

Modular A_4 invariance and leptogenesis

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ABSTRACT: We consider a model with three right-handed neutrinos in which Yukawa coupling constants and Majorana masses are obtained by requiring the modular A_4 symmetry. It has been shown that the model can explain mass hierarchies and mixing patterns of charged leptons and neutrinos with the seesaw mechanism. In this article we investigate the leptogenesis by decays of right-handed neutrinos in this model. It is shown that masses of right-handed neutrinos are about 10^{13} GeV in order to account for the observed baryon asymmetry of the universe. Furthermore, the positive sign of the baryon asymmetry is obtained only for the limited ranges of mixing angles and CP violation phases of active neutrinos, which can be tested by future neutrino experiments.

KEYWORDS: Cosmology of Theories beyond the SM, Neutrino Physics

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1 Introduction

The standard model (SM) is well established by the discovery of the Higgs boson. There are, however, various unsolved problems, and the flavor puzzle is one of those problems. One interesting approach to the origin of flavor structure is to impose a flavor symmetry on a theory. Especially, to explain the large mixing angles in the lepton sector, lepton flavor models with non-Abelian discrete symmetry such as S_3 , A_4 , S_4 , A_5 , and other groups have been studied [1–5].

Recently, lepton flavor model with A_4 being a subgroup of modular group has been proposed [6]. Modular symmetry is a geometrical symmetry of a two-dimensional torus T^2 , and the modulus is a complex field deciding the shape of the torus. Modular group induces S_3 , A_4 , S_4 , or A_5 as the finite quotient group [7]. In this framework, Yukawa couplings are written in terms of modular forms, which are non-trivial representation of flavor symmetry and constrained as explicit functions of the modulus. These features of Yukawa couplings enable us to construct flavor models without flavons. The lepton models with S_3 [8], A_4 [6, 8–15], S_4 [16–18], and A_5 [19, 20] have been studied. Moreover, quark model [21], combination of lepton and quark models [22], and GUT model [23, 24] have also been studied.

Modular symmetry is also interesting in the viewpoint of superstring theory. The torus compactification is a simple compactification of the extra dimensions, and which leads modular symmetry as explained above. Moreover, the orbifold compactification as well as magnetized torus compactification leads flavor symmetry including modular group or its finite subgroups [25–30]. In this sense, the modular symmetry or its finite subgroups can be expected as geometrical symmetries of extra dimensions.

Interestingly, models with the modular symmetry can predict the patterns of masses and mixing angles of charged leptons and neutrinos by using a very limited number of parameters. It has been discussed that the neutrino masses are generated by introducing

the Weinberg's dimension five operators or right-handed neutrinos with the seesaw mechanism, and both possibilities have been shown to be successful. It is a natural question to investigate whether right-handed neutrinos in the models can also explain the baryon asymmetry of the universe (BAU) through the leptogenesis.

The BAU is now measured very precisely by the cosmic microwave background radiation as [31]

$$Y_B = \frac{n_B}{s} = (0.852 - 0.888) \times 10^{-10}, \quad (1.1)$$

where the BAU at the present universe, Y_B , is defined by the ratio between the number density of baryon asymmetry n_B and the entropy density s . This asymmetry should be generated before the beginning of the big-bang nucleosynthesis after the primordial inflation ends. One of the most studied scenarios for baryogenesis is the canonical leptogenesis scenario [32] in which the decays of right-handed neutrinos can generate the lepton asymmetry that is partially converted into the baryon asymmetry [33] via the sphaleron process [34]. The sign and magnitude of the BAU are predicted by the masses and Yukawa coupling constants of right-handed neutrinos.

The absolute mass scales of right-handed neutrinos cannot be determined by the data of the neutrino oscillations and the BAU. When their masses are hierarchical, the lightest one must be larger than $\mathcal{O}(10^9)$ GeV [35, 36] to explain the BAU. It can be, however, small as TeV scale if right-handed neutrinos are quasi-degenerate in mass [37].

Furthermore, the sign of the BAU is controlled by the CP violation pattern in leptonic sector. Note that the sign of the BAU cannot be predicted uniquely even if the CP violations associated with active neutrinos (i.e., the Dirac and Majorana phases in the mixing matrix of active neutrinos) are determined. This is because there exist one or more additional phases associated with right-handed neutrinos which decouple from the low energy phenomena if they are sufficiently heavy. Under these situations, it is interesting to investigate the sign and magnitude of the BAU in the models with the modular symmetry, since there are non-trivial relations between the properties of right-handed neutrinos and the low energy observables of neutrino physics due to the symmetry. As our first work, we shall discuss the leptogenesis in the model with A_4 symmetry [10] simply because the model has a small number of free parameters and then very predictive.

The paper is organized as follows. In section 2, we briefly review the modular symmetry in the framework of the theory with extra dimensions which is compactified on a torus. We then explain the model with the A_4 symmetry in section 3. The leptogenesis in the model is discussed in section 4. We present in section 5 the results of the analysis, namely the sign and magnitude of the BAU predicted by the model. The final section is devoted to conclusions.

2 Modular group and its finite quotient subgroups

In this section, we give a brief review on the modular symmetry on a torus. A two-dimensional torus T^2 can be constructed by \mathbb{R}^2/Λ , where Λ denotes a two-dimensional lattice. We use the complex coordinate on \mathbb{R}^2 and denote basis vectors of Λ as $\alpha_1 = 2\pi R$

and $\alpha_2 = 2\pi R\tau$, where R is real and τ is a modulus belonging to upper-half complex plane $\text{Im } \tau > 0$. There is some ambiguity in choice of the basis vectors. The same lattice can be spanned by the following basis vectors,

$$\begin{pmatrix} \alpha'_2 \\ \alpha'_1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{SL}(2, \mathbb{Z}), \quad (2.1)$$

where

$$\text{SL}(2, \mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in \mathbb{Z}, \quad ad - bc = 1 \right\} \equiv \Gamma. \quad (2.2)$$

This transformation of basis vectors is written in terms of the modulus $\tau \equiv \alpha_2/\alpha_1$ by

$$\tau \rightarrow \tau' = \gamma\tau = \frac{a\tau + b}{c\tau + d}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{SL}(2, \mathbb{Z}). \quad (2.3)$$

The modular group is the transformation group acts on the modulus preserving the lattice Λ . Since γ and $-\gamma$ transform τ in the same way in (2.3), the modular group is isomorphic to $\text{SL}(2, \mathbb{Z})/\{\mathbb{I}, -\mathbb{I}\} \equiv \bar{\Gamma}$. The modular group is generated by two generators S and T ,

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad (2.4)$$

In terms of the modulus, they induce the transformations, $S : \tau \rightarrow -1/\tau$ and $T : \tau \rightarrow \tau + 1$. We can easily see that they satisfy the following algebraic relations, $S^2 = \mathbb{I}$ and $(ST)^3 = \mathbb{I}$. We introduce a series of groups $\Gamma(N)$, $N = 1, 2, 3, \dots$ called principal congruence subgroups,

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{SL}(2, \mathbb{Z}), \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}. \quad (2.5)$$

We also define $\bar{\Gamma}(N) = \Gamma(N)/\{\mathbb{I}, -\mathbb{I}\}$ for $N = 1, 2$ and $\bar{\Gamma}(N) = \Gamma(N)$ for $N > 2$. The groups $\bar{\Gamma}(N)$ are infinite subgroups of the modular group. The quotient groups defined $\Gamma_N \equiv \bar{\Gamma}/\bar{\Gamma}(N)$ are finite subgroups of the modular group, called finite modular groups. In the finite modular groups Γ_N , generators obey additional, algebraic relation $T^N = \mathbb{I}$. The groups Γ_N with $N = 2, 3, 4, 5$ are isomorphic to S_3, A_4, S_4 , and A_5 , respectively [7].

Modular forms $f(\tau)$ of weight k and level N are holomorphic functions transforming under the $\Gamma(N)$ as

$$f(\gamma\tau) = (c\tau + d)^k f(\tau), \quad \gamma \in \Gamma(N), \quad (2.6)$$

where k is even and non-negative value and called modular weight. In the case of $\Gamma_3 \simeq A_4$, the explicit form of A_4 triplet modular forms of weight 2, $Y^{A_4}(\tau) = (Y_1(\tau), Y_2(\tau), Y_3(\tau))$, is obtained as [6]

$$\begin{aligned} Y_1(\tau) &= \frac{i}{2\pi} \left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - \frac{27\eta'(3\tau)}{\eta(3\tau)} \right), \\ Y_2(\tau) &= \frac{-i}{\pi} \left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega^2 \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} \right), \\ Y_3(\tau) &= \frac{-i}{\pi} \left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega^2 \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} \right). \end{aligned} \quad (2.7)$$

where the Dedekind eta-function $\eta(\tau)$ is given by $\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n)$, $q = e^{2\pi i\tau}$.

Under the modular transformation (2.3), chiral superfields $\phi^{(I)}$ transform as [38],

$$\phi^{(I)} \rightarrow (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \phi^{(I)}, \quad (2.8)$$

where $-k_I$ is the modular weight and $\rho^{(I)}(\gamma)$ denotes an unitary representation matrix of $\gamma \in \Gamma_N$. A coupling constant for the n -th order term between $\phi^{(I_1)}, \dots, \phi^{(I_n)}$ should be a modular form of weight $k_Y(n)$ and a representation of Γ_N transformed as

$$Y_{I_1, I_2, \dots, I_n}(\gamma\tau) = (c\tau + d)^{k_Y(n)} \rho(\gamma) Y_{I_1, I_2, \dots, I_n}(\tau), \quad (2.9)$$

where $\rho(\gamma)$ is representation of γ for the modular form, and a modular invariant superpotential W is written by

$$W = Y_{I_1, I_2, \dots, I_n}(\tau) \phi^{(I_1)} \phi^{(I_2)} \dots \phi^{(I_n)} \quad (2.10)$$

satisfying $k_Y(n) = \sum_n k_{I_n}$ and $\rho(\gamma) \prod_n \rho^{(I_n)}(\gamma) = \mathbb{I}$.

We study the model which field content is the same as the minimal supersymmetric standard model (MSSM) extended by right-handed neutrinos in the following sections. The superpotential of our model has vanishing modular weight. We note that Yukawa couplings as well as higher order couplings depend on modulus and can have non-vanishing modular weights. The breaking scale of supersymmetry (SUSY) can be between $\mathcal{O}(1)\text{TeV}$ and the compactification scale. Here we take the breaking scale is sufficiently high, namely it is much higher than the masses of right-handed neutrinos, for simplicity. Then, the lepton flavor physics and the leptogenesis can be discussed without SUSY.¹ The modular symmetry is broken by the vacuum expectation value of τ at the compactification scale which is the Planck scale or slightly lower scale order.

3 Lepton flavor model with modular A_4 symmetry

The A_4 flavor models with flavon field have been discussed in the lepton sector [1–5]. On the other hand, a modular invariant flavor model with the A_4 symmetry can explain the large mixing angles of lepton flavors without flavons. One of the authors (THT) has already obtained a successful result of the lepton sector in A_4 modular symmetry [10]. In order to clarify the difference in the flavor structure of mass matrices between the quarks and leptons, we briefly summarize the previous results of the lepton sector and add discussions of the feature of the lepton model.

It is supposed that the three left-handed lepton doublets L_i are compiled in a triplet of A_4 . The three right-handed neutrinos N_i^c are compiled in a triplet of A_4 . On the other hand, the Higgs doublets, $H_{u,d}$, are supposed to be singlets of A_4 . The three right-handed charged leptons are assigned for three different singlets of A_4 as $(e_1^c, e_2^c, e_3^c) = (e^c, \mu^c, \tau^c) = (1, 1'', 1')$. Therefore, there are three independent couplings α, β and γ , in the superpotential of the charged lepton sector. Those coupling constants can be adjusted to the observed charged lepton masses. The assignments of representations and modular weights to the MSSM fields and right-handed neutrino superfields are presented in table 1.

¹In our scenario we assume that SUSY is broken at Planck scale or close to that. In this condition the masses of SUSY particles including gravitino are around Planck scale. Thus, the stringent constraint on the reheating temperature of the inflation from the gravitino problem can be avoided.

	L	e^c, μ^c, τ^c	N^c	H_u	H_d
$SU(2)_L$	2	1	1	2	2
A_4	3	1, 1'', 1'	3	1	1
$-k_I$	-1	-1	-1	0	0

Table 1. The charge assignment of $SU(2)_L$, A_4 , and the modular weight $-k_I$.

The modular invariant mass terms of leptons are given as the following superpotentials:

$$W_e = \alpha e^c H_d (LY^{A_4})_{\mathbf{1}} + \beta \mu^c H_d (LY^{A_4})_{\mathbf{1}'} + \gamma \tau^c H_d (LY^{A_4})_{\mathbf{1}''}, \quad (3.1)$$

$$W_D = g_1 (N^c H_u (LY^{A_4})_{\mathbf{3s}})_{\mathbf{1}} + g_2 (N^c H_u (LY^{A_4})_{\mathbf{3a}})_{\mathbf{1}}, \quad (3.2)$$

$$W_N = \Lambda (N^c N^c Y^{A_4})_{\mathbf{1}}, \quad (3.3)$$

where sums of the modular weights vanish. The parameters α, β, γ , and $g_{1,2}$ are coupling constants, and Λ is a mass parameter for the Majorana masses for right-handed neutrinos. Following ref. [10], we take g_1 and g_2 as real and complex parameters, respectively.

$$g_2 = |g_2| e^{i\phi_g}. \quad (3.4)$$

The functions $Y^{A_4}(\tau)$ are A_4 triplet modular forms of weight 2 which components are shown in eq. (2.7). As for the field contents and the basis for A_4 group, see discussions in ref. [10].

The superpotential (3.1) leads to the following charged leptons mass matrix:

$$M_E = v_d \text{diag}[\alpha, \beta, \gamma] \begin{pmatrix} Y_1 & Y_3 & Y_2 \\ Y_2 & Y_1 & Y_3 \\ Y_3 & Y_2 & Y_1 \end{pmatrix}_{RL}, \quad (3.5)$$

where $v_d = \langle H_d \rangle$. Note that we should evaluate the mass matrix at the SUSY-breaking scale and also include the corrections due to the RGE evolution and SUSY-breaking. In this analysis we neglect these effects without specifying the scale and the mediation mechanism of SUSY-breaking. Such corrections have been discussed, for example in ref. [9]. The coefficients α, β , and γ are taken to be real positive by rephasing right-handed charged lepton fields without loss of generality. Those parameters can be written in terms of the modulus τ and the charged lepton masses together with v_d . The superpotential (3.2) gives the Dirac neutrino mass matrix:

$$M_D = v_u \begin{pmatrix} 2g_1 Y_1 & (-g_1 + g_2) Y_3 & (-g_1 - g_2) Y_2 \\ (-g_1 - g_2) Y_3 & 2g_1 Y_2 & (-g_1 + g_2) Y_1 \\ (-g_1 + g_2) Y_2 & (-g_1 - g_2) Y_1 & 2g_1 Y_3 \end{pmatrix}_{RL}, \quad (3.6)$$

where $v_u = \langle H_u \rangle$. On the other hand, the right-handed Majorana neutrino mass matrix is obtained from the superpotential (3.3):

$$M_N = \Lambda \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix}_{RR}. \quad (3.7)$$

observable	3σ range
$\sin^2 \theta_{12}$	0.275–0.350
$\sin^2 \theta_{23}$	0.427–0.609
$\sin^2 \theta_{13}$	0.02046–0.02440
Δm_{21}^2	$(6.79\text{--}8.01) \times 10^{-5} \text{ eV}^2$
Δm_{31}^2	$(2.432\text{--}2.618) \times 10^{-3} \text{ eV}^2$

Table 2. The 3σ ranges of neutrino oscillation parameters for the normal hierarchy case from NuFIT 4.1 (2019) [40, 41].

Finally, the effective neutrino mass matrix is obtained through the type I seesaw as follows:

$$M_\nu = -M_D^T M_N^{-1} M_D. \quad (3.8)$$

The masses of active neutrinos, m_i , are found by diagonalizing M_ν , and the lepton mixing matrix U in the charged current is also found by the diagonalization of M_ν and M_E . The matrix U is parameterized as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\text{CP}}} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta_{\text{CP}}} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta_{\text{CP}}} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta_{\text{CP}}} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta_{\text{CP}}} & c_{23}c_{13} \end{pmatrix} \quad (3.9)$$

$$\times \text{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}),$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. δ_{CP} is the Dirac CP violating phase, and α_{21} and α_{31} are the Majorana phases.

It is notable that the model can reproduce the observed values of the mixing angles ($\sin^2 \theta_{23}$ is predicted to be larger than 0.54.) and the mass squared differences $\Delta m_{ij}^2 = m_i^2 - m_j^2$ [10]. Furthermore, the model is very predictive, e.g., the normal hierarchy of neutrino masses is predicted, the Dirac phase is in the range $\delta_{\text{CP}} = \pm(50^\circ - 180^\circ)$, the effective neutrino mass in the neutrinoless double beta decay is around 22 meV, and the sum of neutrino masses is larger than 145 meV. See the details in ref. [10].

It is natural to verify whether the model can explain the BAU or not, since it contains all the essential ingredients for the leptogenesis, i.e., right-handed neutrinos, lepton number violation by the Majorana masses and CP violation in the modulus field and the coupling constants. The yield of the BAU depends on the masses M_i and Yukawa coupling constants of right-handed neutrinos. Since these parameters are highly restricted due to the symmetry in the model, we can expect non-trivial relations between the BAU and the observables in the active neutrino physics, which are the main outcomes of the present article.

Before discussing the leptogenesis, we shall summarize the properties of right-handed neutrinos inferred from the neutrino oscillation data. For this purpose, we reanalyze the numerical study of the model following ref. [10]. We use this time the charged lepton masses in ref. [39] and update the neutrino oscillation parameters in refs. [40, 41] (See table 2.). In addition, we require $\sum m_i < 160 \text{ meV}$ [31]. We find no qualitative difference from the previous analysis. Here we show only the results which are essential in the leptogenesis.

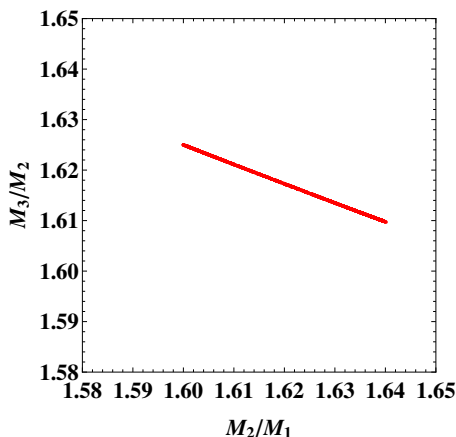


Figure 1. The allowed region of mass ratios between right-handed neutrinos.

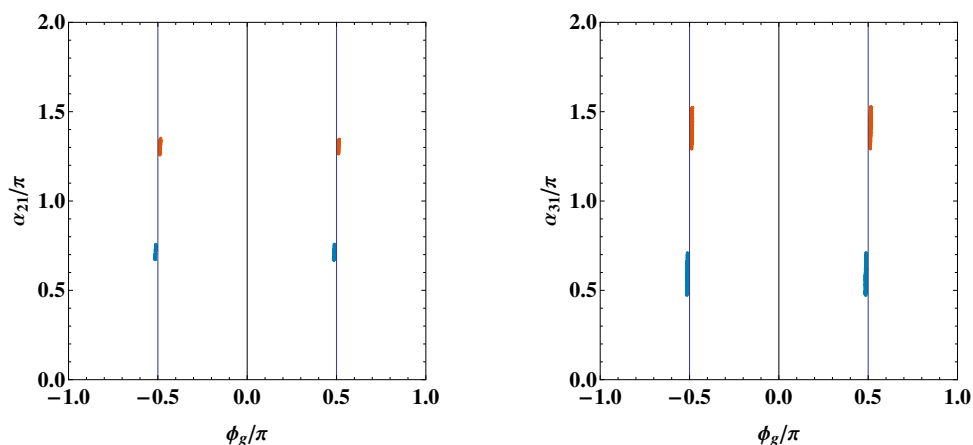


Figure 2. The allowed regions of α_{21} and ϕ_g (left) and that of α_{31} and ϕ_g (right). At the orange (blue) points the sign of Y_B is positive (negative).

First, the allowed range of the mass ratios of right-handed neutrinos is shown in figure 1. It is seen that M_2/M_1 and M_3/M_2 are both about 1.6. Notice that the absolute values of right-handed neutrino masses cannot be determined from the oscillation data, however, as we will show in section 5, the order of magnitude of them can be found from the BAU. There are two consequences to the leptogenesis; (1) All three right-handed neutrinos should be taken into account in the leptogenesis dynamics. (2) The resonant production of the lepton asymmetry by the decays [37] is less effective.

Next, we consider the CP violating parameters in the present model. There are two such parameters τ and g_2 , which are relevant for baryogenesis. Note that they induce the CP violations for active neutrinos (that are observables at low energies) and also for right-handed neutrinos. Interestingly, we observe the strong correlations between the Majorana phases $\alpha_{21,31}$ and the phase of g_2 , ϕ_g , which is represented in figure 2. The allowed value of the Majorana phase changes whether ϕ_g is slightly larger or smaller than $\pm\pi/2$. No strong correlation is found between the Majorana phases and τ . On the other hand, the Dirac phase δ_{CP} depends non-trivially on τ and ϕ_g , and there is no correlation between these parameters.

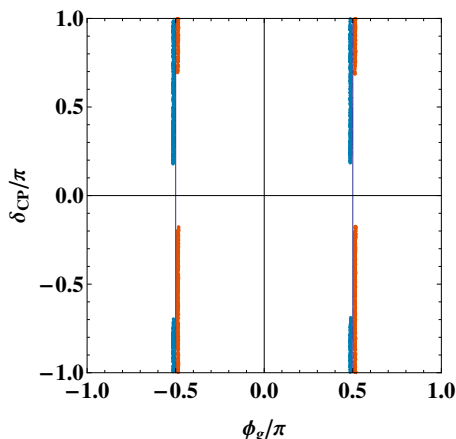


Figure 3. The allowed regions of δ_{CP} and ϕ_g . At the orange (blue) points the sign of Y_B is positive (negative).

4 Leptogenesis

Now we are at the point to discuss the leptogenesis by decays of right-handed neutrinos in the model. As explained before, the mass ratios of right-handed neutrinos are not so large as ~ 1.6 , and then we have to include the effects of all three right-handed neutrinos to the leptogenesis. Here we assume for simplicity that the reheating temperature of inflation is sufficiently higher than the mass of the heaviest right-handed neutrino and that the initial abundances of all right-handed neutrinos are zero. On the other hand, the mass differences of right-handed neutrinos are not so small, and then the resonant enhancement of the leptogenesis [37] does not occur. Thus, we shall use the formalism based on the Boltzmann equations for the estimation of the asymmetries. Furthermore, as shown below, the required masses of right-handed neutrinos are $\mathcal{O}(10^{13})$ GeV, and hence the simple one-flavor approximation of the leptogenesis can be applied and we only consider the (total) lepton asymmetry neglecting the so-called flavor effect [42–49].

We solve the Boltzmann equations for the number densities n_{N_I} for right-handed neutrinos and the lepton asymmetry density n_L .

$$\frac{dY_{N_I}}{dz} = \frac{-z}{sH(M_1)} \left\{ \left(\frac{Y_{N_I}}{Y_{N_I}^{eq}} - 1 \right) \left(\gamma_{N_I} + 2\gamma_{tI}^{(3)} + 4\gamma_{tI}^{(4)} \right) + \sum_{J=1}^3 \left(\frac{Y_{N_I} Y_{N_J}}{Y_{N_I}^{eq} Y_{N_J}^{eq}} - 1 \right) \left(\gamma_{N_I N_J}^{(2)} + \gamma_{N_I N_J}^{(3)} \right) \right\}, \quad (4.1)$$

$$\frac{dY_L}{dz} = \frac{-z}{sH(M_1)} \left\{ \sum_{I=1}^3 \left[\left(1 - \frac{Y_{N_I}}{Y_{N_I}^{eq}} \right) \varepsilon_I \gamma_{N_I} + \frac{Y_L}{Y_\ell^{eq}} \frac{\gamma_{N_I}}{2} \right] + \frac{Y_L}{Y_\ell^{eq}} \left(2\gamma_N^{(2)} + 2\gamma_N^{(13)} \right) + \frac{Y_L}{Y_\ell^{eq}} \sum_{I=1}^3 \left[\frac{Y_{N_I}}{Y_{N_I}^{eq}} \gamma_{tI}^{(3)} + 2\gamma_{tI}^{(4)} + \frac{Y_{N_I}}{Y_{N_I}^{eq}} \left(\gamma_{WI}^{(1)} + \gamma_{BI}^{(1)} \right) + \gamma_{WI}^{(2)} + \gamma_{WI}^{(3)} + \gamma_{BI}^{(2)} + \gamma_{BI}^{(3)} \right] \right\}, \quad (4.2)$$

where $z = M_1/T$. The yields are defined by $Y_{N_I} = n_{N_I}/s$ and $Y_L = n_L/s$ with the entropy density of the universe s . The superscript “*eq*” represents its equilibrium value. We take the yield for a massless particle with one degree of freedom in equilibrium as Y_ℓ^{eq} . Here we apply the Boltzmann approximation and $Y_\ell^{eq} = 45/(2\pi^4 g_{*s})$, with $g_{*s} = 110.75$. Our notations of the reaction densities correspond to those in ref. [50]. The CP asymmetry parameter for the leptogenesis ε_I is defined by

$$\varepsilon_I = \frac{\Gamma(N_I \rightarrow L + \bar{H}_u) - \Gamma(N_I \rightarrow \bar{L} + H_u)}{\Gamma(N_I \rightarrow L + \bar{H}_u) + \Gamma(N_I \rightarrow \bar{L} + H_u)}. \quad (4.3)$$

The explicit form of the reaction density for the N_I decay is given by

$$\gamma_{N_I} = \frac{(Y_\nu Y_\nu^\dagger)_{II}}{8\pi^3} M_1^4 a_I^{3/2} \frac{K_1(\sqrt{a_I} z)}{z}, \quad (4.4)$$

where $z = M_1/T$, $a_I = (M_I/M_1)^2$, and $K_1(x)$ is the modified Bessel function of the second kind. Note that Y_ν is the Yukawa coupling matrix of neutrinos in the base where the mass matrices of charged leptons and right-handed neutrinos are diagonal. The reaction density for the process $A + B \rightarrow C + D$ is given by

$$\gamma(A + B \rightarrow C + D) = \frac{T}{64\pi^4} \int_{(m_A+m_B)^2}^{\infty} ds \hat{\sigma}(s) \sqrt{s} K_1\left(\frac{\sqrt{s}}{T}\right), \quad (4.5)$$

where m_A and m_B are masses of the initial particles and $\hat{\sigma}(s)$ denotes the reduced cross section for the process. As for the $\Delta L = 1$ processes induced through top Yukawa interaction, the $\Delta L = 2$ scattering processes and the annihilation processes of right-handed neutrinos, the expressions of the reduced cross sections are found in ref. [50]. Note that the correct subtraction of the N_I on-shell contribution for $L\bar{H}_u \rightarrow \bar{L}H_u$ process gives [36]

$$\begin{aligned} \hat{\sigma}_N^{(2)}(x) = & \frac{1}{2\pi} \left[\sum_I (Y_\nu Y_\nu^\dagger)_{II}^2 \frac{a_I}{x} \left\{ \frac{x}{a_I} + \frac{x}{D_I} - \left(1 + \frac{x+a_I}{D_I}\right) \log\left(\frac{x+a_I}{a_I}\right) \right\} \right. \\ & + \sum_{I>J} \text{Re}[(Y_\nu Y_\nu^\dagger)_{IJ}^2] \frac{\sqrt{a_I a_J}}{x} \left\{ \frac{x^2 + x(D_I + D_J)}{D_I D_J} \right. \\ & + (x+a_I) \left(\frac{2}{a_J - a_I} - \frac{1}{D_J} \right) \ln\left(\frac{x+a_I}{a_I}\right) \\ & \left. \left. + (x+a_J) \left(\frac{2}{a_I - a_J} - \frac{1}{D_I} \right) \ln\left(\frac{x+a_J}{a_J}\right) \right\} \right], \quad (4.6) \end{aligned}$$

where $c_I = (\Gamma_{N_I}/M_1)^2$ and $D_I = [(x - a_I)^2 + a_I c_I]/(x - a_I)$, in which Γ_{N_I} is the total decay rate of right-handed neutrino N_I . The reduced cross sections for $\Delta L = 1$ processes through the $SU(2)_L$ SM gauge interaction are [36, 37]

$$\begin{aligned} \hat{\sigma}_{WI}^{(1)}(x) = & \frac{3g_2^2 (Y_\nu Y_\nu^\dagger)_{II}}{16\pi x^2} \left[-2x^2 + 6a_I x - 4a_I^2 + (x^2 - 2a_I x + 2a_I^2) \ln\left|\frac{x - a_I + a_L}{a_L}\right| \right. \\ & \left. + \frac{x(a_L x + a_L a_I - a_W a_I)(a_I - x)}{a_L(x - a_I + a_L)} \right], \quad (4.7) \end{aligned}$$

$$\hat{\sigma}_{WI}^{(2)}(x) = \frac{3g_2^2(Y_\nu Y_\nu^\dagger)_{II}}{8\pi x(x-a_I)} \left[2a_I x \ln \left| \frac{x-a_I+a_H}{a_H} \right| + (x^2+a_I^2) \ln \left| \frac{x-a_I-a_W-a_H}{-a_W-a_H} \right| \right], \quad (4.8)$$

$$\begin{aligned} \hat{\sigma}_{WI}^{(3)}(x) = & \frac{3g_2^2(Y_\nu Y_\nu^\dagger)_{II} a_I}{16\pi x^2} \left[\frac{x^2-4a_I x+3a_I^2}{a_I} \right. \\ & \left. + 4(x-a_I) \ln \left| \frac{x-a_I+a_H}{a_H} \right| - \frac{x(4a_H-a_W)(x-a_I)}{a_H(x-a_I+m_H)} \right]. \end{aligned} \quad (4.9)$$

Here $\hat{\sigma}_{WI}^{(1)}$, $\hat{\sigma}_{WI}^{(2)}$ and $\hat{\sigma}_{WI}^{(3)}$ are the reaction densities for the processes $N_I L \rightarrow H_u W$, $N_I W \rightarrow \bar{L} H_u$ and $N_I \bar{H}_u \rightarrow \bar{L} W$, respectively. We have used $a_{L,H,W,B} = m_{L,H_u,W,B}^2/M_1^2$ where m_X with $X = L, H_u, W, B$ are thermal masses of lepton doublets, up-type Higgs, $SU(2)_L$ gauge bosons and $U(1)_Y$ gauge boson, respectively. The reaction densities for the $\Delta L = 1$ processes through $U(1)_Y$ gauge interaction are obtained by substituting $a_W \rightarrow a_B$ and $\frac{3}{2}g_2^2 \rightarrow \frac{1}{4}g_Y^2$ in $\hat{\sigma}_{WI}^{(i)}$.

For the estimation of the reaction densities, we have taken into account the one-loop RGE evolutions of couplings and the renormalization scale is taken as $\mu = 2\pi T$. The important effect is the suppression of top Yukawa coupling at high temperatures due to the RGE effect, which reduces the washout of the produced lepton asymmetry and enlarges a viable parameter space.

The Boltzmann equations are then solved numerically and the total lepton asymmetry Y_L from the decays of right-handed neutrinos is estimated. The present baryon asymmetry can be estimated as $Y_B = -8/23Y_L$,² where we have taken into account for the effect of the two Higgs doublets.

Now, since the lightest right-handed neutrino is sufficiently heavy, we can neglect the flavor effect of the leptogenesis [42–49]. In this case, the final baryon asymmetry becomes insensitive to the PMNS mixing matrix of active neutrinos. However, in the considering model, the phases in the PMNS matrix and the high energy phases associated with right-handed neutrinos are originated in the limited complex parameters τ and g_2 . In this situation, there may exist the correlations between the phases in the PMNS matrix and the yield of the BAU.

5 Sign and magnitude of baryon asymmetry

Let us then show the results of the BAU by right-handed neutrinos in the model. We begin with the sign of the BAU produced by right-handed neutrinos in the model.

The first important result is that the sign of the BAU is determined by the phase ϕ_g of the complex coupling g_2 . This point is represented in figure 4. The positive BAU is obtained when ϕ_g is slightly larger than $\pi/2$ or $-\pi/2$. On the other hand, as shown in figure 5, there is no strong correlation between the BAU sign and the complex parameter τ .³

²Here, we assume that two-Higgs doublet survive at sphaleron freeze-out temperature. On the other hand, $Y_B = -28/79Y_L$ for the one-Higgs doublet case.

³The mass ratios between right-handed neutrinos are determined by τ as shown in eq. (3.7). It is then found from figure 5 there is no correlation between mass hierarchy of right-handed neutrinos and the sign of the BAU.

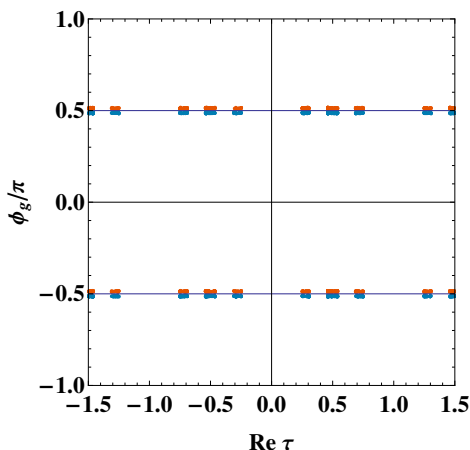


Figure 4. The allowed regions of $\text{Re } \tau$ and phase ϕ_g . Red and blue points correspond to the positive and negative signs of the BAU, respectively.

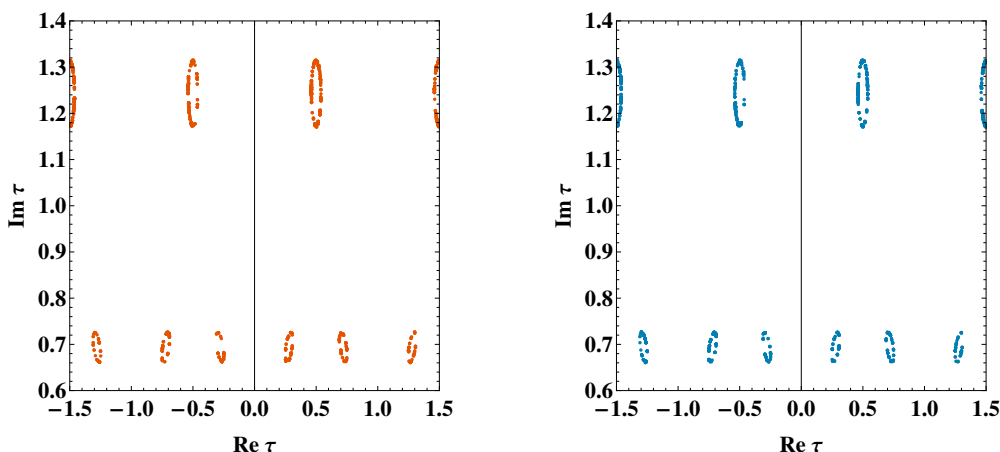


Figure 5. The allowed regions of $\text{Re } \tau$ and $\text{Im } \tau$ when the sign of the BAU is positive (left) or negative (right).

The phase ϕ_g is strongly correlated with Majorana phases α_{21} and α_{31} , and then the positive BAU is possible only for the restricted range of Majorana phases, which is shown in figure 6. We can see that two regions in the Majorana phases are allowed by the observational data about active neutrinos, which are related as $\alpha_{21,31} \simeq 2\pi - \alpha_{21,31}$. The positive BAU is, however, realized only when $\alpha_{21} \sim 1.3\pi$ and $\alpha_{31} \sim 1.5\pi$. This is an important prediction of the leptogenesis in the model although the experimental measurements of Majorana phases are very difficult. Notice that the range of the effective neutrino mass in the $0\nu\beta\beta$ decay is the same for the both cases $Y_B > 0$ and $Y_B < 0$, as shown in figure 7.

Second, we find from figure 6 that the dependence of the BAU sign on Dirac phase is different depending on $\sin^2 \theta_{23}$. For $\sin^2 \theta_{23} \lesssim 0.58$ the positive BAU can be obtained for $\delta_{\text{CP}} < 0$. On the other hand, for $\sin^2 \theta_{23} \gtrsim 0.58$ the positive BAU is possible for both $\delta_{\text{CP}} < 0$ and $\delta_{\text{CP}} > 0$. This shows that the precise measurements of $\sin^2 \theta_{23}$ and δ_{CP} provide a crucial test for the correct sign of the BAU in the considering baryogenesis scenario.

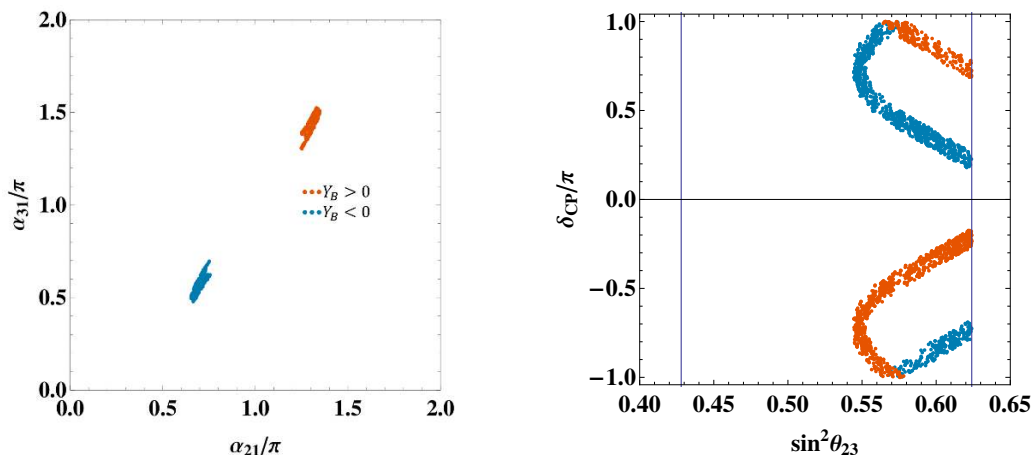


Figure 6. α_{21} and α_{31} (left), and $\sin^2 \theta_{23}$ and δ_{CP} (right). Y_B is positive (negative) at the orange (blue) points.

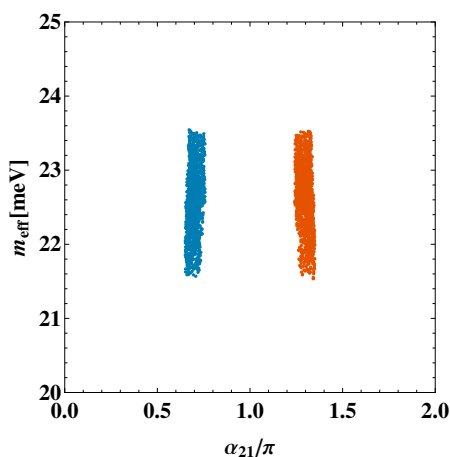


Figure 7. The allowed ranges of α_{21} and m_{eff} .

Next, we discuss the magnitude of the BAU yield. We find that the yield can be at most the same order of the observed value of the BAU (1). This is because the model predicts a relatively large value of the effective neutrino mass of the leptogenesis \tilde{m}_1 which is defined as $\tilde{m}_1 = (Y_\nu Y_\nu^\dagger)_{11} v_u^2 / M_1$. We find numerically $\tilde{m}_1 \simeq 54 - 57$ meV, and then the strong wash-out effect is inevitable. This leads to an important consequence that the lightest right-handed neutrino should be in the mass range $M_1 \simeq (1.5 - 10) \times 10^{13}$ GeV. As can be seen from figure 8, we find that the dependence of the magnitude of the BAU on the lightest right-handed neutrino mass changes at $M_1 \simeq 4.0 \times 10^{13}$ GeV. At $M_1 \lesssim 4.0 \times 10^{13}$ GeV, the larger M_1 is, the larger the magnitude of the generated BAU is. On the other hand at $M_1 \gtrsim 4.0 \times 10^{13}$ GeV, the larger M_1 is, the smaller the magnitude of the generated BAU is. This is because the larger M_1 is, the more the wash-out effect of the $\Delta L = 2$ processes is important. Thereby, the lightest right-handed neutrino mass is restricted to the specific range ($M_1 \simeq (1.5 - 10) \times 10^{13}$ GeV) in order to explain the observed BAU.

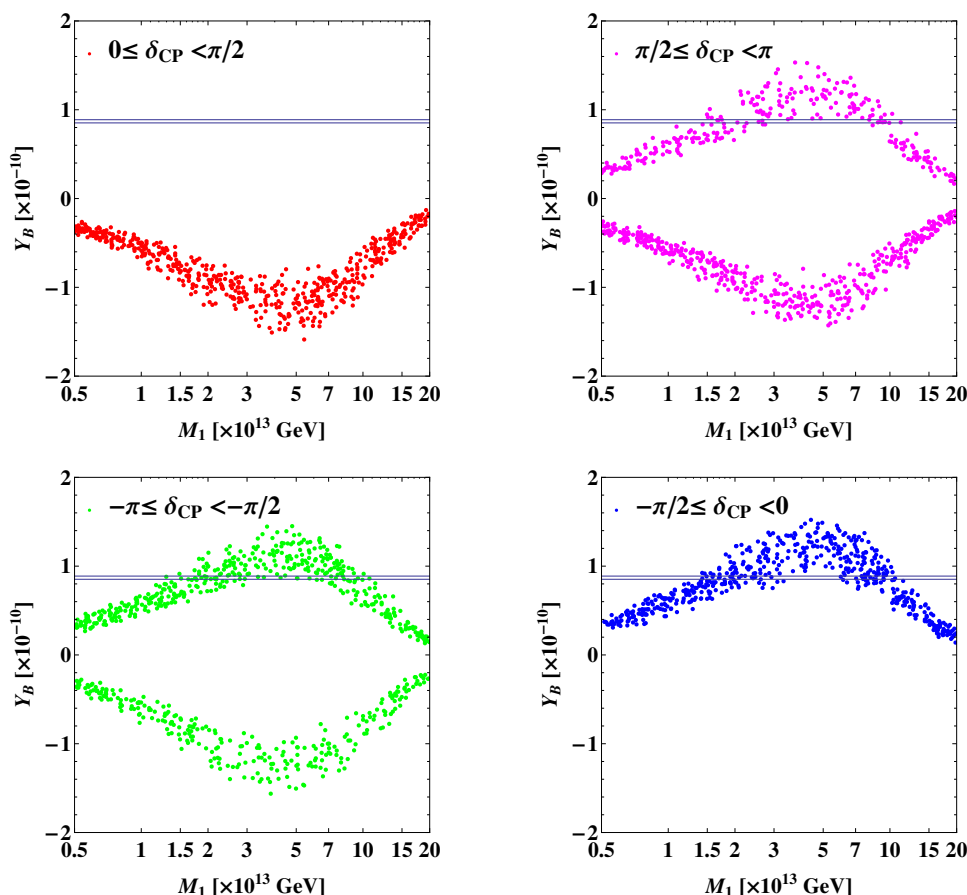


Figure 8. The yields of the BAU in terms of the mass of the lightest right-handed neutrino for different values of δ_{CP} . δ_{CP} is taken in the range $[0, \pi/2]$ (top-left), $[\pi/2, \pi]$ (top-right), $[-\pi, -\pi/2]$ (bottom-left) and $[-\pi/2, 0]$ (bottom-right). The solid lines are upper or lower bound for the observational value of Y_B .

6 Conclusions

We have considered the leptogenesis in the model with three right-handed neutrinos introducing the modular A_4 invariance. The model is very predictive in the sense that all the parameters apart from the overall scale of right-handed neutrino masses are determined within the limited ranges in order to be consistent with the observed values of charged lepton masses as well as mixing angles and masses of active neutrinos. We have shown that the observed value of the BAU can be explained when the mass of the lightest right-handed neutrino is $(1.5 - 10) \times 10^{13}$ GeV. This means that the successful baryogenesis determines the absolute masses of all right-handed neutrinos.

We have also shown that the sign of the BAU is strongly related with the CP violating parameters, Majorana and Dirac phases, since the possible breaking pattern of the CP symmetry is very limited in the considering model. In fact, the positive sign of the BAU is realized only for the unique range of Majorana phases, namely $\alpha_{21} \sim 1.3\pi$ and $\alpha_{31} \sim 1.5\pi$. Moreover, we have shown that the precise measurements of $\sin^2 \theta_{23}$ and δ_{CP} provide a crucial test for the correct sign of the BAU in the considering baryogenesis scenario.

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References

- [1] G. Altarelli and F. Feruglio, *Discrete flavor symmetries and models of neutrino mixing*, *Rev. Mod. Phys.* **82** (2010) 2701 [[arXiv:1002.0211](https://arxiv.org/abs/1002.0211)] [[INSPIRE](#)].
- [2] H. Ishimori et al., *Non-Abelian discrete symmetries in particle physics*, *Prog. Theor. Phys. Suppl.* **183** (2010) 1 [[arXiv:1003.3552](https://arxiv.org/abs/1003.3552)] [[INSPIRE](#)].
- [3] H. Ishimori et al., *An introduction to non-Abelian discrete symmetries for particle physicists*, *Lect. Notes Phys.* **858** (2012) 1 [[INSPIRE](#)].
- [4] S.F. King and C. Luhn, *Neutrino mass and mixing with discrete symmetry*, *Rept. Prog. Phys.* **76** (2013) 056201 [[arXiv:1301.1340](https://arxiv.org/abs/1301.1340)] [[INSPIRE](#)].
- [5] S.F. King et al., *Neutrino mass and mixing: from theory to experiment*, *New J. Phys.* **16** (2014) 045018 [[arXiv:1402.4271](https://arxiv.org/abs/1402.4271)] [[INSPIRE](#)].
- [6] F. Feruglio, *Are neutrino masses modular forms?*, [arXiv:1706.08749](https://arxiv.org/abs/1706.08749) [[INSPIRE](#)].
- [7] R. de Adelhart Toorop, F. Feruglio and C. Hagedorn, *Finite modular groups and lepton mixing*, *Nucl. Phys. B* **858** (2012) 437 [[arXiv:1112.1340](https://arxiv.org/abs/1112.1340)] [[INSPIRE](#)].
- [8] T. Kobayashi, K. Tanaka and T.H. Tatsuishi, *Neutrino mixing from finite modular groups*, *Phys. Rev. D* **98** (2018) 016004 [[arXiv:1803.10391](https://arxiv.org/abs/1803.10391)] [[INSPIRE](#)].
- [9] J.C. Criado and F. Feruglio, *Modular invariance faces precision neutrino data*, *SciPost Phys.* **5** (2018) 042 [[arXiv:1807.01125](https://arxiv.org/abs/1807.01125)] [[INSPIRE](#)].
- [10] T. Kobayashi et al., *Modular A_4 invariance and neutrino mixing*, *JHEP* **11** (2018) 196 [[arXiv:1808.03012](https://arxiv.org/abs/1808.03012)] [[INSPIRE](#)].
- [11] P.P. Novichkov, S.T. Petcov and M. Tanimoto, *Trimaximal neutrino mixing from modular A_4 invariance with residual symmetries*, *Phys. Lett. B* **793** (2019) 247 [[arXiv:1812.11289](https://arxiv.org/abs/1812.11289)] [[INSPIRE](#)].
- [12] T. Nomura and H. Okada, *A modular A_4 symmetric model of dark matter and neutrino*, *Phys. Lett. B* **797** (2019) 134799 [[arXiv:1904.03937](https://arxiv.org/abs/1904.03937)] [[INSPIRE](#)].
- [13] T. Kobayashi et al., *New A_4 lepton flavor model from S_4 modular symmetry*, [arXiv:1907.09141](https://arxiv.org/abs/1907.09141) [[INSPIRE](#)].
- [14] G.-J. Ding, S.F. King and X.-G. Liu, *Modular A_4 symmetry models of neutrinos and charged leptons*, *JHEP* **09** (2019) 074 [[arXiv:1907.11714](https://arxiv.org/abs/1907.11714)] [[INSPIRE](#)].

- [15] T. Kobayashi et al., A_4 lepton flavor model and modulus stabilization from S_4 modular symmetry, *Phys. Rev. D* **100** (2019) 115045 [[arXiv:1909.05139](#)] [[INSPIRE](#)].
- [16] J.T. Penedo and S.T. Petcov, Lepton masses and mixing from modular S_4 symmetry, *Nucl. Phys. B* **939** (2019) 292 [[arXiv:1806.11040](#)] [[INSPIRE](#)].
- [17] P.P. Novichkov, J.T. Penedo, S.T. Petcov and A.V. Titov, Modular S_4 models of lepton masses and mixing, *JHEP* **04** (2019) 005 [[arXiv:1811.04933](#)] [[INSPIRE](#)].
- [18] S.F. King and Y.-L. Zhou, Trimaximal TM_1 mixing with two modular S_4 groups, *Phys. Rev. D* **101** (2020) 015001 [[arXiv:1908.02770](#)] [[INSPIRE](#)].
- [19] P.P. Novichkov, J.T. Penedo, S.T. Petcov and A.V. Titov, Modular A_5 symmetry for flavour model building, *JHEP* **04** (2019) 174 [[arXiv:1812.02158](#)] [[INSPIRE](#)].
- [20] G.-J. Ding, S.F. King and X.-G. Liu, Neutrino mass and mixing with A_5 modular symmetry, *Phys. Rev. D* **100** (2019) 115005 [[arXiv:1903.12588](#)] [[INSPIRE](#)].
- [21] H. Okada and M. Tanimoto, CP violation of quarks in A_4 modular invariance, *Phys. Lett. B* **791** (2019) 54 [[arXiv:1812.09677](#)] [[INSPIRE](#)].
- [22] T. Kobayashi et al., Finite modular subgroups for fermion mass matrices and baryon/lepton number violation, *Phys. Lett. B* **794** (2019) 114 [[arXiv:1812.11072](#)] [[INSPIRE](#)].
- [23] F.J. de Anda, S.F. King and E. Perdomo, SU(5) Grand Unified Theory with A_4 modular symmetry, [arXiv:1812.05620](#) [[INSPIRE](#)].
- [24] T. Kobayashi et al., Modular S_3 invariant flavor model in SU(5) GUT, [arXiv:1906.10341](#) [[INSPIRE](#)].
- [25] D. Cremades, L.E. Ibáñez and F. Marchesano, Computing Yukawa couplings from magnetized extra dimensions, *JHEP* **05** (2004) 079 [[hep-th/0404229](#)] [[INSPIRE](#)].
- [26] T. Kobayashi and S. Nagamoto, Zero-modes on orbifolds: magnetized orbifold models by modular transformation, *Phys. Rev. D* **96** (2017) 096011 [[arXiv:1709.09784](#)] [[INSPIRE](#)].
- [27] T. Kobayashi et al., Modular symmetry and non-Abelian discrete flavor symmetries in string compactification, *Phys. Rev. D* **97** (2018) 116002 [[arXiv:1804.06644](#)] [[INSPIRE](#)].
- [28] T. Kobayashi and S. Tamba, Modular forms of finite modular subgroups from magnetized D-brane models, *Phys. Rev. D* **99** (2019) 046001 [[arXiv:1811.11384](#)] [[INSPIRE](#)].
- [29] A. Baur, H.P. Nilles, A. Trautner and P.K.S. Vaudrevange, Unification of flavor, CP and modular symmetries, *Phys. Lett. B* **795** (2019) 7 [[arXiv:1901.03251](#)] [[INSPIRE](#)].
- [30] Y. Kariyazono, T. Kobayashi, S. Takada, S. Tamba and H. Uchida, Modular symmetry anomaly in magnetic flux compactification, *Phys. Rev. D* **100** (2019) 045014 [[arXiv:1904.07546](#)] [[INSPIRE](#)].
- [31] PLANCK collaboration, Planck 2018 results. VI. Cosmological parameters, [arXiv:1807.06209](#) [[INSPIRE](#)].
- [32] M. Fukugita and T. Yanagida, Baryogenesis without grand unification, *Phys. Lett. B* **174** (1986) 45 [[INSPIRE](#)].
- [33] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, On the anomalous electroweak baryon number nonconservation in the early universe, *Phys. Lett. B* **155** (1985) 36.
- [34] F.R. Klinkhamer and N.S. Manton, A saddle point solution in the Weinberg-Salam theory, *Phys. Rev. D* **30** (1984) 2212 [[INSPIRE](#)].

- [35] S. Davidson and A. Ibarra, *A lower bound on the right-handed neutrino mass from leptogenesis*, *Phys. Lett. B* **535** (2002) 25 [[hep-ph/0202239](#)] [[INSPIRE](#)].
- [36] G.F. Giudice et al., *Towards a complete theory of thermal leptogenesis in the SM and MSSM*, *Nucl. Phys. B* **685** (2004) 89 [[hep-ph/0310123](#)] [[INSPIRE](#)].
- [37] A. Pilaftsis and T.E.J. Underwood, *Resonant leptogenesis*, *Nucl. Phys. B* **692** (2004) 303 [[hep-ph/0309342](#)] [[INSPIRE](#)].
- [38] S. Ferrara, D. Lüst, A.D. Shapere and S. Theisen, *Modular invariance in supersymmetric field theories*, *Phys. Lett. B* **225** (1989) 363 [[INSPIRE](#)].
- [39] PARTICLE DATA GROUP collaboration, *Review of particle physics*, *Phys. Rev. D* **98** (2018) 030001 [[INSPIRE](#)].
- [40] NuFIT 4.1, <http://www.nu-fit.org> (2019).
- [41] I. Esteban et al., *Global analysis of three-flavour neutrino oscillations: synergies and tensions in the determination of θ_{23} , δ_{CP} and the mass ordering*, *JHEP* **01** (2019) 106 [[arXiv:1811.05487](#)] [[INSPIRE](#)].
- [42] A. Abada et al., *Flavor issues in leptogenesis*, *JCAP* **04** (2006) 004 [[hep-ph/0601083](#)] [[INSPIRE](#)].
- [43] E. Nardi, Y. Nir, E. Roulet and J. Racker, *The importance of flavor in leptogenesis*, *JHEP* **01** (2006) 164 [[hep-ph/0601084](#)] [[INSPIRE](#)].
- [44] A. Abada et al., *Flavour matters in leptogenesis*, *JHEP* **09** (2006) 010 [[hep-ph/0605281](#)] [[INSPIRE](#)].
- [45] S. Blanchet and P. Di Bari, *Flavor effects on leptogenesis predictions*, *JCAP* **03** (2007) 018 [[hep-ph/0607330](#)] [[INSPIRE](#)].
- [46] S. Pascoli, S.T. Petcov and A. Riotto, *Connecting low energy leptonic CP-violation to leptogenesis*, *Phys. Rev. D* **75** (2007) 083511 [[hep-ph/0609125](#)] [[INSPIRE](#)].
- [47] S. Pascoli, S.T. Petcov and A. Riotto, *Leptogenesis and low energy CP-violation in neutrino physics*, *Nucl. Phys. B* **774** (2007) 1 [[hep-ph/0611338](#)] [[INSPIRE](#)].
- [48] K. Moffat, S. Pascoli, S.T. Petcov and J. Turner, *Leptogenesis from Low energy CP violation*, *JHEP* **03** (2019) 034 [[arXiv:1809.08251](#)] [[INSPIRE](#)].
- [49] A. De Simone and A. Riotto, *On the impact of flavour oscillations in leptogenesis*, *JCAP* **02** (2007) 005 [[hep-ph/0611357](#)] [[INSPIRE](#)].
- [50] M. Plümacher, *Baryon asymmetry, neutrino mixing and supersymmetric SO(10) unification*, Ph.D. thesis, Hamburg University, Hamburg, Germany (1998), [hep-ph/9807557](#) [[INSPIRE](#)].