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The Semantics of Gradability, Vagueness, and Scale Structure

Modified fractions, granularity and scale structure

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Abstract Pragmatic enrichments arising from the use of modified fractions have been little studied, but offer interesting insights into the subtleties of scale structure and granularity. In this chapter I present some new experimental data on the interpretation of these expressions. I argue that these data suggest that modified fractions, like modified integers, give rise to pragmatic enrichments which are conditioned by scale granularity, but that we need to refine the notion of granularity somewhat to extend it to this domain. There is also evidence for enrichments that are not easily captured in classical quantity implicature terms, but which I suggest could be explained by appeal to typicality effects.

1 Introduction

Expressions of numerical quantity have been the focus of much study in experimental semantics and pragmatics. In many cases, this research is inspired by the realisation that there is an incompatibility between the "expected" meaning of expressions, based on mathematical considerations, and their communicative meaning in linguistic interactions. Consider, for example, (1)-(3).

(1) Most of the American population is female. (Solt 2016)

(2) A hexagon has at most 10 sides. (Nouwen 2010)

(3) London has more than 1000 inhabitants. (Cummins, Sauerland and Solt 2012)

In each case, we can offer a plausible account of the semantics of the expressions by appeal to mathematical considerations. We could argue that "most X is Y" means that the quantity of X that is Y exceeds the quantity of X that is not Y; that "at most 10 X are Y" means that the quantity of entities that are both X and Y does not exceed 10; and that "more than 1000 X are Y" means that the quantity of entities that are both X and Y exceeds 1000.

Under analyses of this kind, (1)-(3) are true. However, these examples are not unanimously judged as true by actual users of language, who consider them anomalous. As in (1), "most" is not considered to be felicitous when referring to values just a little above 50%. As in (2), "at most *n*" is not considered to be felicitous when referring to values clearly and invariably below *n*. As in (3), "more than *n*" is not considered to be felicitous when used out of the blue and referring to values far in excess of *n*.

In these cases, and many others besides, there are broadly two possible approaches to explaining the lack of felicity. One is to argue that the "mathematical" intuition about the semantic analyses is wrong, and that in fact these expressions have a more complex semantics (Geurts and Nouwen 2007, Solt 2016). The other is to argue that the anomalies are pragmatic, and arise for principled reasons that have nothing to do with the semantics *per se* (Cummins and Katsos 2010).

Of course, it is quite possible in principle that both explanations are correct, i.e. that the semantic analyses are more complex than initially supposed, but these meanings are also subject to pragmatic enrichment. None of the specific accounts offered appears to rely on both semantic and pragmatic effects – perhaps it would not be parsimonious to do so – but nevertheless it seems to be common ground that pragmatic enrichments are somewhat widespread in the domain of number.

Most strikingly, the interpretation of numerals itself has been widely argued to rely upon pragmatic factors. Unmodified numerals can convey either a "punctual" or a "lower-bound" meaning – that is, (4) can be interpreted as equivalent in meaning either to (5) or (6). In this case, our intuition might be that (5) is strongly preferred, but (6) is still available, for instance in a context such as (7).

(4) John has three A-levels.

(5) John has exactly three A-levels.

(6) John has at least three A-levels.

(7) You need three A-levels to be considered for the job. Is John eligible?

In one class of accounts, the semantics of numerals is lower-bounding and the punctual interpretation arises because of a quantity implicature (Gazdar 1979, Levinson 1983, Horn 1989): broadly, if the speaker of (4) knew that John had four A-levels, she would typically say so. On other accounts (e.g. Carston 1998, Geurts 2006, Breheny 2008) the semantics of numerals is punctual or underspecified, and pragmatic inference is required either to select a meaning or to obtain the lower-bound reading where required.

Other categories of numerical expression also appear to give rise to pragmatic enrichments, but the problem of determining which alternatives are in play is a more complex one. For instance, as discussed by Fox and Hackl (2006), (8) does not implicate (9); however, Cummins, Sauerland and Solt (2012) show that items such as (10) are widely judged to convey meanings such as (11) (but not (12)).

(8) John has more than four children.

(9) John does not have more than five children.

(10) There's room for more than 80 people.

(11) There is not room for more than 100 people.

(12) There is not room for more than 81 people.

One potential explanation for this difference (Cummins 2012) is that the availability of the alternatives depends upon the properties of the numbers involved as well as on the information content of the sentence. On this account, (8) is only felicitous if the speaker is ignorant as to the truth or falsity of informationally stronger alternatives, or the precise issue of whether John has four or more children is currently under discussion. In either case, the stronger assertion ("…more than five…") is out of court as an alternative, and hence the implicature (9) fails to arise. By contrast, (10) could be felicitous even for a knowledgeable speaker, as it is a convenient approximation that uses a round number (putatively accessible at a lower cognitive cost; cf. Krifka 2002). The corresponding sentence with "…more than 100…" would be a viable alternative, but that with "…more than 81…" would not, as this uses a costlier non-round number.

Whether or not this particular explanation is along the right lines, it seems inevitable that we have to consider the distinct properties of different numbers in order to understand their pragmatic behaviour in full. With respect to the issue of unmodified numerals and their meaning, it may be possible simply to construe the number line as a homogeneous sequence of equally-spaced scale points. However, research on the psychology of number itself (Dehaene 1997, Butterworth 1999) clearly indicates that our cognitive representation of integers is much more structured than this: some numbers (10, 20, 50, 100, …) are major reference points, while others (7, 13, 101, …) are not. And the existence of this structure is known to have linguistic consequences: round numbers are more widely used than non-round numbers (Jansen and Pollmann 2001), and round numbers are capable of being used to express approximate values (Krifka 2002).

2 Expressing fractional quantities

There are several ways in which we can quantify the size of the subdivisions of a whole.[1] We can do this using scalar quantity expressions (a few/little, some, many/much, most, all), percentages, fractions, or derived expressions such as "two out of every five". And these latter numerically-based categories of expression can themselves be further modified by expressions such as "more/less than", "at least/most", "up to", "about", and so on.

One immediate question that arises is how the availability of these distinct means of expression bears upon their perceived meaning. Returning to example (1), repeated below, we can see that this could be expressed in various other ways, such as (13)-(15).

(1) Most of the American population is female.

(13) More than 50% of the American population is female.

(14) More than half of the American population is female.

(15) More than one out of every two Americans is female.

[1] As an anonymous reviewer pointed out, we can also use fractions to quantify over parts of things that are not obviously characterised as "wholes" – for instance, "half a kilogram of…". The examples in this chapter all concern proportions of a finite quantity, and consequently involve proper fractions (those that lie between 0 and 1). Given that most uses of fractions for quantities above 1 also involve proper fractions, in combination with integers – we usually say "two and a quarter" rather than "nine quarters" – I would expect the observations here to apply to the broader class of fractional expressions of quantity.

(15) seems potentially anomalous – perhaps “more than one” is initially interpreted as “at least two” – but (13) and (14) appear to be valid alternatives to (1). We might then ask whether these three options are semantically equivalent, and if they are, whether this has pragmatic consequences (for instance, whether one option is marked and thus gives rise to some form of markedness implicature). This question is explored in detail, for “most” vs. “more than half”, by Solt (2016).

2.1 Inferences from modified fractions

A further question of interest, both for pragmatics and for the nature of the interface between number and language, is how the various different expressions of fractional quantity relate to one another. Is it the case, for instance, that the use of one modified fraction implicates the falsity of another? Consider (16) and (17).

(16) More than one-fifth of the participants were literature students.

(17) More than two-fifths of the participants were literature students.

It seems reasonable to conjecture that the use of (16) by an informed and cooperative speaker could be held to implicate the falsity of (17). But this is not obvious on theoretical grounds. Note that – to recall again Fox and Hackl’s (2006) observation – (18) does not implicate the falsity of (19).

(18) More than one of the participants was a literature student.

(19) More than two of the participants were literature students.

Setting aside the question of precisely why this is so, it is conceivable that (16) could pattern with (18), if we consider (16) to be effectively quantifying over the number of “fifths of the participants that were literature students”. Thus, one immediate question is whether the numerators of modified fractions behave like numerals, for pragmatic purposes. If so, then at least some modified fractions will fail to give rise to quantity implicatures that would theoretically be predicted, while others may give rise to a restricted class of implicatures, negating only a subset of the informationally stronger alternatives. Concretely, for instance, we would expect “more than two-fifths” potentially to implicate “not more than three-fifths” on this account, but not to implicate “not more than half”, as a half does not correspond to a whole number of fifths.

Considering the whole class of proper fractions, it is clear that we will need some way to constrain the set of alternatives that are to be considered in the calculation of quantity implicatures. Given any proper fraction, we can identify proper fractions that are arbitrarily close to it (in either direction): for a fraction p/q, consider for instance the set {3p/2q, 4p/3q, 5p/4q, …}. If “more than p/q” were to implicate the falsity of the corresponding expression with any other member of this set, it would, in the limit, convey “not more than p/q”, which is clearly absurd. In practice we could argue that these quantities are not all calculable by speaker and hearer, and not easily expressible, and for one of these reasons the problematic implicatures are ruled out of court. However, the question of where to draw the line between what can and cannot be inferred is not a straightforward one to resolve on principled grounds.

In suggesting (17) as a potentially consequential candidate alternative to (16), I am implicitly acknowledging the potential relevance of granularity considerations, in the sense of Krifka (2009), in determining which alternatives are pragmatically active. The notion of granularity, traceable to Curtin (1995), captures the fact that measured quantity can be reported at various different levels of precision, the levels differing specifically in the density of representation points. For instance, in the domain of time reporting, "hours" form a coarse-grained scale, with "minutes" forming a finer-grained scale, and "quarters" (units of 15 minutes) constituting an intermediate scale. (16) and (17) could be argued to be matched in granularity, as they are both expressions at the "fifths" level.

Cummins et al. (2012) argue that granularity is relevant to scalar implicatures, and specifically that modified coarse-grained numerals do not implicate the falsity of modified finer-grained alternatives. Empirically, it is an open question whether the same claim holds for fractions, but in principle the same arguments should apply: the alternatives of finer granularity enable the speaker to formulate more informative expressions, but these expressions incur a greater cognitive cost, both for the speaker and the hearer. Consequently, the speaker's failure to use a more informative finer-grained alternative can be attributed to considerations of cost, rather than being interpreted as a signal that the speaker is not in a position to commit to the more informative assertion that would have arisen. Applying this reasoning to fractions, we might expect that the use of a coarse-grained modified fraction will not give rise to pragmatic enrichments based on the existence of finer-grained alternatives. However, we might expect the use of such an expression to give rise to pragmatic enrichments related to equally (or more) coarse-grained alternatives.

2.2 The granularity of fractions, and its consequences

Given the definition of granularity, it would be natural to suppose that fractions' fineness of granularity increases with the increasing size of the denominator. If so, the claim articulated above can be reformulated as follows: fractions with small denominators will not give rise to implicatures concerning alternatives with larger denominators. For instance, the Cummins et al. (2012) argument would seem to suggest that (20) should not implicate the falsity of (21).

(20) More than three-quarters of the participants were literature students.

(21) More than nine-tenths of the participants were literature students.

However, there are problems with interpreting the notion of granularity in such an intuitive way for the case of fractions. Notably, it raises a potential conflict with Krifka's (2009) observations about the construction of granularity scales. He makes two observations: that scales are optimal (in expressive power) if their scale points are distributed in a systematic way (for instance, equidistant, or logarithmically distributed), and that "scales of different granularity levels should align".

The status of Krifka's (2009) latter observation, about the alignment of scales, is not made entirely clear. It could be read as a desideratum in order for granularity scales to be easily usable, or it could be read as a requirement in order for two scales to coexist on the same underlying domain. All the examples that Krifka discusses involve scales that align in this way, but we can readily imagine candidate pairs of scales that do not: for instance, we might count eggs in sixes or twelves, at one granularity level, and in hundreds, at another level. The hundreds are not all scale points on the sixes scale. For that matter, we might measure distance in miles, at a coarse granularity level, or metres, at a fine granularity level, in which case the scale points will (almost) never precisely coincide[2].

The former observation, that scale points should be sensibly distributed, makes tacit appeal to the idea that we wish to be able to describe the quantities that we want to talk about efficiently in terms of scale points. If the scale points – with which convenient expressions are associated – are clustered unevenly and do not span the full range of values that we wish to discuss, they are less helpful to us. The appropriate distribution of scale points clearly depends on the distribution of the values that we wish to discuss. For instance, a five-point rating scale with the options <*OK*, *good*, *very good*, *excellent*, *superb*> would be helpful if most of the things we want to rate are good to a greater or lesser extent, but unhelpful in permitting us to distinguish between things that are variously bad. In the case of expressions of proportion, we might reasonably suppose that we would like to be able to discuss all values across the range (0, 1) with similar levels of acuity[3].

If we consider proper fractions with a single, fixed denominator q, this criterion is satisfied, as they are uniformly distributed between zero and one. If we add to this system a further set of fractions with a denominator that is a multiple of q, both of Krifka's conditions are met. However, if we add to the system a set of fractions of a different denominator that is not a multiple (or factor) of q, both criteria will be violated: the scale points will not be evenly distributed over the range (0, 1), nor will the scale points align.

To take a concrete example, if our scales are based around halves, quarters and eighths, there will be seven equally-distributed scale points between 0 and 1 on the "eighths" scale, three of which are also scale points on the "quarters" scale; one of these is also a scale point on the "halves" scale. If our scales are quarters and thirds, there are three scale points on the quarters scale and two scale points on the thirds scale between 0 and 1, none of which coincide. Consecutive scale points in this case are unevenly spaced: the gaps between them are 1/4, 1/12, 1/6, 1/6, 1/12 and 1/4.

To illustrate the potential limitations of such a system, imagine that speaker and hearer were committed to using a system that relied upon thirds and quarters, that each possible expression within this system was equally costly to use, and that the speaker was known to be fully knowledgeable and cooperative. A description "more than a quarter" would be highly informative in such a system: it would convey that the value in question lay between a quarter and a third. By contrast, "more than three-quarters" would be much less informative: it would only convey that the value lay in the (three times larger) range between three-quarters and 1. This underscores the point that the efficiency of scales depends upon systematic distribution of the scale points. Unless it is particularly important for some reason that values in the

[2] The International Yard and Pound Agreement of 1959 defines the yard as exactly 0.9144 metres, so in fact a mile is exactly 1609.344 metres and 125 miles is thus 201,168 metres. This latter distance is the first point at which the scale points for mile and metre coincide.

[3] This assumption may not always be tenable: if we are talking predominantly about rare events, we might find it more useful to be able to distinguish events occurring with probability 0.001 and probability 0.01 than to be able to distinguish events with probability 0.5 and probability 0.6. However, a simple system of fractions with small denominators would perform very badly by this criterion too.

middle of the range (0, 1) are especially easy to describe economically and accurately, this arrangement is inefficient.

Intuitively, it seems clear that some denominators that would cause problems in such a system – e.g. sevenths – are seldom used. However, it seems very plausible that speakers could use a system that employed both quarters and tenths, or halves and thirds, despite such a system violating both Krifka's (2009) criteria. If so, this would suggest either that granularity is not an appropriate construct for capturing alternatives in the domain of fractions, or that the notion of granularity must be generalised somewhat from Krifka's definition in order to be applied here.

If we were to allow the notion of granularity to be elaborated or generalised to treat the domain of fractions, it is natural to consider whether we should make similar arrangements for other systems of quantity. The scale structure of fractions presumably reflects some kind of cognitive preference on the part of humans. Jansen and Pollmann (2001: 200) conjecture that "doubling and halving (sometimes followed by halving again) are basic means to manipulate quantities", which predicts a central role for halves and quarters in the organisation of the system. Among other researchers on the topic, Sigurd (1988) argues for the relevance of the base system to numerical cognition, and if applied to the case of fractions, this suggests that tenths and hundredths should also have some kind of conceptual primacy. Taking these considerations simultaneously into account, we might also predict a role for fifths and twentieths in the system. However, if fifths are represented as "pairs of tenths", we might expect them to be cognitively less accessible than tenths, which runs counter to the prediction that we would make about fifths and tenths based on standard considerations of granularity (i.e. that fifths are coarser-grained and therefore more accessible than tenths).

Moreover, although it seems plausible that the operation of halving is more cognitively salient than any other operation of division, it seems perfectly feasible to conceptualise entities such as thirds by direct division. If the operation of dividing by three is a great deal more complex than that of dividing by two, we might expect thirds to be less accessible than quarters, again running counter to a straightforward granularity-based account.

The above discussion has entirely concerned the denominators of fractions, but we might also expect the complexity of the numerators to bear upon how fractions are treated by speakers. We could think of a fraction p/q as being represented via a series of stages, in which the whole is first divided into q equal parts (perhaps via a series of distinct operations of division) and then collections of p of these parts are considered. If this is so, we would expect unit fractions – those of the form 1/q, for integer q – to be preferred over other fractions with the same denominator. If an individual's system of fractions effectively comprises a large number of unit fractions, plus a few full sets of fractions with special denominators such as two, three and ten, this system will have a particularly uneven distribution of scale points: specifically, scale points will be clustered near zero. Such a system would be justified if it is particularly important to be able to distinguish small proportions from one another with a high resolution.

Could the granularity of fractions have any implications for other expressions of quantity? There is clearly a potential interplay, as discussed above in the case of the decimal system: if the preference for a particular kind of scale structure arises in for number in general, we could reasonably expect that to carry over to the construction of fractions. The existence of the decimal system makes powers of ten especially important in representing and manipulating expressions of numerical quantity, and that might also influence our preferences as to how we use fractions. Non-decimal systems might also play a part: for instance, the way clock time is represented might encourage us to use the operations of dividing by four and dividing by 60, and practice with these operations might promote use of the relevant fractions.

In the other direction, if we have internalised a particular system of fractions, presumably we will be inclined to apply this when dealing with quantities that – unlike clock time – do not have pre-established points of division. If, for argument's sake, eighths are more salient than thirds, we would expect to find eighths being used preferentially as a way of partitioning up quantity in a novel domain. Moreover, if it were to transpire that (for instance) three-eighths is a more salient fraction than seven-eighths, we might be more likely to talk about units of three-eighths in a novel domain than about units of seven-eighths.

2.3 Testing the predictions by appeal to implicature

Granularity has been argued to have several implications for the way in which quantity expressions are understood. Krifka (2002) argues that coarse-grained numerals attract approximate interpretations, while fine-grained numerals are restricted to precise interpretations. In a situation in which 98 people are present, (22) can be judged as true, although interestingly (23) cannot, even when 99 people are present.

(22) There were 100 people there.

(23) There were more than 100 people there.

Conversely, (24) is false if exactly 100 people are present, and (25) is true only on an existential reading, which is hard to access in this case.

(24) There were 102 people there.

(25) There were 98 people there.

We would expect the same to apply to fractions, but with a possible caveat: namely that all the fractions widely used are coarse-grained enough to attract some kind of approximative reading. Supposing that the US population were precisely 320 million, we would not normally expect (26) and (27) to refer respectively to exactly 160 million people and exactly 64 million people.

(26) Half of the US population is clustered in just 146 counties.

(27) By 2050, one fifth of the US population will be aged 65 or over.

However, to the extent that fractions get interpreted as approximations, this is not a unique attribute: numbers used in continuous measurement behave in much the same way. The speaker of (28) is not understood to be referring to a distance of precisely 400cm, let alone

4000.0mm. It is fine-grained cardinal quantities that are the exception, in their apparent preference for exact readings.

(28) The passage into the Mound of the Hostages is four metres in length.

Building upon Krifka's observations, we could attempt to quantify the salience of particular fractions by examining the size of the regions for which they are acceptable approximations (with or without the explicit use of a hedge such as "about" or "around") – that is, the diameter of their pragmatic halos, in the sense of Lasersohn (1999). A potential issue to address in this case would be dealing with overlapping regions. For instance, if the quantity under discussion is 29%, successively less accurate approximations for this would include "two sevenths", "three tenths", "one quarter" and "one third", and each of these could be presented bare or with a hedging modifier. It is not entirely clear on theoretical grounds whether these should all be acceptable, or whether there is some implied trade-off in which the acceptability of one term is associated with a decrease in acceptability for the others. This might hamper our ability to use data pertaining to the "approximative power" of a fraction as a measure of its salience.

In this chapter I adopt a different approach: I ask participants for range interpretations of modified fractions. If these fractions are able to convey quantity implicatures, as argued above, then the ranges obtained will depend upon the presence of salient alternatives. Acceptance that a range could extend beyond a particular value will thus indicate that that value it is not considered to be a sufficiently salient alternative to mandate its usage, even in cases where doing so would yield expressions that were semantically true. So, for instance, if "more than one quarter" is understood to correspond to the range 25-50%, that indicates that "a half" (= two quarters) is a salient alternative to "one quarter", but also that "one third", "two fifths" etc. are not.

A potential challenge in adopting this approach is that the format in which participants are instructed to give their answers might influence their interpretation. For instance, requiring participants to respond in terms of fractions might represent a confound – simpler fractions would presumably be privileged in the responses. In the following, the use of percentages was adopted to give participants more flexibility in their response: however, it should be noted that this also has its limitations, as it might confer an advantage on fractions that are expressible in precise percentage terms (for instance, promoting the use of tenths).

3 Experiments: pragmatic bounds for modified fractions

This section reports a series of short experiments designed to explore whether modified fractions give rise to pragmatic bounds of the type posited in the above discussion. This preliminary research serves partly as a proof of concept and partly as a first attempt to map out some of the terrain, by establishing the pragmatic relations that are judged to hold between different modified fractions within the system.

Four versions of the stimuli were created and administered separately. Details of the specific stimuli are given below. The general procedure was the same for all sets of stimuli, and was as follows.

3.1 Method

The experiments were conducted using the Amazon Mechanical Turk platform. In each version, participants were presented with the following cover story and instructions:

A market research company has conducted a detailed survey on a large group of people, and has written up the results. For instance, "More than 50% of the participants are female", "Less than 20% of the participants own two cars", and so on.

You're now going to read some expressions that have been used to summarise the results from the survey. For each one, please state the range of possible values, in percent, that you think the expression means.

For example, if the expression is "about half", you might say that that means between 45% and 55%, or between 40% and 60%, etc.

There are no 'correct' answers: we're interested in knowing what you think.

The experimental items consisted of a series of modified fractions presented in a pseudorandom order. In each case, the modifier "more than" or "less than" was used, and the fraction was presented in word form rather than numerals (e.g. "more than three fifths"). The entire list was presented on a single page.

Responses were coded as "literal" if they reflected the semantic bound without any implicature (for example, interpreting "more than four fifths" as corresponding to the range 80-100%), "pragmatic" if they reflected an inferred bound (for example, interpreting "more than four fifths" as corresponding to the range 80-90%), and "error" if the responses failed to respect the semantics of the expression (for example, interpreting "more than four fifths" as corresponding to the range 60-80%). Where present, pragmatic bounds were recorded and analysed.

3.2 Participants

For each version of the experiment, 20 participants were recruited from US locations. Each participant was paid $0.50 for participation.

3.3 Experiment 1

Version 1 of the experiment was directed primarily towards establishing whether modified fractions towards the edges of the (0, 1) range attracted literal interpretations, and secondarily towards establishing whether modified fractions towards the middle of the (0, 1) range attracted pragmatic readings that were conditioned by the presence of "one half" in the system.

3.3.1 Materials

The following 15 items were pseudorandomised in order and presented in the context of the cover story shown above:

- less than one third / one quarter / one fifth / one sixth / one seventh / one eighth
- more than two thirds / three quarters / four fifths / five sixths / six sevenths / seven eighths
- more than two fifths / three sevenths / four ninths

Due to a coding error, "more than two thirds" was omitted and "more than two fifths" was repeated within the experiment as administered.

3.3.2 Results

The numbers of error, literal and pragmatic responses for each item, along with the pragmatic responses given, are presented in the following table.

Table 1 Results of experiment 1

Item	Error	Literal	Pragmatic	Pragmatic lower/upper bounds
less than 1/3	1	10	9	5, 5, 10, 10, 11, 15, 25, 25, 26
less than 1/4	2	9	9	5, 10, 10, 10, 12, 15, 15, 19, 20
less than 1/5	2	12	6	5, 10, 10, 10, 11, 15
less than 1/6	1	11	8	5, 5, 5, 5, 5, 9, 10, 12
less than 1/7	3	12	5	5, 5, 10, 10, 10
less than 1/8	6	10	4	5, 5, 8, 10
more than 3/4	0	8	12	80, 80, 84, 85, 85, 85, 90, 90, 90, 90, 90, 95
more than 4/5	3	11	6	89, 90, 90, 90, 95, 95
more than 5/6	10	4	6	89, 90, 90, 95, 95, 97
more than 6/7	13	2	5	90, 90, 90, 92
more than 7/8	4	9	7	90, 90, 90, 93, 95, 95, 98
more than 2/5	9	10	21	40[4], 45, 45, 45, 47, 47, 50, 50, 50, 55, 55, 55, 55, 60, 60, 60, 60, 60, 60, 60, 60
more than 3/7	13	3	4	49, 49, 50, 55
more than 4/9	8	5	7	49, 49, 55, 55, 60, 60, 65
Total	75	116	109	

3.3.3 Discussion

Generally, participants appear to have been competent with the task: although the overall error rate was 75/300 (=25%), the majority of these errors arose in cases involving relatively little-used fractions that are not straightforward to convert into percentages. I will not attempt to interpret the pragmatics of error responses as we cannot assume the participants' competence with respect to those items.

Of the 225 semantically correct responses, 109 (=48.4%) exhibited some form of pragmatic narrowing, which coheres with the prediction that implicatures are available from modified fractions. Most, although not all, of the pragmatic bounds offered by participants correspond to potentially salient alternative fractions. Of the 41 pragmatic responses to the "less than" items, 13 refer to 5% (1/20) and 14 to 10% (1/10), and of the 36 pragmatic responses to the

[4] This corresponds to a "40-40" response, which could reflect error; here I charitably assume that the respondent interprets this expression as ruling out anything as high as 41%.

corresponding “more than” items, 16 refer to 90% (9/10) – in fact, 18 refer to this alternative fraction if we consider 89% also to represent a bound for “less than 9/10”. In the case of “more than 2/5”, we see some responses reflecting the presence of “one half” as an alternative, but with more responses based on the next point on the fifths scale (3/5, i.e. 60%).

In addition to these responses, there are some that cannot be easily understood in terms of alternative fractions. For instance, 55% is attested as a pragmatic upper bound for “more than 3/7” and “more than 2/5”. As a fraction, this could be interpreted as corresponding to either 11/20 or 5/9, neither of which is predicted to be salient (although when 55% occurs as a pragmatic upper bound for “more than 4/9”, it seems natural to attribute that to the salience of 5/9 as an alternative). There are several possibilities as to how these bounds are arising: perhaps the presence of “more than 4/9” in the experiment has made ninths atypically salient as alternatives, or perhaps the value given reflects an impressionistic range interpretation (something more akin to the typicality effects postulated by Geurts and van Tiel (2013)) or a compromise between two possible interpretations. However, with this small sample, it is also highly plausible that the returned value represents an error, with the participant intending to give a percentage value that corresponded to a more salient fraction. We return to this question later.

3.4 Experiment 2

Version 2 of the experiment aimed to test whether the repeated use of terms on a specific scale, e.g. fifths, would cause alternatives drawn from the same scale to become more salient, or whether the presence of coarser-grained alternatives from other scales would condition the pragmatic readings that were obtained.

3.4.1 Materials

The following 14 items were presented in the context of the same cover story as in experiment 1:

- more/less than one quarter / a half / three quarters
- more than one fifth / two fifths
- less than three fifths / four fifths
- more than one tenth / seven tenths
- less than three tenths / nine tenths

The order of presentation was fixed. Participants first saw the items involving quarters and halves, then the items involving fifths, and finally the items involving tenths. Within each subset of items, the order was pseudorandomised.

3.4.2 Results

The numbers of error, literal and pragmatic responses for each item, along with the pragmatic responses given, are presented in the following table.

Table 2 Results of experiment 2

Item	Error	Literal	Pragmatic	Pragmatic lower/upper bounds

more than 1/4	1	5	14	29, 29, 30, 30, 30, 32, 32, 35, 35, 40, 49, 49, 49, 50
less than 1/4	0	15	5	15, 15, 15, 20, 20
more than 1/2	0	9	11	55, 60, 65, 65, 65, 70, 74, 74, 74, 74, 75
less than 1/2	0	9	11	25, 25, 26, 26, 26, 32, 34, 39, 40, 40, 41
more than 3/4	0	14	6	79, 80, 84, 85, 85, 90
less than 3/4	0	7	13	51, 51, 51, 51, 56, 60, 60, 60, 65, 65, 67, 70, 71
more than 1/5	2	7	11	23, 24, 24, 24, 24, 24, 25, 25, 25, 30, 39
more than 2/5	1	8	11	45, 45, 48, 49, 49, 49, 49, 49, 50, 59, 65
less than 3/5	4	7	9	49, 50, 51, 51, 51, 53, 55, 55, 56
less than 4/5	1	8	11	50, 55, 60, 61, 68, 71, 74, 76, 76, 76, 76
more than 1/10	2	8	10	14, 15, 19, 19, 19, 19, 19, 20, 20, 24
less than 3/10	1	8	11	20, 20, 20, 21, 21, 25, 25, 26, 26, 26, 26
more than 7/10	4	7	9	74, 74, 75, 75, 79, 79, 79, 79, 90
less than 9/10	2	7	11	50, 79, 81, 81, 81, 81, 81, 81, 81, 86, 87
Total	18	119	143	

3.4.3 Discussion

Participants generally appeared to find this version of the task easier, and returned a much lower error rate (18/280 = 6.4%). Just over half the total responses reflected pragmatic bounds.

The absence of finer-grained fractions than quarters from the initial part of the item list seems to have made a difference to the interpretation of “less than one quarter” and “more than three quarters”. In experiment 1, these attracted 9 literal to 9 pragmatic responses and 8 literal to 12 pragmatic responses, respectively. In this version, they attracted respectively 15 literal to 5 pragmatic responses and 14 literal to 6 pragmatic responses. Although neither of these differences reaches significance under Fisher’s exact test, the numerical difference is suggestive that participants in version 2 of the experiment are inclined to draw implicatures based on the next quarter, when presented with a series of stimuli involving quarters. However, this is not a hard and fast rule: for instance, most of the responses for “more than one quarter” reflect a tighter pragmatic bound than that provided by “not more than a half”.

It appears that making quarters salient, as scale points, has had some influence on the interpretation of subsequent expressions with fifths. “More than one fifth” attracted nine responses which could be interpreted as relating to the inference “not more than one quarter”. Similarly, “more than two fifths” attracted seven responses which could be interpreted as relating to the inference “not more than half (= two quarters)”, and 9 out of 11 pragmatic responses for this item involved a bound of 50% or lower. This appears to contrast with responses for this item in experiment 1, where the majority of pragmatic responses involved a bound above 50%.

Finally, fifths having been presented, the expressions with tenths gave rise to predictable interpretations: the majority of pragmatic responses made reference to the next scale point on the tenths scale (which in these cases is also a point on the fifths scale). In the case of “less than three tenths” and “more than seven tenths”, responses were split between referring to the next tenths scale point and referring to the next quarter.

These preliminary results suggest that the overall picture is complex. Generally, repeated reference to a scale appears to make its scale points more salient, which is evident in the interpretation of subsequent items referring both to that scale and finer-grained scales. However, even under these circumstances, there is no guarantee that participants will choose to draw implicatures based on separate coarse-grained scales – e.g. referring to quarters when cognising about values expressed in tenths – and may instead draw weaker inferences based on the current term's scalemates.

3.5 Experiment 3

Version 3 of the experiment was designed to test whether the repeated use of a less salient scale would exert any effect on the interpretation of subsequent expressions, either using that scale or using a different scale.

3.5.1 Materials

The following 16 items were presented in the context of the same cover story as in experiment 1:

- more/less than one sixth / one third / two thirds / five sixths
- more/less than one tenth / three tenths / seven tenths / nine tenths

The order of items was manipulated such that those involving sixths or thirds were presented first, then those with tenths. The order of presentation was pseudorandomised within each group of items.

3.5.2 Results

The numbers of error, literal and pragmatic responses for each item, along with the pragmatic responses given, are presented in the following table.

Table 3 Results of experiment 3

Item	Error	Literal	Pragmatic	Pragmatic lower/upper bounds
more than 1/6	7	2	11	19, 20, 20, 20, 25, 25, 25, 30, 30, 40, 50
more than 1/3	7	3	10	40, 40, 40, 40, 40, 45, 49, 50, 50, 67
more than 2/3	6	5	9	70, 74, 74, 74, 75, 75, 75, 80, 90
more than 5/6	10	4	6	90, 90, 90, 90, 95, 95
less than 1/6	7	6	7	5, 5, 7, 10, 10, 12, 12
less than 1/3	9	4	7	10, 15, 20, 25, 25, 26, 26
less than 2/3	6	4	10	40, 40, 44, 50, 55, 55, 55, 55, 60, 60
less than 5/6	14	2	4	60, 70, 78, 80
more than 1/10	3	6	11	15, 15, 15, 15, 15, 16, 20, 20, 20, 20, 25
more than 3/10	6	6	8	35, 39, 39, 40, 40, 40, 45, 45
more than 7/10	3	6	11	74, 75, 78, 79, 79, 79, 80, 80, 80, 90, 90
more than 9/10	5	11	4	92, 95, 97, 98
less than 1/10	1	15	4	4, 5, 5, 7
less than 3/10	5	6	9	10, 20, 20, 20, 20, 21, 25, 26, 26
less than 7/10	4	6	10	55, 60, 60, 60, 60, 60, 61, 65, 65, 67
less than 9/10	3	5	12	10, 70, 70, 70, 79, 80, 80, 81, 81, 85, 85, 85

Total	96	91	133	

3.5.3 Discussion
Unlike the case of quarters (experiment 2), the repeated use of sixths does not appear to elicit many pragmatic bounds that refer to thirds and sixths. Within the sixths scale itself, participants who derive pragmatic bounds tend to prefer more informative bounds, often referring to tenths. When then presented with expressions with tenths, participants do not tend to infer bounds referring to thirds or sixths, even in cases where these would be more informative than the bounds actually inferred. For instance, "less than seven tenths" attracts a modal lower bound of 60%, rather than 67%; all of the eight pragmatic upper bounds offered for "more than three tenths" exceed 34%. This suggests that the pragmatic influence of thirds and sixths is relatively weak in the participants' systems of fractions, compared to the influence of tenths. In addition, there are high error rates in the conditions involving "five sixths" in particular, perhaps suggesting that participants in this experiment had difficulty in evaluating this in percentage terms. As noted earlier, this particular experimental setup, relying on percentage responses, may be promoting the use of tenths – and perhaps disadvantaging the use of thirds and sixths – to an atypical extent.

3.6 Experiment 4

Version 4 of the experiment was intended as a control for version 3, reversing the order of presentation, to see whether influence could spread between scales in the opposite direction.

3.6.1 Materials
The following 16 items were presented in the context of the same cover story as in experiment 1:

- more/less than one tenth / three tenths / seven tenths / nine tenths
- more/less than one sixth / one third / two thirds / five sixths

The order of items was manipulated such that those involving tenths were presented first, then those with sixths and thirds. The order of presentation was again pseudorandomised within each group of items.

3.6.2 Results
One participant failed to finish the task, and a further participant gave decimal responses without a clear system, so their results are omitted. The numbers of error, literal and pragmatic responses for each item, along with the pragmatic responses given, are presented in the following table.

Table 4 Results of experiment 4

Item	Error	Literal	Pragmatic	Pragmatic lower/upper bounds
more than 1/10	3	3	12	14, 15, 15, 15, 19, 19, 19, 20, 20, 20, 20, 40
more than 3/10	3	3	12	35, 36, 39, 39, 39, 40, 40, 40, 40, 50, 55, 65

more than 7/10	1	4	13	74, 75, 75, 77, 79, 79, 79, 79, 80, 80, 80, 90, 90
more than 9/10	0	14	4	95, 95, 95, 98
less than 1/10	0	11	7	5, 5, 5, 6, 6, 6, 7
less than 3/10	6	4	8	5, 10, 17, 20, 20, 21, 25, 25
less than 7/10	3	4	11	20, 45, 60, 60, 60, 61, 61, 65, 66, 66, 67
less than 9/10	1	4	13	50, 55, 75, 76, 80, 80, 80, 80, 80, 81, 81, 82, 86
more than 1/6	5	3	10	19, 21, 25, 25, 25, 30, 31, 32, 35, 35
more than 1/3	6	3	9	39, 45, 45, 48, 49, 50, 50, 65, 65
more than 2/3	5	6	7	72, 74, 75, 75, 75, 80, 85
more than 5/6	4	7	7	89, 90, 90, 90, 92, 95, 95
less than 1/6	5	8	5	8, 10, 11, 12, 14
less than 1/3	3	6	9	5, 15, 19, 20, 25, 25, 25, 26, 29
less than 2/3	5	4	9	34, 34, 50, 51, 51, 55, 55, 60, 60
less than 5/6	6	4	8	70, 70, 70, 75, 75, 75, 76, 78
Total	56	88	144	

3.6.3 Discussion

The pattern of responses in this experiment closely mirrors that of experiment 3, suggesting that the order of presentation makes relatively little difference for these items. Again, there is little evidence of thirds and sixths being used as pragmatically relevant alternatives. By contrast, tenths continue to be pragmatically relevant when dealing with terms on the thirds/sixths scale, but there is no indication that the prior mention of tenths has promoted inferences involving tenths. In this case, terms involving “five sixths” did not appear to present any particular difficulty to the participants.

3.7 General discussion

The results from these small experiments strongly suggest that people are inclined to interpret modified fractions in a pragmatically restricted way, and that a lot of the readings they obtain are predictable on the basis of a quantity implicature analysis that considers other salient fractions as alternatives. More specifically, the results indicate that quarters and tenths are especially pragmatically relevant alternatives, under such an analysis, while the presence of other potential scale points (such as thirds) does not give rise to any striking pragmatic effects. The presence of literal responses, especially in cases where the pragmatically stronger alternatives are not obvious, suggests that participants have not felt obliged to give pragmatic responses under these experimental conditions.

There is also potential evidence here against an account of fractions in which we consider them simply to be quantifying over parts – for instance, taking “more than two-fifths” to mean “more than two of the fifths”. This approach would explain why some participants obtain interpretations involving the negation of the next point on the scale corresponding to this denominator (in this case, “not more than three-fifths”). However, it fails to predict interpretations also attested in these data which seem to rely upon scale points from other scales. Some participants appear to interpret “more than two-fifths” as implicating “not more than half” (and similarly for other expressions under test). To explain this purely in terms of quantifying over parts, we would need to read this as an instance of "more than two (fifths)" implicating "not more than two-and-a-half (fifths)", an enrichment which is not generally predicted to be available. Thus I take these results to cast doubt on the usefulness of an analysis of fractions in which the nominator is treated as though it were a free-standing numeral. Having said that, the availability of the weaker ("not more than three-fifths") implicature does suggest that the denominator actually present in the numeral is privileged in some weaker sense, and that the use of a particular fraction at least heightens the salience of alternative expressions that share this denominator.

Generally, if these experimental results are an accurate reflection of the reality, we can observe that the concept of granularity requires some modification to be applied to the case of fractions. As discussed earlier, it appears that major and minor scale points are not necessarily aligned in this domain; nor is it clear that coarse-grained scales (defined in terms of the distance between successive representation points) are necessarily less cognitively costly to work with than finer-grained alternative scales. This raises the broader question of whether Krifka's (2009) criteria for granularity scales should be seen as hard-and-fast rules or merely generalisations that admit potential exceptions.

That said, we should exercise some caution in interpreting the experimental results presented here as evidence that tenths, in particular, are necessarily salient scale points. Recall that participants were asked to respond using percentages: this system draws attention to the possibility of divisibility by ten, and could be argued to make the tenths scale points more salient than would otherwise be the case (because they correspond to round numbers in percentage terms). Similarly, this methodology could be argued to privilege quarters and fifths, whose scale points are expressed as round integer percentages, over thirds, sixths, sevenths etc., whose scale points are not.

Even with this caveat, the experimental findings support a view of the fraction system – as represented by hearers – that is more complex than would be supposed based on a naïve application of granularity criteria. Participants are able to access readings of modified fractions that appear to rely on alternatives of the same or coarser granularity, but also sometimes able to access readings that rely on alternatives of finer granularity.

The availability of such alternatives is for instance, crucial in obtaining restricted readings for expressions such as "more than (a) half", as was achieved by a majority of participants in experiment 2. It is striking that these participants all favoured interpretations in which "more than a half" conveyed maximally 75% (in most cases, considerably less). Speculatively, we might note a potential point of contact here with the literature on "most" (e.g. Solt 2016). As discussed earlier, a crucial observation is that "most" is not judged felicitous when referring to quantities that are just over 50%, as in (1), repeated below.

(1) Most of the American population is female.

If it is the case that "more than (a) half" attracts a narrower interpretation than would be predicted on its semantics, this might suggest that "more than (a) half" is a particularly good competitor to "most" for values within its pragmatically typical range (i.e. a little over 50%). This in turn might suggest that the distribution of the meanings or interpretations of "most" should skew higher. Of course, one could counter that "most" would then run into competition from other alternatives ("more than two thirds", "more than three quarters") –

but it is perhaps reasonable to suppose that they are not such salient options as “more than a half”, and consequently that “most” is more likely to be the optimal expression when we are dealing with values in that somewhat higher range (assuming that the speaker does not wish to commit to a precise value).

Finally, it is interesting to note that a substantial minority of the responses elicited appear to reflect a pragmatic enrichment that does not appear to correspond to a specific and highly salient alternative – for instance, “more than two fifths” being judged to convey a value of 40-47% (two responses in experiment 1). Apart from error on the part of the participants, there are several possible reasons for this. It could be that the participants are giving an impressionistic response based on something like a typicality effect associated with the expression that was used (for instance, having a notion that a 7% range is somehow “about right” for this expression). Alternatively, their response may reflect some kind of compromise between competing possible enrichments, based on different alternatives. A third possibility is that the participants are indeed drawing quantity implicatures based on salient alternatives, but that these are not the kinds of alternatives that have been considered in this chapter – for instance, participants might think that something above 47% would have been better described as “about half”. In order fully to understand the possibility of readings of this latter type, we would need to explore the domain of expressions of proportion in much greater generality. However, it is perhaps reasonable to assume that the availability of such alternatives will be attenuated in experiments which consistently omit these alternatives, as in this case (although note that “about half” was presented as an example in the cover story). We might therefore hope that suitably designed experiments will enable us fairly straightforwardly to control for any interference effects from alternative expressions from outside the domain of interest.

4 Conclusion

The relatively understudied domain of fractions appears to exhibit a complex structure which can be seen as a reflection of cognitive preferences about divisibility and salience. Modified fractions give rise to pragmatic enrichments that resemble quantity implicatures, and these indicate the presence of salient alternatives within the system. In the experiments presented here, quarters and tenths are shown to constitute especially salient scale points, and can potentially be seen as the “coarse-grained” representation points in the domain of fractions, although the normal rules of granularity do not straightforwardly apply here. Future work will aim to map the salience of fractions more thoroughly, and also take into account their relation to other expressions of proportional quantity.

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