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Modulated Soliton Solution of the Modified Kuramoto-Sivashinsky's Equation

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Abstract We constructed in this work a modulated soliton solution. This solution is a multiform soliton prototype modulated by the very small parameter ε. In practice it can represent, a model of soliton capable of changing its form with respect to the obstacle in its medium of propagation without any loss in its initial energy. The nonlinear partial differential equation used in the construction of this solution is that of modified Kuramoto-Sivashinsky.

Keywords Kuramoto-Sivashinsky, Soliton Solution, BDK Method, Kink, Pulse, Multiform Soliton

1. Introduction

All physical phenomena are in most cases governed by differential equations and especially by the nonlinear partial differential equations (NPDE) for most complicated cases. Among these NPDEs, the most regular are that of Schrödinger, Ginzburg-Landau, KdV and Kuramoto-Sivashinsky $[1-4]$. The essential thing is not to obtain of NPDEs, but to propose possible solutions. It is in this light that many resolution techniques have been proposed $[5-32]$. In our recent works we proposed a new method of construction of solutions of NPDEs named the method of identification of coefficients of hyperbolic functions or Bogning-Djeumen Tchaho-Kofané method (BDKm) $[33-36]$. In this work, we use the BDKm to construct the soliton solutions formed by combining solutions of type kink and pulse according to the degree of dominion of the parameter ε. It is necessary to say that here we are enlivened by the desire to construct a type of solitary wave solution that is the combination of several shapes of solitary waves. If we can already confess that on the mathematical plan it is possible, on the other hand we cannot already say explicitly what such a solution can represent in the practice or in the physics in general. While we think that in the case where the broadcast of such a signal would be possible in practice, it would be a solitary wave that will be able to change shape according to the characteristic properties of the propagation medium or merely of the met obstacle. To reach our goal, we needed a differential equation where cohabits the scattering and the non linearity. For this reason we chose a differential equation presenting a very strong non linearity. Thus, the construction of this multiform solution is supported by Kuramoto-Sivashinsky's equation. This method has been chosen due to the fact that it is more adapted for the construction or amelioration of soliton solution in NPDEs.

This work is organized as follows:

In section 2, we are going to present the BDK method and in section 3 construct or propose a solution which is closer to the form envisaged. In section 4 we will polish up our work.

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2. The BDK Method

This method is based on the construction of soliton solutions of certain types of nonlinear equations of the form $[33-36]$

$$\gamma_i \sum_i \left(\frac{\partial u}{\partial x_i}\right) + b_i \sum_i \frac{\partial^2 u}{\partial x_i^2} + ... + c_i \sum_i \frac{\partial^l u}{\partial x_i^l} + d_i \sum_{n,m} \frac{\partial^n u \partial^m u}{\partial x_i^l \partial x_i^m} + f\left(u, |u|^2\right) = 0 \quad (1)$$

where γ_i, b_i, c_i, d_i are constants, f a linear function of u and $|u|^2$ and u the variable to determine. Knowing that majority of soliton solutions have their analytic forms constituted by functions $\exp$, $\sinh$, $\cosh$, $\arctan$, $\tanh$, $\sec h$, $\cos ech$ ..., we have imagined the form of solutions capable of bringing together the different functions seen above. Among all general forms of solutions which have come across our minds, the most adapted is

$$u = a_{ij} \sinh^j \alpha x \sec h^i \alpha x, \quad (2)$$

where a_{ij} are constants to determine, α considered as known constant, i and j are natural integers. Why have we chosen solution (Eq.(2))? Simply because it covers effectively the majority of soliton solutions that we come across depending on the variation of integers i and j.

The methodic principle is based on determining the constants a_{ij}. If we propose the construction of solutions of Eq.(1) in the form

$$u=\sum_{ij} a_{ij}\sinh^{j}\alpha x\operatorname{sec}h^{i}\alpha x, \tag{3}$$

taking into account Eq.(3) in Eq.(1) we get an equation of the form indicated below

$$\begin{aligned}&\sum_{i,j,n}F\left(a_{ij}\right)\operatorname{sec}h^{n}\alpha x+\sum_{i,j,m}G\left(a_{ij}\right)\sinh\alpha x\operatorname{sec}h^{m}\alpha x\\&+\sum_{i,j,k}H\left(a_{ij}\right)\cosh^{k}\alpha x+\sum_{i,j,l}T\left(a_{ij}\right)\cosh^{l}\alpha x\sinh\alpha x\\&+\sum_{i,j}W\left(a_{ij}\right)=0.\end{aligned}\tag{4}$$

In the relation given by Eq.(4), the factors $\operatorname{sec}h^{n}\alpha x$, $\sinh\alpha x\operatorname{sec}h^{m}\alpha x$, $\cosh^{k}\alpha x$ and $\cosh^{l}\alpha x\sinh\alpha x$ are considered as a sort of simple elements of Eq.(4). We wish simply to mention the fact that on introducing Eq.(3) in Eq.(1) we obtain hyperbolic functions which are not directly simple elements obtained in Eq.(4). To come back to the form Eq.(4), it is necessary to use adequate transformations $[33-36]$.On identifying the different coefficients of simple elements of Eq.(4) to zero, we obtain a range of equations in a_{ij} to determine. In all these equations those which have exact values and those which have values more or less closes to exact values respect a certain order of priority (high value of n and m). It is convenient to mention here that the choice of the solution to be made is fundamental. When the choice is not appropriate, the results will be contradictory.

In the lines that follow we will go beyond the classical ways of calculation as seen in our previous works $[33-36]$, construction of modulated soliton solution by the parameter ε.

3. Solutions of the Shape

$$\psi=a\operatorname{sec}h^{2}\alpha\xi+b\tanh\alpha\xi+\varepsilon\left(\beta\operatorname{sec}h^{4}\alpha\xi+\gamma\tanh^{2}\alpha\xi\right)$$

The modified Kuramoto-Sivashinsky's equation is given by $[33]$

$$\begin{aligned}&c\psi'+L_1\psi''+L_2\psi'^2+L_3\psi'\psi'''+L_4\psi''''\\&+L_5\psi'^2\psi''+L_6\psi''^2=0,\end{aligned}\tag{5}$$

where $\psi(x,t)=\psi(\xi)$, $\xi=x-ct$, ψ', ψ'', ψ''' and ψ'''' represent respectively the first derivative, the second derivative, the third derivative and the fourth derivative of ψ with respect to ξ, L_ν with $\nu=1,2,3,4,5,6$ are the constants, c the group velocity.

The solutions of the shape

$$\begin{aligned}&\psi(\xi)=a\operatorname{sec}h^{2}\alpha\xi+b\tanh\alpha\xi\\&+\varepsilon\left(\beta\operatorname{sec}h^{4}\alpha\xi+\gamma\tanh^{2}\alpha\xi\right)\end{aligned},\tag{6}$$

Introduced in Eq.(5) leads to the relation

$$\begin{aligned}&\alpha\left(cb+4\alpha L_2a^2\right)\operatorname{sec}h^{2}\alpha\xi\\&+\alpha^2\left(L_2b^2+\alpha^2L_6b^2+4\alpha^2L_3b^2-2L_1a\right)\operatorname{sec}h^{4}\alpha\xi\\&+2\alpha^2\begin{pmatrix}4\alpha^2L_5ab^2-4\alpha^2L_5a^3\\-2\alpha^2L_6b^2-2a^2L_2-3\alpha^2L_3b^2\end{pmatrix}\operatorname{sec}h^{6}\alpha\xi\\&+2\alpha^4L_3ab\operatorname{sec}h^{7}\alpha\xi\\&+2\alpha^4a\left(2L_6a-5L_5b^2-40\beta L_5a\right)\operatorname{sec}h^{8}\alpha\xi\\&+8\alpha^4a^3L_5\operatorname{sec}h^{10}\alpha\xi+80\alpha^4a^2\beta L_5\operatorname{sec}h^{12}\alpha\xi\\&-2\alpha\left(ca+\left(\alpha L_1+4\alpha^3L_4\right)b\right)\sinh\alpha\xi\operatorname{sec}h^{3}\alpha\xi\\&+4\alpha^2\begin{pmatrix}2\alpha^2L_4-L_2a-2\alpha^2L_3a\\-2\alpha^2L_5a^2\end{pmatrix}b\sinh\alpha\xi\operatorname{sec}h^{5}\alpha\xi\\&-40\alpha^4L_4a\sinh\alpha\xi\operatorname{sec}h^{6}\alpha\xi\\&+2\alpha^4\left[\left(6L_3+4L_6\right)ab-L_5b^3\right]\sinh\alpha\xi\operatorname{sec}h^{7}\alpha\xi\\&-16a^2\alpha^4L_3\sinh\alpha\xi\operatorname{sec}h^{8}\alpha\xi\\&+\varepsilon\left\{+2\alpha^2\begin{bmatrix}3\left(L_1+20\alpha^2L_4\right)\gamma\\+8\left(L_1+16\alpha^2L_4\right)\beta\\-4\left(L_2+2\alpha^2L_3\right)\gamma a\\-8\alpha^2L_5a^2\gamma\end{bmatrix}\operatorname{sec}h^{4}\alpha\xi\right.\\&-4\alpha^2\gamma\left(L_1+4\alpha^2L_4\right)\operatorname{sec}h^{2}\alpha\xi\\&+4\alpha^2\begin{bmatrix}\alpha^2L_5\left(6\gamma a^2+16\beta a^2-3\gamma b^2\right)\\+4\left(L_2+8\alpha^2L_3\right)\beta a\\+2\left(L_2+8\alpha^2L_3\right)\gamma a\\-5\left(L_1+52\alpha^2L_4\right)\beta\\-30\alpha^2L_4\gamma\end{bmatrix}\operatorname{sec}h^{6}\alpha\xi\end{aligned}$$

$$
\begin{aligned}
&+2\alpha^2\begin{bmatrix}420\alpha^2 L_4\beta a\\ -8\left(L_2+23\alpha^2L_3+4\alpha^2L_6\right)\beta a\\ -3\alpha^2\left(2L_3+L_6\right)\gamma a\\ +\alpha^2L_5\left(16\gamma a^2+16\beta b^2+3\gamma b^2\right)\end{bmatrix}\sec h^8\alpha\xi\\
&+16\alpha^4\left[9L_3-2L_5\left(\gamma+2\beta\right)ba+5L_6\beta b\right]\sec h^9\alpha\xi\\
&+8\alpha^4\begin{bmatrix}10\left(3L_3+L_6\right)\beta a\\ -L_5\left(5\gamma a^2+12\beta a^2+4\beta b^2\right)\end{bmatrix}\sec h^{10}\alpha\xi\\
&+32a^2\beta\alpha^4L_5\sec h^{12}\alpha\xi+2\alpha c\gamma\sinh\alpha\xi\sec h^3\alpha\xi\\
&+4\alpha\left[\alpha\left(L_2+4\alpha^2\left(L_3+L_6\right)\gamma\beta-\beta c\right)\right]\sinh\alpha\xi\sec h^5\alpha\xi\\
&+4\alpha^2\begin{bmatrix}8\alpha^2L_5\gamma ba-3\alpha^2\left(3L_3+2L_6\right)\gamma b\\ -2\left(L_2+20\alpha^2L_3+16\alpha^2L_6\right)\beta b\end{bmatrix}\sinh\alpha\xi\sec h^7\alpha\xi\\
&+16\alpha^4L_3\gamma a\sinh\alpha\xi\sec h^8\alpha\xi\\
&-32\alpha^4\left(2L_3\beta a+2L_5\beta ba+L_5\gamma ba\right)\sinh\alpha\xi\sec h^{10}\alpha\xi\\
&+96\alpha^4L_5\beta ab\sinh\alpha\xi\sec h^{12}\alpha\xi\Big\}\\
&+32\alpha^4L_5\beta ab\sinh\alpha\xi\sec h^{12}\alpha\xi\Big\}\\
&+\varepsilon^2\Big\{\;4\alpha^2\begin{pmatrix}L_2+4\alpha^2\gamma^2L_3+4\alpha^2\gamma^2L_6\\ -8\alpha^2L_3\gamma\beta\end{pmatrix}\sec h^4\alpha\xi\\
&+4\alpha^2\begin{bmatrix}8\alpha^2L_5\gamma^2a-\left(L_2+16\alpha^2L_3+12\alpha^2L_6\right)\gamma^2\\ -4\left(L_2+6\alpha^2L_3+8\alpha^2L_6\right)\beta\gamma\end{bmatrix}\sec h^6\alpha\xi\\
&+4\alpha^2\begin{bmatrix}\left(4L_2+64\alpha^2L_3+64\alpha^2L_6\right)\beta^2\\ +\left(9\alpha^2L_6+12\alpha^2L_3\right)\gamma^2\\ +4\left(L_2+29\alpha^2L_3+22\alpha^2L_6\right)\beta\gamma\\ -2\alpha^2L_5\left(11\gamma^2+24\gamma\beta\right)a\end{bmatrix}\sec h^8\alpha\xi\\
&+4\alpha^2\begin{bmatrix}2\alpha^2L_5\left(60\beta\gamma+7\gamma^2+32\beta^2\right)a\\ -4\left(L_2+40\alpha^2L_6+46\alpha^2L_3\right)\beta^2\\ -12\alpha^2\left(7L_3+5L_6\right)\beta\gamma\end{bmatrix}\sec h^{10}\alpha\xi\\
&+16\alpha^4\begin{bmatrix}\left(25L_6+30L_3\right)\beta^2\\ -\left(18L_5\beta\gamma+16L_5\beta^2\right)a\end{bmatrix}\sec h^{12}\alpha\xi\\
&+192\alpha^4L_5\beta^2b\sec h^{13}\alpha\xi\\
&+352\alpha^4L_5\beta^2a\sec h^{14}\alpha\xi\\
&-24\alpha^4L_5\gamma^2b\sinh\alpha\xi\sec h^7\alpha\xi\\
&+24\alpha^4L_5\alpha^4\left(\gamma^2+4\gamma\beta\right)\sinh\alpha\xi\sec h^9\alpha\xi\\
&-160\alpha^4L_5b\left(\beta\gamma+\beta^2\right)\sinh\alpha\xi\sec h^{11}\alpha\xi\Big\}\\
&+8\alpha^4L_5b\left(4\beta\gamma+\gamma^2\right)\sinh\alpha\xi\sec h^{14}\alpha\xi\Big\}\\
&+\varepsilon^3\Big\{-16\alpha^4L_5\gamma^3\sec h^6\alpha\xi\\
&+\alpha^4L_5\left(128\beta\gamma^2+40\gamma^3\right)\sec h^8\alpha\xi\\
&-8\alpha^4L_5\left(40\gamma\beta^2+38\beta\gamma^2+3\gamma^3\right)\sec h^{10}\alpha\xi\\
&+16\alpha^4L_5\left(46\beta^2\gamma+11\beta\gamma^2+16\beta^3\right)\sec h^{12}\alpha\xi\\
&-32\alpha^4L_5\left(13\beta^2\gamma+18\beta^3\right)\sec h^{14}\alpha\xi\\
&+320\alpha^4L_5\beta^3\sec h^{16}\alpha\xi\Big\}=0.
\end{aligned}\tag{7}
$$

While identifying the different coefficients of $\sinh^i\alpha\xi\sec h^j\alpha\xi$ where $i=0,1$ $\quad j=0,1,2,\ldots$ and according to the powers of ε which is infinitely small, we get

Term in $\left(\varepsilon^0,\sec h^2\alpha\xi\right)$,

$$cb+4\alpha L_2a^2=0.\tag{8}$$

Term in $\left(\varepsilon^0,\sec h^4\alpha\xi\right)$,

$$\left(L_2+\alpha^2L_6\right)b^2+4\alpha^2L_3b^2-2L_1a=0.\tag{9}$$

Term in $\left(\varepsilon^0,\sec h^6\alpha\xi\right)$

$$2\alpha^2\left(2L_5ab^2-2L_5a^3-L_6b^2\right)-2a^2L_2-3\alpha^2L_3b^2=0.\tag{10}$$

Term in $\left(\varepsilon^1,\sec h^2\alpha\xi\right)$,

$$L_1+4\alpha^2L_4=0.\tag{11}$$

Term in $\left(\varepsilon^1,\sec h^4\alpha\xi\right)$,

$$
\begin{aligned}
&3\left(L_1+20\alpha^2L_4\right)\gamma+8\left(L_1+16\alpha^2L_4\right)\beta\\
&-4\left(L_2+2\alpha^2L_3\right)\gamma a-8\alpha^2L_5a^2\gamma=0.
\end{aligned}\tag{12}
$$

Term in $\left(\varepsilon^1,\sec h^6\alpha\xi\right)$,

$$
\begin{aligned}
&\alpha^2L_5\left(6\gamma a^2+16\beta a^2-3\gamma b^2\right)+4\left(L_2+8\alpha^2L_3\right)\beta a\\
&+2\left(L_2+8\alpha^2L_3\right)\gamma a-5\left(L_1+52\alpha^2L_4\right)\beta\\
&-30\alpha^2L_4\gamma=0.
\end{aligned}\tag{13}
$$

Term in $\left(\varepsilon^1,\sec h^8\alpha\xi\right)$,

$$
\begin{aligned}
&420\alpha^2L_4\beta a-8\left(L_2+23\alpha^2L_3+4\alpha^2L_6\right)\beta a\\
&-3\alpha^2\left(2L_3+L_6\right)\gamma a\\
&+\alpha^2L_5\left(16\gamma a^2+16\beta b^2+3\gamma b^2\right)=0.
\end{aligned}\tag{14}
$$

Term in $\left(\varepsilon^1, \sec h^9 \alpha\xi\right)$,

$$9L_3 - 2L_5\left(\gamma + 2\beta\right)ba + 5L_6\beta b = 0. \tag{15}$$

Term in $\left(\varepsilon^1, \sec h^{10} \alpha\xi\right)$,

$$\begin{aligned} &10\left(3L_3 + L_6\right)\beta a \\ &-L_5\left(5\gamma a^2 + 12\beta a^2 + 4\beta b^2\right) = 0. \end{aligned} \tag{16}$$

The combination of the equations Eq.(15) and Eq.(16) gives a quadratic equation in a of the form

$$ra^2 + sa + t = 0, \tag{17}$$

where

$$r = 4\alpha^2 L_2 L_5 + 4\alpha^4 L_5\left(L_6 + L_3\right),$$

$$s = 2\left(L_2^2 + \alpha^2 L_2 L_6 + \alpha^2 L_2 L_3 - 4\alpha^2 L_1 L_5\right),$$

$$t = 4\alpha^2 L_1 L_6 + 6\alpha^2 L_1 L_3.$$

The resolution of Eq.(17) requires a discussion around the parameters r, s and t.

For $r = 0$ i.e. $L_2 / \left(L_6 + L_3\right) = -\alpha^2$, Eq.(17) becomes an equation of second order and admits the solution

$$a = -\alpha^2 L_1\left(2L_6 + 3L_3\right) / \begin{pmatrix} L_2^2 + \alpha^2 L_2\left(L_6 + L_3\right) \\ -4\alpha^2 L_1 L_5 \end{pmatrix} \tag{18}$$

Inserting Eq.(18) in Eq.(9) we get

$$b = \left\{ -2\alpha^2 L_1^2\left(2L_6 + 3L_3\right) / \left(\begin{bmatrix} L_2 + \alpha^2\left(L_6 + 4L_3\right) \end{bmatrix} \begin{bmatrix} L_2^2 + \alpha^2 L_2\left(L_6 + L_3\right) \\ -4\alpha^2 L_1 L_5 \end{bmatrix} \right) \right\}^{\frac{1}{2}}. \tag{19}$$

For $r \neq 0$ i.e. $L_2 / \left(L_6 + L_3\right) \neq -\alpha^2$, Eq.(17) is of second order in a. The resolution of Eq.(17) in these conditions gives:

For $\Delta' \geq 0$, we obtain the solution

$$a = \left(-\frac{s}{2} \pm \sqrt{\Delta'}\right) / r, \tag{20}$$

and

$$b = \left[2L_1\left(-\frac{s}{2} \pm \sqrt{\Delta'}\right) / \left(L_2 + \alpha^2 L_6 + 4\alpha^2 L_3\right)r\right]^{\frac{1}{2}} \tag{21}$$

where $\Delta' = \left(s^2/4\right) - rt$.

- For $\Delta' \prec 0$, we obtain the following roots

$$a = \left(-\frac{s}{2} \pm i\sqrt{-\Delta'}\right) / r \tag{22}$$

and

$$b = \left[2L_1\left(-\frac{s}{2} \pm i\sqrt{-\Delta'}\right) / \left(L_2 + \alpha^2 L_6 + 4\alpha^2 L_3\right)r\right]^{\frac{1}{2}}. \tag{23}$$

On the other hand, while combining equations Eq.(15) and Eq.(16), one gets β and γ as functions of a and b

$$\beta = 45L_3 a / \begin{pmatrix} 60L_3 a - 5L_6 a \\ -4L_5 a^2 - 8L_5 b^2 \end{pmatrix} b, \tag{24}$$

and

$$\gamma = \begin{bmatrix} \left(270L_3^2 + 90L_6 L_3\right)a \\ -108L_3 L_5 a^2 \\ -36L_5 L_3 b^2 \end{bmatrix} / \begin{bmatrix} \left(60L_3 L_5 - 5L_6 L_5\right)a^2 b \\ -4L_5^2 a^3 b - 8L_5^2 ab^3 \end{bmatrix} \tag{25}$$

where a and b are values given respectively by equations Eq.(18), Eq.(19), Eq.(20), Eq.(21), Eq.(22) and Eq.(23).

Taking into account equations Eq.(18), Eq. (19),..., Eq. (25) in Eq. (6) we get three great families of solutions as seen below.

For $r = 0$, i.e. $L_2 / \left(L_6 + L_3\right) = -\alpha^2$, the first family is given by

$$\begin{aligned} \psi(\xi) = &\left[-\alpha^2 L_1\left(2L_6 + 3L_3\right) / L_2^2 + \alpha^2 L_2\left(L_6 + L_3\right) - 4\alpha^2 L_1 L_5\right]\sec h^2 \alpha\xi \\ &+ \left\{-2\alpha^2 L_1^2\left(2L_6 + 3L_3\right) / \left[L_2 + \alpha^2\left(L_6 + 4L_3\right)\right]\left[L_2^2 + \alpha^2 L_2\left(L_6 + L_3\right) - 4\alpha^2 L_1 L_5\right]\right\}^{\frac{1}{2}} \tanh h\alpha\xi \\ &+\varepsilon \begin{bmatrix} \left[45L_3 a / \left(60L_3 a - 5L_6 a - 4L_5 a^2 - 8L_5 b^2\right)b\right]\sec h^4 \alpha\xi \\ + \left\{ \begin{matrix} \left[\left(270L_3^2 + 90L_6 L_3\right)a - 108L_3 L_5 a^2 - 36L_5 L_3 b^2\right] \\ / \left[\left(60L_3 L_5 - 5L_6 L_5\right)a^2 b - 4L_5^2 a^3 b - 8L_5^2 ab^3\right] \end{matrix} \right\} \tanh^2 \alpha\xi \end{bmatrix}, \end{aligned} \tag{26}$$

where a and b are given by Eq.(18) and Eq.(19).

For $r \neq 0$, i.e. $L_2 / \left(L_6 + L_3\right) \neq -\alpha^2$, we obtain the last two families of solutions

$$\psi(\xi)=\left[\left(-\frac{s}{2}\pm\sqrt{\Delta'}\right)/r\right]\sec h^2\alpha\xi+\left[2L_1\left(-\frac{s}{2}\pm\sqrt{\Delta'}\right)/\left(L_2+\alpha^2L_6+4\alpha^2L_3\right)r\right]^{\frac{1}{2}}\tanh\alpha\xi$$
$$+\varepsilon\left\{\begin{array}{l}\left[45L_3a/\left(60L_3a-5L_6a-4L_5a^2-8L_5b^2\right)b\right]\sec h^4\alpha\xi\\+\left\{\begin{array}{l}\left[\left(270L_3^2+90L_6L_3\right)a-108L_3L_5a^2-36L_5L_3b^2\right]\\/\left[\left(60L_3L_5-5L_6L_5\right)a^2b-4L_5^2a^3b-8L_5^2ab^3\right]\end{array}\right\}\tanh^2\alpha\xi\end{array}\right\}, \quad (27)$$

where $\Delta'\geq 0$, a and b given by Eq.(22) and Eq.(23). We also have

$$\psi(\xi)=\left[\left(-\frac{s}{2}\pm i\sqrt{-\Delta'}\right)/r\right]\sec h^2\alpha\xi+\left[2L_1\left(-\frac{s}{2}\pm i\sqrt{-\Delta'}\right)/\left(L_2+\alpha^2L_6+4\alpha^2L_3\right)r\right]^{\frac{1}{2}}\tanh\alpha\xi$$
$$+\varepsilon\left\{\begin{array}{l}\left[45L_3a/\left(60L_3a-5L_6a-4L_5a^2-8L_5b^2\right)b\right]\sec h^4\alpha\xi\\+\left\{\begin{array}{l}\left[\left(270L_3^2+90L_6L_3\right)a-108L_3L_5a^2-36L_5L_3b^2\right]\\/\left[\left(60L_3L_5-5L_6L_5\right)a^2b-4L_5^2a^3b-8L_5^2ab^3\right]\end{array}\right\}\tanh^2\alpha\xi\end{array}\right\}, \quad (28)$$

where $\Delta'\prec 0$, a and b are given by Eq.(22) and Eq.(23).

To better understand the notion of dominant and less dominant parts of wave $\psi=a\sec h^2\alpha\xi+b\tanh\alpha\xi+\varepsilon\left(\beta\sec h^4\alpha\xi+\gamma\tanh^2\alpha\xi\right)$, we are engaged in the representation of the two main parts that constitute ψ. If we state that $\psi_1=a\sec h^2\alpha\xi+b\tanh\alpha\xi$ and $\psi_2=\beta\sec h^4\alpha\xi+\gamma\tanh^2\alpha\xi$, the wave becomes $\psi=\psi_1+\varepsilon\psi_2$. Figure 1 shows the representation of ψ_1, which is the profile of a soliton wave of a kink type. Figure 2 and Figure 3 show the representation of ψ_2 for a few values of β and γ. The two profiles obtained in Figure 2 and Figure 3 are soliton waves of pulse nature. Figure 4 gives the representation of solution in general $\psi=\psi_1+\varepsilon\psi_2$ for ε very small ($\varepsilon=0.01$). The profile obtained is that of a kink. We simply realize that ψ can take the form of a kink or the form of a pulse depending on the value of ε. As regard our research work, we have chosen for our dominant part ψ_1. The practical interpretation that we can give to this solution is that of a soliton wave solution which changes its form with respect to the environment in which it happens to be or to the obstacle to overcome in its medium of propagation. In the example considered hereafter the soliton ψ can lose its kink or pulse form depending on the conditions under which it is subjected.

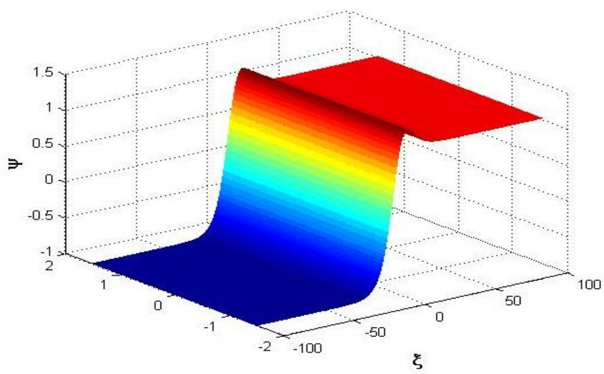

Figure 1. Representative curve of the soliton profile of the solution $\psi_1=a\sec h^2\alpha\xi+b\tanh\alpha\xi$, for $a=1$, $b=1$, $\alpha=0.1$.

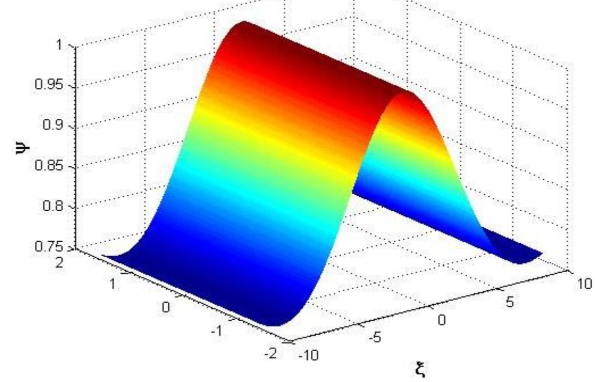

Figure 2. Representative curve of the soliton profile of $\psi_2=\beta\sec h^4\alpha\xi+\gamma\tanh^2\alpha\xi$, for $a=1$, $b=1$, $\alpha=0.1$. and and in the case where $\xi\in[-100,100]$, $\psi\in[-1.5,1.5]$

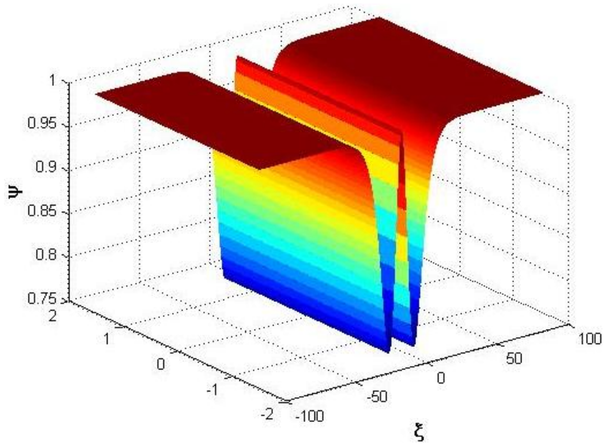

Figure 3. Representative curve of the soliton profile of $\psi_2 = \beta \sec h^4 \alpha\xi + \gamma \tanh^2 \alpha\xi$, for $a=1$, $b=1$, $\alpha = 0.1$ and in the case where $\xi \in [-10,10]$, $\psi \in [-1.5,1.5]$

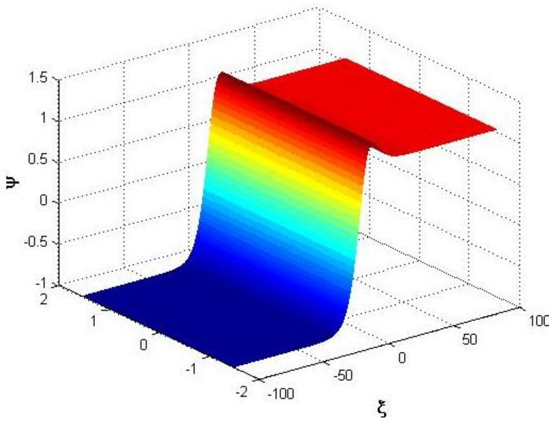

Figure 4. Representative curve of the soliton profile of the combined solution $\psi = a \sec h^2 \alpha\xi + b \tanh \alpha\xi + \varepsilon\left(\beta \sec h^4 \alpha\xi + \gamma \tanh^2 \alpha\xi\right)$, $a=1$, $b=1$, $\alpha=0.1$, $\beta=1$, $\gamma=1$ and $\varepsilon = 0.01$

4. Conclusions

We have just polished up this work by putting forward a solution to Kuramoto-Sivashinsky's equation in the form of a combine soliton wave. In the course of our work we have made up a solution of the form $\psi(\xi) = \psi_1(\xi) + \varepsilon\psi_2(\xi)$, where ψ_1 and ψ_2 are solitary waves. Through the BDK innovative method mentioned above, we want to prove that it is possible to make up modulated soliton solutions of the form $\psi = \psi_1(\xi) + \varepsilon\psi_2(\xi) + \varepsilon^2\psi_3(\xi) + ... + \varepsilon^n\psi_n$, where ψ_1, ψ_2, ..., ψ_n represent solitary waves. This form that we are putting forward as regard all calculations and analysis undertaken up to date, helps us to confirm our satisfaction as regard the usage of the BDK method in making up modulated soliton solution of Kuramoto-Sivashinsky's equation.

On the whole, projecting our reasoning rays beyond its limit, multi-soliton put into evidence experimentally will be an object of attraction for all scientists.

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