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Modified F-Contractions via α-Admissible Mappings and Application to Integral Equations

Hassen Aydi[a,b], Erdal Karapinar[c], Habib Yazidi[d]

[a] *University of Dammam, Department of Mathematics. College of Education of Jubail, P.O: 12020, Industrial Jubail 31961. Saudi Arabia.*
[b] *Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan.*
[c] *Atilim University, Department of Mathematics, 06836, İncek, Ankara, Turkey.*
[d] *Jizan University, College of Sciences, Department of Mathematics, Saudi Arabia.*

Abstract. In this paper, we introduce the concept of a modified F-contraction via α-admissible mappings and propose some theorems that guarantee the existence and uniqueness of fixed point for such mappings in the frame of complete metric spaces. We also provide some illustrative examples. Moreover, we consider an application solving an integral equation.

1. Introduction and Preliminaries

In 2012, Wardowski [15] defined a new concept of F-contractions as follows.

Definition 1.1. *Let (X, d) be a metric space. A self-mapping $T : X \to X$ is said to be an F-contraction if there exists $\tau > 0$ such that*

$$d(Tx, Ty) > 0 \Rightarrow \tau + F(d(Tx, Ty)) \leq F(d(x, y)), \tag{1}$$

for all $x, y \in X$ where $F : \mathbb{R}^+ \to \mathbb{R}$ is a mapping satisfying the following conditions:

(F_1) *F is increasing, i.e., for all $\alpha, \beta \in \mathbb{R}^+$ such that $\alpha < \beta$, $F(\alpha) < F(\beta)$,*

(F_2) *For any sequence $\{\alpha_n\}_{n=1}^{\infty}$ of positive real numbers, $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$.*

(F_3) *There exists $k \in (0, 1)$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.*

We set $\mathfrak{F}$ as the collections of all functions F satisfying (F_1)-(F_3). Consider $F(y) = \ln(y)$, $G(y) = \ln(y) + y$ and $H(y) = -\frac{1}{\sqrt{y}}$ for $y > 0$. It is clear that $F, G, H \in \mathfrak{F}$. For more examples and details, see e.g. [15].

In what follows we recollect the main result of Wardowski [15] which is a generalization of the Banach Contraction Mapping Principle [5]:

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Corresponding author: Hassen Aydi
Email addresses: hmaydi@uod.edu.sa (Hassen Aydi), erdalkarapinar@yahoo.com, ekarapinar@atilim.edu.tr (Erdal Karapinar), yazidi2001@yahoo.com (Habib Yazidi)

Theorem 1.1. *Let (X, d) be a complete metric space and $T : X \to X$ be an F-contraction. Then T has a unique fixed point.*

The concept of an F-contraction as well as Theorem 1.1 have been generalized in many directions, see e.g. [8, 9, 12, 13].

Let Ψ be the family of functions $\psi : [0, \infty) \to [0, \infty)$ satisfying the following conditions:

i) ψ is nondecreasing;

(ii) $\sum_{n=1}^{+\infty} \psi^n(t) < \infty$ for all $t > 0$.

If $\psi \in \Psi$, then it is called c-comparison function. It is easy to show that $\psi(t) < t$ for all $t > 0$ and ψ is continuous at 0.

In 2012, Samet *et al.* [11] introduced the class of α-admissible mappings.

Definition 1.2. *[11] Let $\alpha : X \times X \to [0, \infty)$ be given mapping where $X \neq \emptyset$. A selfmapping T is called α-admissible if for all $x, y \in X$, we have*

$$\alpha(x, y) \geq 1 \Longrightarrow \alpha(Tx, Ty) \geq 1. \tag{2}$$

In what follows we extend the notion of F-contraction.

Definition 1.3. *Let (X, d) be a metric space. A self-mapping $T : X \to X$ is said to be a modified F-contraction via α-admissible mappings if there exists $\tau > 0$ such that*

$$d(Tx, Ty) > 0 \Rightarrow \tau + F(\alpha(x, y)d(Tx, Ty)) \leq F(\psi(d(x, y))), \tag{3}$$

for all $x, y \in X$, where the mapping $F \in \mathfrak{F}$ and $\psi \in \Psi$.

If we let $F(t) = \ln(t)$ for $t > 0$, the contraction form (3) becomes

$$\alpha(x, y)d(Tx, Ty) \leq e^{-\tau}\psi(d(x, y)) \leq \psi(d(x, y)) \quad \text{for all } x, y \in X, \ Tx \neq Ty. \tag{4}$$

(4) is considered as an $\alpha - \psi$-contraction which was introduced by Samet *et al.* [11].

In this paper, we prove some fixed point results of certain contractions whose frames are drawn above. The obtained theorems shall be supported by a concrete example. An application of the observed result is considered in the frame of integral equation theory.

2. Main Results

The following theorem is the first main result of this paper:

Theorem 2.1. *Let (X, d) be a complete metric space and $T : X \to X$ be a modified F-contraction via α-admissible mappings. Suppose that*

(i) T is α-admissible;

(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;

(iii) T is continuous.

Then T has a fixed point.

Proof. By assumption (ii), there exists a point $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. We define a sequence $\{x_n\}$ in X by $x_{n+1} = Tx_n = T^{n+1}x_0$ for all $n \geq 0$. Suppose that $x_{n_0} = x_{n_0+1}$ for some n_0. So the proof is completed. Now, we assume that

$$x_n \neq x_{n+1} \text{ for all } n. \tag{5}$$

Since $\alpha(x_0, x_1) = \alpha(x_0, Tx_0) \geq 1$ and T is α-admissible, we get

$$\alpha(x_n, x_{n+1}) \geq 1, \text{ for all } n = 0, 1, \ldots. \tag{6}$$

From (3) and (5), we have

$$\tau + F(\alpha(x_{n-1}, x_n)d(Tx_{n-1}, Tx_n)) \leq F(\psi(d(x_{n-1}, x_n))).$$

On account of (F_1) and (6), we find

$$\tau + F(d(x_n, x_{n+1})) \leq F(d(x_{n-1}, x_n)) \quad \text{for all } n \geq 1.$$

By letting $d_n = d(x_n, x_{n+1})$, the inequalities above infer that

$$F(d_n) \leq F(d_{n-1}) - \tau \leq F(d_0) - n\tau \quad \text{for all } n \geq 1.$$

Consequently, we obtain

$$\lim_{n\to\infty} F(d_n) = -\infty.$$

By a property (F_2), we have

$$\lim_{n\to\infty} d_n = 0. \tag{7}$$

Now, due to (F_3), we have $\lim\limits_{n\to\infty} d_n^k F(d_n) = 0$, where $k \in (0, 1)$. By (7), the following holds for all $n \geq 0$

$$0 \leq d_n^k F(d_n) - d_n^k F(d_0) \leq d_n^k (F(d_0) - n\tau) - d_n^k F(d_0) = -n\tau d_n^k \leq 0. \tag{8}$$

Letting $n \to \infty$ in (8), we find that

$$\lim_{n\to\infty} n d_n^k = 0.$$

So there exists $n_1 \in \mathbb{N}$ such that $d_n \leq \frac{1}{n^{\frac{1}{k}}}$, for all $n \geq n_1$. For $m, n \in \mathbb{N}$ with $m > n \geq n_1$, we have

$$d(x_n, x_m) \leq d_n + d_{n+1} + \ldots + d_{m-1} \leq \sum_{i=n}^{\infty} \frac{1}{i^{\frac{1}{k}}}.$$

Since $\sum\limits_{i\geq 1} \frac{1}{i^{\frac{1}{k}}}$ converges, the sequence $\{x_n\}$ is Cauchy in (X, d). From the completeness of X, there exists $u \in X$ such that $\lim\limits_{n\to\infty} x_n = u$. Finally, the continuity of T yields $Tu = u$, which completes the proof. $\square$

Theorem 2.1 remains true if we replace the continuity hypothesis by the following property:

(H) If $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$, then there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha(x_{n(k)}, x) \geq 1$ for all k.

Theorem 2.2. *Let (X, d) be a complete metric space and $T : X \to X$ be a modified F-contraction via α-admissible mappings. Suppose that*

- (i) *T is α-admissible;*
- (ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;*
- (iii) *(H) holds.*

Then there exists $u \in X$ such that $Tu = u$.

Proof. Following the lines in the proof of Theorem 2.1, we construct a sequence $\{x_n\}$ in (X, d) which is Cauchy and converges to some $u \in X$.

Suppose that there exists an increasing sequence $\{n(k)\} \subset \mathbb{N}$ such that $x_{n(k)} = Tu$ for all $k \in \mathbb{N}$. Letting $k \to \infty$, by the uniqueness of the limit, we find $Tu = u$. Hence, the proof is completed. As a result, we shall assume that there exists $k_0 \in \mathbb{N}$ such that $x_{n(k)} \neq Tu$ for all $k \in \mathbb{N}$ with $k \geq k_0$. Consequently, we have $Tx_{n(k)-1} \neq Tu$ for all $k \geq k_0$.

Therefore, by (3), we have

$$\tau + F(\alpha(x_{n(k)-1}, u)d(Tx_{n(k)-1}, Tu)) \leq F(\psi(d(x_{n(k)-1}, u))).$$

Regarding $\alpha(x_{n(k)-1}, x) \geq 1$ and (F_1),

$$d(x_{n(k)}, Tu) = d(Tx_{n(k)-1}, Tu) \leq \psi(d(x_{n(k)-1}, u)).$$

Since ψ is continuous at 0 and $d(x_{n(k)-1}, u) \to 0$,

$$\lim_{k\to\infty} \psi(d(x_{n(k)-1}, u)) = 0.$$

Thus, $\lim_{k\to\infty} d(x_{n(k)+1}, Tu) = 0$. By the uniqueness of limit, $Tu = u$. □

We provide the following example.

Example 2.1. *Take $X = \{0, 1, 2\}$ and $T : X \to X$ such that*

$$T0 = 0 \quad \text{and } T1 = T2 = 1. \tag{9}$$

Consider $\alpha : X \times X \to [0, \infty)$ as

$$\alpha(1, 2) = \alpha(2, 1) = \alpha(1, 1) = 1,$$

and 0 otherwise. Clearly, if $\alpha(x, y) = 1$, $\alpha(Tx, Ty) = 1$. Then T is α-admissible. Notice also that $\alpha(1, T1) = \alpha(1, 1) = 1$.

Let $x, y \in X$ such that $Tx \neq Ty$, so (x, y) is equal to $(0, 1)$, $(0, 2)$, $(1, 0)$ or $(2, 0)$. For these four cases, $\alpha(x, y) = 0$, so (4) holds. In other words, (3) holds for $F(t) = \ln(t)$ and for any $\psi \in \Psi$ and any metric d. It is obvious also that the hypothesis (H) is satisfied. Thus, applying Theorem 2.2, the mapping T has a fixed point. Here, we have two fixed points which are $u = 0$ and $u = 1$.

Here, we underline the fact that the mapping considered in above example has two fixed points, 0 and 1. Notice also that $\alpha(0, 1) = 0 < 1$. For the uniqueness, we need an additional condition:

(U) For all $x, y \in Fix(T)$, we have $\alpha(x, y) \geq 1$, where $Fix(T)$ denotes the set of fixed points of T.

Theorem 2.3. *Adding condition (U) to the hypotheses of Theorem 2.1 (resp. Theorem 2.2), we obtain that u is the unique fixed point of T.*

Proof. We shall prove the uniqueness by the method of *Reductio and Absurdum.*

Suppose, on the contrary, that there exist $u, v \in X$ such that $u = Tu$ and $v = Tv$ with $u \neq v$. Then $Tu \neq Tv$, so by (3), we get

$$\tau + F(\alpha(u, v)d(Tu, Tv)) \leq F(\psi(d(u, v))),$$

that is,

$$\tau + F(\alpha(u, v)d(u, v)) \leq F(\psi(d(u, v))) < F(d(u, v)).$$

Again by (F_1), we have

$$\tau + F(d(u, v)) \leq F(\psi(d(u, v))) < F(d(u, v)),$$

which is a contradiction. Thus, $u = v$ which completes the proof. □

The following corollaries are immediate.

Corollary 2.1. *Let (X, d) be a complete metric space and $T : X \to X$ be a given mapping. Suppose there exists $\tau > 0$ such that*

$$d(Tx, Ty) > 0 \Rightarrow \tau + F(d(Tx, Ty)) \leq F(\psi(d(x, y))), \tag{10}$$

for all $x, y \in X$ where F satisfies $(F_1) - (F_3)$. Then T has a unique fixed point.

Proof. It sufficient to take $\alpha(x, y) = 1$ in Theorem 2.3. □

Corollary 2.2. *Let (X, d) be a complete metric space and $T : X \to X$ be a given mapping. Suppose there exists $\tau > 0$ such that*

$$d(Tx, Ty) > 0 \Rightarrow \tau + F(d(Tx, Ty)) \leq F(cd(x, y))), \tag{11}$$

for all $x, y \in X$ where F satisfies $(F_1) - (F_3)$ and $c \in (0, 1)$. Then T has a unique fixed point.

Proof. It follows from Corollary 2.1 with $\psi(t) = ct$. □

The investigation of existence of fixed points on metric spaces endowed with a partial order was initiated by Turinici [14]. Then, several interesting and valuable results appeared in this direction, see e.g. [1–4, 7, 10].

Definition 2.1. *Let $(X, \preceq)$ be a partially ordered set and $T : X \to X$ be a given mapping. We say that T is nondecreasing with respect to $\preceq$ if*

$$x, y \in X,\ x \preceq y \Longrightarrow Tx \preceq Ty.$$

Furthermore, a sequence $\{x_n\} \subset X$ is said to be nondecreasing with respect to $\preceq$ if $x_n \preceq x_{n+1}$ for all n.

Definition 2.2. *Let $(X, \preceq)$ be a partially ordered set and d be a metric on X. We say that $(X, \preceq, d)$ is regular if for every nondecreasing sequence $\{x_n\} \subset X$ such that $x_n \to x \in X$ as $n \to \infty$, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $x_{n(k)} \preceq x$ for all k.*

Under the set-up of partially ordered metric spaces, we have the following result.

Corollary 2.3. *Let $(X, \preceq)$ be a partially ordered set and d be a metric on X such that (X, d) is complete. Let $T : X \to X$ be a nondecreasing mapping with respect to $\preceq$. Suppose that there exist $\tau > 0$, $\psi \in \Psi$ and $F \in \mathfrak{F}$ such that*

$$\tau + F(d(Tx, Ty)) \leq F(\psi(d(x, y)),$$

for $x, y \in X$ with $x \succeq y$ and $Tx \neq Ty$. Suppose also that the following conditions hold:

(i) there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;

(ii) either T is continuous

(ii)′ or $(X, \preceq, d)$ is regular.

Then T has a fixed point.

Proof. Define the mapping $\alpha : X \times X \to [0, \infty)$ by

$$\alpha(x, y) = \begin{cases} 1 \text{ if } x \preceq y \text{ or } x \succeq y, \\ 0 \text{ otherwise.} \end{cases}$$

Clearly, T is a modified F-contractive mapping via α-admissible mappings, that is,

$$\tau + F(\alpha(x, y)d(Tx, Ty)) \leq F(\psi(d(x, y))),$$

for all $x \geq y$ with $Tx \neq Ty$. From condition (i), we have $\alpha(x_0, Tx_0) \geq 1$. Moreover, for all $x, y \in X$, from the monotone property of T, we have

$$\alpha(x, y) \geq 1 \Longrightarrow x \geq y \text{ or } x \leq y \Longrightarrow Tx \geq Ty \text{ or } Tx \leq Ty \Longrightarrow \alpha(Tx, Ty) \geq 1.$$

Thus, T is α-admissible. Now, if T is continuous, then the existence of a fixed point follows from Theorem 2.1. Suppose now that $(X, \preceq, d)$ is regular. Let $\{x_n\}$ be a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x \in X$ as $n \to \infty$. From the regularity hypothesis, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $x_{n(k)} \preceq x$ for all k. This implies from the definition of α that $\alpha(x_{n(k)}, x) \geq 1$ for all k. In this case, the existence of a fixed point follows from Theorem 2.2. □

Now, we present an example which guarantees the uniqueness of the fixed point.

Example 2.2. *Let $X = [0, \infty)$ and $d(x, y) = |x - y|$ for all $x, y \in X$. Take $\tau > 0$. Consider the mapping $T : X \to X$ given by*

$$Tx = \begin{cases} e^{-\tau}\frac{3}{4}x \text{ if } x \in [0,1], \\ e^{-\tau}\frac{3}{4} \text{ if } x > 1. \end{cases}$$

T is continuous in (X, d). Define the mapping $\alpha : X \times X \to [0, \infty)$ by

$$\alpha(x, y) = \begin{cases} 1 \text{ if } x, y \in [0,1], \\ 0 \text{ otherwise.} \end{cases}$$

Consider the function $\psi : [0, \infty) \to [0, \infty)$ by

$$\psi(t) = \begin{cases} \frac{3}{4}t \text{ if } t \in [0,1], \\ \frac{2}{5}t \text{ otherwise.} \end{cases}$$

Let $x, y \in X$ such that $\alpha(x, y) \geq 1$, so $x, y \in [0, 1]$. Then $Tx, Ty \in [0, 1]$, that is, $\alpha(Tx, Ty) = 1$. Hence, T is α-admissible. Mention that $\psi \in \Psi$ and $\alpha(0, T0) = 1$. In the case where $x, y \in [0, 1]$ such that $Tx \neq Ty$, we have

$$\alpha(x, y)d(Tx, Ty) = d(Tx, Ty) = e^{-\tau}\frac{3}{4}|x - y| \leq e^{-\tau}\psi(d(x, y)).$$

In the other case where x or y is not in $[0, 1]$, $\alpha(x, y) = 0$, so the above inequality is satisfied for all $x, y \in X$ with $Tx \neq Ty$. Thus, (3) is satisfied with $F(t) = \ln(t)$ for $t > 0$. Moreover, it is easy that the hypothesis (U) is true. Thus, applying Theorem 2.3, the mapping T has a unique fixed point, which is $u = 0$.

3. Application

In this section, we present an application on existence of a solution of an integral equation. In particular, inspired from [6] and using Corollary 2.3, we will prove the existence of a solution of the following integral equation.

$$x(t) = g(t) + \int_0^1 S(t, s)\, K(t, x(s))\, ds, \tag{12}$$

where $K : [0, 1] \times \mathbb{R} \to \mathbb{R}$ and $g : [0, 1] \to \mathbb{R}$ are continuous functions, and $S : [0, 1] \times [0, 1] \to [0, \infty)$ is a function such that $S(t, .) \in L^1([0, 1])$ for all $t \in [0, 1]$.

Throughout this section, let $X = C([0, 1], \mathbb{R})$ be the set of real continuous functions defined on $[0, 1]$. Take the metric $d : X \times X \to [0, \infty)$ be defined by

$$d(x, y) = \|x - y\|_\infty = \max_{s \in [0,1]} |x(s) - y(s)|.$$

It is known that (X, d) is a complete metric space.

Now, take the operator $T : X \to X$ defined by

$$Tx(t) = g(t) + \int_0^1 S(t,s)\,K(t,x(s))\,ds. \tag{13}$$

Mention that (12) has a solution if and only if the operator T has a fixed point.

Theorem 3.1. *Assume that*

- *(i) there exist $\lambda \in (0,1)$, $\tau > 0$, $\nu : X \times X \to [0,\infty)$ and $\alpha : X \times X \to [0,\infty)$ such that if $\alpha(x,y) \geq 1$ for all $x, y \in X$ then, for every $s \in [0,1]$, we have*

$$0 \leq |K(s,x(s)) - K(s,y(s))| \leq \nu(x,y)|x(s) - y(s)|,$$

and

$$\|\int_0^1 S(t,s)\nu(x,y)\,ds\|_\infty \leq \lambda\, e^{-\tau}.$$

- *(ii) K is non-decreasing with respect to its second variable;*
- *(iii) there exists $x_0 \in X$ such that $x_0(t) \leq g(t) + \int_0^1 S(t,s)\,K(t,x(s))\,ds$.*

Then T has a fixed point in X.

Proof. Define the mapping $\alpha : X \times X \to [0,\infty)$ by

$$\alpha(x,y) = \begin{cases} 1 \text{ if } x \leq y, \\ 0 \text{ otherwise.} \end{cases}$$

Take $\psi(t) = \lambda t$. We define

$$x, y \in X,\ x \preceq y \quad \text{if and only if} \quad x(t) \leq y(t) \quad \text{for all } t \in [0,1],$$

where $\leq$ denotes the usual order of real numbers. If $x \preceq y$, then $\alpha(x,y) \geq 1$. By condition (i)

$$\begin{aligned}
|A(x)(t) - A(y)(t)| &\leq \int_0^1 S(t,s)|K(t,s,x(s)) - K(t,s,y(s))|\,ds \\
&\leq \int_0^1 S(t,s)\nu(x,y)|x(s) - y(s)|\,ds \\
&\leq \|x - y\|_\infty \int_0^1 S(t,s)\nu(x,y)\,ds \\
&\leq e^{-\tau}\lambda\,\|x - y\|_\infty \\
&= e^{-\tau}\psi(\|x - y\|_\infty).
\end{aligned}$$

We deduce for all $x, y \in X$ such that $x \preceq y$ and $Tx \neq Ty$

$$\alpha(x,y)d(Tx,Ty) = d(Tx,Ty) = \|Tx - Ty\|_\infty \leq e^{-\tau}\psi(d(x,y)). \tag{14}$$

It is clear that if $Tx = Ty$, then (14) holds.

Since K is non-decreasing with respect to its second variable, so for all $x, y \in X$ with $x \preceq y$, we get $Tx(t) \leq T(y)(t)$ for all $t \in [0,1]$, that is, if $\alpha(x,y) \geq 1$, $\alpha(Tx,Ty) \geq 1$. Moreover, the condition (iii) yields that there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. Therefore, the integral operator A satisfies all the hypotheses of Corollary 2.3 with $F(t) = \ln(t)$ for $t > 0$. Consequently, T has a fixed point, that is, the integral equation (12) has a solution $x \in X$. $\square$

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