that the age of the Universe in this model depends on δ. In table I we present the values of $H_0 t_0$ for different δ and fixed value of Ω_Λ^0. We observe that with increasing the value of δ, the age of the universe decreases. Besides, from table II, we see that for a fixed value of δ, the age of the universe increases with increasing Ω_Λ^0.

δ	0	0.2	0.4	0.6
$H_0 t_0$	0.9468	0.8725	0.7970	0.7199

TABLE I: Numerical results for $H_0 t_0$ in modified Barrow cosmology for $\Omega_\Lambda^0 = 0.68$ and different values of δ.

Ω_Λ^0	0.68	0.69	0.70	0.71
$H_0 t_0$	0.7970	0.8052	0.8136	0.8224

TABLE II: Numerical results for $H_0 t_0$ in modified Barrow cosmology for $\delta = 0.4$ and different values of Ω_Λ^0.

IV. CLOSING REMARKS

The cosmological field equations govern the evolution of the universe get modified, due to the quantum-gravitational deformation effects of the apparent horizon. In this work, we have investigated the cosmological consequences of the modified Friedmann equations when the entropy associated with the apparent horizon is in the form of Barrow entropy (1). We showed that in the presence of cosmological constant, this model can explain the current accelerated universe, although the transition from decelerated phase ($q > 0$) to the accelerated phase ($q < 0$) takes place in the lower redshifts, compared to the standard cosmology. We obtained the scale factor, Hubble parameter and deceleration parameter in the presence/absence of cosmological constant which depend on the Barrow exponent δ. We have also estimated the age of the universe in this model and observed that comparing to the standard cosmology ($\delta = 0$), the age of the universe decreases with increasing δ. On the other hand for each value of δ, the age of the universe increases with increasing Ω_Λ^0.

In conclusion, the main result obtained in the present work is that, in modified Barrow cosmology without cosmological constant ($\Lambda = 0$), one cannot deduce a thermal history of the universe compatible with observations and one needs to take into account the cosmological constant to reproduce an accelerated universe. In other words, the exponent δ in Barrow entropy cannot reproduce any term in the dynamical cosmological equations which may act as the dark energy sector.

It is also interesting to study the profile of the growth of density perturbation in the context of Barrow cosmology. The details of investigations on the density perturbation as well as the gravitational collapse in the background of Barrow cosmology will be addressed in the future projects.

Appendix: Comparing with "Modified cosmology through spacetime thermodynamics and Barrow horizon entropy" [4]

Here we review the modified cosmology based on Barrow entropy discussed in [4] and compare the present work with it in more details. Taking into account the entropy associated with the apparent horizon in the form of Barrow entropy given in Eq. (1), the author of [4] applies the first law of thermodynamics, $-dE = TdS$, on the apparent horizon and derives the modified Friedmann equations in Barrow cosmology. Here $-dE$ is the energy flux crossing the apparent horizon within an infinitesimal internal of time dt. While in the present work, we take the first law of thermodynamics as $dE = TdS + WdV$, where dE is now the change in the energy inside the apparent horizon. Besides, in [4] the apparent horizon radius $\tilde{r}_A$ has been assumed to be fixed. Thus, the temperature of apparent horizon can be approximated to $T = 1/(2\pi\tilde{r}_A)$ and there is no the term of volume change in it. But, here, we have used the matter energy $E = \rho V$ inside the apparent horizon and the apparent horizon radius changes with time. This is the reason why we have included the term WdV in the first law (4).

For a flat universe the modified Friedmann equations, based on Barrow entropy, derived in [4], are given by

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_{DE}\right) \tag{51}$$

$$\dot{H} = -4\pi G\left(\rho_m + p_m + \rho_{DE} + p_{DE}\right), \tag{52}$$

where the energy density and pressure of the effective dark energy are defined as [4]

$$\rho_{DE} = \frac{3}{8\pi G}\left\{\frac{\Lambda}{3} + H^2\left[1 - \frac{\beta(\delta + 2)}{2 - \delta}H^{-\delta}\right]\right\}, \tag{53}$$

$$p_{DE} = -\frac{1}{8\pi G}\left\{\Lambda + 2\dot{H}\left[1 - \beta\left(1 + \frac{\delta}{2}\right)H^{-\delta}\right]\right.$$
$$\left. + 3H^2\left[1 - \frac{\beta(2+\delta)}{2-\delta}H^{-\delta}\right]\right\}, \tag{54}$$

where β and Λ are constants [4]. It is clear that the Friedmann equations (51) and (52) differ from the modified

Friedmann equations we derived in Eqs. (10) and (11) of the present work. According to approach [4] the gravity side of the Friedmann equation is not modified and the Barrow entropy acts as an effective dark energy sector in the right hand side of the field equations. However, in our work, we keep the energy content of the universe in the form of ordinary matter and radiation and the left hand side of the Friedmann equations get modified due to the correction to the entropy. We believe this is more reasonable, since basically the entropy expression depends on the geometry of spacetime (gravity part of the action). Any modification to the entropy expression should influence the gravity side (left hand side) of the field equations.

Considering the effective dark energy in the Friedmann equations (51) and (52), one can define the effective EoS parameter as

$$w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}} = -1 - \frac{2\dot{H}\left[1 - \beta\left(1 + \frac{\delta}{2}\right)H^{-\delta}\right]}{\Lambda + 3H^2\left[1 - \frac{\beta(2+\delta)}{2-\delta}H^{-\delta}\right]}. \tag{55}$$

The cosmological implications of the modified Friedmann equations (51) and (52) were studied in [4]. It was shown that for $\Lambda = 0$ the EoS parameter of the dark sector behaves as $w_{DE} = \delta/(2 - \delta)\Omega_{DE}^{-1}$ which is always positive ($w_{DE} > 0$) in the allowed range of exponent $0 \leq \delta \leq 1$. This means that exponent δ in Barrow entropy cannot reproduce any term which may play the role of dark energy. In other words, in order to obtain the thermal history of the universe in agreement with observations and reproduce the late time acceleration in the context of Barrow cosmology, one needs to consider an additional dark energy (cosmological constant) in the energy content of the universe. This is consistent with the results obtained in the present work.

[1] John D.Barrow, *The area of a rough black hole*, Phys. Lett. B **808**, 135643 (2020), [arXiv:2004.09444].

[2] E. N. Saridakis, *Barrow holographic dark energy*, Phys. Rev. D **102**, 123525 (2020), [arXiv:2005.04115].

[3] F. K. Anagnostopoulos, S. Basilakos, E. N. Saridakis, *Observational constraints on Barrow holographic dark energy*, Eur. Phys. J. C **80**, 826 (2020), [arXiv:2005.10302].

[4] E. N. Saridakis, *Modified cosmology through spacetime thermodynamics and Barrow horizon entropy*, JCAP **07**, 031 (2020), [arXiv:2006.01105].

[5] S. Nojiri, S. D. Odintsov and V. Faraoni, *From nonextensive statistics and black hole entropy to the holographic dark universe*, Phys. Rev. D **105**, 044042 (2022), [arXiv:2201.02424].

[6] S. Nojiri, Sergei D. Odintsov, T. Paul, *Early and late universe holographic cosmology from a new generalized entropy*, Phys. Lett. B **831**, 137189 (2022), [arXiv:2205.08876].

[7] E. N. Saridakis and S. Basilakos, *The generalized second law of thermodynamics with Barrow entropy*,

[arXiv:2005.08258].

[8] E. M. C. Abreu, et. al., *Barrow's black hole entropy and the equipartition theorem*, Eur. Phys. Lett. **130**, 40005 (2020), [arXiv:2005.11609].

[9] A. A. Mamon, et. al., *Dynamics of an Interacting Barrow Holographic Dark Energy Model and its Thermodynamic Implications* , Eur. Phys. J. P. **136**, 134 (2021), [arXiv:2007.16020].

[10] E. M. C. Abreu, J. A. Neto, *Barrow black hole corrected-entropy model and Tsallis nonextensivity*, Phys. Lett. B **810**, 135805 (2020), [arXiv:2009.10133].

[11] J. D. Barrow, S.S Basilakos, E. N. Saridakis, *Big Bang Nucleosynthesis constraints on Barrow entropy*, Phys. Lett. B **815**, 136134 (2021), [arXiv:2010.00986].

[12] S. Srivastava, U. K. Sharma, *Barrow holographic dark energy with Hubble horizon as IR cutoff,* Int. J. Geom. Methods Mod. Phys. Vol. **18**, No. 1, 2150014 (2021), [arXiv:2010.09439].

[13] B. Das, B. Pandey, *A study of holographic dark energy models using configuration entropy*, [arXiv:2011.07337].

[14] U. K. Sharma, G. Varshney, V. C. Dubey, *Barrow agegraphic dark energy.* Int. J. Mod. Phys. D Vol. **30**, No. 03, 2150021 (2021), [arXiv:2012.14291].

[15] A. Dixit, V. K. Bhardwaj, A. Pradhan, *Barrow HDE model for Statefinder diagnostic in nonflat FRW universe*, Chin. J. Phys. **77**, 646 (2022), [arXiv:2101.00176].

[16] S. Nojiri, S. D. Odintsov and T. Paul, *Different Faces of Generalized Holographic Dark Energy*, Symmetry **13**, 928 (2021), [arXiv:2105.08438].

[17] S. Nojiri, S. D. Odintsov and T. Paul, *Barrow entropic dark energy: A member of generalized holographic dark energy family*, Phys. Lett. B **825**, 136844 (2022), [arXiv:2112.10159].

[18] A. Sheykhi, *Barrow entropy corrections to Firedmann equations*, Phys. Rev. D. **103**, 123503 (2021), [arXiv:2102.06550].

[19] M. Akbar and R. G. Cai, *Thermodynamic behavior of the Friedmann equation at the apparent horizon of the FRW universe*, Phys. Rev. D **75**, 084003 (2007), [arXiv:hep-th/0609128].

[20] A. Sheykhi, *Friedmann equations from emergence of cosmic space*, Phys. Rev. D **87**, 061501(R) (2013), [arXiv:1304.3054].

[21] B. Wang, Y. Gong, E. Abdalla, *Thermodynamics of an accelerated expanding universe*, Phys. Rev. D **74**, 083520 (2006), [arXiv:gr-qc/0511051].

[22] J. Zhou, B. Wang, Y. Gong, E. Abdalla, *The generalized second law of thermodynamics in the accelerating universe*, Phys. Lett. B **652**, 86 (2007), [arXiv:0705.1264].

[23] R.G. Cai and S.P. Kim, *First law of thermodynamics and Friedmann equations of Friedmann-Robertson-Walker universe*, JHEP **02**, 050 (2005) [arXiv:hep-th/0501055].

[24] R. G. Cai, L. M. Cao and Y. P. Hu, *Corrected entropy-area relation and modified Friedmann equations,* JHEP **08**, 090 (2008), [arXiv:0807.1232].

[25] A. Sheykhi, *Thermodynamics of interacting holographic dark energy with the apparent horizon as an IR cutoff*, Class. Quantum Grav. **27** (2010) 025007, [arXiv:0910.0510].

[26] A. Sheykhi, *Entropic corrections to Friedmann equations*, Phys. Rev. D **81**, 104011 (2010), [arXiv:1004.0627].

[27] S. A. Hayward, *Unified first law of black-hole dynamics and relativistic thermodynamics*, Class. Quant. Grav. **15**, 3147 (1998), [arXiv:gr-qc/9710089].

[28] A. Sheykhi, *New explanation for accelerated expansion and flat galactic rotation curves*, Eur. Phys. J. C **80** (2020) 25, [1912.08693].

[29] C. Tsallis, L. J. L. Cirto, *Black hole thermodynamical entropy*, Eur. Phys. J. C **73**, 2487 (2013), [arXiv:1202.2154].

[30] M. Tavayef a, A. Sheykhi a,b, Kazuharu Bamba c, H. Moradpour, *Tsallis holographic dark energy*, Phys. Lett. B **781**, 195 (2018), [arXiv:1804.02983].

[31] M. A Zadeh, A Sheykhi, H Moradpour, K Bamba,*Note on Tsallis holographic dark energy*, Eur. Phys. J. C **78**, 940 (2011), [arXiv:1806.07285].

[32] S. Weinberg, *Gravitation and cosmology, principles and applications of the General theory of Relativity* (John Wiley, 1972);
N. Riazi, *Gravitation and Cosmology* (World Scientific, 2009).