GEOPHYSICAL RESEARCH LETTERS, VOL. 31, L22608, doi:10.1029/2004GL021621, 2004

Modified sidereal filtering: Implications for high-rate GPS positioning

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Received 27 September 2004; accepted 28 October 2004; published 30 November 2004.

[1] High-rate (1-Hz) Global Positioning System (GPS) data are beginning to be used for a variety of geophysical monitoring purposes, including seismology. Improving the precision of high-rate GPS position estimates will increase the value of these 1-Hz GPS monitoring systems. One technique that has been used to improve high-rate GPS positioning takes advantage of the ground track repeat period of the satellites. This study investigates the GPS orbital repeat period and determines that it varies for each satellite and differs significantly from the generally assumed sidereal period. Orbit repeat periods are calculated and used to filter 1-Hz GPS position estimates. Using the calculated orbit repeat period significantly reduces low frequency (0.001–0.04 Hz) errors in 1-Hz GPS position estimates. *INDEX TERMS:* 1242 Geodesy and Gravity: Seismic deformations (7205); 1241 Geodesy and Gravity: Satellite orbits; 1294 Geodesy and Gravity: Instruments and techniques; 7212 Seismology: Earthquake ground motions and engineering. **Citation:** Choi, K., A. Bilich, K. M. Larson, and P. Axelrad (2004), Modified sidereal filtering: Implications for high-rate GPS positioning, *Geophys. Res. Lett.*, *31*, L22608, doi:10.1029/2004GL021621.

1. Introduction

[2] Solid Earth geophysicists primarily use the Global Positioning System (GPS) to measure receiver positions at daily averaging periods using data recorded at 30 second intervals. In the past 5 years there has been increased interest in applying GPS techniques to measure geophysical signals at subdaily periods, including volcanic deformation [*Larson et al.*, 2001], ice sheet flow [*Zwally et al.*, 2002], and seismic waves [*Nikolaidis et al.*, 2001]. Improving the precision of subdaily GPS position estimates will depend strongly on better modeling of the errors which in traditional GPS analyses are mitigated by averaging. The errors that dominate high-precision, high-rate GPS solutions on time scales of 10–600 s are largely site and satellite geometry specific. Because the GPS satellite orbits were established and are maintained to achieve a daily repeating (sidereal) ground track, these errors are highly repeatable from day to day. This is the basis for the sidereal filtering concept suggested by *Bock* [1991]. In this paper we seek to modify this technique by more carefully considering the ground track repeat of the GPS satellites.

2. Sidereal Filtering

[3] High-rate GPS positioning precision is influenced by GPS measurement noise (∼5 mm), the number and location of the satellites, and the ability to model errors associated with orbits, satellite and receiver clocks, atmospheric delays, antenna effects, and multipath. It was noted by *Bock* [1991] and *Genrich and Bock* [1992] that since some of these errors are related to satellite-receiver geometry, one could take advantage of the fact that the GPS satellite period is designed to be exactly half a sidereal day to mitigate these errors. For a receiver on the Earth, this means that satellites visible in the sky today should be visible in the exact same location 23 h 56 m 4 s (the sidereal day) later.

[4] The sidereal filtering concept has two steps and implementation requires data from two or more days. First, 1-Hz positions are estimated on day 1, assuming that the ground has not moved. These day 1 positions are low-pass filtered to remove high-frequency noise unrelated to the satellite-receiver geometry, shifted by the sidereal period (23 h 56 m 4 s) and subtracted from the estimated positions on the second day. This sidereal filtering is the basis for precise subdaily positioning results shown by *Nikolaidis et al.* [2001] and *Bock et al.* [2004].

[5] Research groups have also discussed the correlation of GPS position estimates with respect to the sidereal day for multipath research [*Elósegui et al.*, 1995; *Seeber et al.*, 1997; *Ding et al.*, 1999; *Radovanovic*, 2000; *Wübbena et al.*, 2001; *Forward et al.*, 2003; *Park et al.*, 2004]. With the exception of *Seeber et al.* [1997] and *Ding et al.* [1999], these authors assumed that GPS satellites have a sidereal repeat period. *Seeber et al.* [1997] were the first to notice that the satellite repeat period was both not sidereal and varied for different satellites. *Ding et al.* [1999] also commented on the non-sidereal orbit repeat period, but they could not isolate its true repeat time because they were collecting data at 15 s intervals. To better understand the impact of orbit repeat period variability on the efficacy of sidereal filtering, we first calculate satellite repeat periods for the past year; we then assess the true repeat period's impact on high-rate positioning precision.

3. GPS Orbit Repeat Period

[6] The repeat period of GPS satellites can be assessed in multiple ways. The pseudorange multipath observable, for example, is sensitive to orbit repeat period (Figure S1)[1]. We used the GPS broadcast ephemeris values and Kepler's Third Law to determine the orbit repeat period of individual satellite. Because a recent paper about the San Simeon earthquake (19:15 UTC, 22 December 2003) [*Ji et al.*, 2004] evaluated the precision of unfiltered 1-Hz GPS positions, we use the same GPS sites [*Langbein and Bock*, 2004] in this study. We focus on the 7 satellites visible at

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[1] Auxiliary material is available at ftp://ftp.agu.org/apend/gl/2004GL021621.

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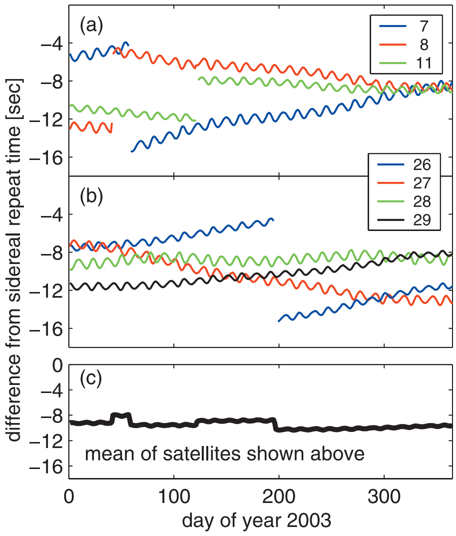

Figure 1. (a–b) Orbit repeat times for GPS (PRN) satellites for the year 2003. Only those satellites that can be viewed from Parkfield at 19:15 UTC, 22 December 2003 are shown. (c) Mean orbit repeat time for satellites shown in Figures 1a and 1b.

19:15 UTC, 22 December 2003 to illustrate the variability of orbit repeat times.

[7] Figure 1 shows that the repeat periods of the satellites are not precisely sidereal, but rather are offset from the sidereal repeat period and distinct for each satellite. The secular drifts in repeat period are due to resonance effects with the tesseral harmonics in the Earth's gravity field [*Chao and Schmitt*, 1991], and the small amplitude oscillations are perturbations produced by lunar gravity (George Rosborough, personal communication, 2004). The large, abrupt changes in the repeat period are the result of satellite maneuvers. In addition to the satellites shown, other satellites in the constellation had much larger variations. For example, the repeat period of satellite 23 was 86 s less than sidereal before it was maneuvered.

[8] Why do the GPS satellites not have a sidereal repeat period? The operational constraint used by the Department of Defense is not that the orbit periods be sidereal but that the GPS ground tracks be fixed. Specifically, the Department of Defense keeps the longitude of the ascending node within ±2° of its nominal value [*Chao and Schmitt*, 1991]. The orbital period of the satellites is set ∼4 s faster than half-sidereal to compensate for the dominant nodal drift rate caused by J2. This puts the GPS satellite over the same longitude every day. Thus, we should expect the orbit repeat time for GPS satellites to generally be ∼8 s earlier than sidereal and it is. The satellite maneuvers shown in Figure 1 were made to reposition satellite orbits that have drifted outside of the specified tolerance.

[9] In sidereal filtering, the position series are shifted by the sidereal period. How should we shift position estimates when, for this example, there are seven distinct orbit repeat periods? As long as the satellites used in the position estimates do not change, the repeat period used for the "modified" sidereal filter is the mean of the individual orbit repeat periods. For the satellites shown in Figure 1 at 19:15 UTC, 22 December 2003, the mean orbit repeat time is ∼9 s less than sidereal, so the position estimates should be shifted by 23 h 55 m 55 s (see supplement, Figure S2).

4. Positioning Improvement

[10] It was previously demonstrated that filtering using a sidereal repeat period does improve high-rate positioning precision at the 30 s data rate [*Bock et al.*, 2000]. Here we assess the impact of using a modified sidereal filter on higher-rate data, where the mean orbit repeat period is used to determine the appropriate time shift. We used data from

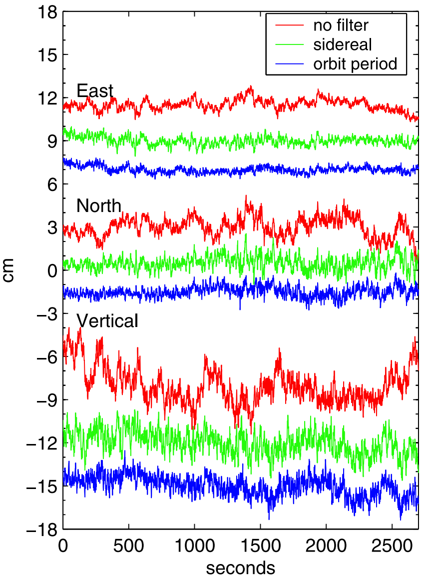

Figure 2. East, north, and vertical position estimates for Parkfield station HUNT. Filtered results are also shown for a sidereal and orbit repeat period. The time period shown is 18:30–19:15 UTC 22 December 2003.

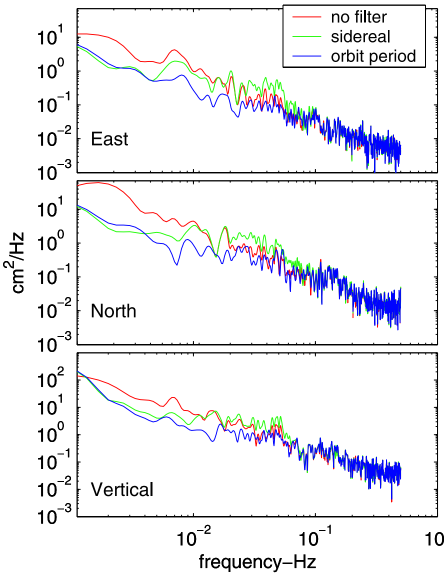

Figure 3. Power spectral density for east, north, and vertical position estimates for station HUNT: unfiltered, sidereal filtered, and orbit repeat time filtered.

the 12 1-Hz GPS sites in the Parkfield array [*Langbein and Bock*, 2004]. A two hour time period (18:15–20:15 UTC) of GPS data on 21 and 22 December 2003 was analyzed using the techniques described by *Larson et al.* [2003]. Orbit parameters are held fixed to their IGS values [*Beutler et al.*, 1994]. The position of one site (WHYT) outside the array is tightly constrained to agree with its ITRF2000 value [*Altamimi et al.*, 2002]; the positions of the other sites are estimated every second. Other estimated parameters include satellite and receiver clocks, constant zenith troposphere delays, and carrier phase ambiguities. Ambiguities were resolved using the pseudorange widelaning technique of *Blewitt* [1989].

[11] We use the time period 18:30–19:15 UTC to evaluate our filters because a constant set of 7 satellites was tracked during this time. Estimated positions on 21 December 2003 were box-car filtered (7 and 11 seconds, for the horizontal and vertical components, respectively) to reduce high frequency noise, shifted by the mean orbit repeat period, and then subtracted from the estimated positions on 22 December 2003. We also computed filtered solutions using the sidereal period. Figure 2 shows estimated positions for site HUNT. The improvement in noise level for both filters is clear, particularly in the north and vertical components. To evaluate the improvement of the filtered solutions at different frequencies, we averaged three 15 minute segments and computed the power spectral density (Figure 3). We can see that both filters significantly improve position estimates at frequencies below 0.004 Hz. Using the modified sidereal filter removes more noise between frequencies of 0.004–0.04 Hz than the standard sidereal filtering technique.

[12] To compare with *Ji et al.* [2004], we also calculated the standard deviation of the station positions for the 500 seconds before 19:15 UTC, 22 December 2003. Figure 4 compares the standard deviations from the two filtering techniques and demonstrates that although the error in the sidereal filter is small, it is clearly less precise than the orbit repeat period filtering results. Specific position improvements for each site in the Parkfield array are given in Figure S3 (see supplement).

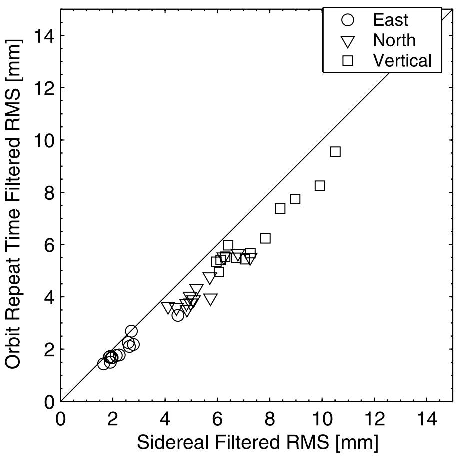

Figure 4. Standard deviations for 1-Hz position estimates at 12 Parkfield GPS sites that have been sidereally filtered compared with positions filtered by the mean orbit repeat period.

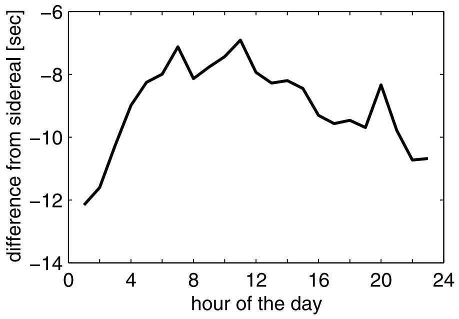

Figure 5. Mean orbit repeat time - relative to the sidereal period - for satellites above an elevation angle of 15° as observed from Parkfield 22 December 2003.

5. Discussion

[13] Our demonstration focused on calculating and evaluating the mean orbit repeat period for 19:15 UTC 22 December 2003. This orbit repeat time is not valid for other times of that same day. As shown in Figure 5, the mean orbit repeat time varies by 5 seconds during the day. This simply reflects that different satellites are visible at different times of the day, and thus the mean orbit repeat period varies as well. This demonstrates that high-rate GPS users should evaluate the mean orbit repeat time at subdaily intervals for highest precision results. Although the orbit repeat period for the GPS constellation can be determined and applied after the fact, the need for using the correct orbit repeat period is particularly important for those who have implemented real-time sidereal GPS filtering.

6. Conclusions

[14] We investigated the orbit repeat time of the GPS constellation in order to determine its impact on filtering 1-Hz GPS positions. Using the data previously discussed by *Ji et al.* [2004], we determined that the generally assumed sidereal repeat period is incorrect and the orbit repeat time is ~9 s earlier for these data. Using this repeat period significantly improves the precision of 1-Hz GPS positions at frequencies from 0.004–0.04 Hz. This result will be helpful as 1-Hz positions are used in more seismic applications [*Miyazaki et al.*, 2004]. A more accurate assessment of the orbit repeat period also has useful implications for hazards monitoring and ionosphere and troposphere studies.

[15] **Acknowledgments.** Support from NSF grant EAR-0337206 and from UNAVCO is gratefully acknowledged. JPL developed and licenses the GIPSY software. We used IGS precise orbits and the SOPAC archive. We acknowledge SCIGN and its funding agencies (W. M. Keck Foundation, NASA, NSF, USGS, SCEC) as the source of the GPS data. We thank Stephen Esterhuizen, James Gidney, John Berg, Jim Davis and an anonymous reviewer for helpful comments and discussions.

References

Altamimi, Z., P. Sillard, and C. Boucher (2002), ITRF2000: A new release of the International Terrestrial Reference Frame for Earth science applications, *J. Geophys. Res.*, *107*(B10), 2214, doi:10.1029/2001JB000561.
Beutler, G., I. Mueller, and R. Neilan (1994), The International GPS Service for Geodynamics, *Bull. Geod.*, *68*, 39–70.

Blewitt, G. (1989), Carrier phase ambiguity resolution for the Global Positioning System applied to geodetic baselines up to 2000 km, *J. Geophys. Res.*, *94*(B8), 10,187–10,203.

Bock, Y. (1991), Continuous monitoring of crustal deformation, *GPS World*, *2*(6), 40–47.

Bock, Y., R. M. Nikolaidis, P. J. de Jonge, and M. Bevis (2000), Instantaneous geodetic positioning at medium distances with the Global Positioning System, *J. Geophys. Res.*, *105*(B12), 28,223–28,253.

Bock, Y., L. Prawirodirdjo, and T. I. Melbourne (2004), Detection of arbitrarily dynamic ground motions with a dense high-rate GPS network, *Geophys. Res. Lett.*, *31*(B10), L06604, doi:10.1029/2003GL019150.

Chao, C., and D. L. Schmitt (1991), Eliminating GPS stationkeeping maneuvers by changing the orbital altitude, *J. Astronaut. Sci.*, *39*(2), 141–153.

Ding, X., Y. Chen, J. Zhu, and D. Huang (1999), Surface deformation detection using GPS multipath signals, paper presented at ION GPS-99, Satell. Div. of the Inst. of Navig., Nashville, Tenn., 53–62.

Elósegui, P., J. L. Davis, R. T. K. Jaldehag, J. M. Johansson, A. E. Niell, and I. I. Shapiro (1995), Geodesy using the Global Positioning System: The effects of signal scattering on estimates of site position, *J. Geophys. Res.*, *100*(B6), 9921–9934.

Forward, T., M. P. Stewart, and M. Tsakiri (2003), GPS data stacking for small scale GPS deformation monitoring applications, paper presented at 11th International FIG Symposium on Deformation Measurements, Int. Fed. of Surv., Santorini, Greece.

Genrich, J. F., and Y. Bock (1992), Rapid resolution of crustal motion at short ranges with the Global Positioning System, *J. Geophys. Res.*, *97*, 3261–3269.

Ji, C., K. M. Larson, Y. Tan, K. W. Hudnut, and K. Choi (2004), Slip history of the 2003 San Simeon earthquake constrained by combining 1-Hz GPS, strong motion, and teleseismic data, *Geophys. Res. Lett.*, *31*, L17608, doi:10.1029/2004GL020448.

Langbein, J., and Y. Bock (2004), High-rate real-time GPS network at Parkfield: Utility for detecting fault slip and seismic displacements, *Geophys. Res. Lett.*, *31*(15), L15S20, doi:10.1029/2003GL019408.

Larson, K., P. Cervelli, M. Lisowski, A. Miklius, P. Segall, and S. Owen (2001), Volcano monitoring using Global Positioning System: Filtering strategies, *J. Geophys. Res.*, *106*(B9), 19,453–19,464.

Larson, K. M., P. Bodin, and J. Gomberg (2003), Using 1-Hz GPS data to measure deformations caused by the Denali fault earthquake, *Science*, *300*(5624), 1421–1424.

Miyazaki, S., K. M. Larson, K. Choi, K. Hikima, K. Koketsu, P. Bodin, J. Haase, G. Emore, and A. Yamagiwa (2004), Modeling the rupture process of the 2003 September 25 Tokachi-Oki (Hokkaido) earthquake using 1-Hz GPS data, *Geophys. Res. Lett.*, *31*, L21603, doi:10.1029/2004GL021457.

Nikolaidis, R. M., Y. Bock, P. J. de Jonge, D. C. Agnew, and M. Van Domselaar (2001), Seismic wave observations with the Global Positioning System, *J. Geophys. Res.*, *106*(B10), 21,897–21,916.

Park, K. D., R. S. Nerem, M. S. Schenewerk, and J. L. Davis (2004), Site-specific multipath characteristics of global IGS and CORS GPS sites, *J. Geod.*, *77*, 799–803.

Radovanovic, R. S. (2000), High accuracy deformation monitoring via multipath mitigation by day-to-day correlation analysis, paper presented at ION GPS 2000, Satell. Div. of the Inst. of Navig., Salt Lake City, Utah, 35–44.

Seeber, G., F. Menge, C. Völksen, G. Wübbena, and M. Schmitz (1997), Precise GPS positioning improvements by reducing antenna and site dependent effects, paper presented at IAG Symposium No. 115, Int. Assoc. of Geod., Rio de Janeiro, Brazil, 237–244.

Wübbena, G., A. Bagge, G. Boettcher, and M. Schmitz (2001), Permanent object monitoring with GPS with 1 millimeter accuracy, paper presented at ION GPS 2000, Satell. Div. of the Inst. of Navig., Salt Lake City, Utah, 1000–1008.

Zwally, J., W. Abdalati, T. Herring, K. Larson, J. Saba, and K. Steffen (2002), Surface melt-induced acceleration of Greenland ice-sheet flow, *Science*, *297*(5579), 218–222.

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