MODIFIED POWER LAW EQUATIONS FOR VERTICAL WIND PROFILES

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ABSTRACT

Equations are presented for calculating power law exponents from wind speed and surface roughness data. Results are evaluated by comparison with wind profile date measured at a variety of sites.

INTRODUCTION

The power law equation is a simple yet useful model of the vertical wind profile which was first proposed by Hellman (1916), according to Simiu and Scanlan (1978). The general from of this equation is

$$V_2 = V_1(z_2/z_1)^{\alpha} \qquad (1)$$

in which V_1 and V_2 are simultaneous steady wind speeds over level terrain at elevations z_1 and z_2, respectively. The exponent α is determined experimentally. For example, early work by Von Karman (1921) showed that under certain conditions α is equal to 1/7, indicating a correspondence between wind profiles and fluid flow over flat plates (Schlicting, 1968). In the general case, however, α is a highly variable quantity. Sisterson and Frenzen (1978) measured wind profile exponents which changed from 1/7 during the day to 1/2 at night, over the same terrain. Golding (1955) describes α as an exponent which varies with height, time of day, season of the year, nature of the terrain, wind speed, and temperature. Most investigators agree that a constant value of α is an oversimplification, and that α must be treated as a statistical parameter.

In spite of the variable nature of the exponent α, Equation (1) is simple enough in form to permit adding some of the factors listed by Golding without losing the engineering convenience of the power law. Two of these factors will be considered here: (1) the nature of the terrain, in terms of its surface roughness, and (2) the wind speed.

Up until now, the effects of wind speed and surface roughness have only been considered separately. As shown in Figure 1, several functional relationships have been proposed for the variation of the exponent α with wind speed. These functions are the constant 1/7 law, a step function (Fales, 1967), a linear function for wind speeds exceeding 27 meters per second (ASCE, 1961), a power function (Fichtl & Smith, 1977), and a logarithmic function (Justus & Mikhail, 1976). All of the variable functions show a

decrease in the exponent with increasing wind speed, but none includes the effect of surface roughness. Frost et al (1978) and Justus (1978) give equivalent values of the exponent α and the well-known surface roughness length z_0, but without specifying the wind speed.

Thus, information on the separate effects of wind speed and surface roughness is available, but a wind profile model which combines these two effects is lacking. The objective of this paper, then, is to present equations relating the mean value of α to both the surface roughness and the steady wind speed. Other factors are assumed to be responsible for statistical variations about this mean value.

The equations presented in this paper are exploratory in nature and should be reevaluated with more data, when such data are available.

SUMMARY OF EQUATIONS

The Justus-Mikhail equation shown graphically in Figure 1 was selected as a starting point for this study because of its relative simplicity and the fact that it has been shown to be compatible with wind distribution data. This equation for the wind profile exponent α in Equation (1) can be written as follows:

$$\alpha = 0.37 - 0.203 \log V_r \qquad (2)$$

in which V_r is the wind speed at a reference evaluation of 10 meters.

Equation (2) can be generalized to include a surface roughness parameter and statistical variations. Assume α is a random variable which is normally distributed about a mean value $\bar{\alpha}$ with standard deviation σ. Assume further that both $\bar{\alpha}$ and σ are functions of the steady (non-gusting) wind speed and $\bar{\alpha}$ is also dependent on the surface roughness. Then, a proposed equation for $\bar{\alpha}$, based on Equation (2) is

$$\bar{\alpha} = \alpha_o \frac{1 - \log V_1/\log V_h}{1 - \alpha_o \log (z_1/z_r)/\log V_h} \qquad (3a)$$

and

$$\alpha_o = (z_0/z_r)^{0.2} \qquad (3b)$$

in which

α_o	surface roughness exponent
V_h	homogeneous wind speed ($\bar{\alpha} = 0$), m/s
V_1	steady wind speed at elevation z_1, m/s
z_r	reference elevation, 10m
z_0	surface roughness length, m

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Equation (3b) is an empirical relationship which will be discussed later. The standard deviation $\sigma(V_1)$ must be determined from site data.

Equations (3) are shown graphically in Figures 2 and 3. The terrain descriptions in Figure 3 follow Frost et al (1978) and Justus (1978).

Weibull distribution parameters C and k and the wind profile parameters α_o and V_h are related, as shown by the following equations:

$$P(V_1 > V) = \exp\left[-(V/C_1)^{k_1}\right] \tag{4}$$

in which

$P(V_1 > V)$	probability that the steady wind speed at elevation z_1 will exceed V
C_1	Weibull scale factor at elevation z_1, m/s
k_1	Weibull shape factor at elevation z_1

Weibull distribution parameters at an elevation z_2 can be calculated from those at elevation z_1 by means of the following equations:

$$C_2 = C_1 (z_2/z_1)^{\alpha_{c,1}} \tag{5a}$$

$$\alpha_{c,1} = \alpha_o \frac{1 - \log C_1 / \log V_h}{1 - \alpha_o \log (z_1/z_r)/\log V_h} \tag{5b}$$

$$k_2 = k_1 \frac{1 - \alpha_o \log (z_1/z_r)/\log V_h}{1 - \alpha_o \log (z_2/z_r)/\log V_h} \tag{5c}$$

The Justus-Mikhail equation, (2), is a special case of Equation (3a) with α_o equal to 0.37 and V_h equal to 67 meters per second. Equations equivalent to Equations (3a) and (5) with these specific values of α_o and V_h were also used by Justus et al (1976) to calculate reference wind speeds.

CALCULATED AND OBSERVED WIND PROFILES

Mean Wind Speed Profiles

Wind profile data (WSSI, 1976 to 1978) from wind turbine sites or potential sites selected by the Department of Energy were used for a preliminary evaluation of Equations (3). Table I lists calculated roughness exponents α_o for five such sites and also for the Justus-Mikhail reference site. A homogeneous wind speed of 67 meters per second was assumed for all sites

in this preliminary evaluation. Figure 4 illustrates the procedure for calculating α_o, using data for Clayton, NM. Exponents calculated from monthly average wind speeds at several elevations are plotted versus the logarithm of the reference elevation wind speed, V_r. A straight line through the centroid of these data points and the homogeneous point ($V_r = V_h = 67$ m/s and $\alpha = 0$) defines α_o, at a wind speed of one meter per second.

The roughness exponents in Table I vary from 0.1 to almost 0.5, in qualitative agreement with the site characteristics. Higher α_o values occur at rougher sites and lower values at smoother sites. The Justus-Mikhail constant of 0.37 appears to be somewhat too large to be used as an average roughness exponent. However, more profile data would have to be analyzed before a better reference value can be established. Based on these preliminary calculations, an average α_o of about 0.30 is indicated.

Equation 3(b) is an empirical relationship between α_o and z_0, which is the conventional measure of surface roughness. As shown in Figure 5, the exponent of 0.2 was obtained by curve-fitting equivalent values of α and z_0 from the literature (Frost, et al, 1978, and Justus, 1978). Calculated values of α_o from Table I then established the coefficient as $(z_1)^{-0.2}$.

Peak Wind Speed Profiles

As shown in Figure 2, the proposed profile model is based on the assumption that wind profiles become uniform or "homogeneous" at high wind speeds, for all values of surface roughness. This concept represents an extrapolation of the Justus-Mikhail equation as originally proposed, and additional verification is required. To do this, profile exponents were calculated for monthly peak winds measured at 11 DOE sites (WSSI, 1976 to 1978), to determine if there was a correlation between high winds and low profile exponents.

Typical results of the peak wind analysis are shown in Figure 4, for the Clayton NM, site. Values of α calculated from monthly peak winds which averaged 23 meters per second were found to be significantly lower than exponents calculated from monthly average winds of about 6 meters per second. The assumed homogeneous wind speed of 67 meters per second is consistent with the Clayton peak wind data.

Table II summarizes the analysis of peak wind profile data. This table lists mean peak wind speeds, mean profile exponents, and standard deviations, for each of 11 DOE sites. A composite average is also given which indicates that mean profile exponents less than 0.07 can be expected at wind speeds higher than 22 meters per second. To predict extreme wind loads, the "mean plus 3σ" value of α would be used, which is 0.27 for the composite average site. This compares closely with the ASCE recommended value of 0.30 shown in Figure 1, for a wind speed of 22 meters per second. Thus, equations (3a) and (3b) are representative of the mean values of the wind profile exponent, and should not be used in extreme winds analysis. More conservative values should be used.

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Mean profile exponents in Table II are shown graphically in Figure 6. Also shown in this figure are mean profile exponents calculated from monthly average wind speeds measured at five DOE sites (see Table I). A band representing predicted values of $\overline{\alpha}$ for flat, open terrain is also shown for comparison. At low wind speeds surface roughness is an important consideration in determining α. Exponents for the two rough sites (Plum Brook and Block Island) fall above the band, and that for a smooth hillside site (Culebra) falls well below. At high wind speeds surface roughness is no longer a significant variable and mean exponents for all sites are low.

CONCLUDING REMARKS

Equations have been presented and evaluated for a wind profile model which incorporates both roughness and wind speed effects while retaining the basic simplicity of the Hellman power law. Moreover, these equations recognize the statistical nature of wind profiles and are compatible with existing analytical models and recent wind profile data.

Predictions of energy output based on the proposed profile equations are 10 percent to 20 percent higher than those made with the 1/7 power law. In addition, correlation between calculated and observed blade loads is significantly better at higher wind speeds when the proposed wind profile model is used instead of a constant power model.

It is recommended that statistical analysis of site wind data include a bivariate distribution of profile exponent versus wind speed. This would permit more accurate calculation of the parameters in the proposed model and permit estimates of standard deviation as a function of wind speed. Statistical information of this type would be extremely useful to wind turbine designers for predicting wind load spectra, fatigue life of components, and energy output.

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Table I. - Surface roughness exponents for five candidate wind turbine sites.

Site	Type of Terrain	Surface roughness [a] exponent, α_o
Plum Brook, OH	flat; high woods	0.48
Block Island, RI	hilly; medium woods	0.44
Reference (Justus and Mikhail, 1976)	relatively flat	0.37
Clayton, NM	flat; open	0.30
Russell, KS	flat; open	0.30
Culebra, PR	shoreline hillside; bushes	0.10

[a] $\overline{\alpha} = \alpha_o(1 - \log V_r/\log V_h)$, with $V_h = 67$ m/s

Table II. - Wind profile exponents for monthly peak winds at candidate wind turbine sites.

| Site	No. of readings	Mean peak speed at 10m, m/s	Wind profile exponent, α	
			Mean, $\overline{\alpha}$	Std. dev., σ
Amarillo, TX	26	22.3	0.053	0.045
Block Island, RI	30	19.5	.114	.052
Clayton, NM	24	22.8	.054	.044
Cold Bay, AK	9	24.1	.042	.044
Culebra, PR	30	17.4	.010	.078
Huron, SD	28	20.6	.094	.056
Kingsley Dam, NA	29	24.6	.061	.073
Medicine Bow, WY	5	29.0	.077	.027
Point Arena, CA	23	20.1	.072	.144
Russell, KS	29	23.4	.059	.040
San Gorgonio, CA	33	23.5	.088	.053
Composite Site	266	22.2	0.067	0.068
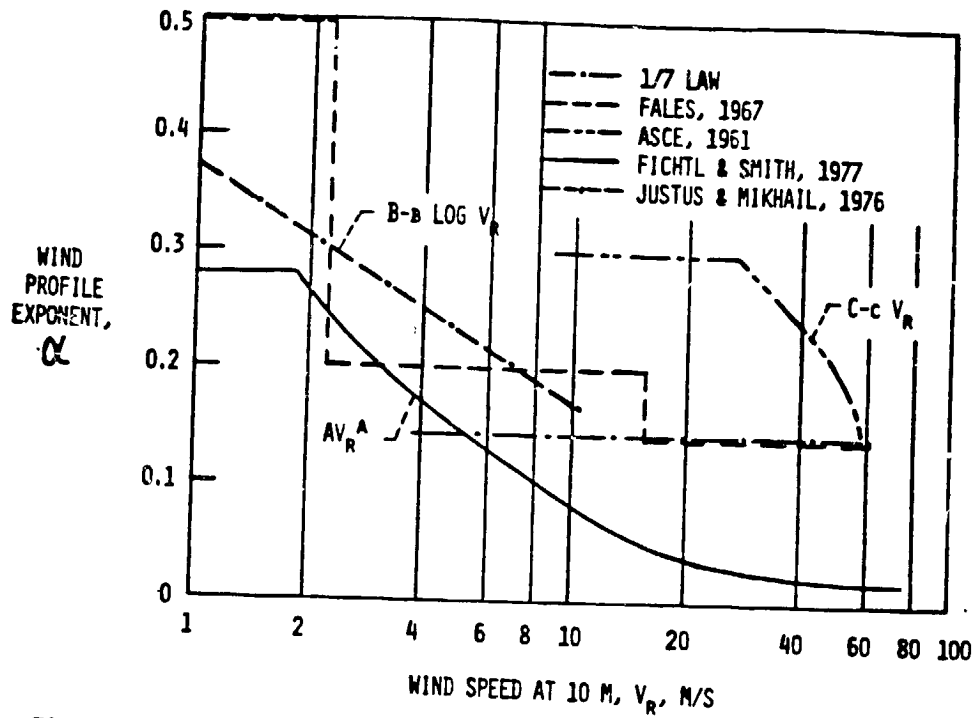
Figure 1 - Various recommendations for the relationship between wind profile exponent and wind speed.
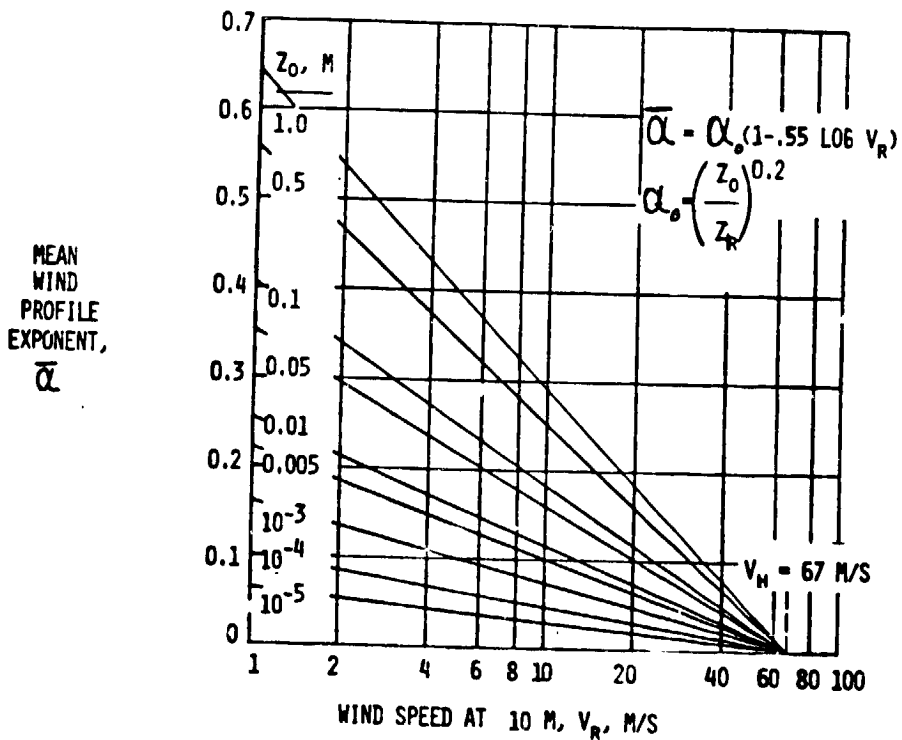

Figure 2 - Graph of proposed equation for wind profile exponent in terms of surface roughness and wind speed.
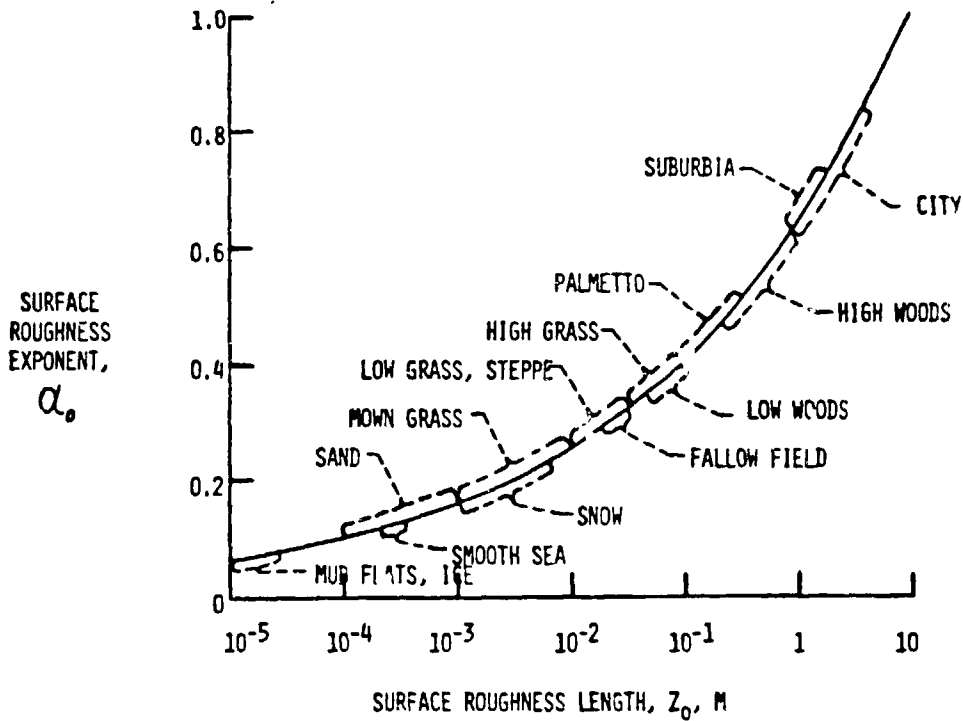
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Figure 3 - Graph of relationship between surface roughness exponent α_o, surface roughness length, z_0 and terrain description.
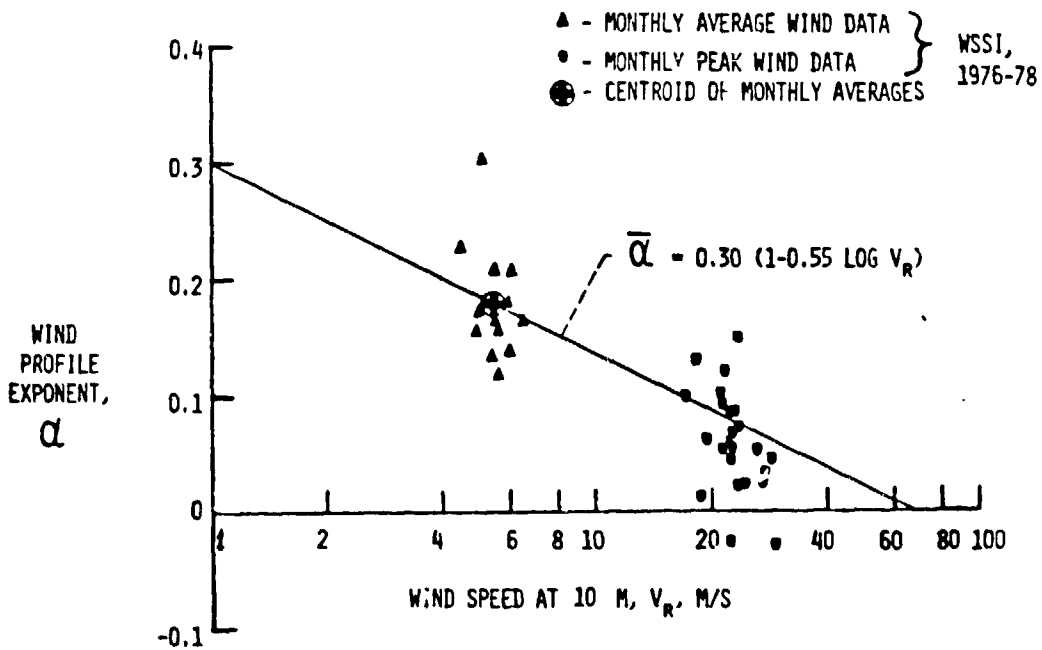

Figure 4 - Comparison of observed and calculated wind profile exponents for Clayton, New Mexico site.
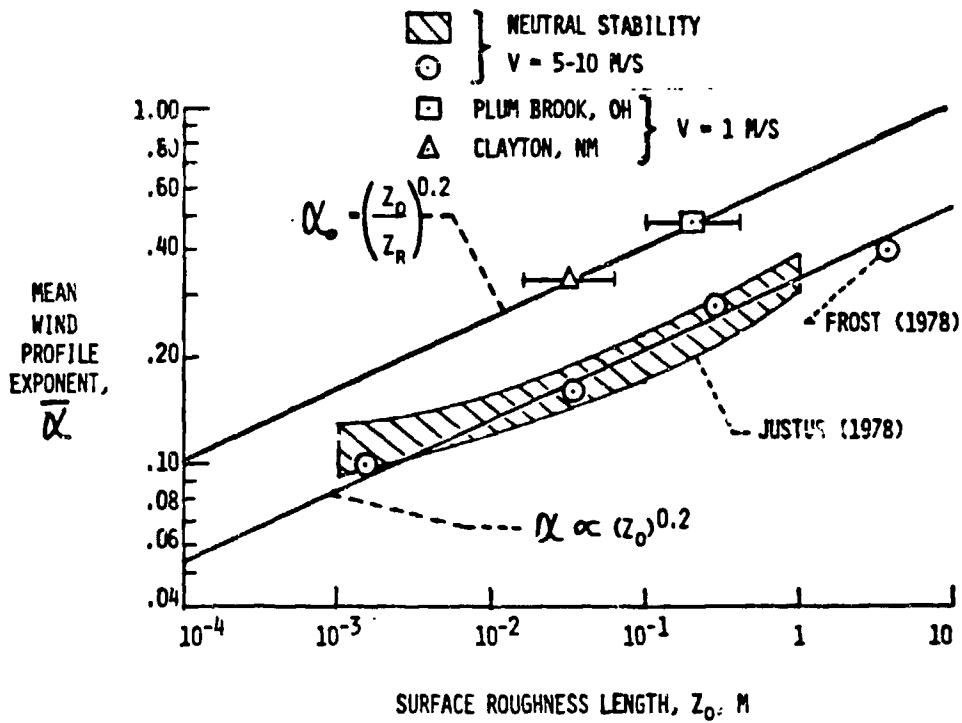
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Figure 5 - Curve-fit of data relating wind profile exponent $\overline{\alpha}$ to surface roughness length.
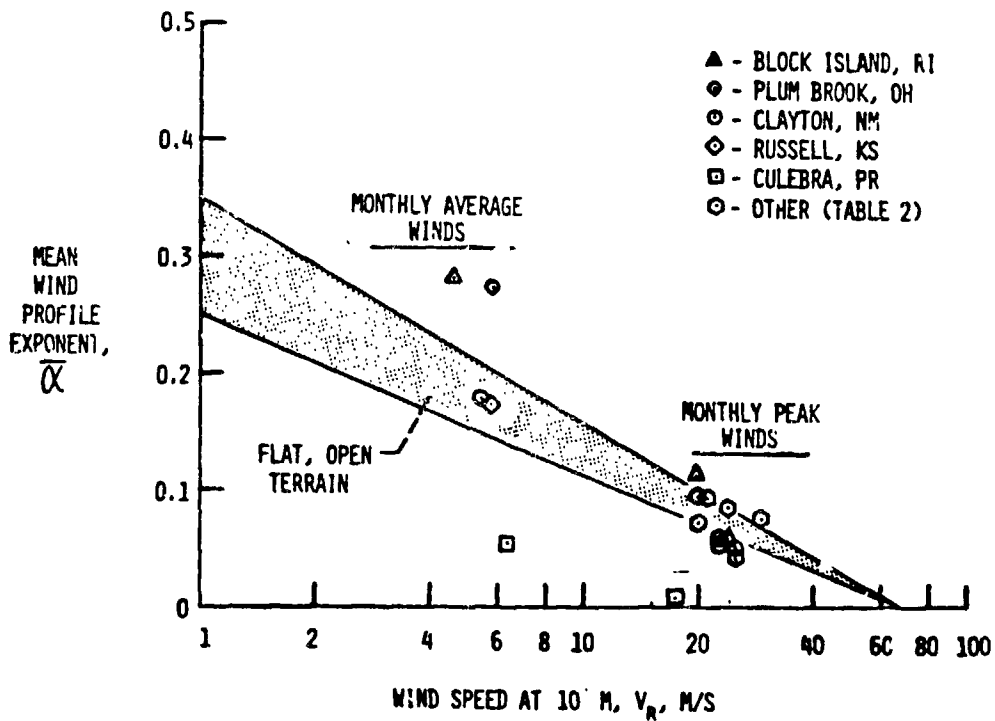

Figure 6 - Comparison of observed and calculated wind profile exponents for various sites.

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