Modified iterated simulated annealing for optimal structural design

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Abstract

A modified iterated simulated annealing (MISA) method is presented for optimal design of structural systems. The method uses a random sequence of designs to determine the optimal one. Automatic reduction of the feasible region and sensitivity analysis for the design variables are also used. Comparisons between classical optimization methods and MISA show that the letter can provide accurate results even when infeasible initial designs are attempted, and is advantageous for problems with dynamic constraints.

Introduction

Automated optimization designs originated from many fields and several methods have been proposed in recent years (Haug and Arora,[1] and Schmit [2]). In 1983 Kirkpatrick *et al.*[3] combined statistical mechanics with optimization and introduced simulated annealing. The simulated annealing method involves random sequences of candidate designs with a probabilistic acceptance criterion of a better design at each subsequent iteration. Ackley[4] developed the iterated simulated annealing method (ISA) and the stochastic hillclimbing method (SHC). In the SHC method the probability evaluation of a new design is held constant for the duration of the search. In the ISA method the probability evaluation of a new design starts at a high value and is reduced by a decay rate during the search. The simulated annealing method has been used by Salama *et al.*,[5] Chen *et al.*,[6] and Balling.[7]

44   Structural Optimization

Review of existing methods

The optimal design of structural systems with static or dynamic constraints can be stated as follows: Find the design vector of the cross-sectional areas of the members ($\overline{A}$) that minimizes the structural volume $V(\overline{A})$, subject to displacement constraints $\overline{u}$, stress constraints $\overline{\sigma}$, and side constraints $\overline{A}_l, \overline{A}_u$, on the design variables. Mathematically, this is expressed as

$$\min_{\overline{A}} V(\overline{A}) \tag{1}$$

Subject to

$$\overline{u}(\overline{A},t) \leq \overline{u}_{all,j} \quad ; \qquad j = 1,...\overline{n}; \quad t \geq 0 \tag{2}$$

$$\overline{\sigma}(\overline{A},t) \leq \overline{\sigma}_{all,i} \quad ; \qquad i = 1,...\overline{m}; \quad t \geq 0 \tag{3}$$

$$A_{l,i} \leq A \leq A_{u,i} \quad ; \qquad i = 1,...\overline{m} \tag{4}$$

where $\overline{u}_{all}$ is the allowable displacement at certain nodes of the structure ($\overline{n}$), and $\overline{\sigma}_{all}$ is the allowable stress in the members of the structure ($\overline{m}$). Note that if the problem is statically constrained, the quantities on the left in eqns. (2) and (3) become time-independent. The static problem can be solved using various existing algorithms. The dynamically constrained problem is more difficult because of the time-dependent nature of the constraints which usually created disjoint design space.

Since 1983, when the simulated annealing method was introduced,[3] two developments evolved related to the method presented in this paper. These are Ackley's ISA and SHC methods,[4] and Balling's simulated annealing method.[7]

Ackley's methods[4]
Both the ISA and SHC methods consider a random change to the current design, and accept the change with a probability, p, determined by[4]

$$p = \frac{1}{1 + e^{(V_a - V_c)/T}} \tag{5}$$

where V_c is the current value of the objective function from the preselected point, V_a is a candidate value of the objective function which is produced by an adjacent point randomly selected, and T is an adjustable parameter described as "temperature". In the SHC method, the temperature T is held constant for the duration of the search. In the ISA technique, the temperature is variable; it starts at a high value and is reduced by a decay rate during the search.

Balling's method[7]
The probabilistic acceptance criterion for determining whether the candidate design should replace the current design or be rejected is formed with a probability, p, developed by Balling[7]

$$p = e^{\left(-D/_{CT}\right)} \tag{6}$$

where D is the difference in the value of the objective function between the candidate design and the current design, C is a normalization constant which is the running average of D, and T is the strategy temperature which decreases according to a "cooling factor", f, defined as

$$f = \left(\frac{\log(p_s)}{\log(p_f)} \right)^{1/N-1} \tag{7}$$

where p_s, p_f are the starting and final acceptance probabilities for an average $D = C$, and N is the number of cycles.

In Balling's procedure, the acceptance criterion allows worse designs to be accepted in the initial stages of the optimization. Two strategies are used: the first is a strategy used to obtain a base design, and the second is a simulated annealing strategy which is used to obtain a global optimal design. Both strategies use an approximate method for calculating constraints, developed by Vanderplaats and Salajegheh.[8]

Modified iterated simulated annealing (MISA) method

Based on Ackley's ISA algorithm regarding the probability of acceptance and portions of Balling's algorithm, a modified iterated simulated annealing method is proposed for optimal structural design (Figure 1). The design variables are determined by a random reference number that is requested by the program based on the current time of the computer clock. Two ranges of iterations are performed which are defined as M and N. M is the maximum value of m which is the counter for the inner loop. The inner loop determines the search direction and M is usually a small number less than 10. N is the maximum value of n which is the counter for the outer loop. The outer loop determines the best design for different annealing probabilities, based on different values of T (temperature); n_1 is the minimum number of runs of the outer loop; k is an integer, and $k \times n_2$ is the number of times required, in addition to n_1, for the outer loop to converge; n_1 and n_2 must be determined by the user for a particular application.

After the outer loop runs ($n_1 + n_2$) times, the designs at the n_1 and ($n_1 + n_2$) iterations are compared. When these two designs are identical the program stops. Otherwise, the outer loop is repeated n_2 times, and comparison is made between the designs at ($n_1 + n_2$) and ($n_1 + 2n_2$). The procedure is repeated k times until the two designs converge.

Reduction of the feasible region is achieved as follows in MISA: when one candidate relative minimum value of the objective function is found, such as point O in Figure 2, the feasible region is reduced to exclude points outside an enlarged region which contains the relative minimum. This procedure is performed at stage "A" in Figure 1. The range for X_r and Y_r in Figure 2 is

46   Structural Optimization

chosen as 15 percent of X and Y respectively. Since the next point is chosen randomly, two situations are possible. First, the new design point could end up in the infeasible region in which case the design is rejected and there is no need for reduction of the feasible region. The second possibility is that the new design point is inside the current search range and the new candidate design has a smaller objective function value than the current design. In this case, a new search range will be found as described above.

Sensitivity analysis is used in MISA as follows: if a given displacement violates the constraint, the neighboring structural members are identified by sensitivity analysis[9] and the area of those members is increased by a random number. The purpose of the sensitivity analysis is to identify the design variables that must be modified in the new design, in order to decrease the magnitude of a certain displacement in the most economical way. This sensitivity analysis is performed at stage "B" in Figure 1. Even though the updated values of the design variables are random, the identification of which variables must be modified is done using deterministic sensitivity analysis.[9] It was found that by using sensitivity analysis, the expense of multiple trials is avoided and the efficiency of the MISA method is improved.

In the MISA method, design iterations use random sequences of candidate designs. This is advantageous in dynamically constrained problems where the feasible region is usually disjoint. Standard optimization methods may not find sequential feasible designs because of the disjoint feasible region.

Example: optimal design of a two-story frame with dynamic constraints

A case with dynamic constraints for a two-story frame is used to test the performance of the MISA method. The design variables are defined as the cross-sectional areas of the structural members; their moment of inertia, I, can be obtained using the relations:[1]

$$I_i = Z_i \times s_i$$

$$Z_i = \beta_i \left(\frac{A_i}{\alpha_i} \right)^{3/2} \; ; \quad s_i = \chi_i \left(\frac{A_i}{\alpha_i} \right)^{1/2} \tag{8}$$

where Z_i, s_i, and A_i are the section modulus, the least radius of gyration, and the cross-sectional area of the ith element; α_i, β_i, and χ_i are constants.

The dynamic degrees of freedom are reduced from 12 to 2 (Figure 3(a)) by the Guyan reduction method,[10] and the lumped mass procedure is used for the solution of the dynamic equations of motion. The allowable stress for each member is assumed as 150 MPa, and the allowable drift for each floor as $h/180$ (2.54 cm). The strong-column weak-beam philosophy is implemented by computing the strength ratio of column to beam, which reflects current code requirements for earthquake design. In addition, elastic response of the frame is assumed throughout the iteration history.

Four design variables as shown in Figure 3(b) are used. The lumped mass for each floor is given as 27,234 kg and 5 percent critical damping is assumed in each of the two vibration modes. The excitation is chosen as the S00E ground acceleration record of the May, 18 1940 El-Centro earthquake.

Table 1 shows the optimization results by using the MISA and the Modified Feasible Directions (MFD)[11] methods. Because of the random search there exist intermediate iterations in MISA for which the volume increases. For both methods, the drift of the second floor is active. The volume of the two-story frame obtained by the MISA method is 0.7 percent more than the volume obtained by the MFD method. It should be noted that the Sequential Linear Programming method of the DOT program,[11] failed to find the optimum dynamic structure regardless the initial values of the design variables.

Table 1. Optimal design of a two-story frame for dynamic stress and drift constraints

Group of Frame (1)	Cross-sectional area (cm ^{2})		Member No. (4)	Combined stress (MPa)		Floor (7)	Drift (cm)	
	MISA (2)	MFD (3)		MISA (5)	MFD (6)		MISA (8)	MFD (9)
A_1	268.8	270.9	①	111.33	108.97	1st	1.75	1.70
A_2	257.6	263.1	②	111.86	109.47	2nd	2.54	2.54
A_3	211.6	193.1	③	106.71	103.29			
A_4	180.2	190.8	④	67.92	78.97			
Volume			⑤	68.00	79.06			
(cm ^{3})	706,083	700,896	⑥	87.55	81.80			

Conclusions

A modified iterated simulated annealing (MISA) method with sensitivity analysis and automatic reduction of the feasible region is presented. The sensitivity analysis is used to identify which design variables need to be modified in order to decrease a certain displacement in the most economical way. Even though the actual values of the new design variables are determined randomly, the knowledge of which design variables to modify improves the efficiency of MISA. The automatic reduction of the feasible region limits the extend of the search and improves computational efficiency.

The MISA method was found to be advantageous for structural systems with dynamic constraints, when compared to standard mathematical programming methods. For dynamically constrained problems where the feasible region is disjoint, MISA has the advantage of converging to the minimum even when infeasible initial designs are used. The method proceeds to the minimum even when intermediate iterations are worse than previous iterations, which is advantageous in dynamically constrained problems where the feasible region is disjoint.

48   Structural Optimization

Acknowledgments

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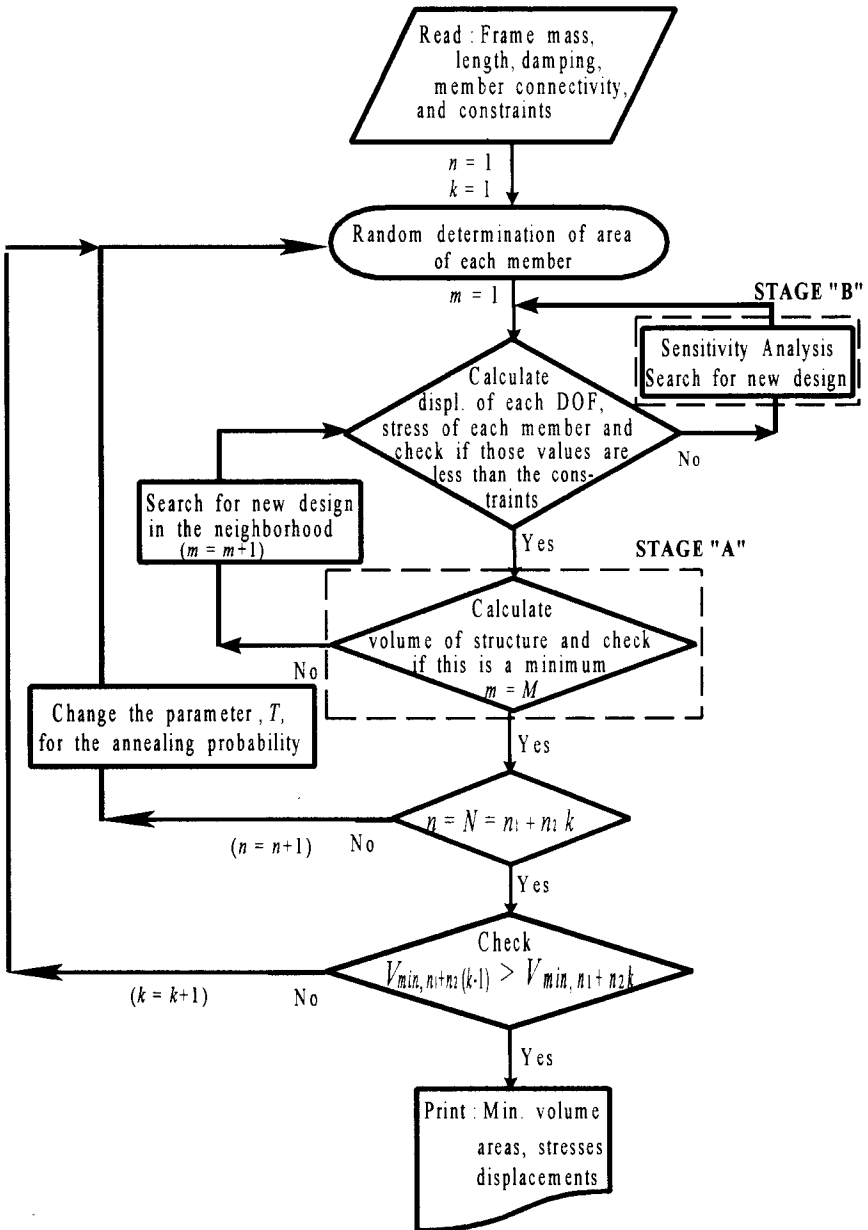
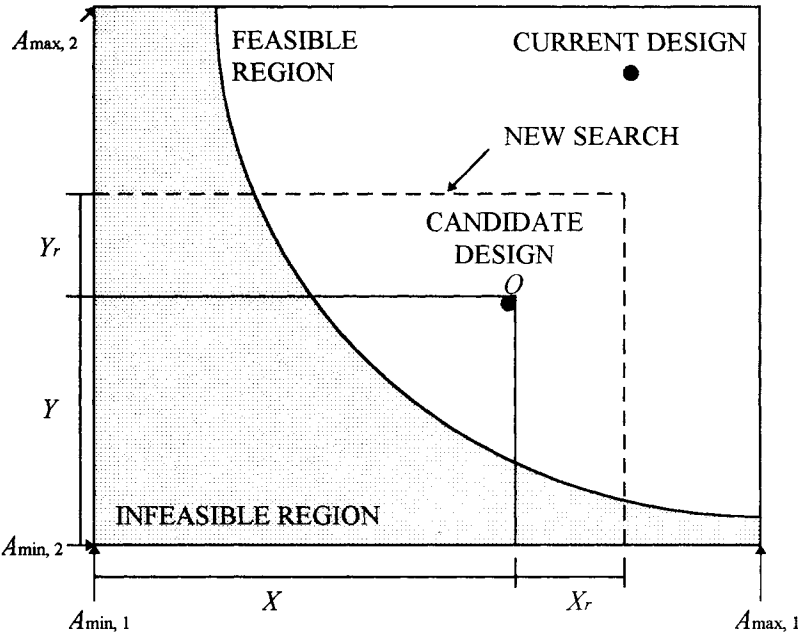Figure 1. Flowchart for modified iterated simulated annealing (MISA) optimal structural design method

Figure 2. Automatic reduction of the feasible region used in the MISA method

$A_1, ..., A_4 =$ Area of Frame Member
①, ..., ⑥ = Member Number
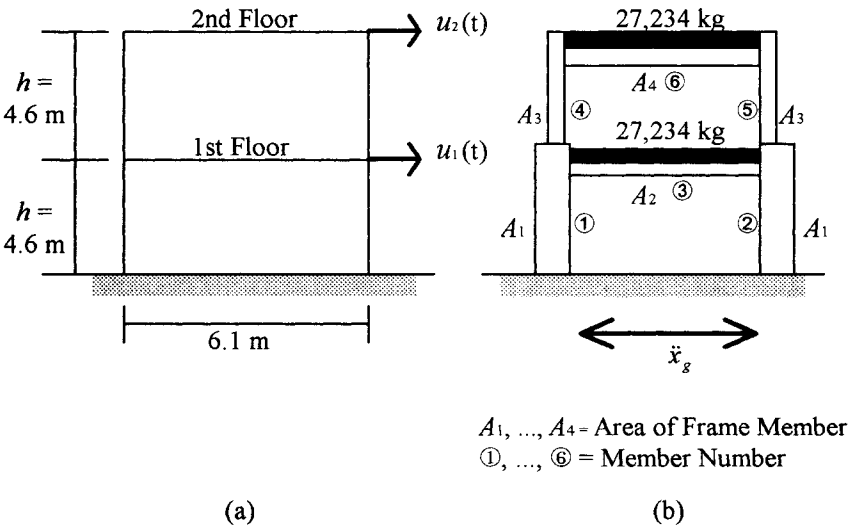
(a)   (b)

Figure 3. Two-story frame for dynamic stress and drift constraints: (a) dimensions and degrees of freedom, (b) loading and design parameters