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MODIFIED NEWTONIAN DYNAMICS AS AN ALTERNATIVE TO DARK MATTER

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■ **Abstract** Modified Newtonian dynamics (MOND) is an empirically motivated modification of Newtonian gravity or inertia suggested by Milgrom as an alternative to cosmic dark matter. The basic idea is that at accelerations below $a_o \approx 10^{-8}$ cm/s^2 $\approx cH_o/6$ the effective gravitational attraction approaches $\sqrt{g_n a_o}$, where g_n is the usual Newtonian acceleration. This simple algorithm yields flat rotation curves for spiral galaxies and a mass-rotation velocity relation of the form $M \propto V^4$ that forms the basis for the observed luminosity–rotation velocity relation—the Tully-Fisher law. We review the phenomenological success of MOND on scales ranging from dwarf spheroidal galaxies to superclusters and demonstrate that the evidence for dark matter can be equally well interpreted as evidence for MOND. We discuss the possible physical basis for an acceleration-based modification of Newtonian dynamics as well as the extention of MOND to cosmology and structure formation.

INTRODUCTION

The appearance of discrepancies between the Newtonian dynamical mass and the directly observable mass in large astronomical systems has two possible explanations: either these systems contain large quantities of unseen matter, or gravity (or the response of particles to gravity) on these scales is not described by Newtonian theory. Most attention has focused on the first of these explanations. An intricate paradigm has been developed in which nonbaryonic dark matter plays a central role—not only in accounting for the traditional dynamical mass of bound gravitational systems (Faber & Gallagher 1979, Blumenthal et al. 1984) but also in promoting the formation of structure via gravitational collapse beginning in the highly homogeneous ionized universe (Peebles 1982, Vittorio & Silk 1984). The paradigm of cold dark matter (CDM) is widely purported to be successful in this cosmological context, particularly in predicting the scale-dependence of density

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fluctuations. Moreover, with the development of cosmic N-body simulations of high precision and resolution, this hypothesis has gained predictive power on the scale of galaxies—a power that considerably restricts the freedom to arrange dark matter as one would wish in order to explain the form and magnitude of the discrepancy in any particular system (Navarro et al. 1996).

It is in this second aspect, the distribution of dark matter in galactic systems, that the CDM paradigm encounters observational difficulties (McGaugh & de Blok 1998a, Sellwood & Kosowsky 2001). It is not the purpose of this review to discuss the possible problems with the CDM hypothesis. We only comment that, as of the date of this review, candidate dark matter particles have not yet been detected by any means independent of their putative global gravitational effect. So long as this is the only evidence for dark matter, its presumed existence is not independent of the assumed law of gravity or inertia on astronomical scales. If a physical law, when extended to a regime in which it has never before been tested, implies the existence of a medium (e.g., an ether) that cannot be detected by any other means, then it would not seem unreasonable to question that law.

Of course, if one chooses to modify Newtonian dynamics or gravity in an ad hoc fashion, then the set of alternative possibilities is large. It is a simple matter to claim that Newton's law of gravity fails on galactic scales and then to cook up a recipe that explains a particular aspect of the observations—such as flat rotation curves of spiral galaxies. To be credible, an empirically based alternative to dark matter should at least provide a more efficient description of the phenomenology. Any viable alternative should account for various aspects of the observations of astronomical systems (such as global scaling relations) with as few additional parameters as possible. A second, but less immediate, requirement is that the suggested alternative should have some basis in sensible physics—it should make contact with familiar physical principles or at least a reasonable extrapolation of those principles.

To date, the only suggestion that goes some way toward meeting these requirements (particularly the former) is Milgrom's modified Newtonian dynamics (MOND) (Milgrom 1983a,b,c). The empirical successes of this hypothesis on scales ranging from dwarf spheroidal galaxies to super-clusters, and its possible physical basis and extension to a cosmological context, is the subject of this review. It may be argued that this is a speculative topic for review in this series. In our opinion the subject of dark matter (Trimble 1987) is, in the absence of its direct detection, no less speculative, particularly considering that the standard model of particle physics does not predict the existence of candidate dark matter particles with the necessary properties. Reasonable extensions of the standard model (e.g., supersymmetry) can, with an appropriate adjustment of parameters, accommodate such particles, but this also requires an extrapolation of known physics (e.g., Griest et al. 1990). Here we demonstrate that the evidence for dark matter can be equally well interpreted as evidence for modified dynamics.

There have been other attempts to modify gravity in order to account for astronomical mass discrepancies without invoking dark matter. We mention some of these efforts, but it is fair to say that none of these alternatives has enjoyed the phenomenological success of MOND or been as extensively discussed in the literature. A considerable lore on MOND has emerged in the past two decades—with contributions not only by advocates reporting phenomenological successes but also by critics pointing out possible problems or inconsistencies. There have also been several contributions attempting to formulate MOND either as a covariant theory in the spirit of General Relativity, or as a modified particle action (modified inertia). Whereas none of these attempts has, so far, led to anything like a satisfactory or complete theory, they provide some insight into the required properties of generalized theories of gravity and inertia.

In the absence of a complete theory, MOND cannot be unambiguously extended to problems of cosmology and structure formation. However, by making certain reasonable assumptions, one may speculate on the form of a MOND cosmology. We discuss the efforts that have been made in this direction. The general expectation is that, because MOND results in effectively stronger gravity for low peculiar accelerations, the rapid growth of structure is possible even in a low-density purely baryonic Universe.

BASICS OF MODIFIED NEWTONIAN DYNAMICS

An Acceleration Scale

The phenomenological basis of MOND consists of two observational facts about spiral galaxies: (*a*) The rotation curves of spiral galaxies are asymptotically flat (Shostak 1973, Roberts & Whitehurst 1975, Bosma 1978, Rubin et al. 1980), and (*b*) there is a well-defined relationship between the rotation velocity in spiral galaxies and the luminosity—the Tully-Fisher (TF) law (Tully & Fisher 1977, Aaronson et al. 1982). This latter implies a mass-velocity relationship of the form $M \propto V^{\alpha}$, where α is in the neighborhood of 4.

If one wishes to modify gravity in an ad hoc way to explain flat rotation curves, an obvious first choice would be to propose that gravitational attraction becomes more like 1/r beyond some length scale that is comparable to the scale of galaxies. So the modified law of attraction about a point mass M would read

$$F = \frac{GM}{r^2} f(r/r_o), \tag{1}$$

where r_o is a new constant of length on the order of a few kpc, and $f(x)$ is a function with the asymptotic behavior: $f(x) = 1$, where $x \ll 1$ and $f(x) = x$, where $x \gg 1$. Finzi (1963), Tohline (1983), Sanders (1984), and Kuhn & Kruglyak (1987) have proposed variants of this idea. In Sanders' (1984) version the Newtonian potential is modified by including a repulsive Yukawa term ($e^{-r/r_o}/r$), which can yield a flat rotation velocity over some range in r. This idea keeps reemerging with various modern justifications (e.g., Eckhardt 1993, Hadjimichef & Kokubun 1997, Drummond 2001, Dvali et al. 2001).

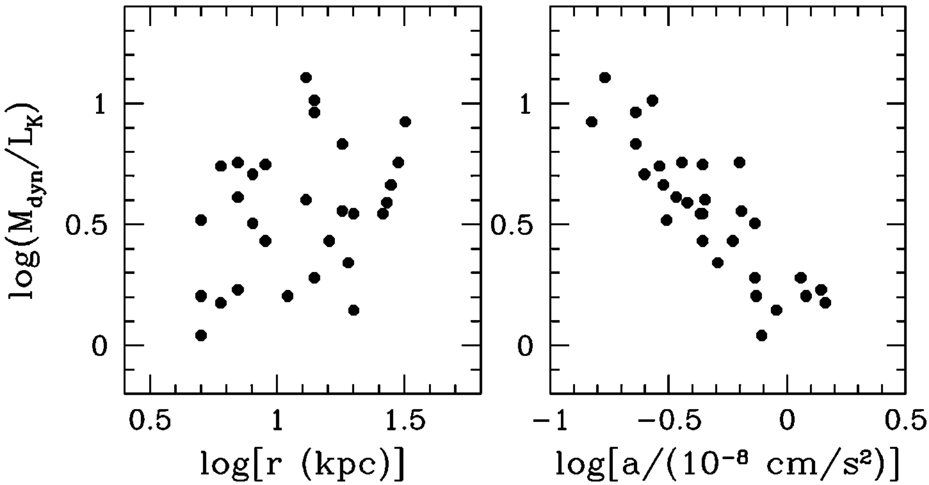


Figure 1 The global Newtonian mass-to-K′-band-luminosity ratio of Ursa Major spirals at the last measured point of the rotation curve plotted first against the radial extent of the rotation curve (*left*) and then against the centripetal acceleration at that point (*right*).

All of these modifications attached to a length scale have one thing in common: Equating the centripetal to the gravitational acceleration in the limit $r > r_o$ would lead to a mass-asymptotic rotation velocity relation of the form $v^2 = GM/r_o$. Milgrom (1983a) realized that this was incompatible with the observed TF law, $L \propto v^4$. Moreover, any modification attached to a length scale would imply that larger galaxies should exhibit a larger discrepancy (Sanders 1986). This is contrary to the observations. There are very small, usually low surface brightness (LSB) galaxies with large discrepancies, and very large high surface brightness (HSB) spiral galaxies with very small discrepancies (McGaugh & de Blok 1998a).

Figure 1 illustrates this. At the left is a log-log plot of the dynamical $M/L_{K'}$ vs. the radius at the last measured point of the rotation curve for a uniform sample of spiral galaxies in the Ursa Major cluster (Tully et al. 1996, Verheijen & Sancisi 2001). The dynamical M/L is calculated simply using the Newtonian formula for the mass v^2r/G (assuming a spherical mass distribution), where r is the radial extent of the rotation curve. Population synthesis studies suggest that $M/L_{K'}$ should be about 1, so anything much above 1 indicates a global discrepancy—a "dark matter problem." It is evident that there is not much of a correlation of M/L with size. On the other hand, the Newtonian M/L plotted against centripetal acceleration (v^2/r) at the last measured point (Figure 1, *right*) looks rather different. There does appear to be a correlation in the sense that $M/L \propto 1/a$ for $a < 10^{-8}$ cm/s^2. The presence of an acceleration scale in the observations of the discrepancy in spiral galaxies has been pointed out before (Sanders 1990, McGaugh 1998), and as the data have improved, it has become more evident. Any modification of gravity attached to a length scale cannot explain such observations.

Milgrom's insightful deduction was that the only viable sort of modification is one in which a deviation from Newton's law appears at low acceleration. (Data such as that shown in Figure 1 did not exist at the time of Milgrom's initial papers; an acceleration scale was indicated by the slope of the TF relation.) MOND as initially formulated could be viewed as a modification of inertia or of gravity (this dichotomy remains). In the first case the acceleration of a particle with mass m under the influence of an external force would be given by

$$m\mathbf{a}\mu(a/a_o) = \mathbf{F}, \tag{2}$$

where a_o is a new physical parameter with units of acceleration and $\mu(x)$ is a function that is unspecified but must have the asymptotic form $\mu(x)=x$ when $x \ll 1$ and $\mu(x)=1$ when $x \gg 1$. Viewed as a modification of gravity, the true gravitational acceleration $\mathbf{g}$ is related to the Newtonian gravitational acceleration $\mathbf{g_n}$ as

$$\mathbf{g}\mu(|g|/a_o) = \mathbf{g_n}. \tag{3}$$

Although there are clear differences in principle and practice between these two formulations, the consequence for test particle motion in a gravitational field in the low acceleration regime is the same: The effective gravitational force becomes $g = \sqrt{g_n a_o}$. For a point mass M, if we set g equal to the centripetal acceleration v^2/r, this gives

$$v^4 = GMa_o \tag{4}$$

in the low acceleration regime. Thus, all rotation curves of isolated masses are asymptotically flat, and there is a mass-luminosity relation of the form $M \propto v^4$. These are aspects that are built into MOND, so they cannot rightly be called predictions. However, in the context of MOND, the aspect of an asymptotically flat rotation curve is absolute. MOND leaves rather little room for maneuvering; the idea is in principle falsifiable, or at least it is far more fragile than the dark matter hypothesis. Unambiguous examples of rotation curves of isolated galaxies that decline in a Keplerian fashion at a large distance from the visible object would falsify the idea.

In addition, the mass-rotation velocity relation and implied TF relation is absolute. The TF relation should be the same for different classes of galaxies, independent of surface brightness, and the logarithmic slope (at least of the mass-velocity relation) must be 4.0. Moreover, the relation is essentially one between the total baryonic mass of a galaxy and the asymptotic flat rotational velocity—not the peak rotation velocity but the velocity at large distance. This is the most immediate and most obvious prediction (see McGaugh & de Blok 1998b and McGaugh et al. 2000 for a discussion of these points).

Converting the mass-velocity relation (Equation 4) to the observed luminosity-velocity relation, we find

$$\log(\mathrm{L}) = 4\log(v) - \log(\mathrm{Ga_o}\langle\mathrm{M/L}\rangle). \tag{5}$$

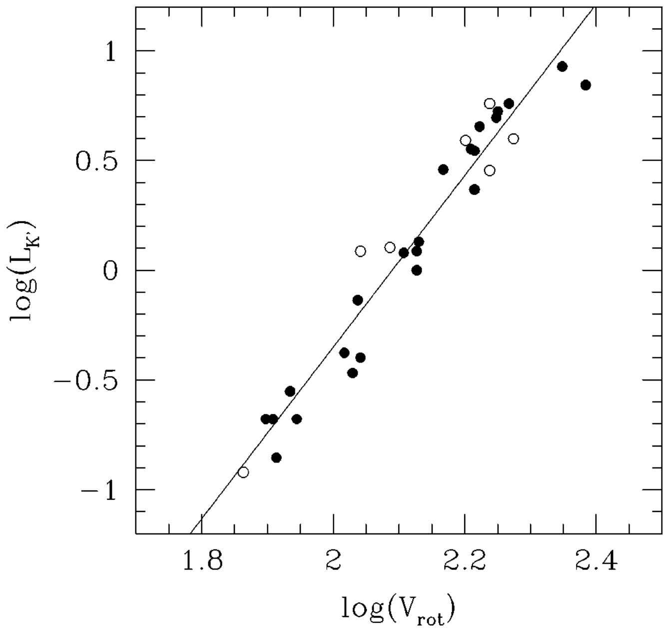


Figure 2 The near-infrared Tully-Fisher relation of Ursa Major spirals (Sanders & Verheijen 1998). The rotation velocity is the asymptotically constant value. The velocity is in units of kilometer/second and luminosity in 10^{10} $L_\odot$. The unshaded points are galaxies with disturbed kinematics. The line is a least-square fit to the data and has a slope of 3.9 ± 0.2.

Figure 2 shows the near-infrared TF relation for Verheijen's UMa sample (Sanders & Verheijen 1998), where the velocity, v, is that of the flat part of the rotation curve. The scatter about the least-square fit line of slope 3.9 ± 0.2 is consistent with observational uncertainties (i.e., no intrinsic scatter). Given the mean M/L in a particular band ($\approx$1 in the K′ band), this observed TF relation Equation 5 tells us that a_o must be on the order of 10^{-8} cm/s^2. Milgrom immediately noticed that $a_o \approx cH_o$ to within a factor of 5 or 6. This cosmic coincidence is provocative and suggests that MOND perhaps reflects the effect of cosmology on local particle dynamics.

General Predictions

There are several other direct observational consequences of modified dynamics—all of which Milgrom explored in his original papers—that do fall in the category of predictions in the sense that they are not part of the propositional basis of MOND.

1. There exists a critical value of the surface density

$$\Sigma_m \approx a_o/G. \tag{6}$$

If a system such as a spiral galaxy has a surface density of matter greater than

Σ_m, then the internal accelerations are greater than a_o, so the system is in the Newtonian regime. In systems with $\Sigma \geq \Sigma_m$ (HSB galaxies) there should be a small discrepancy between the visible and classical Newtonian dynamical mass within the optical disk. In the parlance of rotation curve observers, an HSB galaxy should be well represented by the "maximum disk" solution (van Albada & Sancisi 1986), but in LSB galaxies ($\Sigma \ll \Sigma_m$) there is a low internal acceleration, so the discrepancy between the visible and dynamical mass would be large. Milgrom predicted, before the actual discovery of LSB galaxies, that there should be a serious discrepancy between the observable and dynamical mass within the luminous disk of such systems—should they exist. They do exist, and this prediction has been verified, as is evident from the work of McGaugh & de Blok (1998a,b) and de Blok & McGaugh (1998).

2. It is well known since the work of Ostriker & Peebles (1973) that rotationally supported Newtonian systems tend to be unstable to global nonaxisymmetric modes that lead to bar formation and rapid heating of the system. In the context of MOND these systems would be those with $\Sigma > \Sigma_m$, so this would suggest that Σ_m should appear as an upper limit on the surface density of rotationally supported systems. This critical surface density is 0.2 g/cm^2 or 860 $M_\odot/pc^2$. A more appropriate value of the mean surface density within an effective radius would be $\Sigma_m/2\pi$ or 140 $M_\odot/pc^2$, and taking $M/L_b \approx 2$, this would correspond to a surface brightness of about 22 mag/arc sec^2. There is such an observed upper limit on the mean surface brightness of spiral galaxies, and this is known as Freeman's law (Freeman 1970, Allen & Shu 1979). The point is that the existence of such a maximum surface density (McGaugh et al. 1995, McGaugh 1996) is quite natural in the context of MOND but must be put in by hand in dark matter theories (e.g., Dalcanton et al. 1997).

3. Spiral galaxies with a mean surface density near this limit—HSB galaxies—would be, within the optical disk, in the Newtonian regime. Thus, one would expect that the rotation curve would decline in a near Keplerian fashion to the asymptotic constant value. In LSB galaxies, with mean surface density below Σ_m, the prediction is that rotation curves continuously rise to the final asymptotic flat value. Thus, there should be a general difference in rotation curve shapes between LSB and HSB galaxies. Figure 3 shows the rotation curves of two galaxies, an LSB and an HSB, where we see exactly this trend. This general effect in observed rotation curves was first noted by Casertano & van Gorkom (1991).

4. With Newtonian dynamics, pressure-supported systems that are nearly isothermal have infinite mass. However, in the context of MOND it is straightforward to demonstrate that such isothermal systems are of finite mass with the density at large radii falling approximately as r^{-4} (Milgrom 1984). The equation of hydrostatic equilibrium for an isotropic, isothermal system reads

$$\sigma_r^2 \frac{d\rho}{dr} = -\rho g, \tag{7}$$

where, in the limit of low accelerations, $g = \sqrt{GM_r a_o}/r$. σ_r is the radial velocity

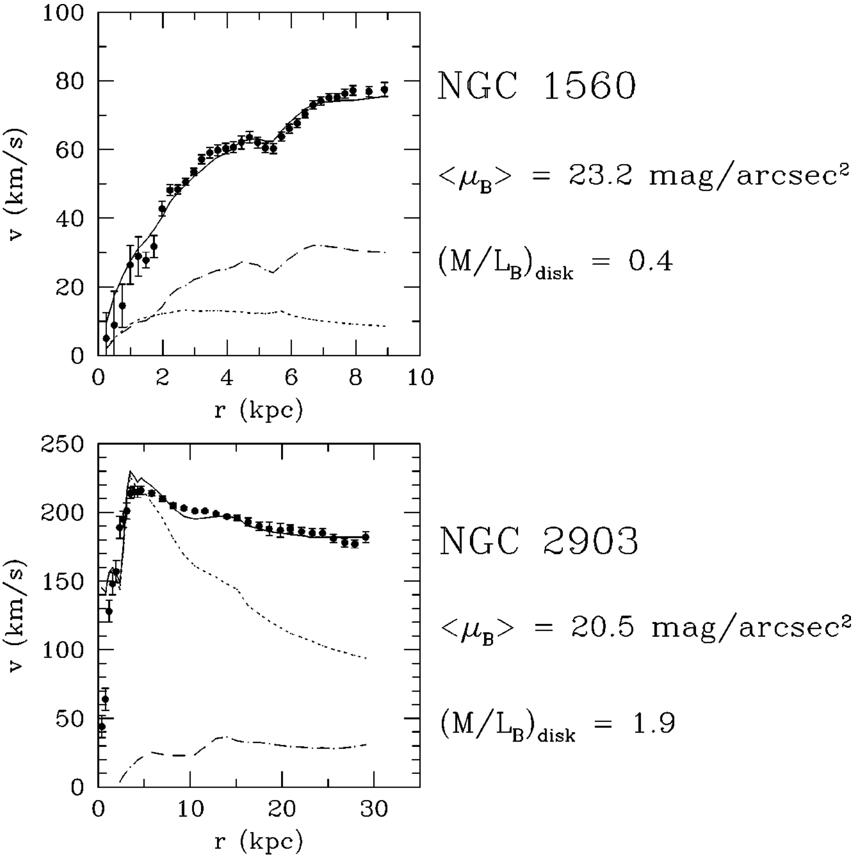


Figure 3 The points show the observed 21-cm line rotation curves of a low surface brightness galaxy, NGC 1560 (Broeils 1992), and a high surface brightness galaxy, NGC 2903 (Begeman 1987). The dotted and dashed lines are the Newtonian rotation curves of the visible and gaseous components of the disk, and the solid line is the MOND rotation curve with $a_o = 1.2 \times 10^{-8}$ cm/s^2—the value derived from the rotation curves of 10 nearby galaxies (Begeman et al. 1991). The only free parameter is the mass-to-light ratio of the visible component.

dispersion, ρ is the mass density, and M_r is the mass enclosed within r. It then follows immediately that, in the outer regions, where $M_r = M =$ constant,

$$\sigma_r^4 = GMa_o\left(\frac{d\ln(\rho)}{d\ln(r)}\right)^{-2}. \tag{8}$$

Thus, there exists a mass-velocity dispersion relation of the form

$$(M/10^{11}M_\odot) \approx (\sigma_r/100\,\mathrm{km\,s^{-1}})^4, \tag{9}$$

which is similar to the observed Faber-Jackson relation (luminosity-velocity dispersion relation) for elliptical galaxies (Faber & Jackson 1976). This means that a MOND near-isothermal sphere with a velocity dispersion of 100–300 km/s will always have a galactic mass. Moreover, the same $M - \sigma$ relation (Equation 8) should apply to all pressure-supported, near-isothermal systems, from globular clusters to clusters of galaxies, albeit with considerable scatter owing to deviations from a strictly isotropic, isothermal velocity field (Sanders 2000).

The effective radius of a near-isothermal system is roughly $r_e \approx \sqrt{GM/a_o}$ (at larger radii the system is in the MOND regime and is effectively truncated). This means that a_o appears as a characteristic acceleration in near-isothermal systems and that Σ_m appears as a characteristic surface density—at least as an upper limit (Milgrom 1984). Fish (1964), on the basis of then-existing photometry, pointed out that elliptical galaxies exhibit a constant surface brightness within an effective radius. Although Kormendy (1982) demonstrated that more luminous ellipticals have a systematically lower surface brightness, when the ellipticals are considered along with the bulges of spiral galaxies and globular clusters (Corollo et al. 1997) there does appear to be a characteristic surface brightness on the order of that implied by Σ_m, i.e., the Fish law is recovered for the larger set of pressure-supported objects.

5. The "external field effect" is not a prediction but a phenomenological requirement on MOND that has strong implications for nonisolated systems. In his original papers Milgrom noted that open star clusters in the Galaxy do not show evidence for mass discrepancies, even though the internal accelerations are typically below a_o. He therefore postulated that the external acceleration field of the Galaxy must have an effect upon the internal dynamics of a star cluster—that, in general, the dynamics of any subsystem is affected by the external field in which that system is found. This implies that the theory upon which MOND is based does not respect the equivalence principle in its strong form. (This in no way implies that MOND violates the universality of free fall—the weak version of the equivalence principle—which is the more cherished and experimentally constrained version.) Milgrom suggested that this effect arises owing to the nonlinearity of MOND and can be approximated by including the external acceleration field, $\mathbf{g_e}$, in the MOND equation, i.e.,

$$\mu(|\mathbf{g_e} + \mathbf{g_i}|/a_o)\,\mathbf{g_i} = \mathbf{g_{n_i}}, \tag{10}$$

where $\mathbf{g_i}$ is the internal gravitational field of the subsystem and $\mathbf{g_{n_i}}$ is the Newtonian field of the subsystem alone. This means that a subsystem with internal accelerations below a_o will exhibit Newtonian dynamics if the external acceleration exceeds a_o ($g_i < a_o < g_e$). If the external and internal accelerations are below a_o and $g_i < g_e < a_o$, then the dynamics of the subsystem will be Newtonian but with a larger effective constant of gravity given by $G/\mu(g_e/a_o)$. If $g_e < g_i < a_o$ the dynamics is MONDian but with a maximum discrepancy given by $[\mu(g_e/a_o)]^{-1}$. In addition the dynamics is anisotropic with dilation along the direction of the external field.

The external field effect would have numerous consequences: It would influence the internal dynamics of globular clusters and the satellite dwarf companions of the Milky Way independently of tides. It may provide an important nontidal mechanism for the maintenance of warps in galaxies owing to the presence of companions. The peculiar accelerations resulting from large-scale structure would be expected to limit the mass discrepancy in any particular galactic system (no object is isolated), and the deceleration (or acceleration) of the Hubble flow may influence the development of large-scale structure.

Dark Halos with an Acceleration Scale

Can the phenomenology of MOND be reproduced by dark halos, specifically the kind of dark halos that emerge from cosmological N-body simulations with an initial fluctuation spectrum given by CDM? This is an important question because the phenomenological success of MOND may be telling us something about the universal distribution of dark matter in galaxies and its relation to the visible component rather than anything about the law of gravity or inertia at low accelerations. This question was first considered by Begeman et al. (1991), who attempted to devise disk-halo coupling rules that could yield a one-parameter fit to rotation curves (M/L of the visible disk) similar to MOND. Without any physical justification, the core radius and asymptotic circular velocity of an isothermal halo were adjusted to the scale length and maximum rotation velocity of the disk to yield a characteristic acceleration. With such coupling rules, the fits to galaxy rotation curves were of lower quality than the MOND fits (particularly for the dwarf systems), and there were numerous ambiguities (e.g., in gas-dominated galaxies what is the proper disk length scale?). Similar ad hoc coupling rules between visible and dark components have also been considered by Giraud (2000).

The idea that the halo might exhibit a characteristic acceleration was carried further by Sanders & Begeman (1994) when the first cosmic N-body calculations with high resolution (Dubinski & Carlberg 1991) indicated that CDM halos were not at all similar to an isothermal sphere with a constant density core. The objects emerging from the simulations exhibited a density law with a r^{-1} cusp that steepened in the outer regions to r^{-4}. Dubinski & Carlberg pointed out that this run of density was well described by the model of Hernquist (1990):

$$\rho(r) = \frac{\Sigma_o}{r}(1 + r/r_o)^{-3}. \qquad (11)$$

This has been subsequently confirmed by the extensive calculations of Navarro et al. (1997), who corrected the outer power law to -3 (this is the famous NFW halo). The reality of the cusp, if not the exact power law, seems well established (Moore et al. 1998) and is due to the fact that there are no phase-space constraints upon the density of a collapsed object composed of CDM.

Sanders & Begeman (1994) pointed out that if Σ_o were fixed with only the characteristic length scale varying from halo to halo, this implied that a fixed acceleration scale could be associated with any halo, $a_h = 2\pi G\Sigma_o$. They demonstrated that a halo density law of this form provided a reasonable fit to rotation curves of several HSB galaxies (comparable to that of MOND), where the length scale of the halo was proportional to the mass of the visible disk. This proportionality would follow if the baryonic mass were a fixed fraction (about 0.03) of the dark mass as is usually assumed, so this appeared to be a natural way to explain MOND phenomenology in the context of CDM.

There are two problems with this idea. First, a fixed Σ_o implies that no galaxy could exhibit an acceleration in the inner regions less than a_o; this is not true for a number of LSB galaxies (McGaugh & de Blok 1998b). The problem was already evident in the fits to the LSB galaxies in the sample of Sanders & Begeman in which the one-parameter fitting scheme broke down. Second, the halos that emerge from the cosmic N-body simulations do not have fixed Σ_o, as is evident from the mass-rotation velocity law of $m \propto v^3$ (NFW). No characteristic surface density or acceleration is evident in these objects.

Semianalytic models for the formation of disk galaxies in the context of CDM (van den Bosch & Dalcanton 2000) can be tuned to give rise to a characteristic acceleration. In such models one starts with a specified dark halo density law (NFW); allows some fraction of the halo mass, presumably baryonic, to collapse by a factor determined by the dimensionless spin parameter of the halo; applies a stability criterion to allow some further fraction of this dissipational component to be converted to stars; and removes gas from the system by an appropriate number of supernovae (feedback). In this procedure there are dimensionless parameters that quantify the feedback mechanism. Because these parameters can be adjusted to produce a TF law of the form $L \propto V^4$, it is not surprising that there is a fixed acceleration connected with these models ($a \approx V^4/GL\langle M/L\rangle$). The exercise is essentially that of modeling complicated astrophysical processes by a set of free parameters; these parameters are then tuned in order to achieve a desired result. The fact that this is possible is interesting but not at all compelling.

The possibility of a characteristic acceleration arising from CDM has been revisited by Kaplinghat & Turner (2002), who offered an explanation for why dark matter appears to become dominant beyond an acceleration numerically comparable to cH_o. They argued that halos formed from CDM possess a one-parameter density profile that leads to a characteristic acceleration profile that is only weakly dependent upon the mass (or comoving scale) of the halo. Then with a fixed collapse factor for the baryonic material, the transition from dominance of dark over baryonic occurs at a universal acceleration, which by numerical coincidence, is on the order of cH_o. Milgrom (2002) responded by pointing out that a_o plays several roles in the context of MOND: It not only is a transition acceleration below which the mass discrepancy appears, but it also defines the asymptotic rotation velocity of spiral galaxies (via Equation 4) and thereby determines the zero point of the TF relation (Equation 5). a_o determines the upper limit on the surface density of spirals (i.e., Σ_m); a_o appears as an effective upper limit upon the gravitational acceleration of a halo component in sensible disk-halo fits to observed rotation curves (Brada & Milgrom 1999a); a_o determines the magnitude of the discrepancy within LSB galaxies (where the ratio of missing to visible mass is a_o/g_i); a_o sets the scale of the Faber-Jackson relation via Equation 8; a_o appears as an effective internal acceleration for pressure-supported, quasi-isothermal systems and determines the dynamics of galaxy systems—global and detailed—ranging from small groups to super-clusters. These roles are independent in the context of dark matter and would each require a separate explanation. The explanation of Kaplinghat & Turner applies only to the first of these and by construction prohibits the existance of objects that are dark matter–dominated within their optical radius (such as LSB galaxies).

The basic problem in trying to explain a fixed acceleration scale in galaxies in terms of galaxy formation rather than underlying dynamics is that the process is stochastic: Each galaxy has its own history of formation-evolution-interaction. One would expect these effects to erase any intrinsic acceleration scale, not enhance it. Dark matter may address the general trends but it cannot account for the individual idiosyncrasies of each rotation curve. In the next section we present the evidence that MOND can do this with a_o as the only additional fixed parameter.

ROTATION CURVES OF SPIRAL GALAXIES

Method and Results of Rotation-Curve Fitting

The measured rotation curves of spiral galaxies constitute the ideal body of data to confront ideas such as MOND (Begeman et al. 1991, Sanders 1996, McGaugh & de Blok 1998b, Sanders & Verheijen 1998). That is because in the absence of dark matter the rotation curve is in principle predictable from the observed distribution of stars and gas. Moreover, the rotation curve as measured in the 21-cm line of neutral hydrogen often extends well beyond the optical image of the galaxy where the centripetal acceleration is small and the discrepancy is large. In the particularly critical case of the LSB galaxies, 21-cm line observations can be supplemented by H_α observations (McGaugh et al. 2001, de Blok & Bosma 2002) and compared in detail with the rotation curve predicted from the distribution of detectable matter. The procedure that has usually been followed is outlined below:

1. One assumes that light traces stellar mass, i.e., M/L is constant in a given galaxy. There are color gradients in spiral galaxies, so this cannot be generally true—or at least one must decide which color band is the best tracer of the mass distribution. The general opinion is that the near-infrared emission of spiral galaxies is the optimal tracer of the underlying stellar mass distribution because the old population of low-mass stars contributes to this emission, and the near-infrared is less affected by dust obscuration. Thus, where available, near-infrared surface photometry is preferable.
2. In determining the distribution of detectable matter one must include the observed neutral hydrogen scaled up with an appropriate correction factor (typically 1.3–1.4) to account for the contribution of primordial helium. The gas can make a dominant contribution to the total mass surface density in some (generally low luminosity) galaxies.

3. Given the observed distribution of mass, the Newtonian gravitational force, g_n, is calculated via the classical Poisson equation. Here it is usually assumed that the stellar and gaseous disks are razor thin. It may also be necessary to add a spheroidal bulge if the light distribution indicates the presence of such a component.
4. Given the radial distribution of the Newtonian force, the true gravitational force, g, is calculated from the MOND formula (Equation 3) with a_o fixed. Then the mass of the stellar disk is adjusted until the best fit to the observed rotation curve is achieved. This gives M/L of the disk as the single free parameter of the fit (unless a bulge is present).

In this procedure, one assumes that the motion of the gas is a coplaner rotation about the center of the given galaxy. This is certainly not always the case because there are well-known distortions to the velocity field in spiral galaxies caused by bars and warping of the gas layer. In a fully two-dimensional velocity field these distortions can often be modeled (Bosma 1978, Begeman 1989), but the optimal rotation curves are those in which there is no evidence for the presence of significant deviations from coplanar circular motion. Not all observed rotation curves are perfect tracers of the radial distribution of force. A perfect theory will not fit all rotation curves because of these occasional problems (the same is true of a specified dark-matter halo). The point is that with MOND, usually, there is one adjustable parameter per galaxy and that is the mass or M/L of the stellar disk.

The preferred value of a_o has been derived from a highly selected sample of large galaxies with well-determined rotation curves (Begeman et al. 1991). Assuming a distance scale, $H_o = 75$ km/s Mpc in this case, rotation-curve fits to all galaxies in the sample were made allowing a_o to be a free parameter. The mean value for nine of the galaxies in the sample, excluding NGC 2841 with a distance ambiguity (see below), is $1.2 \pm 0.27 \times 10^{-8}$ cm/s^2. Having fixed a_o in this way, one is no longer free to take this as a fitting parameter.

There is, however, a relation between the derived value of a_o and the assumed distance scale because the implied centripetal acceleration in a galaxy scales as the inverse of the assumed distance. With respect to galaxy rotation curves, this dependence is not straightforward because the relative contributions of the stellar and gaseous components to the total force vary as a function of distance. For a gas-rich sample of galaxies the derived value of a_o scales as H_o^2, and for a sample of HSB galaxies dominated by the stellar component $a_o \propto H_o$. This is related to a more general property of MOND: a_o in its different roles scales differently with H_o. This fact in itself means that MOND cannot live with any distance scale; to be consistent with MOND, H_o must be in the range of 50–80 km/s-Mpc.

Figure 3 shows two examples of MOND fits to rotation curves. The dotted and dashed curves are the Newtonian rotation curves of the stellar and gaseous disks, respectively, and the solid curve is the MOND rotation curve with the standard value of a_o. Not only does MOND predict the general trend for LSB and HSB galaxies, but it also predicts the observed rotation curves in detail from the observed distribution of matter. This procedure has been carried out for about 100 galaxies, 76 of which are listed in Table 1; the results are given in terms of the fitted mass of the stellar disk (in most cases, the only free parameter) and the implied M/Ls.

Rotation curves for the entire UMa sample of Sanders & Verheijen (1998) are shown in Figure 4, where the curves and points have the same meaning as in Figure 3. As noted above, this is a complete and unbiased sample of spiral galaxies all at about the same distance (here taken to be 15.5 Mpc). The sample includes both HSB galaxies (e.g., NGC 3992) and LSB galaxies (e.g., UGC 7089) and covers a factor of 10 in centripetal acceleration at the outermost observed point (Table 1 and Figure 1). The objects denoted by the asterisk in Figure 4 are galaxies previously designated by Verheijen (1997) as being kinematically disturbed (e.g.,

TABLE 1 Rotation-curve fits

Galaxy (1)	**Type (2)**	L_B **(3)**	L_r **(4)**	M_{HI} **(5)**	V_∞ **(6)**	M_* **(7)**	M_*/L_B **(8)**	M_*/L_r **(9)**	**Ref (10)**
UGC 2885	Sbc	21.0	—	5.0	300	30.8	1.5	—	1
NGC 2841[a]	Sb	8.5	17.9	1.7	287	32.3	3.8	1.8	2
NGC 5533	Sab	5.6	—	3.0	250	19.0	3.4	—	1
NGC 6674	SBb	6.8	—	3.9	242	18.0	2.6	—	1
NGC 3992	SBbc	3.1	7.0	0.92	242	15.3	4.9	2.2	3
NGC 7331	Sb	5.4	18.0	1.1	232	13.3	2.5	0.7	2
NGC 3953	SBbc	2.9	8.5	0.27	223	7.9	2.7	0.9	3
NGC 5907	Sc	2.4	4.9	1.1	214	9.7	3.9	2.0	1
NGC 2998	SBc	9.0	—	3.0	213	8.3	1.2	—	1
NGC 801	Sc	7.4	—	2.9	208	10.0	1.4	—	1
NGC 5371	S(B)b	7.4	—	1.0	208	11.5	1.6	—	1
NGC 5033	Sc	1.90	3.90	0.93	195	8.8	4.6	2.3	1
NGC 3893[b]	Sc	2.14	3.98	0.56	188	4.20	2.0	1.1	3
NGC 4157	Sb	2.00	5.75	0.79	185	4.83	2.4	0.8	3
NGC 2903	Sc	1.53	2.15	0.31	185	5.5	3.6	2.6	2
NGC 4217	Sb	1.90	5.29	0.25	178	4.25	2.2	0.8	3
NGC 4013	Sb	1.45	4.96	0.29	177	4.55	3.1	0.9	3
NGC 3521	Sbc	2.40	—	0.63	175	6.5	2.7	—	1
NGC 4088[b]	Sbc	2.83	5.75	0.79	173	3.30	1.1	0.6	3
UGC 6973[b]	Sab	0.62	2.85	0.17	173	1.69	2.7	0.6	3
NGC 3877	Sc	1.94	4.52	0.14	167	3.35	1.7	0.7	3
NGC 4100	Sbc	1.77	3.50	0.30	164	4.32	2.4	1.2	3

(*Continued*)

TABLE 1 (*Continued*)

Galaxy (1)	Type (2)	L_B (3)	L_r (4)	M_{HI} (5)	V_∞ (6)	M_* (7)	M_*/L_B (8)	M_*/L_r (9)	Ref (10)
NGC 3949	Sbc	1.65	2.33	0.33	164	1.39	0.8	0.6	3
NGC 3726	SBc	2.65	3.56	0.62	162	2.62	1.0	0.7	3
NGC 6946	SABcd	5.30	—	2.7	160	2.7	0.5	—	1
NGC 4051[b]	SBbc	2.58	3.91	0.26	159	3.03	1.2	0.8	3
NGC 3198[c]	Sc	0.90	0.80	0.63	156	2.3	2.6	2.9	2
NGC 2683	Sb	0.60	—	0.05	155	3.5	5.8	—	1
UGC 5999[e]	Im	—	0.13	0.25	155	0.09	—	0.7	4
NGC 4138	Sa	0.82	2.88	0.14	147	2.87	3.5	1.0	3
NGC 3917	Scd	1.12	1.35	0.18	135	1.40	1.3	1.0	3
NGC 4085	Sc	0.81	1.22	0.13	134	1.00	1.2	0.8	3
NGC 2403	Sc	0.79	0.98	0.47	134	1.1	1.4	1.1	2
NGC 3972	Sbc	0.68	1.00	0.12	134	1.00	1.5	1.0	3
UGC 128	Sdm	0.52	0.41	0.91	131	0.57	1.1	1.4	4
NGC 4010	SBd	0.63	1.20	0.27	128	0.86	1.4	0.7	3
F568-V1	Sd	0.22	0.17	0.34	124	0.66	3.0	3.8	4
NGC 3769[b]	SBb	0.68	1.27	0.53	122	0.80	1.2	0.6	3
NGC 6503	Sc	0.48	0.47	0.24	121	0.83	1.7	1.8	2
F568-3	Sd	0.33	0.27	0.39	120	0.44	1.3	1.6	4
F568-1	Sc	0.28	0.21	0.56	119	0.83	3.0	4.0	4
NGC 4183	Scd	0.90	0.73	0.34	112	0.59	0.7	0.8	3
F563-V2	Irr	0.30	—	0.32	111	0.55	1.8	—	4
F563-1	Sm	0.14	0.10	0.39	111	0.40	3.0	4.0	4
NGC 4389[b]	SBbc	0.61	1.22	0.05	110	0.23	0.4	0.2	3
NGC 1003	Scd	1.50	0.45	0.82	110	0.30	0.2	0.7	1
UGC 6917	SBd	0.38	0.42	0.20	110	0.54	1.4	1.3	3
UGC 6930	SBd	0.50	0.40	0.31	110	0.42	0.8	1.0	3
M 33	Sc	0.74	0.43	0.13	107	0.48	0.6	1.1	1
UGC 6983	SBcd	0.34	0.34	0.29	107	0.57	1.7	1.7	3
NGC 247	SBc	0.35	0.22	0.13	107	0.40	1.1	1.8	1
UGC 1230[e]	Sm	0.32	0.22	0.81	102	0.38	1.2	1.7	4
F574-1[d]	Sd	—	0.37	0.49	100	0.26	—	0.7	4
NGC 7793	Scd	0.34	0.17	0.10	100	0.41	1.2	2.4	1
UGC 5005[e]	Im	—	0.15	0.41	99	0.74	—	4.8	4

(*Continued*)

TABLE 1 (*Continued*)

Galaxy (1)	Type (2)	L_B (3)	L_r (4)	M_{HI} (5)	V_∞ (6)	M_* (7)	M_*/L_B (8)	M_*/L_r (9)	Ref (10)
NGC 300	Sc	0.30	—	0.13	90	0.22	0.7	—	1
NGC 5585	SBcd	0.24	0.14	0.25	90	0.12	0.5	0.9	1
NGC 2915[f]	BCD	0.04	—	0.10	90	0.25	6.9	—	1
UGC 6399	Sm	0.20	0.21	0.07	88	0.21	1.0	1.0	3
NGC 55	SBm	0.43	—	0.13	86	0.10	0.2	—	1
UGC 6667	Scd	0.26	0.28	0.08	86	0.25	1.0	0.9	3
UGC 2259	SBcd	0.10	—	0.05	86	0.22	2.1	—	2
F583-1	Sm	0.06	0.06	0.24	85	0.11	1.7	2.0	4
UGC 6446	Sd	0.25	0.14	0.30	82	0.12	0.5	0.9	3
UGC 6923	Sdm	0.22	0.21	0.08	81	0.16	0.8	0.8	3
UGC 7089	Sdm	0.44	0.21	0.12	79	0.09	0.2	0.4	3
UGC 5750[e]	SBdm	—	0.36	0.14	75	0.32	—	0.9	4
UGC 6818[b]	Sd	0.18	0.12	0.10	73	0.04	0.2	0.3	3
F571-V1[e]	Sdm	0.10	0.80	0.164	73	0.67	0.7	0.8	4
NGC 1560	Sd	0.035	0.063	0.098	72	0.034	1.0	0.5	2
F583-4	Sc	—	0.071	0.077	67	0.022	—	0.3	4
IC 2574	SBm	0.080	0.022	0.067	66	0.010	0.1	0.5	1
DDO 170	Im	0.016	—	0.061	64	0.024	1.5	—	2
NGC 3109	SBm	0.005	—	0.068	62	0.005	0.1	—	2
DDO 154	IB	0.005	—	0.045	56	0.004	0.1	—	2
DDO 168	Irr	0.022	—	0.032	54	0.005	0.2	—	1
F565-V2[e]	Im	0.023	0.019	0.084	51	0.050	2.2	2.7	4

Explanation of columns of Table 1: (1) Galaxy name. (2) Morphological Type. (3) *B*-band luminosity in units of $10^{10}\ L_\odot$. (4) Red band luminosity in units of $10^{10}\ L_\odot$. The precise band used depends on the reference: Refs. 1 & 2: *H*-band. Ref. 3: K'-band. Ref. 4: *R*-band. (5) Mass of neutral hydrogen in units of $10^{10}\ M_\odot$ assuming $M_{gas} = M_{HI}$. (6) Asymptotic flat rotation velocity in km s^{-1}. (7) Stellar mass from MOND fit in units of $10^{10}\ M_\odot$. (8) *B*-band stellar mass-to-light ratio in units of $M_\odot/L_\odot$. (9) *R*-band stellar mass-to-light ratio in units of $M_\odot/L_\odot$. Both *B*- and *R*-band mass-to-light ratios refer only to the stars (the gas is not included in the mass) and average over disk and bulge where both components are significant. See original references for further details. (10) References: 1. Sanders (1996). 2. Begeman, Broeils, & Sanders (1991). 3. Sanders & Verheijen (1998). 4. de Blok & McGaugh (1998).

Notes for Table 1: [a]The MOND fit for this galaxy is sensitive to its distance, prefering $D \approx 19$ Mpc (Sanders 1996) to the Hubble flow value of ≈9 Mpc. Macri et al. (2001) give a Cepheid distance of 14 Mpc, which is marginally tolerable given the uncertainties in this galaxy's warp (R. Bottema, J.L.G. Pestanã, B. Rothberg, R.H. Sanders, unpublished manuscript).

[b]Noted as having disturbed kinematics by Verheijen (1997).

[c]The MOND fit for this galaxy is sensitive to its distance, prefering a smaller value than the Cepheid distance of 13.8 Mpc (R. Bottema, J.L.G. Pestanã, B. Rothberg, R.H. Sanders, unpublished manuscript).

[d]The original MOND fit for this galaxy (de Blok & McGaugh 1998) was not very good. The 21-cm observations of this galaxy were severely affected by beam smearing.

[e]Inclination uncertain (de Blok & McGaugh 1998).

[f]Distance uncertain.

nonaxisymmetric velocity field caused by bars or interactions); the derived rotation curves are less secure for these galaxies. (Note that fits to the UMa rotation curves using a revised cluster distance of 18.5 Mpc from the Cepheid-based recalibrated TF relation of Sakai et al. (2000) would imply that a_o should be reduced to 1×10^{-8} cm/s^2.)

This is a fair selection of MOND fits to rotation curves in which the only free parameter is the M/L of the visible disk (no separate bulge components were fitted in these cases). In HSB objects in which the centripetal acceleration remains large out to the last measure point of the rotation curve, such as NGC 3954, there is a very small difference between the Newtonian curve and the predicted

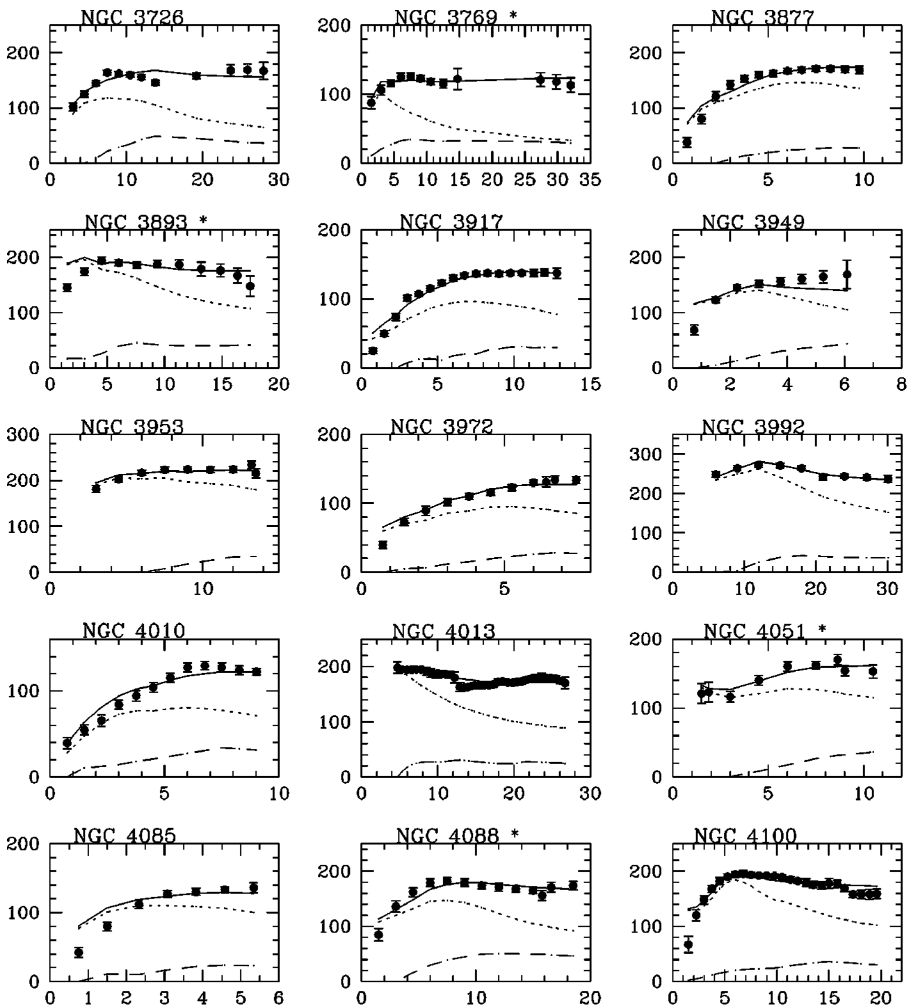


Figure 4 (*Continued*)

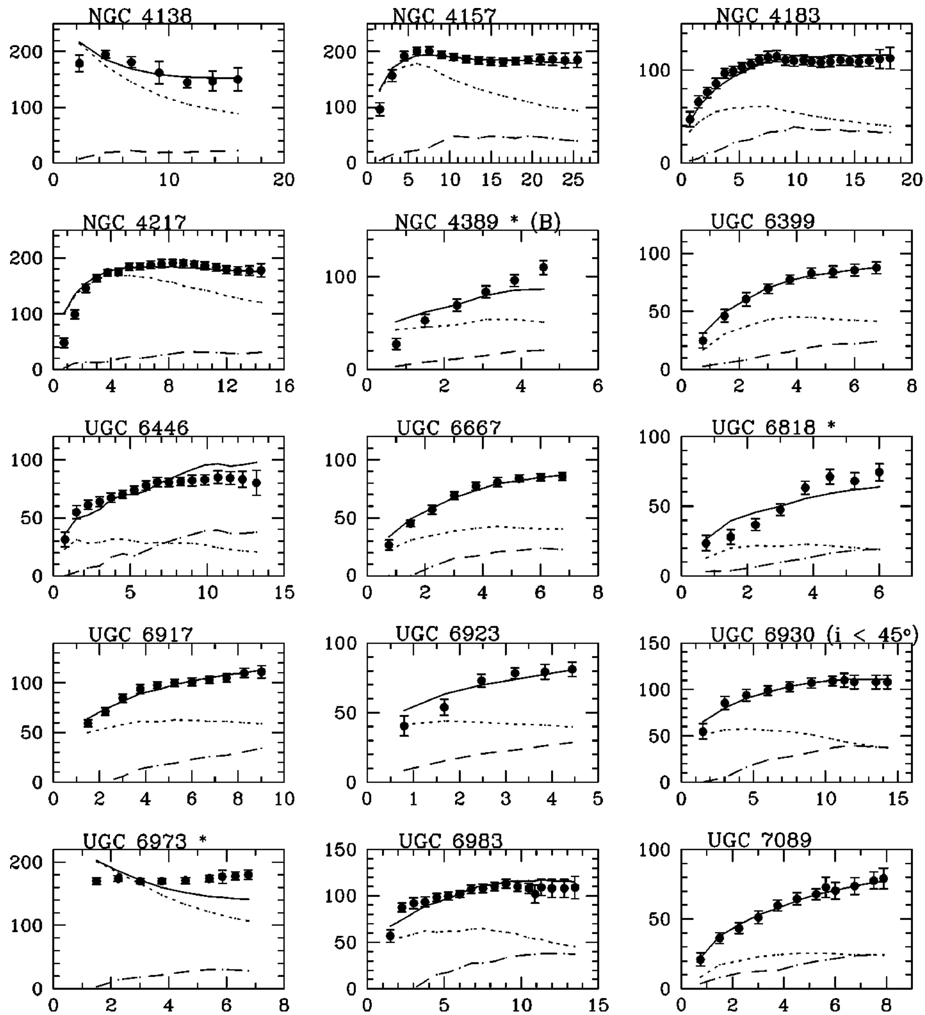


Figure 4 MOND fits to the rotation curves of the Ursa Major galaxies (Sanders & Verheijen 1998). The radius (horizontal axis) is given in kiloparsecs, and in all cases the rotation velocity is in kilometers/second. The points and curves have the same meaning as in Figure 3. The distance to all galaxies is assumed to be 15.5 Mpc, and a_o is the Begeman et al. (1991) value of 1.2×10^{-8} cm/s^2. The free parameter of the fitted curve is the mass of the stellar disk. If the distance to UMa is taken to be 18.6 Mpc, as suggested by the Cepheid-based recalibration of the Tully-Fisher relation (Sakai et al. 2000), then a_o must be reduced to 10^{-8} cm/s^2.

curve; i.e., the observed rotation curve is reasonably well described by Newtonian theory, as expected when accelerations are high. In lower acceleration systems, the discrepancy is larger (e.g., UGC 6667). In gas-rich galaxies, such as UGC 7089, the shape of the rotation curve in the outer regions essentially follows from the shape of the Newtonian rotation curve of the gaseous component, as though the gas surface density were only scaled up by some factor on the order of 10—a property that has been noticed for spiral galaxies (Carignan & Beaulieu 1989, Hoekstra et al. 2001). Empirically, MOND gives the rule that determines the precise scaling.

In general the MOND curves agree well with the observed curves, but there are some cases in which the agreement is less than perfect. Usually these cases have an identifiable problem with the observed curve or its interpretation as a tracer of the radial force distribution. For example NGC 4389 is strongly barred, and the neutral hydrogen is contiguous with the visible disk and bar. Another example is UGC 6818, which is probably interacting with a faint companion at its western edge.

Figure 5 shows a less homogeneous sample of rotation curves. These are curves from the literature based upon observations carried out either at the Westerbork Radio Synthesis Telescope or the Very Large Array (VLA) from Sanders (1996) and McGaugh & de Blok (1998b) and ranked here in order of decreasing circular velocity. These are mostly galaxies with a large angular size, so there are many independent points along the rotation curve. The selection includes HSB and LSB galaxies such as NGC 2403 and UGC 128—two objects with the same asymptotic rotation velocity (≈130 km/s). Here the general trend, predicted by MOND, is evident: The LSB exhibits a large discrepancy throughout the galaxy in contrast to the HSB, where the discrepancy becomes apparent in the outer regions. In several objects, such as NGC 2403, NGC 6503, and M33, the quality of the MOND fit is such that, given the density of points, the fitted curve cannot be distinguished beneath the observations.

The most striking aspect of these studies is the fact that not only general trends but also the details of individual curves are well reproduced by Milgrom's simple formula applied to the observed distribution of matter. In only about 10% of the roughly 100 galaxies considered in the context of MOND does the predicted rotation curve differ significantly from the observed curve.

We have emphasized that the only free parameter in these fits is the M/L of the visible disk, so one may well ask if the inferred values are reasonable. It is useful to consider again the UMa sample because all galaxies are at the same distance and there is K′-band (near infrared) surface photometry of the entire sample. Figure 6 shows the M/L in the B-band required by the MOND fits plotted against B-V color index (*top*) and the same for the K′-band (*bottom*). We see that in the K′-band $M/L \approx 0.8$ with a 30% scatter. In other words if one were to assume a K′-band M/L of about 1 at the outset, most rotation curves would be quite reasonably predicted from the observed light and gas distribution with no free parameters. In the B-band, on the other hand, the MOND M/L does appear to be a function of color in the sense that redder objects have larger M/L values. This is exactly what is expected from population synthesis models, as is shown by the solid lines in both panels (Bell & de Jong 2001). This is quite interesting because there is

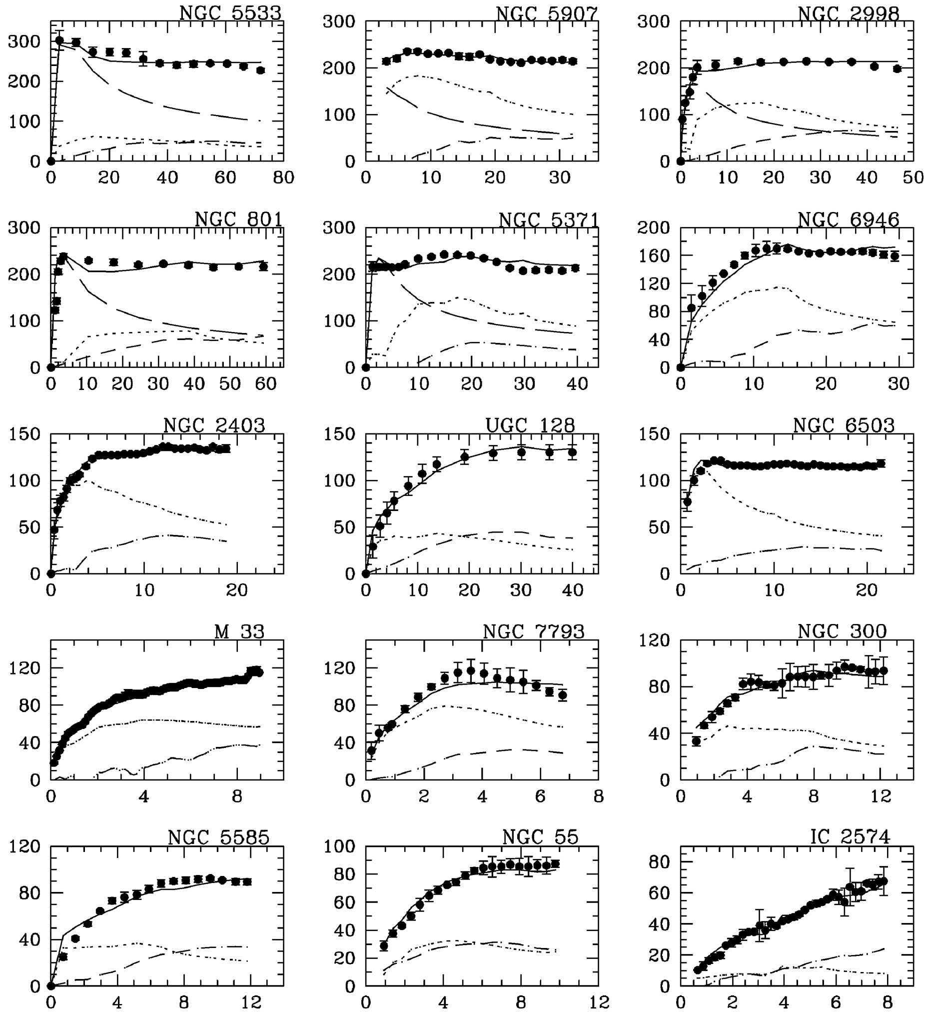


Figure 5 MOND fits to the rotation curves of spiral galaxies with published data, from Sanders (1996) and McGaugh & de Blok (1998). The symbols and curves are as in Figure 4.

nothing built into MOND that would require that redder galaxies should have a higher M/L_b; this simply follows from the rotation-curve fits.

Falsification of Modified Newtonian Dynamics with Rotation Curves

It is sometimes said that MOND is designed to fit rotation curves, so it is not surprising that it does so. This is not only a trivialization of a remarkable phenomenological success, but it is also grossly incorrect. MOND was designed to

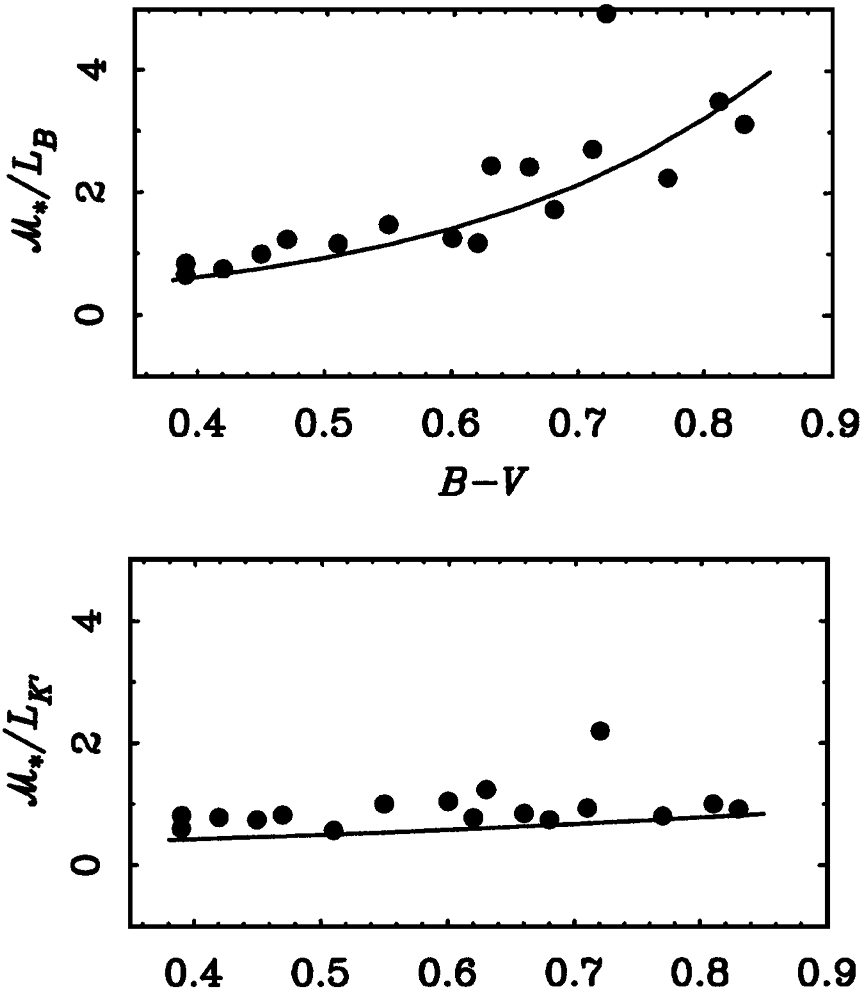


Figure 6 Inferred mass-to-light ratios for the UMa spirals (Sanders & Verheijen) in the B-band (*top*) and the K′-band (*bottom*) plotted against blue-visual (B-V) color index. The solid lines show predictions from population synthesis models by Bell & de Jong (2001).

produce asymptotically flat rotation curves with a given mass-velocity relation (or TF law). It was not designed to fit the details of all rotation curves with a single adjustable parameter—the M/L of the stellar disk (MOND also performs well on galaxies that are gas dominated and have no adjustable parameter). It was certainly not designed to provide a reasonable dependence of fitted M/L on color. Indeed, none of the rotation curves listed in Table 1 were available in 1983; designing a theory to fit data that are not yet taken is called "prediction."

However, MOND is particularly vulnerable to falsification by rotation-curve data. Although there are problems, mentioned above, in the measurement and interpretation of velocity field and photometric data, MOND should not "fail" too often; especially damaging would be a systematic failure for a particular class of objects. In this regard Lake (1989) has claimed that the value of a_o required to fit rotation curves varies with the maximum rotation velocity of the galaxy in the sense that objects with lower rotation velocities (and therefore lower luminosity galaxies) require a systematically lower value of a_o. He supported this claim by rotation-curve fits to six dwarf galaxies with low internal accelerations. If this were true, then it would be quite problematic for MOND, implying at the very least a modification of Milgrom's simple formula. Milgrom (1991) responded to this criticism by pointing out inadequacies in the data used by Lake: uncertainties in the adopted distances and/or inclinations. Much of the rotation-curve data is also of lower quality than the larger galaxies considered in the context of MOND.

R.A. Swaters & R.H. Sanders (2002, unpublished manuscript) reconsidered this issue on the basis of extensive 21-cm line observations of a sample of 35 dwarf galaxies (Swaters 1999). When a_o is taken as an additional free parameter, the effect pointed out by Lake is not seen: There is no systematic variation of a_o with the maximum rotation velocity of a galaxy. There is a large scatter in the fitted a_o, but this is due to the fact that many dwarf galaxies contain large asymmetries or an irregular distribution of neutral hydrogen. Moreover, the galaxies in this sample have large distance uncertainties, the distances in many cases being determined by group membership. The mean a_o determined from the entire sample ($\approx 10^{-8}$ cm/s^2) is consistent with that implied by the revised Cepheid-based distance scale.

Given the well-known uncertainties in the interpretation of astronomical data, it is difficult to claim that MOND is falsified on the basis of a single rotation curve. However, it should generally be possible to identify the cause of failures (i.e., poor resolution, bars, interactions, warps, etc.). An additional uncertainty is the precise distance to a galaxy because, as noted above, the estimated internal acceleration in a galaxy depends upon its assumed distance. For nearby galaxies, such as those of the Begeman et al. (1991) sample, the distances are certainly not known to an accuracy of better than 25%. When the MOND rotation curve is less than a perfect match to the observed curve, it is often possible to adjust the distance, within reasonable limits (i.e., the distance appears as a second free parameter). In principle, precise independent distance determinations place more severe restrictions on this extra degree of freedom and are therefore relevant to rotation-curve tests of MOND.

There are now four galaxies from the original sample of Begeman et al. (1991) with Cepheid-based distances. Three of these (NGC 2403, NGC 3198, and NGC 7331) have been observed as part of the HST Key Project on the extragalactic distance scale (Sakai et al. 2000). For NGC 2403 and NGC 7331 the MOND rotation curve fits precisely the observed curve at the Cepheid-based distances.

For NGC 3198 MOND clearly prefers a distance at least 10% smaller than the Cepheid-based distance of 13.8 Mpc (R. Bottema, J.L.G. Pestanã, B. Rothberg, R.H. Sanders, unpublished manuscript), even with the lower value of a_o implied by the revised distance scale. Given the likely uncertainties in the Cepheid method, and in the conversion of a 21-cm line velocity field to a rotation curve, this cannot be interpreted as problematic for MOND.

A more difficult case is presented by NGC 2841, a large spiral galaxy with a Hubble distance of about 10 Mpc (Begeman et al. 1991). The rotation curve of the galaxy cannot be fit using MOND if the distance is only 10 Mpc; MOND, as well as the TF law, prefers a distance of 19 Mpc (Begeman et al. 1991, Sanders 1996). A Cepheid distance to this galaxy has been determined (Macri et al. 2001) at 14.1 ± 1.5 Mpc. At a distance of 15.6 Mpc the MOND rotation curve of the galaxy still systematically deviates from the observed curve, and the implied M/L is 8.5; thus, this galaxy remains the most difficult case for MOND. It is nonetheless interesting that the Cepheid-calibrated T-F relation (Sakai et al. 2001) implies a distance of about 23 Mpc for NGC 2841, and supernova 1999 by, if a normal type Ia, would imply a distance of 24 Mpc.

Overall, the ability of MOND, as an ad hoc algorithm, to predict galaxy rotation curves with only one free parameter (the M/L of the visible disk) is striking. This implies that galaxy rotation curves are entirely determined by the distribution of observable matter. Regardless of whether or not MOND is correct as a theory, it does constitute an observed phenomenology that demands explanation. Herein lies a real conundrum for the dark matter picture. The natural expectations of dark matter theories for rotation curves do not look like MOND, and hence fail to reproduce a whole set of essential observational facts. The best a dark matter theory can hope to do is contrive to look like MOND and hence reproduce a posteriori the many phenomena that MOND successfully predicts. This gives one genuine pause to consider how science is supposed to proceed.

PRESSURE-SUPPORTED SYSTEMS

General Properties

Figure 7 is a log-log plot of the velocity dispersion versus size for pressure-supported, nearly isothermal astronomical systems. At the bottom left the star-shaped points are globular clusters (Pryor & Meylen 1993, Trager et al. 1993) and the solid points are giant molecular clouds in the Galaxy (Solomon et al. 1987). The group of points (*crosses*) near the center are high–surface brightness elliptical galaxies (Jørgensen et al. 1995a,b, Jørgensen 1999). At the upper right the squares indicate X-ray-emitting clusters of galaxies from the compilation by White et al. (1997). The triangle-shaped points are the dwarf spheroidal systems surrounding the Milky Way (Mateo 1998), and the dashes are compact dwarf ellipticals (Bender et al. 1992). The plotted parameters are taken directly from the relevant observational papers. The measure of size is not homogeneous: For ellipticals and globular

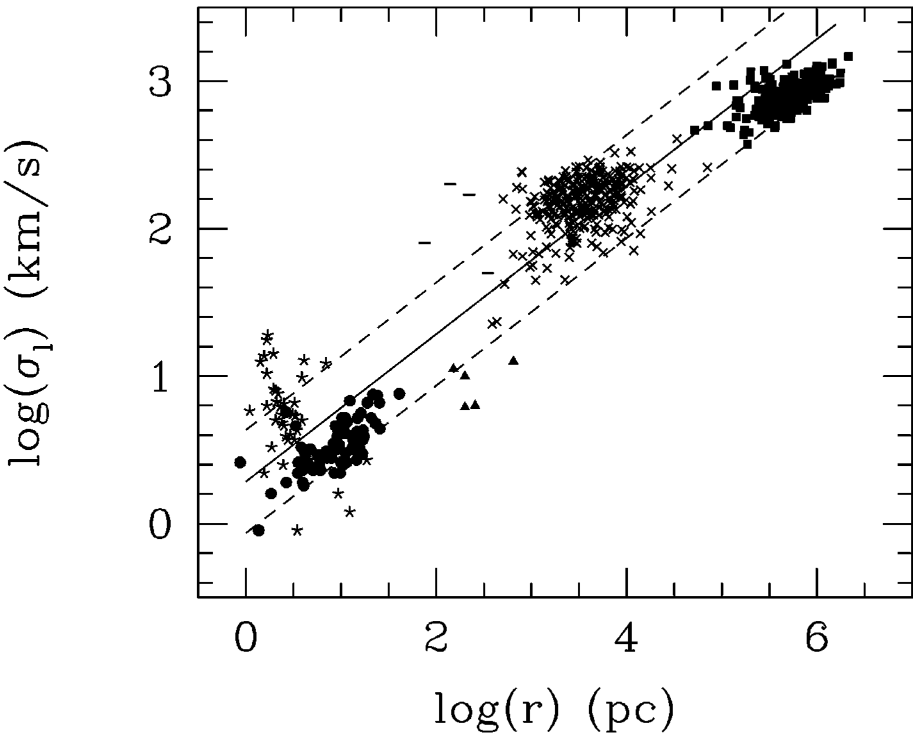


Figure 7 The line-of-sight velocity dispersion vs. characteristic radius for pressure-supported astronomical systems. The star-shaped points are globular clusters (Pryor & Meylen 1993, Trager et al. 1993), the points are massive molecular clouds in the Galaxy (Solomon et al. 1987), the triangles are the dwarf spheroidal satellites of the Galaxy (Mateo 1998), the dashes are compact elliptical galaxies (Bender et al. 1992), the crosses are massive elliptical galaxies (Jørgensen et al. 1995a,b; Jørgensen 1999), and the squares are X-ray-emitting clusters of galaxies (White et al. 1997). The solid line shows the relation $\sigma_l^2/r = a_o$, and the dashed lines a factor of 5 variation about this relation.

clusters it is the well-known effective radius; for the X-ray clusters it is an X-ray intensity isophotal radius; and for the molecular clouds it is an isophotal radius of CO emission. The velocity dispersion refers to the central velocity dispersion for ellipticals and globulars; for the clusters it is the thermal velocity dispersion of the hot gas; for the molecular clouds it is just the typical line width of the CO emission.

The parallel lines represent fixed internal accelerations. The solid line corresponds to $\sigma_l^2/r = 10^{-8}$ cm/s^2, and the parallel dashed lines to accelerations five times larger or smaller than this particular value. It is clear from this diagram that the internal accelerations in most of these systems are within a factor of a few a_o. This also implies that the surface densities in these systems are near the MOND surface density Σ_m.

It is easy to over-interpret such a log-log plot containing different classes of objects covering many orders of magnitude in each coordinate. We do not wish to claim a velocity–dispersion size correlation, although such a relationship has been previously noticed for individual classes of objects—in particular, for molecular clouds (Solomon et al. 1987) and clusters of galaxies (Mohr & Evrard 1997). Probable pressure-supported systems such as super-clusters of galaxies (Eisenstein et al. 1996) and Ly α forest clouds (Schaye 2001) are clearly not on this relation, but there are low-density solutions for MOND isothermal objects (Milgrom 1984) that have internal accelerations far below a_o.

It has been noted above that, with MOND, if certain very general conditions are met, self-gravitating, pressure-supported systems would be expected to have internal accelerations comparable to or less than a_o. The essential condition is that these objects should be approximately isothermal. It is not at all evident how Newtonian theory with dark matter can account for the fact that these different classes of astronomical objects, covering a large range in size and located in very different environments, all appear to have comparable internal accelerations. In the context of MOND the location of an object in this diagram, above or below the $\sigma_l^2/r = a_o$ line, is an indicator of the internal dynamics and the extent to which these dynamics deviate from Newtonian theory.

Now we consider individual classes of objects on Figure 7.

Luminous Elliptical Galaxies

Systems above the solid line in Figure 7, e.g., the luminous elliptical galaxies, are high–surface brightness objects and, in the context of MOND, would not be expected to show a large mass discrepancy within the bright optical object. In other words, if interpreted in terms of Newtonian dynamics, these objects should not exhibit much need for dark matter within an effective radius; this is indicated by analysis of the stellar kinematics in several individual galaxies (e.g., Saglia et al. 1992). MOND isotropic, isothermal spheres have a lower mean internal acceleration within r_e (about one-quarter a_o); i.e., these theoretical objects lie significantly below the solid line in Figure 7. This was noted by Sanders (2000), who pointed out that, to be consistent with their observed distribution in the $r - \sigma_l$ plane, elliptical galaxies cannot be represented by MOND isothermal spheres; these objects must deviate both from being perfectly isothermal (in the sense that the velocity dispersion decreases outward) and from perfect isotropy of the velocity distribution (in the sense that stellar orbits become radial in the outer regions).

The general properties of ellipticals can be matched by representing these objects as high-order, anisotropic polytropic spheres; i.e., objects having a radial velocity dispersion-density relation of the form

$$\sigma_r^2 = A\rho^{1/n}, \tag{12}$$

where A is a constant depending upon n, the polytropic index. In these models the deviation from isotropy toward more radial orbits appears beyond an anisotropy radius, r_a. To reproduce the distribution of ellipticals in the $r_e - \sigma_l$ plane, it must be the case that $12 \leq n \leq 16$ and that $r_a > r_e$ (i.e., the radial orbit anisotropy does not extend within an effective radius). However, the strict homology of models is broken and the mass-velocity relation given above for isotropic, isothermal spheres (Equation 8) is replaced by a more general relation of the form

$$\sigma_l^4 = q(\Sigma/\Sigma_m)\, G a_o M. \tag{13}$$

That is to say, for these mixed Newtonian-MOND objects, the mean surface density enters as an additional parameter; actual elliptical galaxies would comprise a two-parameter family and not a one-parameter family, as suggested by the simple MOND $M - \sigma_l$ relation for a homologous class of objects.

This is consistent with the fact that elliptical galaxies do seem to comprise a two-parameter family, as indicated by the small scatter about the "fundamental plane"—a relation between the luminosity, effective radius, and central line-of-sight velocity dispersion of the form $L \propto \sigma^a r_e^b$, where $a \approx 1.5$ and $b \approx 0.8$ (Dressler et al. 1987, Djorgovsky & Davis 1987). This has generally been attributed to the traditional virial theorem combined with a systematic dependence of M/L upon luminosity (e.g., van Albada et al. 1995). With MOND high-order polytropes are Newtonian in the inner region and MONDian beyond an effective radius. MOND imposes boundary conditions upon the inner Newtonian solution that restrict these solutions to a dynamical fundamental plane, i.e., $M \propto \sigma^\alpha r_e^\gamma$, where the exponents may differ from the Newtonian expectations. The breaking of homology leads to a considerable dispersion in the $M - \sigma$ relation owing to a factor of 10 dispersion in q in Equation 13. This is shown in Figure 8*a*, which is the $M - \sigma$ relation for the anisotropic polytropes covering the required range in n and r_a. A least-square fit yields

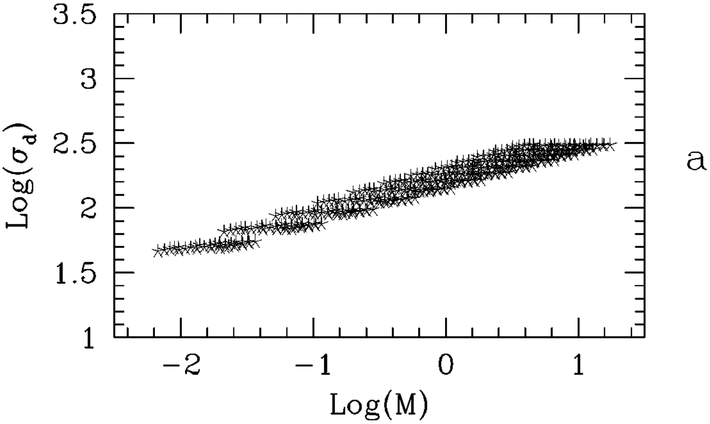
$$M/(10^{11} M_\odot) = 2 \times 10^{-8}[\sigma_d(\mathrm{km/s})]^{3.47}, \tag{14}$$

where σ_d is the velocity dispersion measured within a circular aperture of fixed linear size as defined by Jørgensen et al. (1995a). The fact that q in Equation 13 is a well-defined function of mean surface brightness (roughly a power-law) results in a tighter fundamental plane relation (Figure 8*b*) of the form

$$M/(10^{11} M_\odot) = 3 \times 10^{-5}[\sigma_d(\mathrm{km\,s^{-1}})]^{1.76}[r_e(\mathrm{kpc})]^{0.98} \tag{15}$$

(see Figure 8*b*). With $M/L \propto M^{0.17}$, the fundamental plane in its observed form is reproduced.

The existence of a fundamental plane, in itself, is not a critical test for MOND because Newtonian theory also predicts such a relation via the virial theorem. However, for MOND a single relation of the form of Equation 15 applies for range of nonhomologous models; this is due to the underlying dynamics and not to the details of galaxy formation or subsequent dynamical evolution. A curious aspect of the Newtonian basis for the fundamental plane is the small scatter in the observed relation given the likely deviations from homology—particularly considering a dynamical history that presumably involves multiple mergers. Moreover, Newtonian



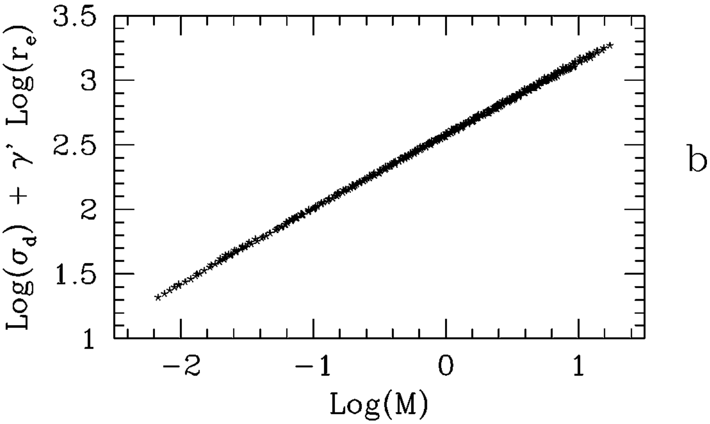


Figure 8 (*a*) The mass-velocity dispersion relation for an ensemble of anisotropic polytropes covering the range necessary to produce the observed properties of elliptical galaxies. Mass is in units of 10^{11} $M_\odot$ and velocity in kilometers/second. (*b*) The result of entering a third parameter; i.e., this is the best-fitting fundamental plane relation. $\log(\sigma_d) + \gamma' \log(r_e)$ is plotted against $\log(M)$ and γ' is chosen to give the lowest scatter (r_e is in kiloparsecs). The resulting slope is about 1.76 with $\gamma' = 0.56$. From Sanders (2000).

theory offers no explanation for the existence of a mass-velocity dispersion relation (even one with large scatter). As noted above, in the context of MOND, a near-isothermal object with a velocity dispersion of a few hundred km/s will always have a galactic mass.

The compact dwarf ellipticals (the dashes in Figure 7) have internal accelerations considerably greater than a_o and a mean surface brightness larger than Σ_m. In the context of MOND this can only be understood if these objects deviate considerably from isothermality. If approximated by polytropic spheres, these objects would have a polytropic index, n, less than about 10 (for $n \leq 5$ Newtonian polytropes no longer have infinite extent and are not necessarily MONDian objects; thus, there are no restrictions upon the internal accelerations or mean surface densities). This leads to a prediction: For such high surface brightness objects the line-of-sight stellar velocity dispersion should fall more dramatically with projected radius than in those systems with $\langle\Sigma\rangle \approx \Sigma_m$. We might also expect the compact dwarfs not to lie on the fundamental plane as defined by the lower surface brightness ellipticals.

Dwarf Spheroidal Systems

With MOND, systems that lie below the solid line in Figure 7, i.e., those systems with low internal accelerations, would be expected to exhibit larger discrepancies. This is particularly true of the dwarf spheroidal systems with internal accelerations ranging down to 0.1 a_o. On the basis of the low surface brightness of these systems Milgrom (1983b) predicted that, when velocity dispersion data became available for the stellar component, these systems should have a dynamical mass 10 or more times larger than that accounted for by the stars. These kinematic data are now available, and, indeed, these systems, when considered in the context of Newtonian dynamics, require a significant dark matter content, as is indicated by M/L values in the range of 10–100 (Mateo 1998).

For a spherically symmetric, isolated, low-density object that is deep in the MOND regime, a general mass estimator is given by

$$M = \frac{81}{4}\frac{\sigma^4}{Ga_o}, \tag{16}$$

where σ is the line-of-sight velocity dispersion (Gerhard & Spergel 1992, Milgrom 1994b). However, in estimating the dynamical mass of dwarf spheroidals with MOND, one must consider the fact that these objects are near the Galaxy, and the external field effect may be important. A measure of the degree of isolation of such an object would be given by

$$\eta = \frac{3\sigma^2/2r_c}{V_\infty^2/R} \approx \frac{g_i}{g_e}, \tag{17}$$

where r_c is the core radius, V_∞ is the asymptotic rotation velocity of the Galaxy ($\approx$200 km/s), and R is the galactocentric distance of the dwarf. For $\eta < 1$ the dwarf spheroidal is dominated by the Galactic acceleration field, and the external field effect must be taken into account. In this case the dynamical mass is simply given by the Newtonian estimate with the effective constant of gravity multiplied by a_o/g_e. In the opposite limit the MOND mass estimator for a system deep in the MOND limit is given by Equation 16.

Gerhard & Spergel (1992) and Gerhard (1994) have argued that MOND M/L values for dwarf spheroidals, based upon these estimators, have a very large range, with, for example, Fornax requiring a global M/L between 0.2 and 0.3, whereas the UMi dwarf requires M/L $\approx$ 10–13; that is to say, although some implied M/L values are unrealistically low, in other cases MOND still seems to require dark matter. Milgrom (1995) has responded to this criticism by pointing out that the kinematic data on dwarf spheroidals is very much in a state of flux and when more recent values for σ are used along with the realistic error estimates, the MOND M/L values for the dwarf spheroidals span a very reasonable range—on the order of one to three. Velocity dispersion data on dwarf spheroidals compiled by Mateo (1998) yield the MOND M/L values shown in Figure 9. In addition, MOND seems to fit the radial variation of velocity dispersion with a plausible amount of anisotropy (comparable to or less than required by dark matter) in the cases for which such data are available (Lokas 2001). It is clear that, for this class of objects, improved data gives MOND estimates for M/L values that are generally consistent with that expected for standard stellar populations.

Globular Clusters and Molecular Clouds

The globular clusters in Figure 7 generally lie well above the solid line; i.e., the internal accelerations are in excess of a_o. This implies that these systems should show no significant mass discrepancy within the half-light radius, as seems to be implied by the very reasonable M/L values based upon Newtonian dynamics. For a set of 56 globular clusters tabulated by Pryor & Meylan (1993) the mean Newtonian M/L_V is 2.4 ± 1. There are several cases of globular clusters with very low internal accelerations (for example NGC 6366 having $g_i/a_o \approx 0.07$), but these are generally cases in which the external Galactic field dominates (i.e., this object is only 4 kpc from the Galactic Center and $g_e > a_o$). Periodic tidal shocks may also affect the internal dynamics of the systems and result in larger core radii than if the systems were completely isolated.

The massive molecular clouds in the Galaxy are a unique class of objects to be considered in this context, in the sense that they are not generally included in discussions of the dark matter problem or global scaling relations. However, we see from Figure 7 that the internal accelerations within these objects are also roughly comparable to a_o—a fact that emerged from the empirically discovered size-line width relation for molecular clouds in the Galaxy (Solomon et al. 1987). Milgrom (1989b) noticed that this also implies that the surface density of molecular clouds is comparable to Σ_m—a property too striking to be entirely coincidental. The suggested explanation is that molecular clouds with $g_i > a_o \approx g_e$ expand via classical internal two-body evaporation until $g_i \approx a_o$, at which point they encounter a barrier to further evaporation; this can be seen as a consequence of the fact that an isolated system in MOND is always bound. If, alternatively, $g_i \ll a_o \approx g_e$, then there is no barrier to tidal disruption in the Galaxy. Thereby, a_o emerges as a preferred internal acceleration for molecular clouds. Regions in a galaxy where $g_e > a_o$ would, as an additional consequence, lack massive molecular clouds (as in the inner 3 kpc of the Galaxy apart from the exceptional Galactic Center clouds). The

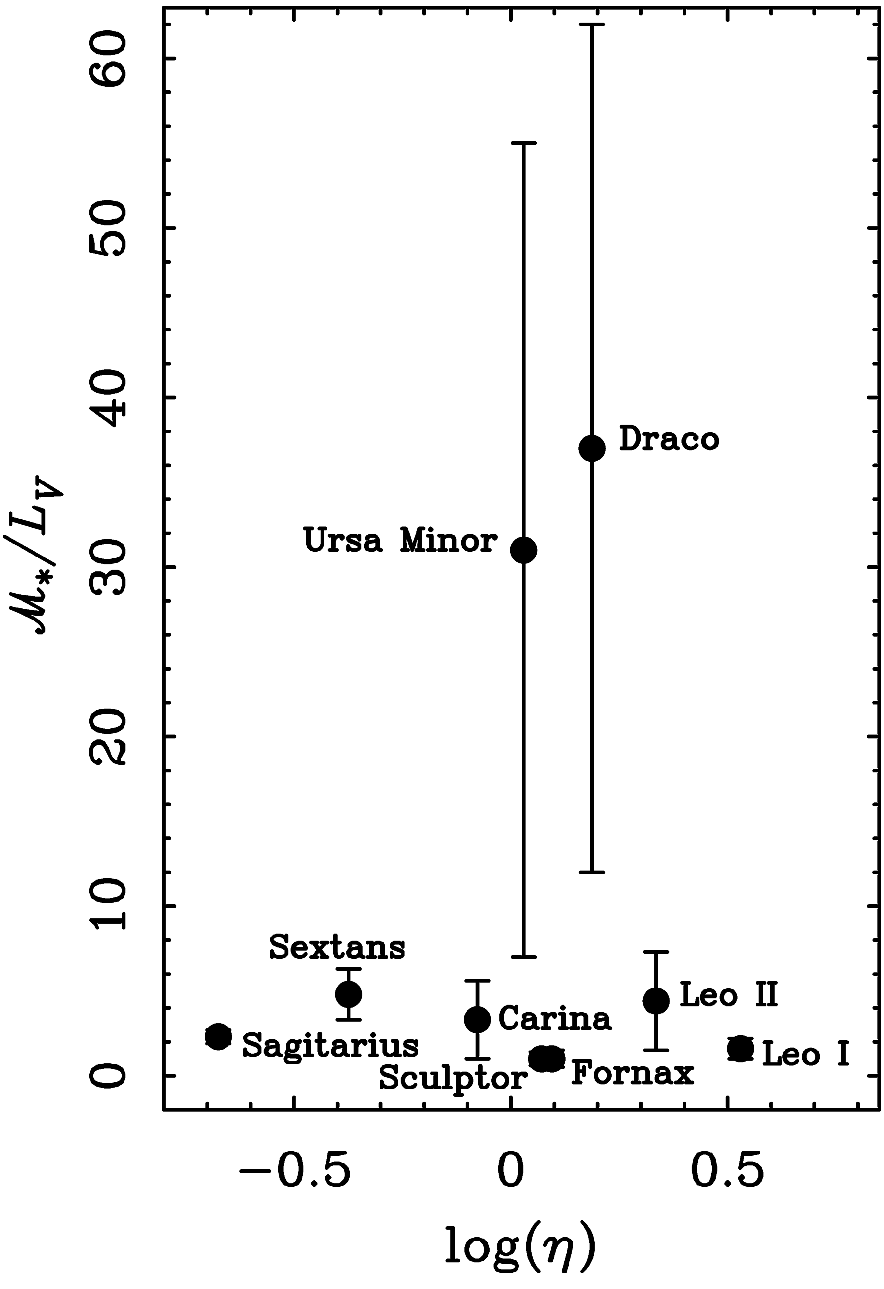


Figure 9 The MOND mass-to-light ratio for dwarf spheroidal satellites of the Galaxy as a function of η, the ratio of the internal to external acceleration. This is the parameter that quantifies the influence of the Galactic acceleration field (the external field effect); when $\eta < 1$ the object is dominated by the external field.

fact that the molecular clouds lie somewhat below the solid line in Figure 7 would also suggest that, viewed in the context of Newtonian dynamics, there should be a dark matter problem for molecular clouds; that is, the classical dynamically inferred mass should be somewhat larger than the mass derived by counting molecules.

Solomon et al. (1987) noted that combining the size–line width relation with the Newtonian virial theorem and an empirical mass-CO luminosity relation for molecular clouds results in a luminosity–line width relation that is analogous to the Faber-Jackson relation for ellipticals. Viewed in terms of MOND, the corresponding mass-velocity dispersion relation is not just analogous: It is the low mass extrapolation of the same relation that applies to all pressure-supported, nearly isothermal systems up to and including clusters of galaxies. If one applies Equation 8 to objects with a velocity dispersion of 4 or 5 km/s (typical of giant molecular clouds), then one deduces a mass of a few times 10^5 $M_\odot$. No explanation of global scaling relations for extragalactic objects in terms of dark matter can accommodate the extension of the relation to such subgalactic objects.

Small Groups of Galaxies

We include small galaxy groups in this section on pressure-supported systems even though this is more properly a small n-body problem. Proceeding from individual galaxies, the next rung on the ladder is binary galaxies, but it is difficult to extract meaningful dynamical information about these systems, primarily because of high contamination by false pairs. The situation improves for small groups because of more secure identification with an increasing number of members. Although uncertainties in the mass determination of individual groups remains large, either in the context of Newtonian dynamics or MOND, it is likely that statistical values deduced for selected samples of groups may be representative of the dynamics.

This problem has been considered by Milgrom (1998), who looked primarily at a catalogue of groups by Tucker et al. (1998) taken from the Las Campanas Redshift Survey. The median orbital acceleration of galaxies in this sample of groups is on the order of a few percent of a_o, so these systems are in the deep MOND regime. Milgrom therefore applied the MOND mass estimator relevant to this limit (Equation 16) and found that median M/L values are reduced from about 100 based on Newtonian dynamics to around 3 with MOND. Given the remaining large uncertainties owing to group identification and unknown geometry, these results are consistent with no dark matter in groups when considered in terms of MOND.

Rich Clusters of Galaxies

Clusters of galaxies lie below the $\sigma_l^2/r = a_o$ line in Figure 7; thus, these objects would be expected to exhibit significant discrepancies. That this is the case has been known for 70 years (Zwicky 1933), although the subsequent discovery of hot X-ray-emitting gas goes some way toward alleviating the original discrepancy. For an isothermal sphere of hot gas at temperature T, the Newtonian dynamical

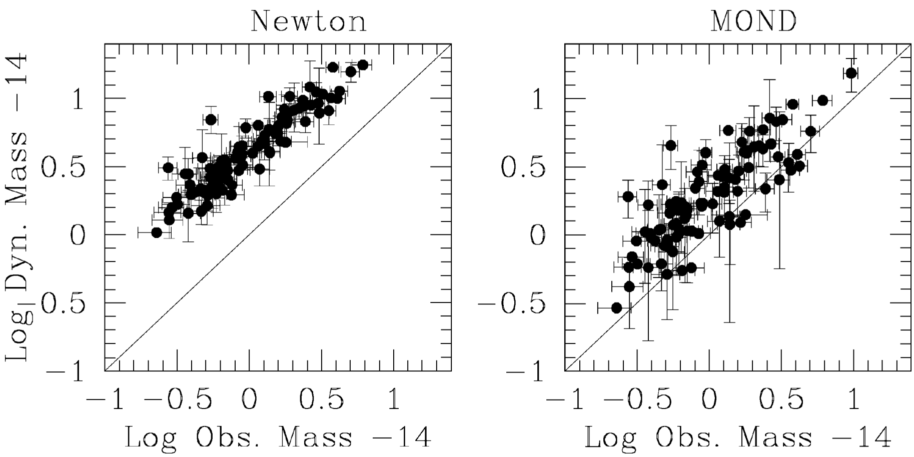


Figure 10 (*Left*) the Newtonian dynamical mass of clusters of galaxies within an observed cutoff radius (r_{out}) vs. the total observable mass in 93 X-ray-emitting clusters of galaxies (White et al. 1997). The solid line corresponds to $M_{dyn} = M_{obs}$ (no discrepancy). (*Right*) the MOND dynamical mass within r_{out} vs. the total observable mass for the same X-ray-emitting clusters. From Sanders (1999).

mass within radius r_o, calculated from the equation of hydrostatic equilibrium, is

$$M_n = \frac{r_o}{G}\frac{kT}{m}\left(\frac{d\ln(\rho)}{d\ln(r)}\right), \tag{18}$$

where m is the mean atomic mass and the logarithmic density gradient is evaluated at r_o. For the X-ray clusters tabulated by White et al. (1997), this Newtonian dynamical mass plotted against the observable mass, primarily in hot gas, is shown in Figure 10 (Sanders 1999), in which we see that the dynamical mass is typically about a factor of 4 or 5 larger than the observed mass in hot gas and in the stellar content of the galaxies. This rather modest discrepancy viewed in terms of dark matter has led to the so-called baryon catastrophe: not enough nonbaryonic dark matter in the context of standard CDM cosmology (White et al. 1993).

With MOND, the dynamical mass [assuming an isothermal gas (Equation 8)] is given by

$$M_m = (Ga_o)^{-1}\left(\frac{kT}{m}\right)^2\left(\frac{d\ln(\rho)}{d\ln(r)}\right)^2, \tag{19}$$

and this is also shown in Figure 10, again plotted against the observable mass. The larger scatter is due to the fact that the temperature and the logarithmic density gradient enter quadratically in the MOND mass determination. Here we see that, using the same value of a_o determined from nearby galaxy rotation curves, the discrepancy is on average reduced to about a factor of 2. The fact that MOND predicts more mass than is seen in clusters has been pointed out previously in the specific example of the Coma cluster (The & White 1988) and for three clusters with measured temperature gradients for which the problem is most evident in the central regions (Aguirre et al. 2001).

The presence of a central discrepancy is also suggested by strong gravitational lensing in clusters, i.e., the formation of multiple images of background sources in the central regions of some clusters (Sanders 1999). The critical surface density required for strong lensing is

$$\Sigma_c = \frac{1}{4\pi}\frac{cH_o}{G}F(z_l, z_s), \tag{20}$$

where F is a dimensionless function of the lens and source redshifts that depends upon the cosmological model (Blandford & Narayan 1992); typically for clusters and background sources at cosmological distances $F \approx 10$ (assuming that a MOND cosmology is reasonably standard). It is then evident that $\Sigma_c > \Sigma_m$, which means that strong gravitational lensing always occurs in the Newtonian regime. MOND cannot help with any discrepancy detected by strong gravitational lensing. Because strong gravitational lensing in clusters typically indicates a projected mass within 200–300 kpc between 10^{13} and 10^{14} $M_\odot$, which is not evidently present in the form of hot gas and luminous stars, it is clear that there is a missing mass problem in the central regions of clusters that cannot be repaired by MOND. This remaining discrepancy could be interpreted as a failure, or one could say that MOND predicts that the baryonic mass budget of clusters is not yet complete and that there is more mass to be detected (Sanders 1999). It would have certainly been devastating for MOND had the predicted mass turned out to be typically less than the observed mass in hot gas and stars; this would be a definitive falsification.

There is an additional important aspect of clusters of galaxies regarding global scaling relations. As pressure-supported, near-isothermal objects, clusters should lie roughly upon the same $M - \sigma$ relation defined by the elliptical galaxies. That this is the case was first pointed out by Sanders (1994), using X-ray observations of about 16 clusters that apparently lie upon the extension of the Faber-Jackson relation for elliptical galaxies. From Equation 8 we find that an object having a line-of-sight velocity dispersion of 1000 km/s would have a dynamical mass of about $0.5 \times 10^{14}\ M_\odot$, which is comparable to the baryonic mass of a rich cluster of galaxies. The fact that the Faber-Jackson relation—albeit with considerable scatter—extends from molecular clouds to massive clusters of galaxies finds a natural explanation in terms of MOND.

Super-Clusters and Lyα Forest Clouds

The largest coherent astronomical objects with the lowest internal accelerations are super-clusters of galaxies, as exemplified by the Perseus-Pisces filament. If one assumes that this object is virialized in a direction perpendicular to the long axis of the filament, then a linear mass density (μ_o) for the filament may be calculated following the arguments given by Eisenstein et al. (1996); by approximating the filament as an infinitely long, axisymmetric, isothermal cylinder, one finds

$$\mu_o = \frac{2\sigma^2}{G}. \tag{21}$$

Applying this relation to Perseus-Pisces, these authors estimated a global M/L in the super-cluster of 450 h—indicating a serious mass discrepancy.

Milgrom (1997b) has generalized the arguments of Eisenstein et al. and found that a relation similar to Equation 21 holds even if one drops the assumptions of axial symmetry and isothermality. He then derived a MOND estimator for the line density of a filament:

$$\mu_o = Q\frac{\langle\sigma_\perp^2\rangle^2}{a_o G r_h}, \tag{22}$$

where $\sigma_\perp$ is the velocity dispersion perpendicular to the filament axis, r_h is the half mass radius, and Q ($\approx$2) depends upon the velocity anisotropy factor. Applying this expression to Perseus-Pisces, Milgrom found an M/L value on the order of 10; once again the MOND M/L seems to require little or no dark matter, even on this very large scale. This is significant because the internal acceleration in this object is on the order of 0.03 a_o, which suggests that the MOND formula applies, at least approximately, down to this very low acceleration.

The diffuse intergalactic clouds resulting in the Ly_α forest absorption lines in the spectra of distant quasars are also apparently objects with internal acceleration very much lower than a_o. These have been considered as self-gravitating objects both in the context of dark matter (Rees 1986) and MOND (Milgrom 1988). There is now evidence that the sizes of individual absorbers may be as large as 100 kpc, as indicated by observations of gravitationally lensed quasars and quasar pairs (Schaye 2001). Given that the widths of the absorption lines are on the order of 10 km/s, then the internal accelerations within these systems may be as small as $3 \times 10^{-4}\, a_o$. Schaye (2001) has argued that the characteristic sizes of the Lyα clouds are most likely to be comparable to the Jeans length. In the context of MOND this would be

$$\lambda_J = \left(\frac{\sigma^4}{G a_o \Sigma}\right)^{\frac{1}{2}}, \tag{23}$$

where Σ is the mean surface density. Because the fractional ionization is likely to be very high (Σ is dominated by protons), one finds that this characteristic size, in terms of MOND should be more on the order of 10 kpc, in contradiction to observations of common lines in quasar pairs. On this basis, Aguirre et al. (2001) have argued that the observed large sizes of the absorbers, perpendicular to the line of sight, are inconsistent with the predictions of MOND.

However, these authors noted that the external field effect provides a possible escape. The implied internal accelerations of the clouds, if they are roughly spherical with sizes of 100 kpc, are likely to be much smaller than the external acceleration field resulting from large-scale structure, which, as we saw above, is on the order of several percent a_o. In this case the Jean's length is given by the traditional Newtonian formula with an effective constant of gravity, which may be 20 or 30 times larger than G, and the sizes can be consistent with the large observed extent.

THE PHYSICAL BASIS OF MOND

The Bekenstein-Milgrom Theory

In spite of its empirical success, MOND remains a largely ad hoc modification of Newtonian gravity or dynamics without connection to a more familiar theoretical framework. This is, at present, the essential weakness of the idea. The original algorithm (Equation 2 or 3) cannot be considered as a theory but as a successful phenomenological scheme for which an underlying theory is necessary. If one attempts to apply Milgrom's original prescription (either as a modification of gravity or inertia) to an N-body system, then immediate physical problems arise, such as nonconservation of linear momentum (Felten 1984).

Bekenstein & Milgrom (1984) recognized this and proposed a nonrelativistic Lagrangian-based theory of MOND as a modification of Newtonian gravity. Given a scalar potential ϕ, the dynamics of the theory is contained in field action

$$S_f = -\int d^3r \left[\rho\phi + (8\pi G)^{-1} a_o^2 F\left(\frac{\nabla\phi^2}{a_o^2}\right)\right]. \tag{24}$$

The particle action takes its standard form. The field equation derived, as usual, under the assumption of stationary action is

$$\nabla \cdot \left[\mu\left(\frac{|\nabla\phi|}{a_o}\right)\nabla\phi\right] = 4\pi G\rho, \tag{25}$$

where the function $\mu(x) = dF/dx^2$ must have the asymptotic behavior required in the simple MOND prescription; i.e., $F(x^2) = (x^2)^{3/2}$ in the MOND limit ($x \ll 1$) and $F(x^2) = x^2$ in the Newtonian limit. The equation of motion for a particle assumes its usual Newtonian form.

Because of the symmetry of the Lagrangian density to space-time translations (and to space rotations), the theory respects the laws of conservation of energy and (angular) momentum. Moreover, Bekenstein & Milgrom demonstrated that, in the context of this theory, the motion of a compound object (e.g., a star or star cluster in the Galaxy) in an external field is independent of its internal structure (or internal accelerations) and may be described in terms of its center-of-mass accelerations; i.e., objects like stars with Newtonian internal accelerations behave like billiard balls in the external field, even in the MOND limit. However, the external acceleration field does affect the internal dynamics of such a subsystem in just the way proposed by Milgrom—the external field effect.

In addition to enjoying the properties of consistency and conservation, this modified Poisson equation has an interesting symmetry property. It is well known that the usual Poisson equation is conformally invariant in two spatial dimensions. Conformal transformations comprise a set of angle-preserving coordinate transformations that represent, in effect, a position-dependent transformation of units of length. Many of the well-known equations of physics (e.g., Maxwell's equations) are invariant under transformations of this form. Milgrom (1997a) discovered that there is a nonlinear generalization of the Poisson equation that is conformally invariant in D spatial dimensions in the presence of a source ρ. This is of the form

$$\nabla \cdot \left\{[(\nabla\phi)^2]^{D/2-1}\nabla\phi\right\} = \alpha_D G\rho. \quad (26)$$

When $D = 2$ Equation 26 becomes the usual Poisson equation, but when $D = 3$ the equation takes the form that is exactly required for MOND phenomenology. In other words, the Bekenstein-Milgrom field equation in the MOND limit is conformally invariant in three spatial dimensions. The full significance of this result is unclear, but it should be recalled that much of modern physics rests upon just such symmetry principles.

The Bekenstein-Milgrom theory is a significant step beyond Milgrom's original prescription. Even though the theory is noncovariant, it demonstrates that MOND can be placed upon a solid theoretical basis and that MOND phenomenology is not necessarily in contradiction with cherished physical principles. Although this is its essential significance, the theory also permits a more rigorous consideration of specific aspects of MOND phenomenology relating to N-body systems—such as the external field effect.

It is, in general, difficult to solve this nonlinear equation except in cases of high symmetry in which the solution reduces to that given by the simple algorithm. Brada & Milgrom (1995) have derived analytic solutions for Kuzmin disks and of their disk-plus-bulge generalizations. The solution can be obtained in the form of a simple algebraic relation between the Bekenstein-Milgrom solution and the Newtonian field of the same mass distribution, and this relation can be extended to a wider class of disk configurations (e.g., exponential disks) where it holds approximately. From this work, it is evident that the simple MOND relation (Equation 3) gives a radial force distribution in a thin disk, which is generally within 10% of that determined by the Bekenstein-Milgrom field equation.

Brada (1997) has developed a numerical method of solution for N-body problems based upon a multigrid technique, and Brada & Milgrom (1999b) used this method to consider the important problem of stability of disk galaxies. They demonstrated that MOND, as anticipated (Milgrom 1989a), has an effect similar to a dark halo in stabilizing a rotationally supported disk against bar-forming modes. However, there is also a significant difference (also anticipated by Milgrom 1989a). In a comparison of MOND and Newtonian truncated exponential disks with identical rotation curves (the extra force in the Newtonian case being provided by a rigid dark halo), Brada & Milgrom found that, as the mean surface density of the disk decreases (the disk sinks deeper into the MOND regime), the growth rate of the bar-forming $m = 2$ mode decreases similarly in the two cases. However, in the limit of very low surface densities the MOND growth rate saturates while

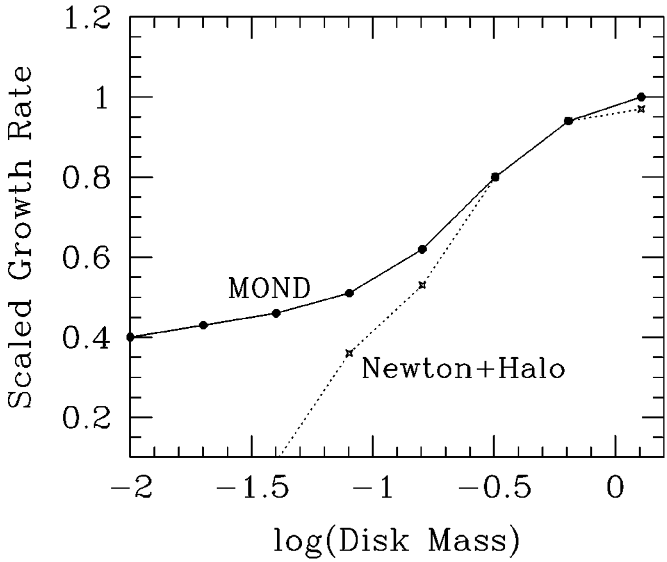


Figure 11 The scaled growth rate of the m = 2 instability in Newtonian disks with dark matter (*dashed line*) and MONDian disks as a function of disk mass. In the MOND case as the disk mass decreases, the surface density decreases and the disk sinks deeper into the MOND regime. In the equivalent Newtonian case, the rotation curve is maintained at the MOND level by supplementing the force with an inert dark halo. From Brada & Milgrom 1999b.

the Newtonian growth rate continues to decrease as the halo becomes more dominant. This effect, shown in Figure 11, may provide an important observational test: With MOND, LSB galaxies remain marginally unstable to bar- and spiral-forming modes, whereas in the dark matter case, halo-dominated LSB disks become totally stable. Observed LSB galaxies do have bars and m = 2 spirals (McGaugh et al. 1995a). In the context of dark matter, these signatures of self-gravity are difficult to understand in galaxies that are totally halo-dominated (Mihos et al. 1997).

The Brada method has also been applied to calculating various consequences of the external field effect, such as the influence of a satellite in producing a warp in the plane of a parent galaxy (Brada & Milgrom 2000a). The idea is that MOND, via the external field effect, offers a mechanism other than the relatively weak effect of tides in inducing and maintaining warps. As noted above, the external field effect is a nonlinear aspect of MOND, subsumed by the Bekenstein-Milgrom field equation; unlike Newtonian theory, even a constant external acceleration field influences the internal dynamics of a system. Brada & Milgrom (2000a) demonstrated that a satellite at the position and with the mass of the Magellanic clouds can produce a warp in the plane of the Galaxy with about the right amplitude and form.

The response of dwarf satellite galaxies to the acceleration field of a large parent galaxy has also been considered (Brada & Milgrom 2000b). It was found that the satellites become more vulnerable to tidal disruption because of the expansion induced by the external field effect as they approach the parent galaxy (the effective constant of gravity decreases toward its Newtonian value). The distribution of satellite orbits is therefore expected to differ from the case of Newtonian gravity plus dark matter, although it is difficult to make definitive predictions because of the unknown initial distribution of orbital parameters.

Although the Bekenstein-Milgrom theory is an important development for all the reasons outlined above, it must be emphasized that it remains, at best, only a clue to the underlying theoretical basis of MOND. The physical basis of MOND may lie completely in another direction—as modified Newtonian inertia rather than gravity. However, a major advantage of the theory is that it lends itself immediately to a covariant generalization as a nonlinear scalar-tensor theory of gravity.

Modified Newtonian Dynamics as a Modification of General Relativity

As noted above, the near coincidence of a_o with cH_o suggests that MOND may reflect the influence of cosmology upon particle dynamics or $1/r^2$ attraction. However, in the context of general relativity there is no such influence of this order, with or without the maximum permissible cosmological term—a fact that may be deduced from the Birkhoff theorem (Nordtvedt & Will 1972). Therefore, general relativity, in a cosmological context, cannot be the effective theory of MOND, although the theory underlying MOND must effectively reduce to general relativity in the limit of high accelerations [see Will (2001) for current experimental constraints on strong field deviations from general relativity].

The first suggested candidate theory (Bekenstein & Milgrom 1984) is an unconventional scalar-tensor theory that is a covariant extension of the nonrelativistic Bekenstein-Milgrom theory. Here the Lagrangian for the scalar field is given by

$$L_s = \frac{a_o^2}{c^4} F\left[\frac{\phi_{,\alpha}\phi^{,\alpha} c^4}{a_o^2}\right], \tag{27}$$

where F(X) is an arbitrary positive function of its dimensionless argument. This scalar field, as usual, interacts with matter jointly with $g_{\mu\nu}$ via a conformal transformation of the metric, i.e., the form of the interaction Lagrangian is taken to be

$$L_I = L_I[\xi(\phi^2) g_{\mu\nu} \ldots], \tag{28}$$

where ξ is a function of the scalar field (this form preserves weak equivalence where particles follow geodesics of a physical metric $\hat{g}_{\mu\nu} = \xi(\phi^2) g_{\mu\nu}$). The scalar field action ($\int L_s \sqrt{-g} d^4x$) is combined with the usual Einstein-Hilbert action of general relativity and the particle action formed from L_I to give the complete theory.

Thus, the covariant form of the Bekenstein-Milgrom field equation becomes

$$(\mu\phi^{,\alpha})_{;\alpha} = \frac{4\pi GT}{c^4}, \tag{29}$$

where, again, $\mu = dF/dX$ and T is the contracted energy-momentum tensor. The complete theory includes the Einstein field equation with an additional source term owing to the contribution of the scalar field to the energy-momentum tensor. Again we require that F(X) has the asymptotic behavior $F(X) \rightarrow X^{\frac{3}{2}}$ in the limit where $X \ll 1$ (the MOND limit) and $F(X) \rightarrow \omega X$ in the limit of $X \gg 1$. Thus, in the limit of large field gradients the theory becomes a standard scalar-tensor theory of Brans-Dicke form (Brans & Dicke 1961); it is necessary that $\omega \gg 1000$ if the theory is to be consistent with local solar system and binary pulsar tests of general relativity (Will 2001). Because of the nonstandard kinetic Lagrangian (Equation 27) Bekenstein (1988) termed this theory the aquadratic Lagrangian or AQUAL theory.

Bekenstein & Milgrom (1984) immediately noticed a physical problem with the theory: Small disturbances in the scalar field propagate at a velocity faster than the speed of light in directions parallel to the field gradient in the MOND regime. This undesirable property appears to be directly related to the aquadratic form of the Lagrangian and is inevitably true in any such theory in which the scalar force decreases less rapidly than $1/r^2$ in the limit of low field gradient. Clearly, the avoidance of causality paradoxes, if only in principle, should be a criterion for physical viability.

The acausal propagation anomaly led Bekenstein (1988a,b) to propose an alternative scalar-tensor theory in which the field is complex

$$\chi = Ae^{i\phi} \tag{30}$$

and the Lagrangian assumes its usual quadratic form

$$L_s = \frac{1}{2}A^2\phi_{,\alpha}\phi^{,\alpha} + A_{,\alpha}A^{,\alpha} + V(A^2), \tag{31}$$

where $V(A^2)$ is a potential associated with the scalar field. The unique aspect of the theory is that only the phase couples to matter (jointly with $g_{\mu\nu}$ as in Equation 28); hence, it is designated "phase coupling gravitation" (PCG). The field equation for the matter coupling field is then found to be

$$(A^2\phi^{,\alpha})_{;\alpha} = \frac{4\pi\eta G}{c^4}T, \tag{32}$$

where η is a dimensionless parameter describing the strength of the coupling to matter. Thus, the term A^2 plays the role of the MOND function μ; A^2 is also a function of $(\nabla\phi)^2$, but the relationship is differential instead of algebraic. Bekenstein noted that if $V(A^2) = -kA^6$ precisely, the phenomenology of MOND is recovered by the scalar field ϕ (here a_o is related to the parameters k and η). Because such a potential implies an unstable vacuum, alternative forms were considered by Sanders (1988), who demonstrated that phenomenology similar to MOND is predicted as long as $dV/dA < 0$ over some range of A. Moreover, in a cosmological context (Sanders 1989) PCG, with the properly chosen bare potential, becomes an effective MOND theory where the cosmological $\dot{\phi}$ plays the role of a_o. Romatka (1992) considered PCG as one of a class of two-scalar plus tensor theories in which the scalar fields couple in one of their kinetic terms and demonstrated that, in a certain limit, Bekenstein's sextic potential theory approaches the original AQUAL theory. This suggests that PCG may suffer from a similar physical anomaly as AQUAL, and indeed, Bekenstein (1990) discovered that PCG apparently permits no stable background solution for the field equations—an illness as serious as that of the acausal propagation that the theory was invented to cure.

A far more practical problem with AQUAL, PCG, or, in fact, all scalar-tensor theories in which the scalar field enters as a conformal factor multiplying the Einstein metric (Equation 28) is the failure to predict gravitational lensing at the level observed in rich clusters of galaxies (Bekenstein & Sanders 1994). If one wishes to replace dark matter by a modified theory of gravity of the scalar-tensor type with the standard coupling to matter, then the scalar field produces no enhanced deflection of light. The reason for this is easy to understand: In scalar-tensor theories particles follow geodesics of a physical metric that is conformally related (as in Equation 28) to the usual Einstein metric. But Maxwell's equations are conformally invariant, which means that photons take no notice of the scalar field (null geodesics of the physical and Einstein metrics coincide). In other words, the gravitational lensing mass of an astronomical system should be comparable to that of the detectable mass in stars and gas and thus much less than the traditional virial mass. This is in sharp contrast to the observations (Fort & Mellier 1994).

A possible cure for this ailment is a nonconformal relation between the physical and Einstein metrics; that is, in transforming the Einstein metric to the physical metric, a special direction is picked out for additional squeezing or stretching (Bekenstein 1992, 1993). To preserve the isotropy of space, this direction is usually chosen to be time-like in some preferred cosmological frame as in the classical stratified theories (Ni 1972). In this way one may reproduce the general relativistic relation between the weak-field force on slow particles and the deflection of light (Sanders 1997). However, the Lorentz invariance of gravitational dynamics is broken, and observable preferred frame effects—such as a polarization of the earth-moon orbit (Müller et al. 1996)—are inevitable at some level. It is of interest that an aquadratic Lagrangian for the scalar field (similar to Equation 27) can provide a mechanism for local suppression of these effects; essentially, the scalar force may be suppressed far below the Einstein-Newton force in the limit of solar system accelerations. On this basis, one could speculate that cosmology is described by a preferred frame theory (there is clearly a preferred cosmological frame from an observational point of view). Then it may be argued that the reconciliation of preferred frame cosmology with general relativistic local dynamics (weak local preferred frame effects) requires MOND phenomenology at low accelerations (Sanders 1997). However, any actual theory is highly contrived at this point.

In summary, it is fair to say that, at present, there is no satisfactory covariant generalization of MOND as a modification of general relativity. But this does not imply that MOND is wrong any more than the absence of a viable theory of quantum gravity implies that general relativity is wrong. It is simply a statement that the theory remains incomplete and that perhaps the tinkering with general relativity is not the ideal way to proceed.

Modified Newtonian Dynamics as a Modification of Newtonian Inertia

A different approach has been taken by Milgrom (1994a, 1999), who considers the possibility that MOND may be viewed as a modification of particle inertia. In such theories, at a nonrelativistic level, one replaces the standard particle action ($\int v^2/2\,dt$) by a more complicated object, $A_m S[\mathbf{r}(t), a_o]$, where A_m depends upon the body and can be identified with the particle mass, and S is a functional of the particles trajectory, $\mathbf{r}(t)$, characterized by the parameter a_o. This form ensures weak equivalence. Milgrom (1994a) proved that if such an action is to be Galilei invariant and have the correct limiting behavior (Newtonian as $a_o \rightarrow 0$ and MONDian as $a_o \rightarrow \infty$), then it must be strongly nonlocal; i.e., the motion of a particle at a point in space depends upon its entire past trajectory. This nonlocality has certain advantages in a dynamical theory: For example, because a particle's motion depends upon an infinite number of time-derivatives of the particle's position, the theory does not suffer from the instabilities typical of higher derivative (weakly nonlocal) theories. Moreover, because of the nonlocality, the acceleration of the center of mass of a composite body emerges as the relevant factor in determining its dynamics (Newtonian or MONDian) rather than the acceleration of its individual components. Milgrom further demonstrated that, in the context of such theories, the simple MOND relation (Equation 2) is exact for circular orbits in an axisymmetric potential (although not for general orbits).

Although these results on the nature of generalized particle actions are of considerable interest, this, as Milgrom stresses, is not yet a theory of MOND as modified inertia. The near coincidence of a_o with cH_o suggests that MOND is, in some sense, an effective theory; that is to say, MOND phenomenology only arises when the theory is considered in a cosmological background (the same may also be true if MOND is due to a modified theory of gravity). However, the cosmology does not necessarily directly affect particle motion; the same agent—a cosmological constant—may affect both cosmology and dynamics. Suppose, for example, that inertia results from the interaction of an accelerating particle with the vacuum. Suppose further that there is a nonzero cosmological constant (which is consistent with a range of observations). Then because a cosmological constant is an attribute of the vacuum, we might expect that it has a nontrivial effect upon particle inertia at accelerations corresponding to $\approx c\sqrt{\Lambda}$.

The phenomenon of Unruh radiation provides a hint of how this might happen (Unruh 1975). An observer uniformly accelerating through Minkowski space sees a nontrivial manifestation of vacuum fields as a thermal bath at temperature

$$kT = \frac{\hbar}{2\pi c} a, \tag{33}$$

where a is the acceleration (this is exactly analogous to Hawking radiation, where a is identified with the gravitational acceleration at the event horizon). In other words, an observer can gain information about his state of motion by using a quantum detector. However, the same observer accelerating through de Sitter space sees a modified thermal bath now characterized by a temperature

$$kT_\Lambda = \frac{\hbar}{2\pi c}\sqrt{a^2 + \frac{c^2\Lambda}{3}} \tag{34}$$

(Narnhofer et al. 1996, Deser & Levin 1997). The presence of a cosmological constant changes the accelerating observer's perception of the vacuum through the introduction of a new parameter with units of acceleration ($c\sqrt{\Lambda}$) and a magnitude comparable to a_o. If the observer did not know about the cosmological constant this would also change his perception of the state of motion.

The Unruh radiation itself is too miniscule to be directly implicated as the field providing inertia: It may be, in effect, a tracer of the particle's inertia. Milgrom (1999) has suggested that inertia may be what drives a noninertial body back to some nearby inertial state—attempting to reduce the vacuum radiation to its minimum value. If that were so then the relevant quantity with which to identify inertia would be $\Delta T = T_\Lambda - T$. In that case one could write

$$\frac{2\pi c}{\hbar} k\Delta T = a\mu(a/a_o), \tag{35}$$

where

$$\mu(x) = [1 + (2x)^{-2}]^{1/2} - (2x)^{-1} \tag{36}$$

with $a_o = 2c(\Lambda/3)^{1/2}$. Inertia defined in this way would have precisely the two limiting behaviors of MOND.

Again, this is not a theory of MOND as modified inertia but only a suggestive line of argument. To proceed further along this line, a theory of inertia derived from interaction with vacuum fields is necessary—something analogous to induced gravity (Sakharov 1968), in which the curvature of space-time modifies the behavior of vacuum fields producing an associated action for the metric field. If this approach is correct, the free action of a particle must be derived from the interactions with vacuum fields.

Gravitational Lensing and No-Go Theorems

We have noted above that the phenomenon of gravitational lensing places strong constraints upon scalar-tensor theories of modified dynamics, specifically upon the relation between the physical metric and the gravitational metric (Bekenstein & Sanders 1994, Sanders 1997). Here we wish to discuss gravitational lensing in a more general sense because it is the only measureable relativistic effect that exists on the scale of galaxies and clusters and therefore is generally relevant to proposed modifications of general relativity that may be only effective on this scale. Indeed, several authors have attempted to formulate no-go theorems for modified gravity on the basis of the observed gravitational lensing.

The first of these was Walker (1994), who considered general metrics of the standard Schwarzschild form $ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega^2$ with the condition that $A = B^{-1}$ and $B - 1 = 2\phi(r)$, where $\phi(r)$ is a general weak field potential of the form $\phi \propto r^n$. With these assumptions he demonstrated that gravity is actually repulsive for photons if $0 < n < 2$—i.e., gravitational lenses would be divergent. He used this argument to rule out the Mannheim & Kazanas (1989) spherical vacuum solution for Weyl conformal gravity in which $\phi(r)$ contains such a linear term. But then Walker went on to consider the mean convergence $\langle\kappa\rangle$ and the variance of the shear σ_γ in the context of $\phi(r) \propto \log r$ which might be relevant to MOND. Again with the condition that $A = B^{-1}$, this form of the potential would imply a mean convergence of $\langle\kappa\rangle \approx 10^7$ while the observations constrain $\kappa < 1$; that is to say, with such an effective potential the optical properties of the Universe would be dramatically different than observed.

The second no-go theorem is by Edery (1999) and is even more sweeping. Basically the claim is that, again assuming a metric in the standard Schwarzschild form with $A = B^{-1}$, any potential that yields flat rotation curves (i.e., ϕ falls less rapidly than 1/r) is repulsive for photons even though it may be attractive for nonrelativistic particles. This was disputed by Bekenstein et al. (2000), who pointed out that solar system tests do not constrain the form of A and B on galactic scale, except in the context of a specific gravity theory. An alternative theory may exhibit $AB = 1$ to high accuracy on a solar system scale but $AB \neq 1$ on a galactic scale; indeed, this is a property of the stratified scalar-tensor theory of Sanders (1997), which predicts enhanced deflection in extragalactic sources but is also consistent with solar system gravity tests at present levels of precision.

Walker's objection would actually seem to be more problematic for MOND; here the estimate of enormous mean convergence due to galactic lenses is independent of the assumption of $AB = 1$. This problem has also emerged in a different guise in the galaxy-galaxy lensing results of Hoekstra et al. (2002). These results imply that galaxy halos have a maximum extent; if represented by isothermal spheres, the halos do not extend beyond 470 h^{-1} on average. With MOND the equivalent halo for an isolated galaxy would be infinite. Walker also noted that for MOND to be consistent with a low mean convergence, the modified force law could only extend to several Mpc at most, beyond which there must be a return to r^{-2} attraction.

The external field effect provides a likely escape from such objections in the context of pure (unmodified) MOND. Basically, no galaxy is isolated. For an L^* galaxy the acceleration at a radius of 470 kpc is about $0.02a_o$. This is at the level of the external accelerations expected from large-scale structure. For lower accelerations one would expect a return to a $1/r^2$ law with a larger effective constant of gravity. For this reason, such objections cannot be considered as a falsification of MOND. It might also be that at very low accelerations the attraction really does return to $1/r^2$, as speculated by Sanders (1986), although there is not yet a compelling reason to modify MOND in this extreme low acceleration regime.

MONDian gravitational lensing in a qualitative sense is also considered by Mortlock & Turner (2001), who after making the reasonable assumption that the relation between the weak field force and deflection in MOND is the same as in general relativity (specifically including the factor 2 over the Newtonian deflection) considered the consequences when the force is calculated from the MOND equation. The first result of interest is that the thin lens approximation (i.e., the deflection in an extended source depends only upon the surface density distribution) cannot be made with MOND; this means that the deflection depends, in general, upon the density distribution along the line of sight. They further pointed out that observations of galaxy-galaxy lensing (taking the galaxies to be point masses) is consistent with MOND, at least within the truncation noted by Hoekstra et al. (2002). Mortlock & Turner proposed that a test discriminating between MOND and dark halos would be provided by azimuthal symmetry of the galaxy-galaxy lensing signal: MOND would be consistent with such symmetry, whereas halos would not. They further noted that gravitational microlensing in the context of MOND would produce a different signature in the light curves of lensed objects (particularly in the wings) and that this could be observable in cosmological microlensing (however, this effect may be limited by the external field of the galaxy containing the microlensing objects).

In general, in extragalactic lenses such as galaxy clusters distribution of shear in background sources (and hence apparent dark mass distribution) should be calculable from the distribution of observable matter; i.e., there should be a strong correlation between the visible and, in terms of general relativity, the dark mass distribution. The theme of correlation between the observable (visible) structure of a lens and the implied shape of the dark matter distribution was taken up by Sellwood & Kosowsky (2002), who emphasized that the observed correlation in position angles between the elongated light distribution and implied mass distribution; Kochanek (2002) argues strongly in favor of some form of modified gravity.

It is clear from these discussions that gravitational lensing may provide generic tests of the MOND hypothesis vs. the dark matter hypothesis and that any more basic theory must produce lensing at a level comparable to that of general relativity with dark matter. This already strongly constrains the sort of theory that may underpin MOND.

COSMOLOGY AND THE FORMATION OF STRUCTURE

Considerations of cosmology in the context of MOND might appear to be premature in the absence of a complete theory. However, there are some very general statements that can be made about a possible MOND universe independently of any specific underlying theory. First of all, the success of the hot Big Bang with respect to predicting the thermal spectrum and isotropy of the cosmic microwave background as well as the observed abundances of the light isotopes (e.g., Tytler et al. 2000) strongly implies that a theory of MOND should preserve the standard model—at least with respect to the evolution of the early hot Universe. This, in fact, may be considered as a requirement on an underlying theory. Second, it would be contrary to the spirit of MOND if there were cosmologically significant quantities of nonbaryonic CDM ($\Omega_{cdm} \ll 1$); i.e., dark matter that clusters on the scale of galaxies. This is not to say dark matter is nonexistent; the fact that Ω_V (luminous matter) is substantially less than the Ω_b (baryons) implied by primordial nucleosynthesis (Fukugita et al. 1998) means that there are certainly as-yet-undetected baryons. Moreover, particle dark matter also exists in the form of neutrinos. It is now clear from the detection of neutrino oscillations (Fukuda et al. 1998) that at least some flavors of neutrinos have mass: The constraints are $.004h^{-2} < \Omega_\nu < 0.1h^{-2}$ (Turner 1999) with the upper limit imposed by the experimental limit on the electron neutrino mass (3 ev). However, it would be entirely inconsistent with MOND if dark matter, baryonic or nonbaryonic, contributed substantially to the mass budget of galaxies. Neutrinos near the upper limit of 3 ev cannot accumulate in galaxies owing to the well-known phase space constraints (Tremaine & Gunn 1979), but they could collect within and contribute to the mass budget of rich clusters of galaxies (which would not be inconsistent with MOND, as noted above). Apart from this possibility it is reasonable to assume that MOND is most consistent with a purely baryonic universe.

This possibility has been considered by McGaugh (1998), who first pointed out that, in the absence of CDM, oscillations should exist in the present power spectrum of large-scale density fluctuations—at least in the linear regime. These oscillations are the relic of the sound waves frozen into the plasma at the epoch of recombination and are suppressed in models in which CDM makes a dominant contribution to the mass density of the Universe (Eisenstein & Hu 1998). McGaugh (1999b) further considered whether or not a cosmology with $\Omega_m \approx \Omega_b \approx 0.02h^{-2}$ would be consistent with observations of anisotropies in the cosmic microwave background, particularly the pattern of acoustic oscillations predicted in the angular power spectrum (e.g., Hu et al. 1997). McGaugh, using the standard CMBFAST program (Seljak & Zaldarriaga 1996), pointed out that in a purely baryonic Universe with vacuum energy density being the dominant constituent the second acoustic peak would be much reduced with respect to the a priori expectations of the concordance ΛCDM model (Ostriker & Steinhardt 1995). The reason for this low amplitude is Silk damping (Silk 1968) in a low Ω_m, pure baryonic universe—the shorter wavelength fluctuations are exponentially suppressed by photon diffusion. When the Boomerang and Maxima results first appeared (Hanany et al. 2000, Lange et al. 2001), much of the initial excitement was generated by the unexpected low amplitude of the second peak. With $\Omega_{total} = 1.01$ and $\Omega_m = \Omega_b$ (no CDM or nonbaryonic matter of any sort), McGaugh (2000) produced a good match to these initial Boomerang results. A further prediction is that the third acoustic peak should be even further suppressed. There are indications from the more complete analysis of Boomerang and Maxima data (Netterfield et al. 2001, Lee et al. 2001) that this may not be the case, but the systematic uncertainties remain large.

The SNIa results on the accelerated expansion of the Universe (Perlmutter et al. 1999) as well as the statistics of gravitational lensing (Falco et al. 1998) seem to exclude a pure baryonic, vacuum energy–dominated universe, although it is unclear that all of the systematic effects are well understood. It is also possible that a MOND cosmology differs from a standard Friedmann cosmology in the low-z Universe, particularly with regard to the angular size distance–redshift relation. At this point it remains unclear whether these observations require CDM. It is evident that such generic cosmological tests for CDM relate directly to the viability of MOND. Nonetheless, cosmological evidence for dark matter, in the absence of its direct detection, is still not definitive, particularly considering that a MOND universe may be non-Friedmannian.

Can we then, in the absence of a theory, reasonably guess what form a MOND cosmology might take? When a theory is incomplete, the way to proceed is to make several assumptions (*Ansätze*) in the spirit of the theory as it stands and determine the consequences. This has been done by Felten (1984) and by Sanders (1998), who following the example of Newtonian cosmology, considered the dynamics of a finite expanding sphere. Here it is assumed that the MOND acceleration parameter a_o does not vary with cosmic time. The second critical assumption is that, in the absence of a relativistic theory, the scale factor of the sphere is also the scale factor of the Universe, but then an immediate contradiction emerges. MONDian dynamics of a sphere permits no dimensionless scale factor. Uniform expansion of a spherical region is not possible, and any such region will eventually recollapse regardless of its initial density and expansion velocity (Felten 1984). In the low acceleration regime the dynamical equation for the evolution of the sphere, the MONDian equivalent of the Friedmann equation, is given by

$$\dot{r} = u_i^2 - \left[2\Omega_m H_o^2 r_o^3 a_o\right]^{1/2} \ln(r/r_i), \tag{37}$$

where r_i is the initial radius of the sphere, r_o is a comoving radius, and u_i is the initial expansion velocity. From the form of Equation 37 it is obvious that the sphere will eventually recollapse. It would also appear that a MONDian universe is inconsistent with the cosmological principle.

However, looking at the Newtonian equations for the dynamics of a spherical region, one finds that, at any given epoch, the acceleration increases linearly with radial distance from the center of the sphere. This means that there exists a critical radius given by

$$r_c = \sqrt{GM_{r_c}/a_o}, \tag{38}$$

where M_{r_c} is the active gravitational mass within r_c. Beyond r_c (which is epoch dependent) the acceleration exceeds a_o; therefore, on larger scales the dynamics of any spherical region is Newtonian and the expansion may be described by a dimensionless scale factor. During this Newtonian expansion, it would appear possible to make the standard assumption of Newtonian cosmology that the scale factor of the sphere is identical to the scale factor of the universe, at least on comoving scales corresponding to M_{r_c} or larger.

Making use of the Friedmann equations we find that

$$r_c = \frac{2a_o}{\Omega_m H_o^2} x^3 \tag{39}$$

during the matter-dominated evolution of the Universe (Sanders 1998). Here x is the dimensionless scale factor with $x = 1$ at present. Therefore, larger and larger comoving regions become MONDian as the Universe evolves. Because in the matter-dominated period the horizon increases as $r_h \propto x^{1.5}$, it is obvious that at some point in the past the scale over which MOND applies was smaller than the horizon scale. This would suggest that, in the past, whereas small regions may have been dominated by modified dynamics, the evolution of the Universe at large is described by the usual Friedmann models. In particular, in the early radiation-dominated universe, r_c is very much smaller than the MONDian Jeans length, $\lambda_j \approx (c/H_o)x^{4/3}$ (the Hubble deceleration is very large at early times), so the expectation is that the dynamical history of the early MOND universe would be identical to the standard Big Bang.

After recombination, the Jeans length of the baryonic component falls very much below r_c, and by a redshift of 3 or 4, r_c approaches the horizon scale; the entire Universe becomes MONDian. Thus, we might expect that the evolution of a postrecombination MOND universe might differ in significant ways from the standard Friedmann-Lemaitre models, particularly with respect to structure formation. One could assume that when the critical radius (Equation 39) grows beyond a particular comoving scale, then the MOND relation (Equation 37) applies for the subsequent evolution of regions on that scale, and recollapse will occur on a time scale comparable to the Hubble time at that epoch. For a galaxy mass object ($10^{11} M_\odot$, $r_c \approx 14$ kpc) this happens at a redshift of about 140, and recollapse would occur on time scale of several hundred million years. Therefore, we might expect galaxies to be in place as virialized objects by a redshift of 10. It is evident that larger scale structure forms later, with the present turn around radius being at about 30 Mpc.

There are two problems with such a scenario for structure formation. The first is conceptual: In a homogeneous Universe what determines the point or points about which such recollapse occurs? Basically in this picture small density fluctuations play no role,whereas we might expect that in the real world structure develops from the field of small density fluctuations as in the standard picture. The second problem is observational: This picture predicts inflow out to scales of tens of megaparsecs; it would have been quite difficult for Hubble to have discovered his law if this were true.

These problems may be overcome in the context of a more physically consistent, albeit nonrelativistic, theory of MOND cosmology (Sanders 2001). Following Bekenstein & Milgrom (1984), one begins with a two-field Lagrangian-based theory of MOND (nonrelativistic) in which one field is to be identified with the usual Newtonian field and the second field describes an additional MOND force that dominates in the limit of low accelerations. The theory embodies two important properties: MOND plays no role in the absence of fluctuations (the MOND field couples only to density inhomogeneities). This means the basic Hubble flow is left intact. Second, although the Hubble flow is not influenced by MOND, it enters as an external field that influences the internal dynamics of a finite-size region. Basically, if the Hubble deceleration (or acceleration) over some scale exceeds a_o, the evolution of fluctuations on that scale is Newtonian.

Figure 12 shows the growth of fluctuations of different comoving scales compared with the usual Newtonian growth. It is evident that MOND provides a considerable boost, particularly at the epoch during which the cosmological constant begins to dominate. This is due to the fact that the external acceleration field vanishes at this point and therefore plays no role in suppressing the modified dynamics. This adds a new aspect to an anthropic argument originally given by

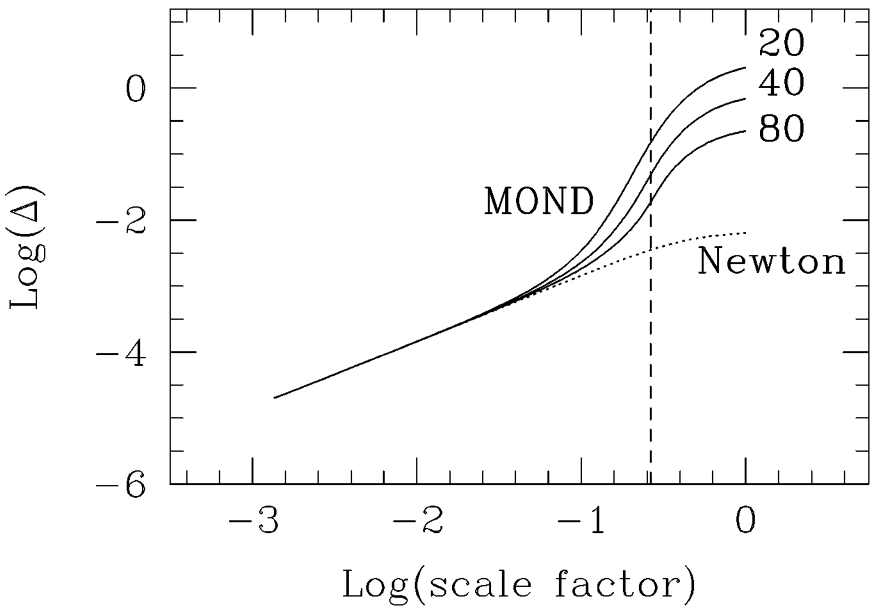


Figure 12 The growth of spherically symmetric over-densities in a low-density baryonic universe as a function of scale factor in the context of a two-field Lagrangian theory of MOND. The solid curves correspond to regions with comoving radii of 20, 40, and 80 Mpc. The dotted line is the corresponding Newtonian growth. With MOND, smaller regions enter the low-acceleration regime sooner and grow to larger final amplitude. The vertical dashed line indicates the epoch at which the cosmological constant begins to dominate the Hubble expansion.

Milgrom (1989c): We are observing the universe at an epoch during which Λ has only recently emerged as the dominant term in the Friedmann equation because that is when structure formation proceeds rapidly. The predicted MOND power spectrum (Sanders 2001) is rather similar in form to the CDM power spectrum in the concordance model but contains the baryonic oscillations proposed by McGaugh (1998). These would be telling but are difficult to resolve in large-scale structure surveys.

It is the external field effect owing to Hubble deceleration that tames the very rapid growth of structure in this scenario. In the context of the two-field theory this effect may be turned off, and then it is only the peculiar accelerations that enter the MOND equation. This case has been considered by Nusser (2002), who finds extremely rapid growth to the nonlinear regime and notes that the final MOND power spectrum is proportional to k^{-1}, independent of its original form. He found that to be consistent with the present amplitude of large-scale fluctuations a_o must be reduced by about a factor of 10 over the value determined from rotation curve-fitting. He has confirmed this with N-body simulations, again applying the MOND equation only in determining the peculiar accelerations. This suggests that the Hubble deceleration should come into play in a viable theory of MOND structure formation.

All of these conclusions are tentative; their validity depends upon the validity of the original assumptions. Nonetheless, it is evident that MOND is likely to promote the formation of cosmic structure from very small initial fluctuations; this, after all, was one of the primary motivations for nonbaryonic cosmic dark matter.

CONCLUSIONS

It is noteworthy that MOND, as an ad hoc algorithm, can explain many systematic aspects of the observed properties of bound gravitating systems: (*a*) the presence of a preferred surface density in spiral galaxies and ellipticals (the Freeman and Fish laws); (*b*) the fact that pressure-supported, nearly isothermal systems ranging from molecular clouds to clusters of galaxies are characterized by specific internal acceleration ($\approx a_o$); (*c*) the existence of a tight rotation velocity-luminosity relation for spiral galaxies (the TF law)—specifically revealed as a correlation between the total baryonic mass and the asymptotically flat rotation velocity of the form $M \propto V^4$; (*d*) the existence of a luminosity-velocity dispersion relation in elliptical galaxies (Faber-Jackson)—a relation that extends to clusters of galaxies as a baryonic mass-temperature relation; and (*e*) the existence of a well-defined two-parameter family of observed properties, the fundamental plane, of elliptical galaxies—objects that have varied formation and evolutionary histories and nonhomologous structure. Moreover, this is all accomplished in a theory with a single new parameter with units of acceleration, a_o, that must be within an order of magnitude of the cosmologically interesting value of cH_o. Further, many of these systematic aspects of bound systems do not have any obvious connection to what has been traditionally called the "dark matter problem." This capacity to connect seemingly unrelated points is the hallmark of a good theory.

Impressive as these predictions (or explanations) of systematics may be, it is the aspect of spiral galaxy rotation curves that is most remarkable. The dark matter hypothesis may, in principle, explain trends, but the peculiarities of an individual rotation curve must result from the unique formation and evolutionary history of that particular galaxy. The fact that there is an algorithm—MOND—that allows the form of individual rotation curves to be successfully predicted from the observed distribution of detectable matter—stars and gas—must surely be seen, at the very least, as a severe challenge for the dark matter hypothesis. This challenge would appear to be independent of whether or not the algorithm has a firm foundation in theoretical physics because science is, after all, based upon experiment and observation. Nonetheless, if MOND is, in some sense, correct then the simple algorithm carries with it revolutionary implications about the nature of gravity and/or inertia—implications that must be understood in a theoretical sense before the idea can be unambiguously extended to problems of cosmology and structure formation.

Does MOND reflect the influence of cosmology on local particle dynamics at low accelerations? The coincidence between a_o and cH_o would suggest a connection. Does inertia result from interaction of an acceleration object with the vacuum as some have suggested? If so, then one would expect a cosmological vacuum energy density to influence this interaction. Are there long-range scalar fields in addition to gravity, which in the manner anticipated by the Bekenstein-Milgrom theory become more effective in the limit of low field gradients? Additional fields with gravitational strength coupling are more or less required by string theory, but their influence must be suppressed on the scale of the solar system (high accelerations); otherwise, they would have revealed themselves as deviations from the precise predictions of general relativity at a fundamental level—violations of the equivalence principle or preferred frame effects. Such suppression can be achieved via the Bekenstein-Milgrom field equation.

Ideally, a proper theory of MOND would make predictions on a scale other than extragalactic; this would provide the possibility of a more definitive test. An example of this is the stratified aquadratic scalar-tensor theory, which predicts local preferred frame effects at a level that should soon be detectable in the lunar laser ranging experiment (Sanders 1997). An additional prediction that is generic of viable scalar-tensor theories of MOND is the presence of an anomalous acceleration, on the order of a_o, in the outer solar system. The reported anomalous acceleration detected by the Pioneer spacecrafts beyond the orbit of Jupiter (Anderson et al. 1998) is most provocative in this regard, but the magnitude (8×10^{-8} cm/s^2) is somewhat larger than would be naively expected if there is a connection with MOND. However, this is an example of the kind of test that, if confirmed, would establish a breakdown of Newtonian gravity or dynamics at low acceleration.

In a 1990 review on dark matter and alternatives Sanders (1990) wrote, "overwhelming support for dark matter would be provided by the laboratory detection of candidate particles with the required properties, detection of faint emission from low mass stars well beyond the bright optical image of galaxies, or the definite observation of 'micro-lensing' by condensed objects in the dark outskirts of galaxies." Now, more than a decade later, a significant baryonic contribution to the halos of galaxies in the form of "machos" or low mass stars seems to have been ruled out (Alcock et al. 2000). Particle dark matter has been detected in the form of neutrinos, but of such low mass—certainly less than 3 ev and probably comparable to 0.15 ev (Turner 1999)—that they cannot possibly constitute a significant component of the dark matter—either cosmologically or on the scale of galaxies. At the same time, the inferred contribution of CDM to the mass budget of the Universe has dropped from 95% to perhaps 30%, and both observational and theoretical problems have arisen with the predicted form of halos (Sellwood & Kosowsky 2001). However, all of this has not deterred imaginative theorists from speculative extrapolations of the standard model to conjure particles having the properties desired to solve perceived problems with dark matter halos. It is surely time to apply Occam's sharp razor and seriously consider the suggestion that Newtonian dynamics may break down in the heretofore unobserved regime of low accelerations.

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