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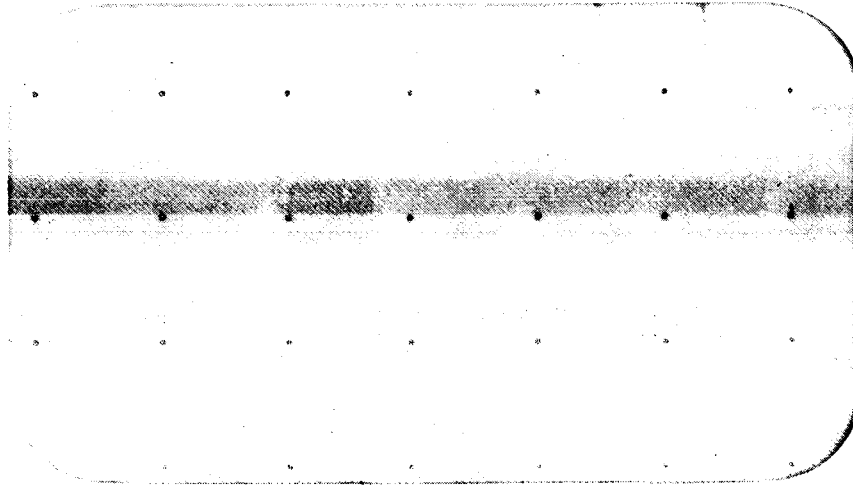
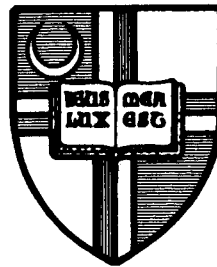# SCHOOL OF ENGINEERING

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MODIFIED INDEPENDENT MODAL SPACE CONTROL

METHOD FOR ACTIVE CONTROL OF

FLEXIBLE SYSTEMS

BY

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ABSTRACT

A Modified Independent Modal Space Control (MIMSC) method is developed for designing active vibration control systems for large flexible structures. The method accounts for the interaction between the controlled and residual modes. It incorporates also optimal placement procedures for selecting the optimal locations of the actuators in the structure in order to minimize the structural vibrations as well as the actuation energy.

The MIMSC method relies on an important feature which is based on "Time Sharing" of a small number of actuators, in the modal space, to control effectively a large number of modes.

Numerical examples are presented to illustrate the application of the method to generic flexible systems.

The obtained results suggest the potential of the devised method in designing efficient active control systems for large flexible structures.

INTRODUCTION

Considerable attention has been directed recently towards the design of active vibration control systems for large flexible structures. The strategies employed in the design of such control systems are based primarily on the modal control methods whereby the flexible structures are controlled by controlling their dominant modes of vibrations. Generally, these modal control strategies belong to either the class of the coupled methods [1-6] or to the class of the independent modal space control (IMSC) method developed by Meirovitch and Coworkers [7-12]. In the first class, the closed-loop equations of the system are coupled via the feedback control such that the optimal computation of the feedback gains requires the solution of a coupled matrix Riccati equation [3-6]. For large flexible structure the solution of the resulting Riccati equation can pose serious difficulties which limit significantly the applicability of the coupled modal control methods. The IMSC method avoids, however, such limitations as the control laws are designed completely, in the modal space maintaining the originally uncoupled open-loop equations of the system as a set of independent second-order equations even after including the modal feedback controllers. Meirovitch el al [7-12] showed, under such conditions, that it is possible to compute, in a close form, the optimal modal feedback gains. This feature makes the IMSC method computationally attractive and lends it suitable for controlling large structures.

However, the present study is initiated to modify the IMSC method to account for the spillover from the controlled modes into the uncontrolled

modes due to the use of fewer actuators than the modeled modes. The IMSC is also modified to incorporate an optimal placement procedure that will enable the selection of the optimal location of the actuators in the structure to ensure minimal amplitudes of oscillation and input control energy. A third modification of the IMSC is to include an efficient algorithm for time sharing a small number of actuators, in the modal space, to control a large number of modes of vibrations.

With these modifications, the MIMSC method would provide more effective and faster control of the vibration of flexible systems.

MODIFIED INDEPENDENT MODAL SPACE CONTROL METHOD

Modal Description of Flexible Systems

Complex flexible systems can be modeled dynamically by a discrete finite element model as follows :

$$M\ddot{\delta} + K\delta = F \tag{1}$$

where M   is the overall mass matrix of the structure

     K   is the overall stiffness matrix of the structure

     δ and $\ddot{\delta}$ are the displacement and acceleration of the nodal points of the structure

     F   is the vector of the external and control forces acting on the structure

Equation (1) is put in the modal space by using the following weighted modal transformation :

$$\delta = \phi U \tag{2}$$

where U is the modal coordinates of the system

ϕ is the weighted modal shape matrix of the eigenvectors of the flexible system

Using such transformation, reduces the coupled equation of motion (1) to the following uncoupled form :

$$\ddot{U} + \lambda U = f \tag{3}$$

where λ is a diagonal matrix of the eigenvalues of the system

f is the modal force matrix given by

$$f = \phi^T F \tag{4}$$

or

$$[f] = \begin{bmatrix} f_c \\ f_R \end{bmatrix} = \begin{bmatrix} \phi_1(l_1)\ldots\ldots\phi_1(l_c) & \phi_1(l_{c+1})\ldots\ldots\phi_1(l_N) \\ \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\ \phi_c(l_1)\ldots\phi_c(l_c) & \phi_c(l_{c+1})\ldots\phi_c(l_N) \\ \phi_{c+1}(l_1)\ldots\phi_{c+1}(l_c) & \phi_{c+1}(l_{c+1})\ldots\phi_{c+1}(l_N) \\ \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\ \phi_N(l_1)\ldots\phi_N(l_c) & \phi_N(l_{c+1})\ldots \phi_N(l_N) \end{bmatrix} * \begin{bmatrix} F_c \\ F_R \end{bmatrix} \tag{5}$$

where $f_{c,R}$ are the modal forces on the controlled and residual modes respectively.

$F_{c,R}$ are the physical forces on the controlled and residual modes.

$\phi_i(l_j)$ is the modal shape at mode i and location l_j.

The above equation can be rewritten as :

$$\begin{bmatrix} f_c \\ f_R \end{bmatrix} = \begin{bmatrix} B_{cc} & B_{cR} \\ B_{Rc} & B_{RR} \end{bmatrix} \begin{bmatrix} F_c \\ F_R \end{bmatrix} \tag{6}$$

If only C modes are controlled with equal number of control forces F_c, then $F_R=0$ and equation (6) reduces to :

$$f_c = B_{cc}F_c \qquad\qquad (7)$$

and

$$f_R = B_{Rc}F_c \qquad\qquad (8)$$

Effect of Control Spillover

In the IMSC method, it is assumed that the control forces F_c will not contribute to the excitation of the residual higher order modes. Accordingly, it was assumed that there is no control spillover from the controlled modes into the uncontrolled modes. Mathematically, this means that the IMSC method assumes that $f_R=0$. This of course can only be true if the number of controlled modes is very large compared to the number of residual modes or when the residual modes are at much higher frequency band than the controlled modes. If these two conditions are not satisfied, then there will be considerable interaction between the controlled and residual modes.

The MIMSC method considers such interaction by calculating the optimal modal control forces [f_c] using the IMSC close form solution of the Riccati Equation such that the control force f_i of the ith mode, as given by [7], is :

$$f_i = -(g_1\omega_i u_i + g_2\dot{u}_i)/R \qquad\qquad (9)$$

where R   is a factor that weighs the importance of minimizing the vibration with respect to the control forces.

ω_i is the resonant frequency at the ith normal mode.

u_i, $\overset{\bullet}{u}_i$ are the modal displacement and velocity respectively.

g_1, g_2 are the modal position and velocity feedback gains given by [7] as :

$$g_1 = -\omega_i R + \sqrt{(\omega_i R)^2 + \omega_i^2 R} \tag{10}$$

$$g_2 = \sqrt{2R\omega_i [-\omega_i R + \sqrt{(\omega_i R)^2 + \omega_i^2 R}] + \omega_i^2 R} \tag{11}$$

Accordingly, the displacement u_i and velocity $\overset{\bullet}{u}_i$ at the ith mode can be feedback and used along with equations (9), (10) and (11) to determine the modal control force f_i.

Once these forces are calculated, equation (7) is solved to give the physically applied control forces F_C as follows :

$$F_C = B_{CC}^{-1} * f_C \tag{12}$$

Then equation (8) is used to calculate the modal forces f_R that would excite the residual modes which are generated by the spillover from the controlled modes. Definitely these f_R are not equal to zero as originally assumed in the IMSC method.

Equations (15) can then be integrated with respect to the time to determine the modal displacements (u_i) and velocities ($\overset{\bullet}{u}_i$) which can, in turn, be used again to compute the modal forces f and so on.

From the modal displacements and velocities, the physical state (δ) of the flexible system can be determined from equation (2). A relationship can therefore be established between the physical state of the system and the physical control forces F_C applied to it.

Optimum Placement of Actuators

It is very important to point out here that the magnitude of the

modal forces f_c depends primarily on the magnitude ω_i's of the controlled modes as well as the modal state variables u and $\dot{u}$. On the other hand, the magnitude of the actual physical control forces F_c depends mainly, for a given controlled mode, on the point of application of these forces as defined by the matrix B_{cc}^{-1}. Therefore, minimizing f_c does not necessarily means that F_c will be minimum inspite of the fact that it is represented as a linear combination of f_c. This is simply because the coefficients of the linear combination, which are elements of the B_{cc}^{-1} matrix, depend on the placement strategy of the control forces F_c. One could still find an optimally placed set of physical control forces F_c such that the physical displacements and control forces would assume minimum value.

This optimum placement of the physical control forces is an important feature of the MIMSC method and will be demonstrated to be essential part of the design of the active control system.

One should stress here also that if all the modes are controlled and there is no residual modes then the conditions for minimizing f_c will make F_c minimum as well. But, in real large structures this will be unlikely to happen as the number of controlled modes is much smaller than the number of modeled modes. Therefore, it is essential to augment the IMSC method with an optimal placement algorithm to guarantee efficient design of the control system.

The optimum placement of the actuators is implemented through the use of the Univariate Search method which varies the location of one actuator at a time in order to :

$$\text{Minimize} \quad \int (\delta^2 + R F_c^2)\, dt \qquad\qquad (13)$$

In other words, the optimal placement algorithm minimizes the weighted sum of the amplitudes of vibration and the generated control forces. The weighing factor R is selected by the designer to emphasize the importance of damping out the vibration over the expended control energy (when R<<1) or vice versa when R>>1. Equal importance of the two parameters is achieved with R=1.

Time Sharing of Actuators in the Modal Space

The MIMSC method incorporates also an extremely important feature which is based on the "TIME SHARING" of a small number of actuators in the modal space to control large number of modes.

Two time sharing control strategies are considered to generate the modal control forces. The first is sequential and the second is based on the modal energy.

In the sequential time sharing strategy, the control forces are computed, at the first time interval, to control the first through the Cth modes using C actuators. Then, at the second time interval, the control signals are computed so as to control the second through the $(C+1)^{th}$ modes followed by commands to control the third through the $(C+2)^{th}$ modes and so on until all the modeled modes are controlled in this sequential fashion. Once all the modeled modes have received their share from the control action the cycle is repeated again to effectively damp out all the modes of vibration with few number of actuators. This strategy will be shown to result in efficient control of the vibration of large structures with relatively small number of actuators when the IMSC

fails to do so.

Better yet, control of the vibration can be achieved when the time sharing is based on the modal energy strategy particularly when the number of controlled modes is very small compared to the uncontrolled modes.

In this strategy, the modes of vibrations of the flexible system are ranked according to their modal energy level. If C actuators are to be used, then these actuators will be dedicated, at any instant of time, to control the C modes that have the highest modal energy. In this way, the actuators will first attenuate the modal energy of the controlled modes. During that time the control spillover will excite the uncontrolled modes. When the modal energy of the uncontrolled modes starts exceeding that of the controlled modes, the actuators are switched to control these high energy modes in order to damp out their vibrations. Such time sharing of the actuators between the modes will eventually bring all these modes under control.

Figure (1) outlines a flowchart of the MIMSC method indicating the main steps of optimal placement and time sharing of the actuators as well as the consideration of the spillover between the controlled and residual modes.

Application of MIMSC

The MIMSC method is utilized to design active vibration controllers for flexible system when subjected to specific external loading and end conditions. The resulting dynamic performance of these systems is compared with their performance when controlled by the IMSC in order to

illustrate the merits and potential of the MIMSC method as a viable and efficient method for actively controlling the vibration of large systems with only few actuators.

NUMERICAL EXAMPLES

I. Multi Spring-Mass System

Figure (2) shows a multi spring-mass system which is considered as a simple example of a flexible system to illustrate the intricacy of the MIMSC method. The main dynamic characteristics of this system are given in Table (1).

A. Control by two actuators with weighting factor R=1

(i) Using IMSC method

The three masses of the flexible system shown in Figure (2) are displaced initially 1,-1 and 0 respectively from their equilibrium positions and then left to vibrate under the action of an IMSC controller with all the states are observed. The controller is designed to control the first two modes of vibrations through the use of two actuators placed at the first and second masses.

Figures (3-a) and (3-b) show the time history of the amplitudes of vibration of the three masses and the associated control forces respectively.

Figure (3-a) indicates that after an initial transition period of about 4 seconds, a state of limit cycle is attained. During this state, the first and the third masses undergo in-phase oscillations which are of the same amplitude and frequency. The second mass vibrates, however, in the opposite direction at the same frequency but at a higher amplitude.
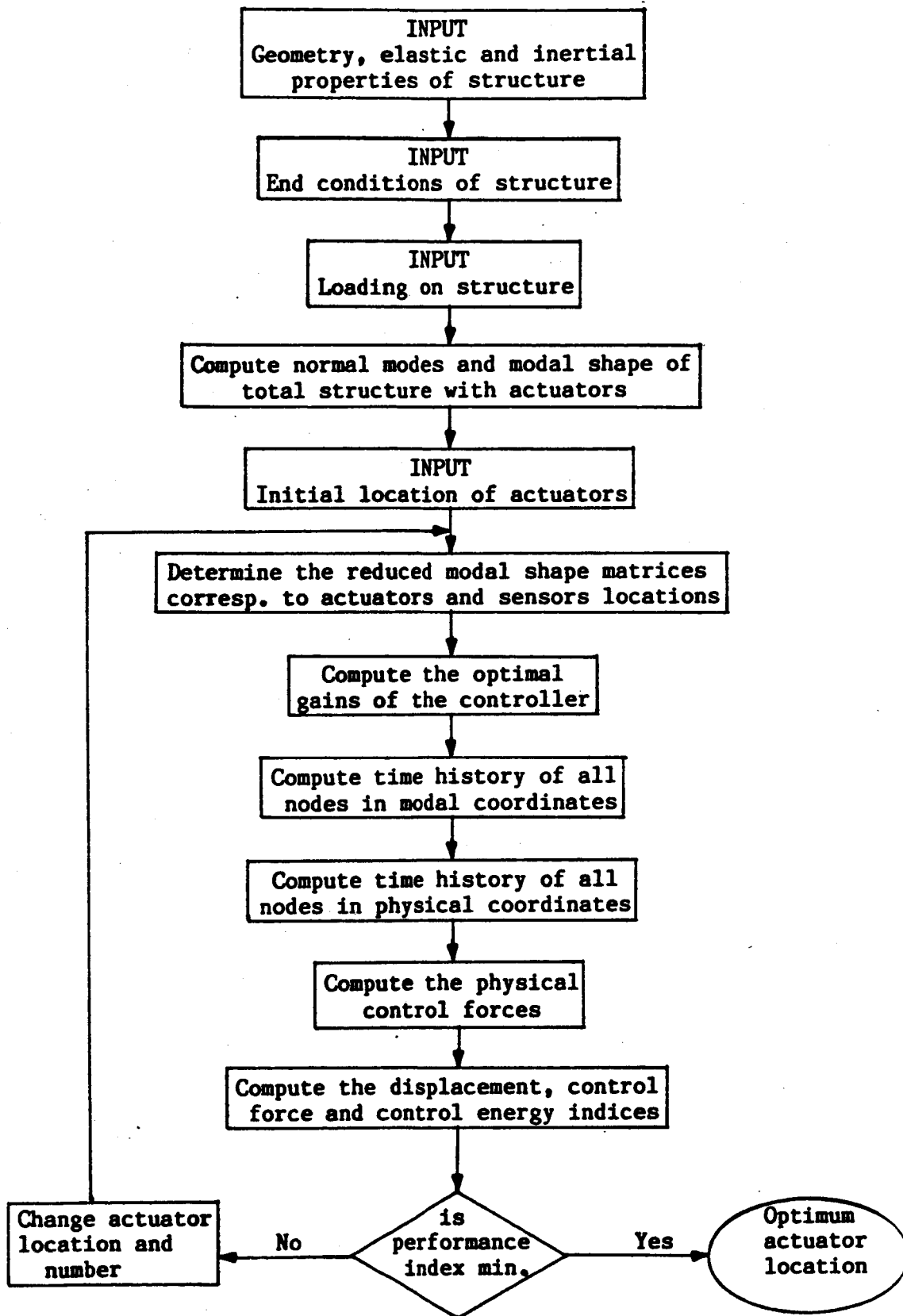
```
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                    │         INPUT           │
                    │ Geometry, elastic and inertial │
                    │   properties of structure │
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                    │         INPUT           │
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                    │   Loading on structure  │
                    └─────────────────────────┘
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                ┌─────────────────────────────────┐
                │ Compute normal modes and modal shape of │
                │     total structure with actuators │
                └─────────────────────────────────┘
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                    │         INPUT           │
                    │ Initial location of actuators │
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              │ Determine the reduced modal shape matrices │
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Figure (1) - Flow chart of the MIMSC Computational Algorithm

$$m_i = 1 \qquad k_i = 1$$
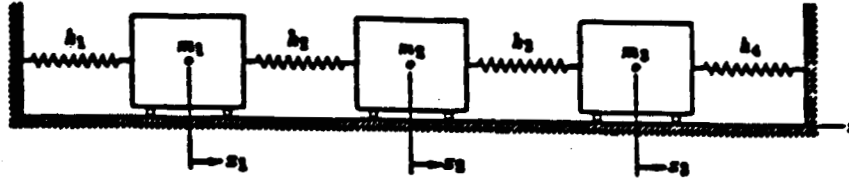
Figure (2) - A spring-mass system

Table(1) - Dynamic characteristics of the spring-mass system

```
--------------------------------
          Stiffness Matrix
--------------------------------

    2.0000    -1.0000     0.0000
   -1.0000     2.0000    -1.0000
    0.0000    -1.0000     2.0000
--------------------------------
            Mass Matrix
--------------------------------

    1.0000     0.0000     0.0000
    0.0000     1.0000     0.0000
    0.0000     0.0000     1.0000
--------------------------------
            Eigenvalues
--------------------------------

    0.5858     2.0000     3.4142
--------------------------------
            Eigenvectors
--------------------------------

    0.5000    -0.7071     0.5000
    0.7071     0.0000    -0.7071
    0.5000     0.7071     0.5000
--------------------------------
```

Relating such an observation to the dynamic characteristics of the system, given in Table (1), one finds that this limiting state corresponds to the third normal mode of vibration. Accordingly, the IMSC method has been successful in damping out, as intended, the first and second modes of vibration during the first 4 seconds. The method fails, however, to reduce the excessive amplitudes of vibration of the three masses at the third mode of vibration. Such drawback can be related directly to the fact that the two actuators have been utilized only to eliminate the first two modes and once this goal has been achieved the two actuators ceased to provide any control action as can be clearly seen from Figure (3-b). In other words, the two actuators became completely idle inspite of the fact that system is still vibrating. This observation constitutes the main motivation for the concept of time sharing the actuators in the modal space which is the basic feature of the MIMSC method.

(ii) Using MIMSC method

With the time sharing concept, the MIMSC utilizes effectively the installed actuators such that these actuators will not cease to operate unless the vibrations of the system is completely damped out. Accordingly, in the considered example, the two actuators are powered by signals to eliminate all the three modes of the system and not only the first two modes as in the IMSC. This is achieved by time sharing the two actuators, among the three modes, either sequentially or based on the maximum modal energy ranking.

Figures (4-a) and (4-b) show the time history of the amplitudes of vibrations of the three masses and the associated control forces of the
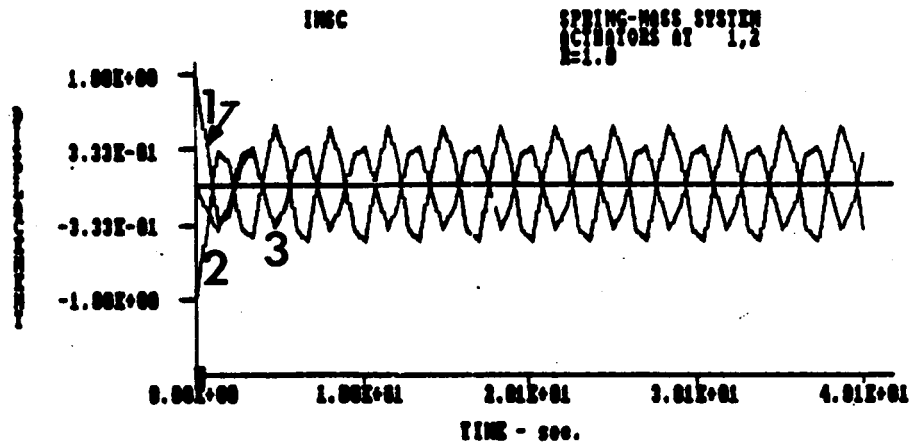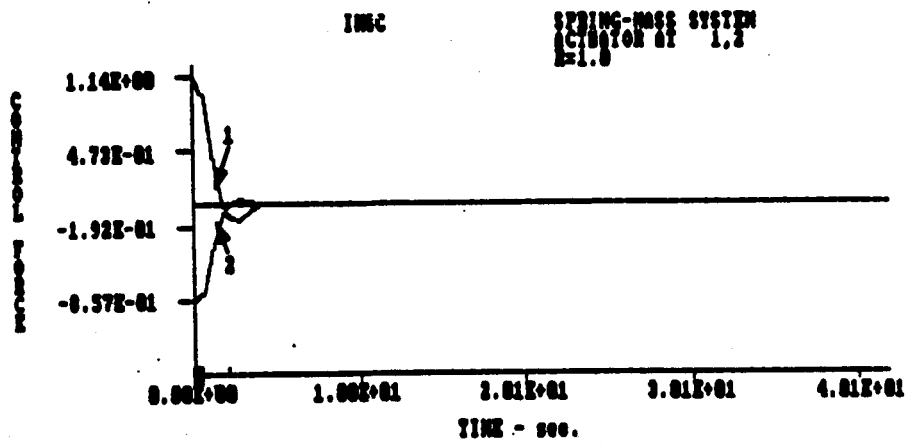
Figure (3) - Time history of amplitude of vibration and control forces
for spring-mass system using IMSC method with two
actuators (R=1)
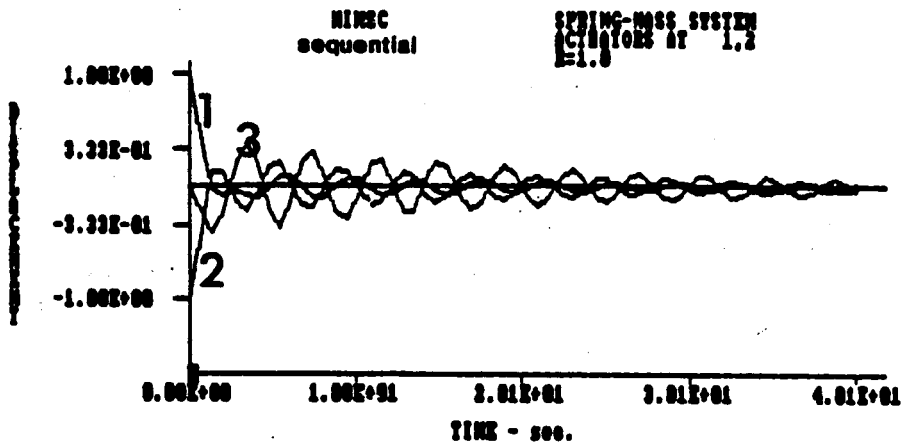
(a)
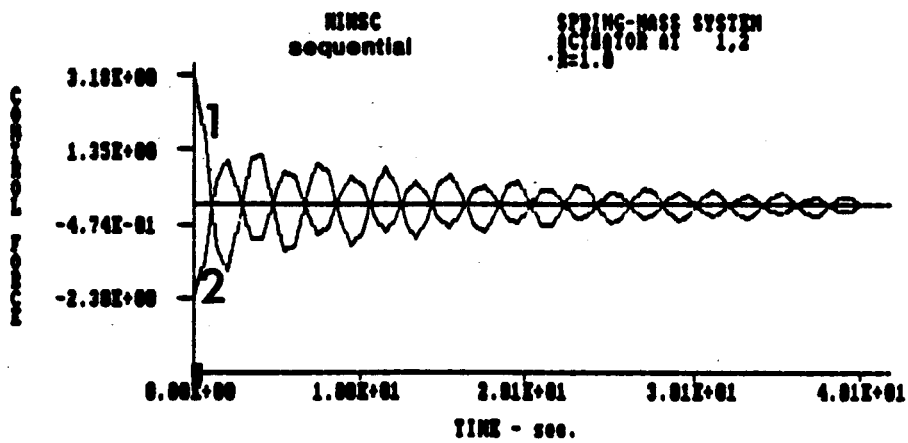
(b)

Figure (4) - Time history of amplitude of vibration and control forces
for spring-mass system using MIMSC with sequential
time sharing of two actuators (R=1)

two actuators respectively when the MIMSC utilizes a sequential time sharing strategy.

Figure (4-a) indicates that sharing the small number of actuators among a larger number of modes has been effective in damping out the amplitudes of vibration of all the modes. Such a process is done by making these actuators work as long as there is vibration to be damped out as can be seen from Figure (4-b). This is unlike the same two actuators which have been only partially utilized by the IMSC method as indicated in Figure (3-b).

When the time sharing strategy is based on dedicating the two actuators to control the two modes that have the highest modal energy $(U^2 + \dot{U}^2)$, at any instant of time, then the resulting time history of the amplitudes of vibrations of three masses and the associated control forces are as shown in Figures (5-a) and (5-b) respectively.

Figure (5-a) demonstrates the effectiveness of the scheme of time sharing based on the ranking of the modal energy in suppressing the vibration of the three-mass system in a fashion which is more efficient than the sequential scheme and definitely than the IMSC method.

A better quantitative comparison between the two schemes can be established based on the displacement, control force, and control energy indices U_d, U_c and U_E which are given by :

$$U_d = \sum_{t=0}^{t=t^*} \sum_{i=1}^{N} {}_i^2 * \Delta t \tag{14}$$

$$U_c = \sum_{t=0}^{t=t^*} \sum_{i=1}^{N} F_i^2 * \Delta t \tag{15}$$

$$U_E = \sum_{t=0}^{t=t^*} \sum_{i=1}^{N} | \delta_i * F_i | \Delta t \qquad (16)$$

where   N is the number of d.o.f. of system

Δt is the integration time increment

t^* is the maximum time limit of integration

Table (2) summarizes the results of such a comparison.

Table (2) - Effect of the strategy of time sharing two actuators on
displacement, control force and control energy indices for
spring-mass system with R=1.

Strategy	Displacement index	Control forces index	Control energy index
Sequential	2.44	32.36	7.14
Modal energy	2.15	13.99	3.04

B.   Control by two actuators with weighting factor R=100

The effect of increasing the weighting factor R to 100 on the time history of the amplitudes of vibration of the three masses and the associated control forces is shown in Figures (6-a) and (6-b) respectively. The figures emphasis the same trends observed for R=1 however and more importantly show that the MIMSC method is still very effective in damping out quickly the vibrations of the three masses but with control forces of much smaller magnitudes. For example, when R=100 the control forces required with sequential and modal energy time sharing assume maximum values of 0.460 and 0.524 respectively. These magnitudes are at least 6 times lower than those computed for R=1 as can be seen in
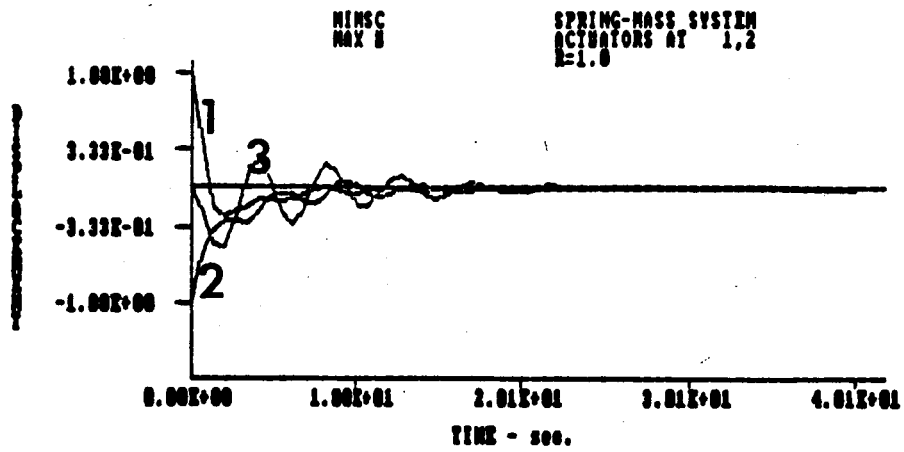
(a)
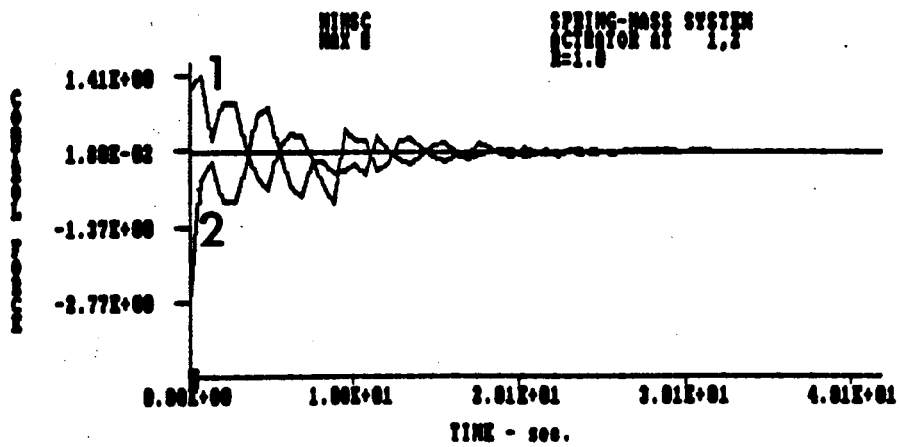
(b)

Figure (5) - Time history of amplitude of vibration and control forces for spring-mass system using MIMSC with modal energy time sharing of two actuators (R=1)
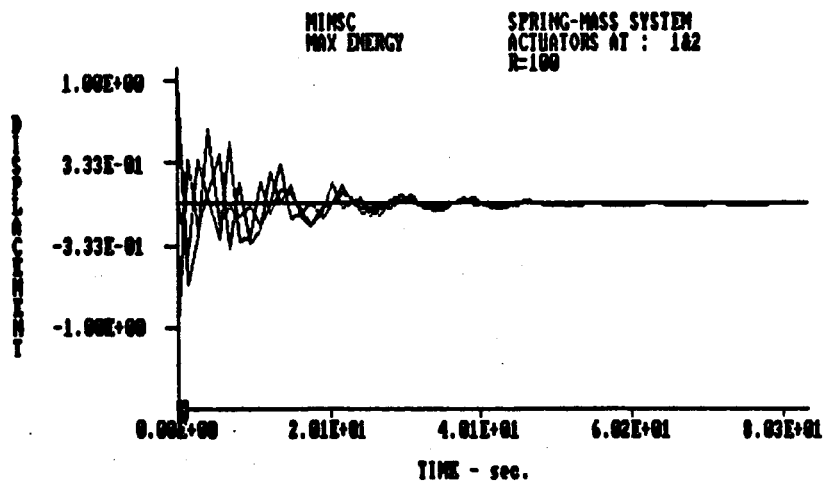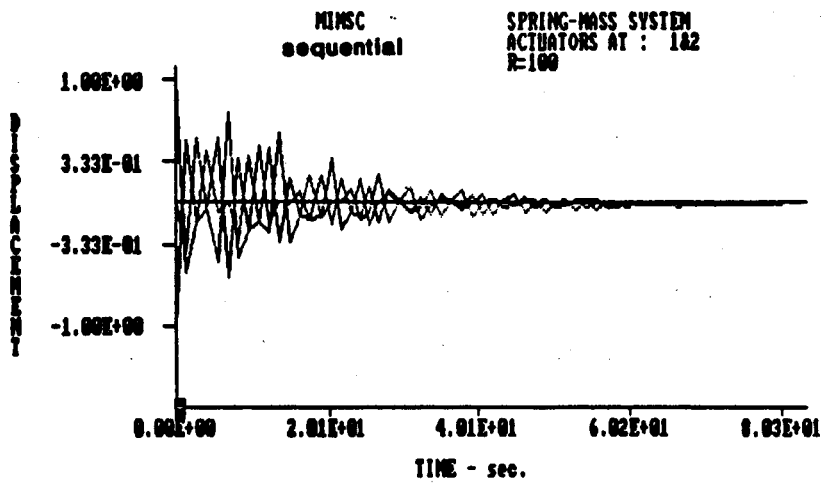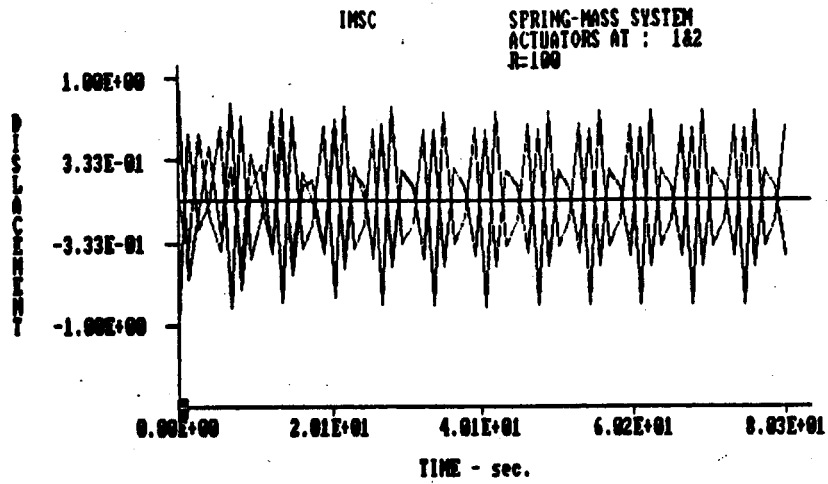
Figure (6-a) -   Time history of the amplitudes of vibration of
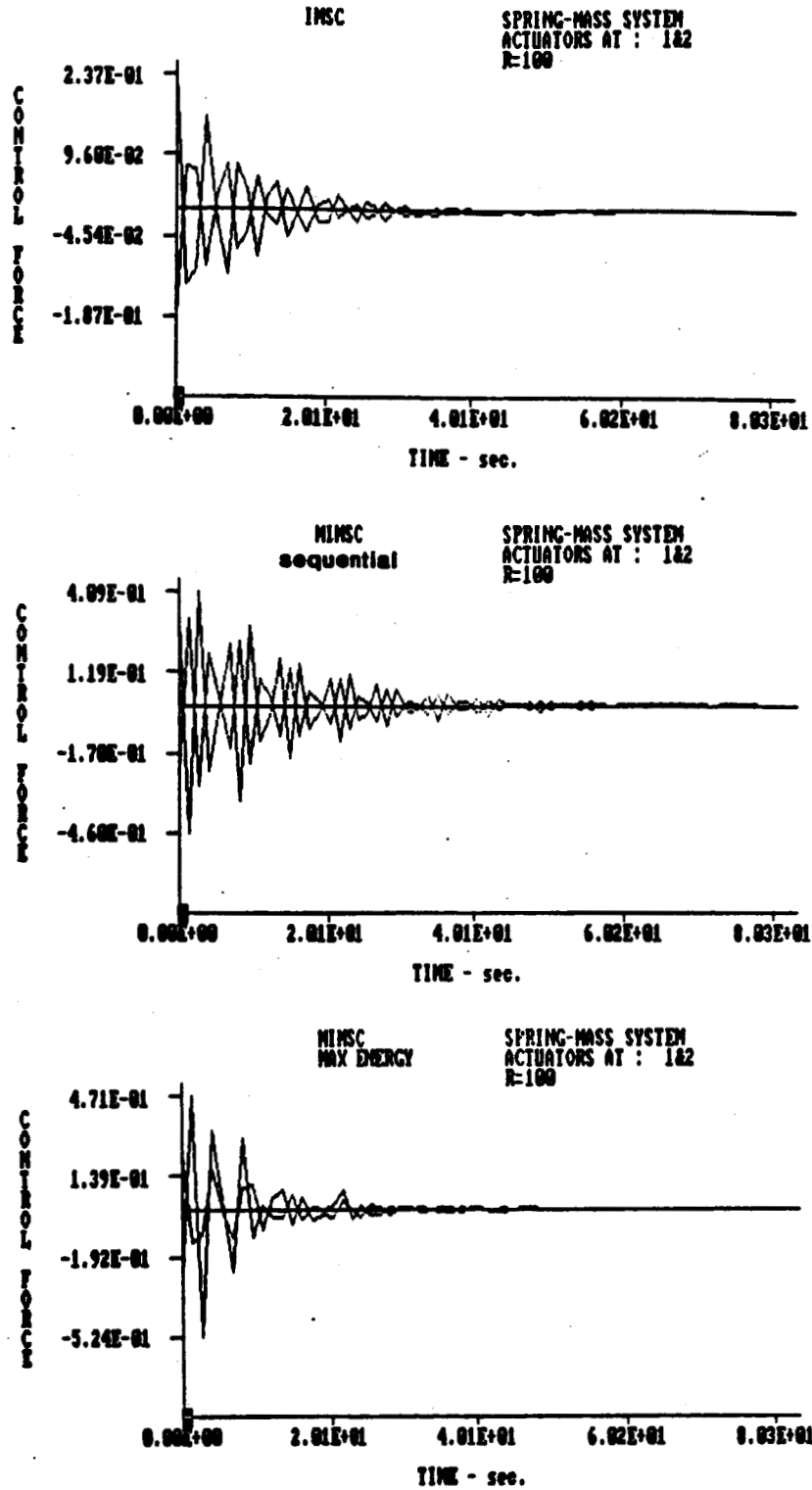spring-mass system with two actuators and R=100

Figure (6-b) - Time history of the control forces on the spring-mass
system with two actuators and R=100

Figures (4-b) and (5-b).

Table (3) lists the effect of increasing R to 100 on the displacement, control force and control energy indices when the time sharing is based on sequential and modal energy strategies.

The table indicates a considerable reduction in the control force and energy indices that are more dominant than the increase in the displacement index.

Table (3) - Effect of the strategy of time sharing two actuators on displacement, control force and control energy indices for spring-mass system with R=100.

Strategy	Displacement index	Control forces index	Control energy index
Sequential	8.40	1.960	5.05
Modal energy	5.03	1.462	4.03

In the case of the IMSC method, changing R did not influence at all the amplitudes of vibration at steady state but it did, however, prolong the duration of the transition time needed to eliminated the first two modes.

Accordingly, with optimally selected weighting factor R the MIMSC method can effectively suppress the vibration without the need for excessively large control forces.

C. Control by one actuator with weighting factor R=100

To demonstrate more dramatically the effectiveness of the MIMSC in controlling the vibration of large number of modes with a small number of actuators a single actuator, placed at mass 1, is used to actively

control the three spring-mass system.

Figures (7-a) and (7-b) show the time history of the amplitudes of vibrations and the associated control forces respectively as obtained by the IMSC and MIMSC methods.

Again the figures emphasis the potential of the MIMSC particularly with its modal energy time sharing strategy as a viable active control.

Table (4) summarizes the results obtained from the analysis of these figures.

Table (4) - Effect of the strategy of time sharing one actuator on displacement, control force and control energy indices for spring-mass system with R=100.

Strategy	Displacement index	Control forces index	Control energy index
Sequential	14.129	2.752	1.753
Modal energy	11.833	2.960	1.289

II. Cantilever Beam

A. The beam system

Figure (8) shows a steel cantilever beam modeled by a 3-finite element modal that has 3 d.o.f. of linear translation and 3 d.o.f. of angular rotations with node 1 fixed. The beam is 0.15m long and has rectangular cross section which is 0.0125m wide and 0.0021m thick. For this beam, the normal modes of vibrations are found to be 73, 382.6, 937.7, 1582.8, 2134.1 and 2519.0 Hz respectively.
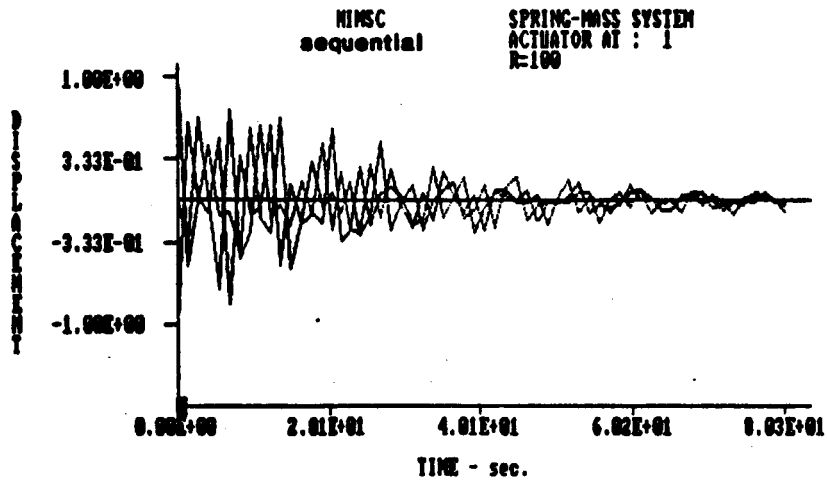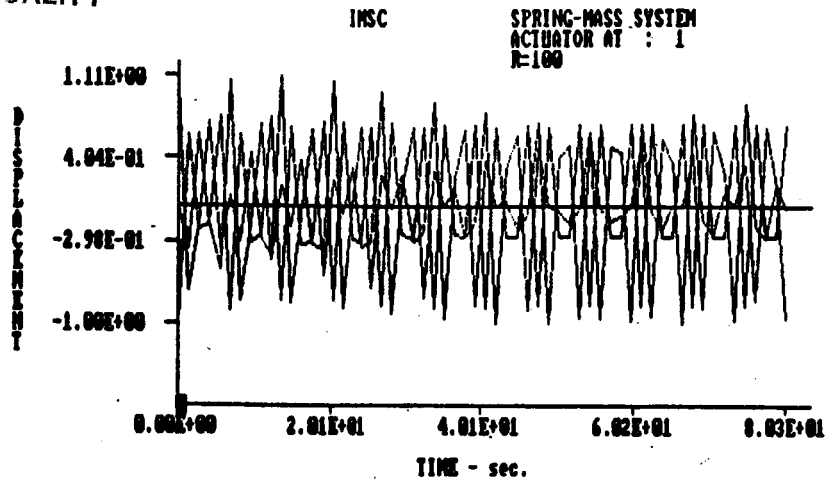
B. Performance with one actuator
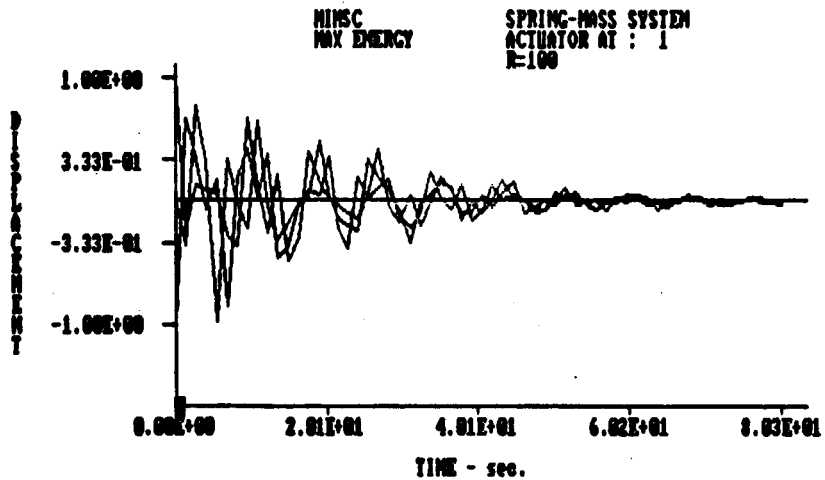
Figure (7-a) - Time history of the amplitudes of vibration of
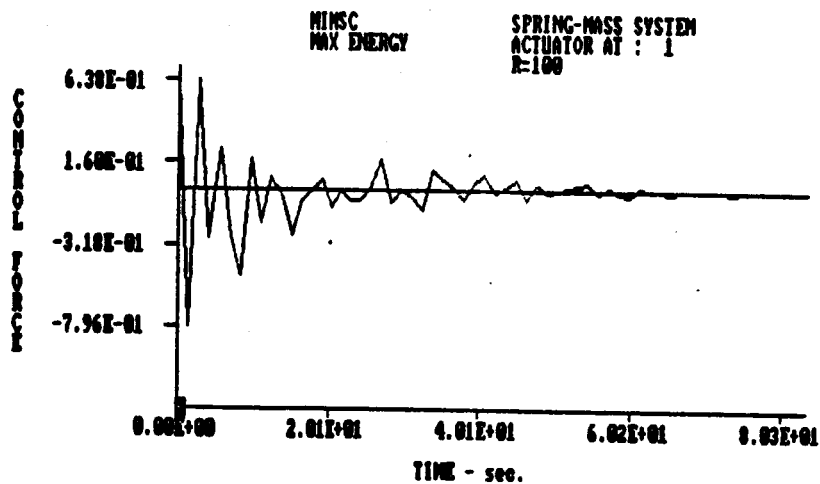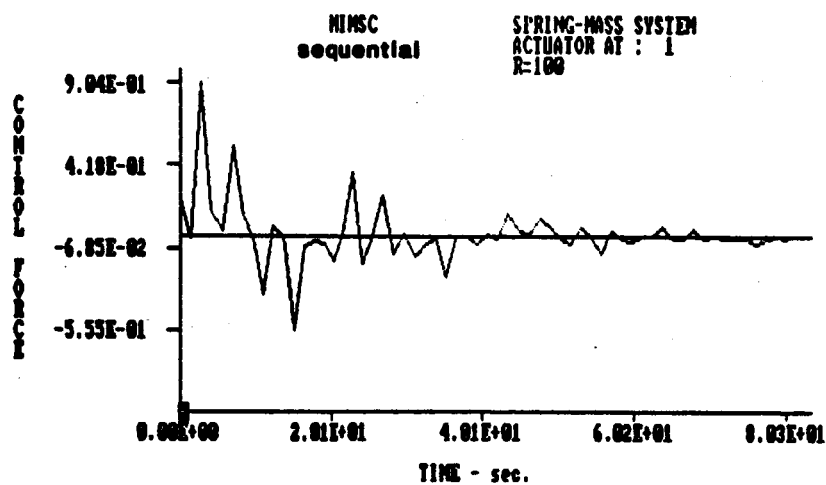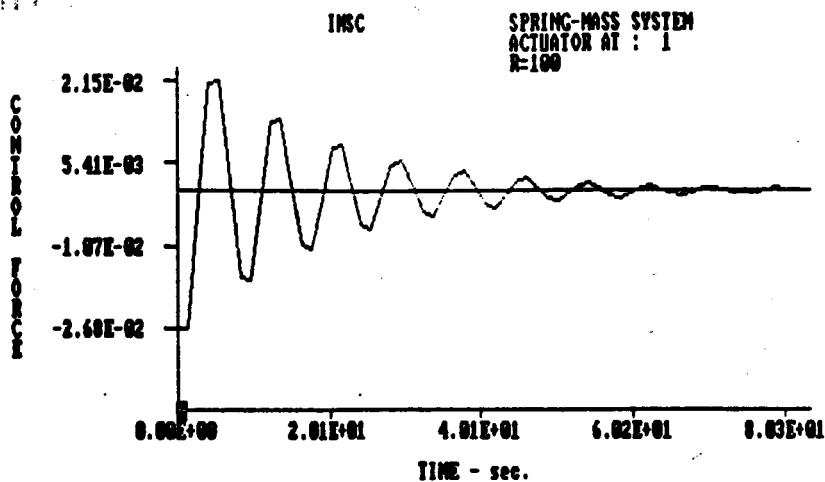spring mass system with one actuator and R=100

Figure (7-b) Time history of the control forces on the spring-mass
system with one actuator and R=100

In this example, the beam is assumed to be controlled by one linear actuator placed at its free end, i.e. at node 4. The beam is subjected to an impulsive load of magnitude 1.0N and duration of 0.1ms.

The IMSC and the MIMSC, with its two time sharing strategies, are utilized to design the active controller of the beam. These methods are compared as far as their effectiveness in damping out the vibration of the beam as shown in Figure (9).

The figure indicates that the IMSC is again successful in suppressing the lowest mode of vibration but all the higher modes remain totally undamped. On the contrary, the MIMSC with the modal energy time sharing exhibits complete control over all the modes and effective damping is demonstrated. Also the maximum amplitude of oscillation of the beam is observed to be, in this case, about 10.7% lower than that obtained with the IMSC method.

Considering, however, the MIMSC method with sequential time sharing it can be seen that this strategy is not as effective as the modal energy strategy or the IMSC method. The reasons are obvious and it is important to cite them. First, the beam being a six-mode system controlled by one actuator, then, it is essential to utilize this actuator in the best possible way. Dedicating it to the first mode, as in the IMSC, is found adequate to damp out large amplitude oscillations but inadequate to take care of the high frequency jitters. Using this actuator to control any mode in any sequence without due consideration to its contribution to the system's vibration or energy is definitely a reason behind the ineffectiveness of the sequential time sharing strategy. But, once the actuator is set to control always the mode that has the highest modal
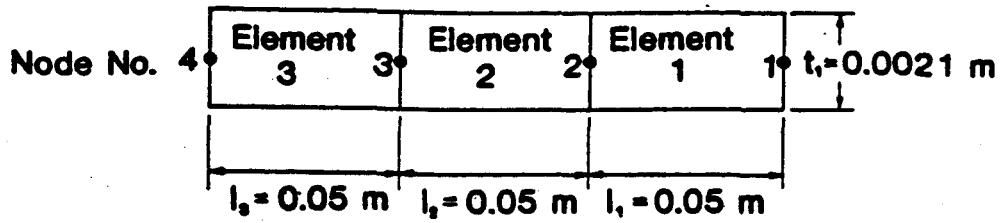
Figure (8) - Layout of a 3-element flexible beam

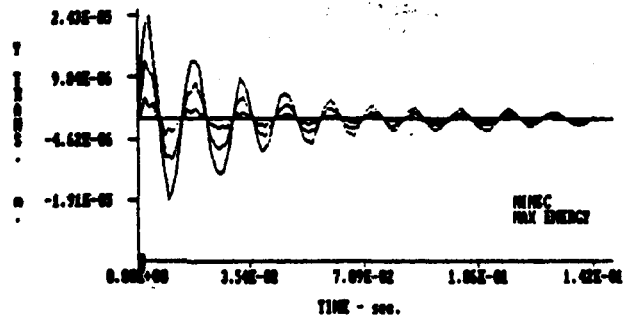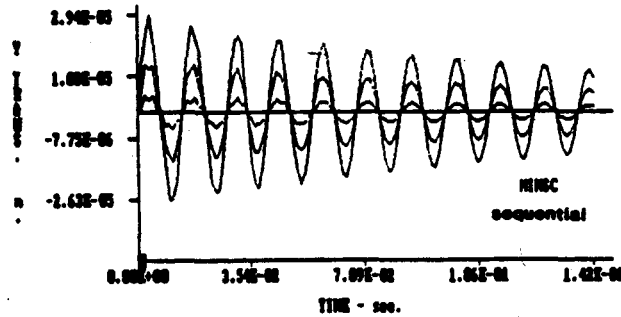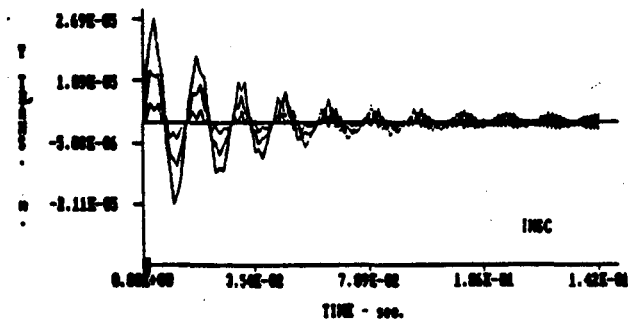Figure (9) - Time history of the amplitudes of transverse vibration
of cantilever beam with one actuator and R=100

energy then the time sharing is efficient and fast. The second reason for the inadequacy of the sequential time sharing in the case of the beam, unlike the three spring-mass system, is that as the actuator is sequential to control the high frequency modes, which may have low energy level, it leaves the high energy modes uncontrolled. Therefore, the more the number of modes between which an actuator is shared, the longer it will take to go through them all and come back to control the low frequency modes that may still have the high energy. The delay period in controlling the high energy modes increase as the number of modes is increased and accordingly this will prolong the time needed to actively bring the structure under control.

Quantitatively, the comparison between the two time sharing strategies is given in Table (5).

Table (5) - Effect of the strategy of time sharing one actuator on displacement, control force and control energy indices for cantilever beam with R=100.

Strategy	Displacement index ($\times 10^9$)	Control forces index ($\times 10^5$)	Control energy index ($\times 10^6$)
Sequential	5.919	0.9081	3.205
Modal energy	1.243	2.9000	1.596

C. Optimum placement of one actuator

The MIMSC method is used to optimally place a single actuator in the considered beam system. Table (6) summarizes the effect of placing the actuator on the displacement, control force and control energy indices.

Table (6) - Effect of actuator location on displacement, control force and control energy indices for cantilever beam.

Actuator at node	Actuator type	Displacement index ($\times 10^9$)	Control force index ($\times 10^5$)	Control energy index ($\times 10^6$)
2	linear	1.553	18.91400	3.169
	rotary	1.555	0.02800	6.305
3	linear	1.579	1.13700	1.495
	rotary	1.242	0.00100	1.306
4	linear	1.243	2.90000	1.596
	rotary	1.324	0.00054	1.461

The obtained results suggest that the actuator should be of the rotary type and be placed at node 3. At this location the displacement as well as the control energy indices are minimal. The table suggests that the location and type of actuator have a considerable effect on the system performance unlike what is stated by the IMSC method.

CONCLUSIONS

This report has presented a modified algorithm of the Independent Modal Space Control method where control spillover as well as optimum placement and time sharing of actuators have been considered.

The considered numerical examples indicate the importance of including such modifications to the IMSC.

It is shown that time sharing a small number of actuators between a large number of modes can be effective in suppressing the vibration if the actuators are dedicated to control the modes that have the highest modal energy at any particular instant.

The presented algorithm has the potential of being a viable means for controlling large flexible structures.

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