Modulation and Detection for Simple Receivers in Rapidly Time Varying Channels

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***Abstract*— We investigate the performance degradation of basic modulation schemes in a rapidly time varying channel using a first order autoregressive channel model. Various performance metrics are used to indicate the relative advantages of each modulation scheme. We find that noncoherent frequency shift keying (FSK) is suitable for operating at very high mobility and high SNR, ideal for some military applications. We then propose a partially coherent detector for FSK and differential phase shift keying (DPSK) that exploits partial channel knowledge to enable the receiver to operate effectively in both fast and slow fading. The maximum likelihood rule (ML) obtained for the partially coherent FSK turns out to be a linear combination of coherent and noncoherent detection rules. Results demonstrate that significant performance improvement can be achieved over the best of coherent and noncoherent FSK detection. The detector is robust to estimation errors present in the channel statistics. We also propose a few adaptive schemes that employ various combinations of modulation schemes to increase the robustness of the system in fast fading.**

***Index Terms*— Fast fading, rapidly time varying channel, partially coherent detection, intra-block adaptation, inter-block adaptation**

I. INTRODUCTION

With the rapid growth of wireless networks and multimedia applications, next generation wireless systems are not only expected to support very high data rates, but also very high quality of service, stressing the need for robustness under all channel conditions. These systems must be able to operate reliably in rapidly fading environments and therefore the detrimental effects of mobility must be mitigated. A mobile traveling at a speed of 75mph (miles per hour) and operating at a carrier frequency of 5GHz can give rise to a Doppler shift as high as 550Hz. There are also scenarios in which an even higher Doppler is encountered such as in satellite communications and some military applications like unmanned airborne vehicles (UAV). This poses a major impediment to many existing wireless systems which would breakdown under such a large Doppler shift. Some of the effects of mobility on major communication blocks are studied in [1].

In this paper, we consider the problems posed by a rapidly time varying channel on modulation and detection in simple receivers. Almost all modulation schemes operating in the band limited regime [2] either require accurate channel estimate at the receiver or at least require the channel to remain invariant for a certain time duration. However these requirements might be very hard to satisfy in a rapidly time varying channel. For a coherent scheme to operate well in a time varying channel, the channel has to be estimated quite frequently leading to spectral efficiency loss. Thus, noncoherent schemes like differential phase shift keying (DPSK) and noncoherent frequency shift keying (FSK) are preferred in a fast fading channel as the cost and complexity associated with channel estimation becomes prohibitive. However, even differential schemes suffer from an error floor [3] in rapidly fading environment when the channel does not remain constant for two symbol periods. For most schemes the rapid channel variations translate to loss in effective SNR. Noncoherent FSK, which is commonly employed in power limited systems [2], finds application in satellite communication systems where it counters a large Doppler and at the same time provides a low power solution. Ideally FSK is not suited for bandlimited systems due to its spectral inefficiency. However, the rapidly fading channel creates a level playing field by degrading the modulation schemes that are suitable for operation in the bandlimited regime. An important question in this context is to determine the relative performance of FSK in bandlimited systems with high mobility. We address this question in Section III.

Nodes that are deployed in wireless sensor and ad-hoc networks can only afford simple processing of the received signal and are required to operate at minimum energy to enhance the longevity of the network. Existing results [4], [5] establish that the system must operate at a very low rate (after modulation and coding) to minimize the energy spent per bit. This result cannot be directly applied in a time varying scenario (without considering coding), as increase in transmission duration may violate the basic requirements of the modulation scheme. Therefore decreasing the rate might not be the direct solution for simple receivers operating in rapidly varying channel. Circuit energy consumption and delay constraints are the other problems [6] that should be dealt with, when increasing the transmission duration.

Many existing works in the literature [2], [7]–[9] approach the detection problem in fading channels under two extreme cases: the coherent case with perfect channel knowledge available at the receiver, and the noncoherent case with absolutely no knowledge of the channel. As the second case is more pertinent in time varying channels, non coherent detection has been a unanimous choice for data detection in time selective channels. However the channel knowledge at the receiver in practical wireless channels lies in between these two extremes. It is not unrealistic to assume partial channel knowledge at the receiver even in a rapidly varying channel and then perform a combination of coherent and noncoherent detection. Partially coherent detection was first proposed in [10] for AWGN channels with phase noise arising from the phase locked loop (PLL). The receiver has imperfect phase estimates with the phase errors assuming Tikhonov densities. This is extended to fading channels in [11] and optimal decision rule found. In both these cases, the optimal rule turns out to be be a linear combination of coherent and noncoherent detection rule. In Section IV, we propose partially coherent detectors for BFSK and DPSK that utilize channel information consisting of both amplitude and phase uncertainties. Interestingly, for BFSK, the optimal maximum likelihood (ML) rule for the proposed partially coherent detector turns out to be a linear combination of coherent and noncoherent ML detectors similar to [10].

Throughout this paper, we identify the parameters that different modulation schemes are sensitive to, and propose some adaptive strategies in a time varying scenario. The choice of the modulation scheme critically depends on the rate at which the channel varies. The varied performances of coherent, differential and noncoherent schemes provide us the opportunity to use these schemes effectively depending on the channel conditions. In Section VII, we propose two ways of adapting the modulation scheme at the transmitter, namely *intra-block adaptation* and *inter-block adaptation*. Results are provided to substantiate the merits of the schemes. Finally, we conclude with Section VIII.

II. System Model

We assume complex baseband notation throughout the paper. Consider a communication link consisting of a single antenna transmitter and receiver that operates in a time selective and frequency nonselective Rayleigh fading environment modeled by a first order autoregressive process.

$$h_k = a h_{k-1} + \sqrt{1-a^2} w_k, \tag{1}$$

where a is the correlation parameter, $0 < a \leq 1$ and w_k, the varying component of the channel is an independent and identically distributed (i.i.d.) random process with density $\mathcal{CN}(0, \sigma_h^2)$. It can be noticed from the above equation that the lower the value of a, the greater is the channel variation rate. The channel realizations become i.i.d. when $a = 0$ while $a = 1$ models quasi-static fading. The relationship between the Doppler frequency and a can be approximated using Jakes autocorrelation model [7] and it is given by

$$a = \mathcal{J}_0(2\pi f_d T_s), \tag{2}$$

where $\mathcal{J}_0(x)$ is the zeroth order Bessel function of the first kind, $f_d = \frac{f_c v}{c} = \frac{v}{\lambda}$ is the Doppler shift and T_s is the symbol duration. A mobile at a velocity of 75 mph results in a Doppler shift of 550 Hz at a carrier frequency of 5 GHz. For a data rate less than 5 kbps, the value of a according to this model is less than 0.999. Note that a decreases with decrease in data rate or increase in carrier frequency or mobile velocity.

The input-output relationship of the single antenna link is given by

$$y_k = h_k x_k + n_k, \tag{3}$$

where n_k is complex additive white Gaussian noise (AWGN) with power spectral density N_0. We assume that an accurate estimate of the channel is obtained at the receiver after every N data symbols. With this information, the channel knowledge at the receiver can be described as a complex Gaussian random process,

$$\widehat{h}_k \backsim \mathcal{CN}(a^k h_0, 1-a^{2k}). \tag{4}$$

Note that $a = 1$ indicates perfect CSI (channel state information) while $a = 0$ denotes no CSI at the receiver. We do not assume any error in estimating h_0. In general, if the estimation error has to be included in the model, the MMSE estimator will be $\widehat{h}_0 = \frac{\sigma_h^2 \sqrt{E_s}}{\sigma_h^2 E_s + N_0} y_0$. In the next subsection, we quantify the performance loss in basic modulation schemes for simple receivers due to channel variations. For ease of exposition, we confine our analysis to binary modulation schemes and we therefore consider coherent and noncoherent detection for BPSK and BFSK modulation.

A. Coherent BPSK

Let the symbol transmitted at the k^{th} symbol duration be

$$x_k = \sqrt{E_s} e^{j\phi_k}, \tag{5}$$

where ϕ_k is the transmitted phase and E_s is the symbol energy. The received symbol at k^{th} symbol duration is given by (3). The transmitted phase ϕ_k takes value from $\{0, \pi\}$, the BPSK constellation set. We assume that channel state information is obtained at the receiver through training and the quality of the estimate depends on the frequency of estimation. In our model, the channel is estimated during first symbol slot of every block and it is used for decoding subsequent symbols in the block. With such a model for channel estimation, the estimation error is dependent on the symbol position and is greater for symbols far apart from the training symbol. The estimation error for the k^{th} data symbol is given by $\sigma_{CE}^2 = 1 - a^{2k}$. With the channel knowledge acquired through training, the distribution of the channel knowledge at the receiver is a time varying Gaussian denoted by $\mathcal{CN}(a^k h_0, 1-a^{2k})$.

Conditioned on the transmitted sequence and partial channel knowledge, the individual received symbols are not independent and therefore the optimal rule to employ will be the maximum likelihood sequence estimation [12]. The complexity associated with this decision rule grows exponentially with the block size. Such a decision rule cannot be implemented in simple receivers and therefore the need arises for symbol by symbol detection, even though it is suboptimal. It should be noted that the error floor is not eliminated even with sequence estimation [10]. For symbol by symbol detection, the optimal rule turns out to be co-phasing of the received symbols with the noisy estimate h_0. The decision variable for the k^{th} received signal obtained by equalizing its phase with the estimate h_0 is

$$\frac{h_0^*}{|h_0|} y_k = a^k |h_0| x_k + z_k, \tag{6}$$

where the effective noise term z_k is Gaussian with variance $(1-a^{2k})E_s+N_0$. Then the instantaneous effective SNR γ_k is given by

$$\gamma_k = \frac{a^{2k}|h_0|^2 E_s}{(1-a^{2k})E_s+N_0}. \tag{7}$$

The average effective SNR for the k^{th} symbol position Γ_k is

$$\Gamma_k = \frac{a^{2k} E_s}{(1-a^{2k})E_s+N_0}. \tag{8}$$

The effective SNR depends on the symbol location and it decreases with time until a new channel estimate is obtained. Each symbol in the block has different average SER. The closer it is to the training symbol the lower the probability of symbol error. The average error probability of the k^{th} symbol position for a coherent BPSK system is given by

$$\overline{P_e}(k) = \frac{1}{2}\left(1-\sqrt{\frac{\Gamma_k}{1+\Gamma_k}}\right) \tag{9}$$

$$= \frac{1}{2}\left(1-a^k\sqrt{\frac{E_s}{E_s+N_0}}\right). \tag{10}$$

The overall average BER for the N-symbol block is

$$\overline{P_e} = \frac{1}{N}\sum_{k=1}^{N} P_e\,(k). \tag{11}$$

Substituting (9) in (11) and upon simplification, we obtain

$$\overline{P_e} = \frac{1}{2}\left[1-\frac{a}{N}\left(\frac{1-a^N}{1-a}\right)\sqrt{\frac{E_s}{E_s+N_0}}\right]. \tag{12}$$

The error floor of coherent BPSK due to constrained channel estimation rate can be calculated from (12) above and is given by

$$\overline{P_e} = \frac{1}{2}\left[1-\frac{a}{N}\left(\frac{1-a^N}{1-a}\right)\right]. \tag{13}$$

From the above equation it can be said that, with very high channel estimation rate, coherent schemes can perform well even in a very rapidly varying channel (lesser value of a) while in a slow fading channel they can still perform poorly if the channel estimation rate is very low.

B. Differential Detection: DPSK

DPSK has found widespread use in wireless communications due to its simplicity and robustness as it eliminates the carrier phase recovery problem commonly associated with fading channels. It also removes the effect of slow phase drift in local oscillators. The modulation scheme works exceedingly well when the channel stays approximately constant over two symbol periods. However in rapidly varying channels, where the channel cannot be assumed to be constant over two consecutive symbol durations, it is susceptible to an error floor as well.

The optimal decoding would involve joint detection of the entire block using the knowledge of the channel statistic a and the estimate h_0. But the simplicity of DPSK will be lost when resorting to sequence detection. Also the encoding window can be increased to improve the performance when joint detection is permitted [13], [14]. Since we are interested in simple receivers, we employ conventional differential detection. We discuss the effect of this operation on the overall capacity in Section III.

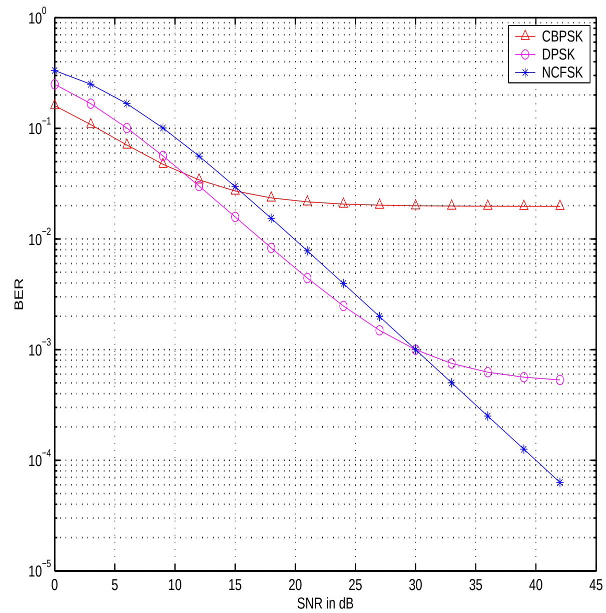

Fig. 1. BER performance of the schemes at a=0.999 & N=100.

For the differential system, the phase of the transmitted symbol during the k^{th} symbol duration is encoded as $\phi_k = \phi_{k-1}+\theta_k$ where θ_k is a point in the BPSK signal constellation. The following operation is performed at the receiver to obtain the decision variable.

$$\begin{aligned} y_k^* y_{k+1} &= (h_k^* x_k^* + n_k^*)\,(h_{k+1}x_{k+1}+n_{k+1}) \\ &= (h_k^* x_k^* + n_k^*)\left(ah_k+\sqrt{1-a^2}w_k x_{k+1}+n_{k+1}\right) \\ &= a\,|h_k|^2\,u_{k+1}+z_{k+1}, \end{aligned}$$

where u_{k+1} is the actual data symbol and z_{k+1} contains all the noise terms. We neglect the product of Gaussian random variables $w_k n_k^*$ and $n_{k+1}n_k^*$ in the calculation of the SNR and the error probability. The instantaneous post detection SNR γ_{dif} is,

$$\gamma_{\text{dif}} = \frac{a^2|h_k|^2 E_s}{(1-a^2)\,E_s+(1+a^2)N_0}. \tag{14}$$

Unlike coherent schemes, the SNR here is independent of the position of the symbol and thus all symbols in the block have the same SNR. The average post detection effective SNR is given by

$$\Gamma_{\text{dif}} = \frac{a^2 E_s}{(1-a^2)\,E_s+(1+a^2)N_0}. \tag{15}$$

The probability of error for a DPSK system [15] with an average SNR Γ_{dif} is given by

$$\overline{P_e} = \frac{1}{2}\left(1-\frac{2a^2E_s}{(1+a^2)\,(E_s+N_0)}\right). \tag{16}$$

From the above equation, the error floor caused due to channel variation within successive symbol durations in DPSK can be obtained as

$$\overline{P_e} = \frac{1}{2}\left(\frac{1-a^2}{1+a^2}\right). \tag{17}$$

For a transmit diversity system employing differential space time codes [16] that require the channel to be constant for two codewords, the effect of rapid variations in channel will be more pronounced.

C. Coherent FSK Detection

Coherent FSK is usually not preferred because there is a 3dB degradation in its performance when compared to BPSK. Apart from that, there is a bandwidth expansion for operating in two orthogonal channels. Furthermore it requires accurate channel estimate at the receiver for reliable operation. The probability of error can be obtained from the probability of error of BPSK in (12), by modifying the noise power.

$$\overline{P_e} = \frac{1}{2}\left[1 - \frac{a}{N}\left(\frac{1-a^N}{1-a}\right)\sqrt{\frac{E_s}{E_s+2N_0}}\right] \tag{18}$$

This expression is valid when the bandwidth of each orthogonal band is equal to the total bandwidth of BPSK scheme. It is shown in [2] that the bands should be separated by at least $\frac{1}{2T_s}$ for orthogonality, resulting in a total bandwidth in excess of $\frac{3}{2T_s}$. The spectral efficiency becomes 0.5 b/s/Hz when there is no overlap between bands. To obtain the probability of error for the same total bandwidth, the value of a should be modified to account for the increase in symbol duration, which results in more pronounced fast fading. The error floor of coherent FSK due to constrained channel estimation rate in a fast fading channel is

$$\overline{P_e} = \frac{1}{2}\left[1 - \frac{a}{N}\left(\frac{1-a^N}{1-a}\right)\right]. \tag{19}$$

It can be noted that this error floor is same as in (13). A unique advantage of FSK is that it can be detected coherently or noncoherently, depending upon the receiver's capability.

D. Noncoherent FSK (NCFSK) Detection

Noncoherent FSK, the most popular form of FSK, is commonly used in power limited systems like satellite communications. Unlike DPSK, it does not exhibit an error floor[1] in rapidly varying channel. The average SNR for a noncoherent BFSK system is

$$\Gamma = \frac{\sigma_h^2 E_s}{N_0}. \tag{20}$$

The average probability of error is then given by

$$\overline{P}_e = \frac{1}{(2+\Gamma)}. \tag{21}$$

The bit error performance of the modulation schemes in a rapidly time varying channel is shown in Fig. 1 for a=0.999 and N =100. It can be seen that FSK is the ideal modulation scheme to employ at high SNR as the other schemes suffer

[1]We do not consider the error floor caused from frequency interference [17] caused by Doppler shifts, which is negligible.

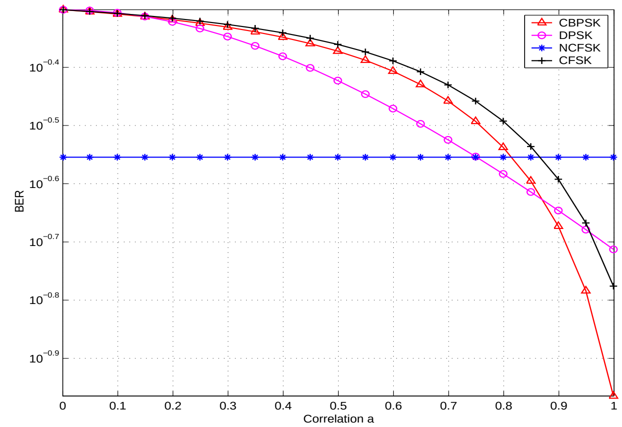

Fig. 2. BER as a function of a at SNR=2 dB and N=5 symbols.

from error floors. At low SNR, the noise power is comparable to the error due to channel variation and hence BPSK and DPSK perform better than NCFSK. In Table I, the best performing modulation scheme for various ranges of the channel variation rate(a) and channel estimation rate(N) is listed. DPSK is the best choice in a slow fading or medium scale fast fading channel when it is difficult to obtain an accurate channel estimate. Similarly BPSK can perform well, even in a fast fading channel when accurate estimates are easier to obtain. However in the "rapidly varying" regime ($a < 0.999$), only NCFSK provides reliable performance.

However it must be noted that the BER plot in Fig. 1 does not capture the data rate mismatch between the schemes as FSK operates at half the data rate of BPSK and DPSK for the same bandwidth. With increase in constellation size, the data rate mismatch between MFSK and MPSK or MQAM widens. To account for the data rate of the modulation schemes, we consider their capacity in the next section.

III. Hard Decision Capacity of Modulation Schemes

We determined in the previous section that orthogonal schemes are effective in mitigating the effects of rapid channel variations and channel uncertainty. This is evident from Figures 1 and 2 where noncoherent BFSK performs well in rapid mobility scenarios while the other schemes suffer from an error floor. However, an important aspect of the modulation schemes is neglected in the analysis: the rate of the modulation schemes. For the same rate, BFSK requires twice the bandwidth of BPSK and DPSK modulation. For the same bandwidth, BFSK operates only at half the data rate of BPSK and DPSK. We therefore need a performance metric that takes into account all the relevant parameters of the modulation schemes. Therefore we investigate the capacity of schemes. While the complexity of capacity achieving codes may be infeasible for simple receivers, a capacity perspective allows a direct comparison between these schemes. Furthermore, a capacity notion will be relevant in a relay network scenario where the intermediate nodes perform only demodulate and forward [18], while the message is encoded at the source and

$a : N$	$N < 40$ (symbols)	$40 < N < 100$	$N > 100$
$a > 0.9999$ (slow fading)	BPSK	BPSK	DPSK
$0.9999 < a < 0.999$ (medium scale fast fading)	BPSK	DPSK	DPSK
$a < 0.999$ (rapidly varying channel)	NCFSK	NCFSK	NCFSK

TABLE I

BEST MODULATION SCHEME (IN TERMS OF ERROR PROBABILITY) AT SNR=20 dB

decoded at the destination. Thus the capacity obtained from this model will be useful in finding the maximum rate of data transfer when coding is employed over the modulation scheme at the transmitter while at the receiver symbol level demodulation is performed followed by full decoding.

To gain a capacity perspective we consider the system shown in Fig. 3.

The effective channel that includes the effect of modulator/demodulator, interleaver/deinterleaver and the physical channel is described by the following relationship:

$$\hat{V}_k = V_k \oplus Z_k, \quad k = 1...N, \tag{22}$$

where $\oplus$ is the binary XOR operation. The distribution of the error variable Z_k depends on the demodulating function $f(Y_i, H_0)$ of the demodulator. For BPSK modulation, the distribution of Z_k is given by

$$Z_k = \begin{cases} 1 \text{ if } \mathfrak{Re}\{H_0^*(\sqrt{E_s}H_k + n_k)\} < 0 \\ 0 \text{ otherwise} \end{cases} \tag{23}$$

We assume that the coded symbols from multiple codewords are interleaved so that different symbols from the same codeword experience independent channels. The scheme is relevant from a practical standpoint as interleaving is often used to avoid bursts of errors. We assume each codeword is decoded in isolation from the rest of the codewords. Under these assumptions, the channel is a memoryless channel from the decoder's perspective, allowing a single letter capacity characterization that can be easily computed. The capacity in this case is expressed as:

$$\begin{aligned} C &= \frac{1}{N}\mathbb{I}(V^N; \hat{V}^N|H_0) && (24)\\ &= \frac{1}{N}\left(\mathbb{H}(\hat{V}^N|H_0) - \mathbb{H}(\hat{V}^N|V^N, H_0)\right) && (25)\\ &= 1 - \frac{1}{N}\mathbb{H}(Z^N|H_0) && (26)\\ &= 1 - \frac{1}{N}\sum_{k=1}^{N}\mathbb{H}(Z_k|H_0, Z^{k-1}) && (27)\\ &= 1 - \frac{1}{N}\sum_{k=1}^{N}\mathbb{H}(Z_k|H_0) && (28)\\ &= 1 - \frac{1}{N}\sum_{k=1}^{N}\mathcal{E}_{H_0}\left[\mathbb{H}_2(\text{Prob}(Z_k = 1|H_0))\right], && (29) \end{aligned}$$

where $\mathbb{H}_2(x) = -x\log_2 x - (1-x)\log_2(1-x)$, the binary entropy function. (28) is obtained from the fact that Z_k is independent of Z^{k-1} due to interleaving across independent channels.

Fig. 4 compares the hard decision capacity of BPSK, DPSK, and coherent and noncoherent BFSK with memoryless interleaving. It can be seen that DPSK is the best performing modulation scheme at high SNR while BPSK seems to be an ideal choice at low SNR. This result is in total contrast with the BER performance plot in Fig. 1, where the bandwidth expansion in BFSK is not accounted for. However, it can be easily predicted that MFSK will fare very poorly in terms of capacity when compared with MQAM or MPSK, due to its spectral inefficiency. However, for extreme cases of mobility, when the channel takes i.i.d values, noncoherent FSK is preferable.

While we are interested in the capacity perspective for the system described above, for the sake of completeness we also address the capacity of this system without interleaving. In the absence of interleaving, the memory in the channel experienced by different symbols of a codeword can be utilized by the decoder to achieve a higher capacity. While the memory of the channel does not allow a direct single letter capacity characterization, such an upperbound can be obtained when perfect channel knowledge is provided by a genie to the decoder. From (27), we have

$$\begin{aligned} C &= 1 - \frac{1}{N}\sum_{k=1}^{N}\mathbb{H}(Z_k|H_0, Z^{k-1}) && (30)\\ &\leq 1 - \frac{1}{N}\sum_{k=1}^{N}\mathbb{H}(Z_k|H_0, H_1..H_k, Z^{k-1}) && (31)\\ &= 1 - \frac{1}{N}\sum_{k=1}^{N}\mathbb{H}(Z_k|H_0, H_1..H_k) && (32)\\ &= 1 - \frac{1}{N}\sum_{k=1}^{N}\mathcal{E}_{H_0,H_k}\left[\mathbb{H}_2(\text{Prob}(Z_k = 1|H_k, H_0))\right], \end{aligned}$$

where the inequality in (31) is obtained by providing additional channel state information to the decoder.

Capacity upperbounds when channel memory is utilized by the decoder are presented in Fig. 5. We see that, providing full channel state information to the decoder helps the coherent schemes more than the noncoherent schemes.

IV. PARTIAL CHANNEL KNOWLEDGE

In a fast fading channel, the channel estimate obtained from training gets outdated so quickly that coherent detection cannot be performed. Nevertheless the outdated channel information can still be utilized in the detection process if it would result in a considerable performance improvement. The channel information in this context (4) has both amplitude and phase

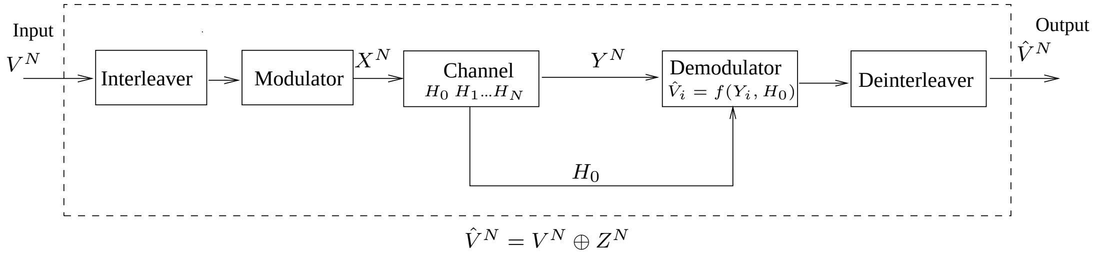

Fig. 3. Block diagram of the system with memoryless interleaving and symbol wise demodulator.

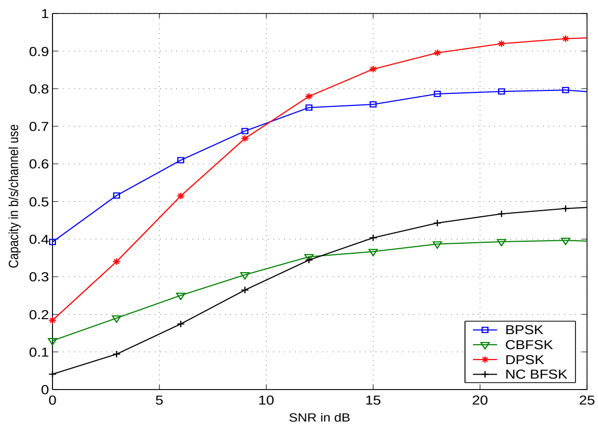

Fig. 4. Hard decision capacity of modulation schemes for the system shown in Fig. 3 for $a = 0.98$, N=25

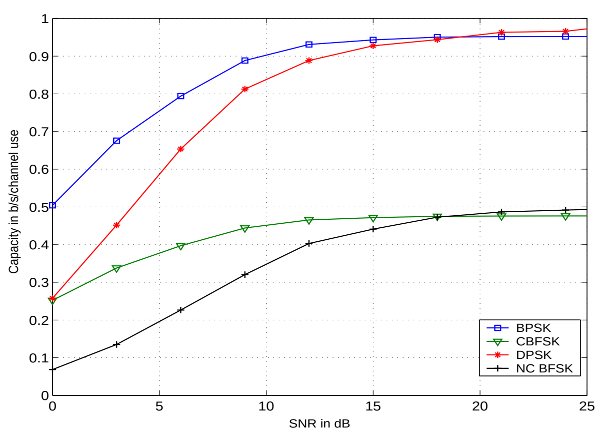

Fig. 5. Capacity upperbounds of modulation schemes for $a = 0.98$, N=25

uncertainties. In this section, we explore ways to utilize this partial channel knowledge in basic noncoherent schemes like FSK and DPSK, without increasing the complexity. Ideally we require the receiver to perform symbol by symbol detection taking into account the channel knowledge. It is obvious that the additional channel knowledge will improve the system performance but what remains interesting to know is the amount of gain that can be obtained and the extra complexity it entails. We find that a gain as much as 2 dB can be obtained over the best of coherent and noncoherent FSK, with partial channel information. Interestingly the ML detection rule turns out to be a linear combination of ML rules of coherent and noncoherent BFSK. However for DPSK, the ML rule remains complex and the partial channel knowledge does not result in significant performance improvement.

V. BFSK WITH PARTIAL CHANNEL KNOWLEDGE

The transmitted symbol $\mathbf{x_k}$ for FSK modulation assumes one of the two possible states $\mathbf{x_k} = \left[x_k^1, x_k^2\right]^T = [0,1]^T$ or $[1,0]^T$ representing a binary symbol d_k. Receivers for FSK have a unique advantage of operating coherently and noncoherently as the transmission is same for both the schemes. Suppose we have the estimate of the channel h_0 obtained at the start of the block, then the coherent detection rule for symbol by symbol demodulation will be

$$\mathfrak{Re}\left(h_0^*\left(y_k^1 - y_k^2\right)\right) \gtrless_1^0 0. \tag{33}$$

Discarding the outdated channel estimate, the noncoherent ML rule will be

$$\left(|y_k^1|^2 - |y_k^2|^2\right) \gtrless_1^0 0, \tag{34}$$

where $\mathfrak{Re}(x)$ and $|x|$ denotes the real part and absolute value of the complex number x respectively. If the receiver is aware of the channel statistic a, the channel estimate h_0 and the symbol position k, a partially coherent detection can be performed. The ML rule is

$$\Pr\left(y_k^1, y_k^2 | a, h_0, d_k = 0\right) \quad \gtrless_1^0 \quad \Pr\left(y_k^1, y_k^2 | a, h_0, d_k = 1\right). \tag{35}$$

$$\Pr\left(y_k^1, y_k^2 | a, h_0, d_k\right) = \Pr\left(y_k^1 | a, h_0, x_k^1\right)\ \Pr\left(y_k^2 | a, h_0, x_k^2\right). \tag{36}$$

It is straightforward to arrive at the following densities.

$$\begin{aligned}
\Pr\left(y_k^1 | a, h_0, d_k = 1\right) &\sim \mathcal{CN}(a^k h_0, 1 - a^{2k} + N_0)\\
\Pr\left(y_k^2 | a, h_0, d_k = 1\right) &\sim \mathcal{CN}(0, N_0)
\end{aligned}$$

$$\begin{aligned}
\Pr\left(y_k^1 | a, h_0, d_k = 0\right) &\sim \mathcal{CN}(0, N_0)\\
\Pr\left(y_k^2 | a, h_0, d_k = 0\right) &\sim \mathcal{CN}(a^k h_0, 1 - a^{2k} + N_0)
\end{aligned}$$

The final decision rule for the k^{th} symbol, obtained after solving (35) and simplifying the terms is

$$\begin{aligned}
2a^k N_0\ \mathfrak{Re}\left(h_0^*\left(y_k^1 - y_k^2\right)\right)\sqrt{E_s}&\\
+ \left(1 - a^{2k}\right) E_s \left(|y_k^1|^2 - |y_k^2|^2\right) &\gtrless_1^0 0.
\end{aligned} \tag{37}$$

The decision rule obtained above is a linear combination of the optimal ML rules coherent and noncoherent detection with

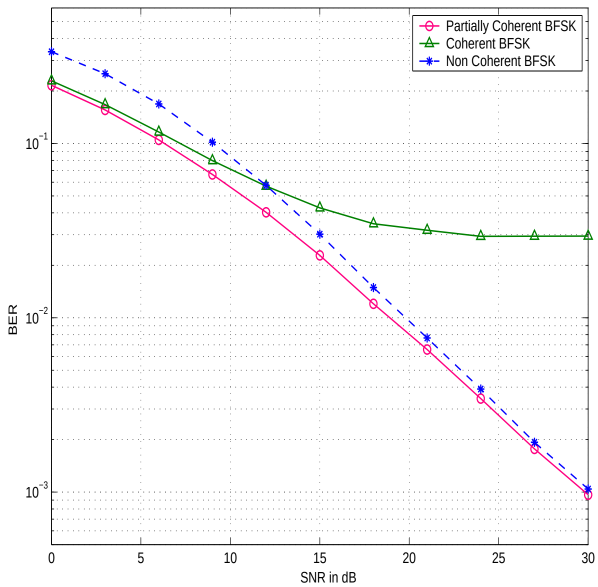

Fig. 6. Performance improvement in FSK with partial CSI for a=0.999 and N=100.

the weights determined by the channel variation rate and SNR. The decision rule is analogous to MRC combining as coherent detection is given more emphasis at slow fading and at low SNR while noncoherent detection is prominent at fast fading and high SNR. It can be noted that $a = 1$ results in a purely coherent detection while complete noncoherent detection takes place at $a = 0$. For intermediate values of a, both coherent and noncoherent detection take place. The knowledge of the channel statistic a and N_0 is required for implementing this rule, which can be obtained by monitoring the reverse link. A significant advantage of the detector is that the quality of the channel estimate i.e. the amount of coherence, does not drastically affect the system performance, unlike PSK systems. The performance of this system along with the conventional coherent and noncoherent BFSK system is shown in Fig. 6. It can be seen that it outperforms the best of coherent and noncoherent BFSK for all SNRs. A gain of 2 dB is obtained over a wide range of SNRs.

A very tight upper bound for the probability of error for this detector can be readily obtained by noting that the detector performs better than the best of coherent and noncoherent detection for any a and N.

$$\begin{aligned}\overline{P_e}(k) &\leq \min\left(\overline{P_e}^{\text{C}}(k), \overline{P_e}^{\text{NC}}\right) \qquad (38)\\ &\leq \min\left(\frac{1}{2}\left(1 - a^k\sqrt{\frac{E_s}{E_s + 2N_0}}\right), \frac{1}{2 + \frac{E_s}{N_0}}\right)\end{aligned}$$

As the quality of the channel estimate degrades with k, noncoherent FSK will outperform coherent FSK after k reaches a threshold. The probability of error $\overline{P_e}(k)$ averaged over k yields

$$\overline{P_e} \leq \frac{N - N_t}{N}\left(\frac{N_0}{2N_0 + E_s}\right) + \frac{N_t}{2N} - \frac{a}{2N}\left(\frac{1 - a^{N_t}}{1 - a}\right)\sqrt{\frac{E_s}{E_s + 2N_0}}. \qquad (39)$$

The value of N_t is chosen such that

$$\overline{P}_e^{\text{C}}(N_t) = \overline{P}_e^{\text{NC}}(N_t). \qquad (40)$$

An adaptive scheme based on this upper bound is discussed in Section VII.

A. Effect of inaccurate channel statistic a

The ideal coherent detector in (37) combines coherent and noncoherent detection in an ideal fashion such that the performance is better than the individual detectors. Hence we obtained the inequality in (39). However, when the detector possesses inaccurate knowledge of a, the ML rule combination is imperfect and results in an increased or decreased weight for both the coherent and noncoherent detection. This leads to a performance degradation with respect to the ideal detector. The greater the deviation of $\hat{a}$, the greater is the performance degradation. In the worst case scenarios, in cases where ($a < \hat{a} = 1$ or $a > \hat{a} = 0$) only one of the detectors will be utilized. Thus, the error probability of the detector can be bounded as

$$\overline{P}_e \leq \max(\overline{P}_e^{\text{C}}, \overline{P}_e^{\text{NC}}).$$

Table II shows the error probability of a non-ideal detector for extreme values of a and $\hat{a}$.

a	$\hat{a}$	$\overline{P}_e$
1	1	$\overline{P}_e^{\text{C}}$
1	0	$\overline{P}_e^{\text{NC}}$
0	1	0.5
0	0	$\overline{P}_e^{\text{NC}}$

TABLE II

ERROR PROBABILITY OF PARTIALLY COHERENT DETECTION FOR EXTREME VALUES OF a AND $\hat{a}$

1) $\hat{a} < a$: When a is underestimated, it results in an overemphasis of noncoherent detection. This suggests that the channel knowledge is not completely utilized by the coherent part of the partially coherent detector. Its effect is more pronounced at low SNR.The performance of the detector is bounded as

$$\overline{P}_e \leq \overline{P}_e^{\text{NC}} = \frac{1}{(2 + \frac{E_s}{N_0})}.$$

The worst case scenario is when the channel is assumed to be invariant when it is i.i.d, i.e $\hat{a} = 0 < a = 1$. It can be readily noticed that the performance loss in this case is 3dB. However such a mismatch is not realistic and the performance loss for practical cases is insignificant as shown by the simulation results in Fig. 7 for $a = 0.985$ and $\hat{a} = 0.999$.

2) $\hat{a} > a$: When the channel correlation is assumed to be higher than the actual value, coherent detection will be

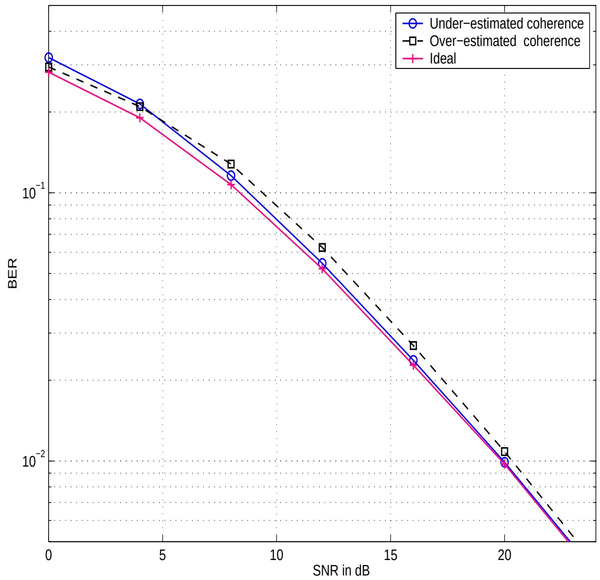

Fig. 7. Performance degradation due to imperfect estimate of $a(0.985)$: over-estimated coherence ($\hat{a} = 0.999$), under-estimated coherence ($\hat{a} = 0.95$) and the ideal case ($\hat{a} = a = 0.985$).

overemphasized by the partially coherent detector. Thus we have,

$$\overline{P}_e \le \overline{P}_e^{\mathrm{C}} = \frac{1}{2}\left[1 - \frac{a}{N}\left(\frac{1-a^N}{1-a}\right)\sqrt{\frac{E_s}{E_s + 2N_0}}\right].$$

The worst case scenario occurs when $\hat{a} = 1 > a = 0$. The corresponding performance loss or the increase in the error probability is given by

$$D = \frac{1}{2} - \frac{1}{(2 + \frac{E_s}{N_0})},$$

where the first term corresponds to the probability of error of the mismatched detector and the second term is the error probability of the ideal detector The effect of this type of error is more pronounced at high SNR.

B. A Practical Scheme

From the preceding discussion, it is quite clear that the impact of inaccurate a is not drastic. The performance loss is insignificant when the correlation estimate is less than the actual value. This leads to a practical scheme in receivers that employ the minimum value of the correlation parameter.

$$\hat{a} = a_{min} = \mathcal{J}_0(2\pi f_d^{max} T_s)$$

where $f_d^{max} = \frac{v^{max} f_c}{C}$ corresponds to the Doppler shift associated with the maximum mobility allowed by the system.

It can be easily noted that assuming lower channel coherence than the actual channel correlation does not result in significant performance loss. The reduced coherence plot in Fig. 7 represents the performance of a partially coherent detector employing this scheme. Significant gain is obtained over the best of the coherent and noncoherent detector and the performance loss is not significant when compared to the ideal detector. Thus the partially coherent detection can be performed even without explicitly determining the channel correlation parameter a.

VI. DPSK WITH PARTIAL CSI

In this section, we derive a partially coherent detector for DPSK. Suppose u_k and x_k represent the actual data and transmitted symbol respectively at the k^{th} time slot. The received symbol can be written as

$$\begin{aligned} y_k &= h_k x_k + n_k && (41)\\ y_{k+1} &= h_{k+1}x_{k+1} + n_{k+1} && (42)\\ &= h_{k+1}x_k u_{k+1} + n_{k+1}. && (43) \end{aligned}$$

The maximum likelihood detection will be

$$\Pr\left(y_k, y_{k+1}|a, h_0, u_{k+1} = -1\right) \gtrless_{1}^{-1} \Pr\left(y_k, y_{k+1}|a, h_0, u_{k+1} = 1\right). \quad (44)$$

Now the required probability is written as a mixture of two Gaussian distributions,

$$\begin{aligned} &\Pr\left(y_k,\ y_{k+1}|a, h_0, u_{k+1} = s_m\right) = \\ &\quad \frac{1}{2}\Pr\left(y_k,\ y_{k+1}|a, h_0, u_{k+1} = s_m, x_k = 1\right) + \\ &\quad\quad \frac{1}{2}\Pr\left(y_k,\ y_{k+1}|a, h_0, u_{k+1} = s_m, x_k = -1\right). \end{aligned} \quad (45)$$

The joint probability distribution of the received vector conditioned on u_{k+1}, x_k, h_0 and a is given by

$$\Pr\left(\mathbf{y}^k|u_{k+1}, x_k, h_0, a\right) = \frac{1}{\pi^2 \det\left(K_y\right)} e^{-\left[\left(\mathbf{y}^k - \mathbf{m}^k\right)^\dagger K_y^{-1}\left(\mathbf{y}^k - \mathbf{m}^k\right)\right]} \quad (46)$$

where $\mathbf{y}^k = \left[y_k, y_{k+1}\right]^T$ and $\mathbf{m}^k = \left[a^k h_0 x_k, a^{k+1} h_0 x_k u_{k+1}\right]^T$.

The covariance matrix K_y^k is obtained as

$$K_y^k = \begin{bmatrix} 1 - a^{2k} + N_0 & \left(a - a^{2k+1}\right) u_{k+1} \\ \left(a - a^{2k+1}\right) u_{k+1} & 1 - a^{2k+2} + N_0 \end{bmatrix}.$$

The detection rule in (44) cannot be further simplified after substituting (45) and therefore it is quite complex to implement. The performance of a DPSK system employing this detection rule is shown in Fig. 8. Although the performance is better than the conventional DPSK for all SNR and fading rate a, the gain achieved from the channel knowledge is at most 1 dB at low SNR range.

VII. ADAPTIVE SCHEMES

It is clear from the previous sections that the performance of modulation schemes is sensitive to many parameters. The varied performance of the modulation schemes in a time varying channel provides us the opportunity to adapt the modulation schemes to the channel conditions. Due to the time varying nature of the channel, the quality of the channel estimate in coherent schemes degrades with symbol position, thereby making the probability of error dependent on the symbol position. This motivates us to employ coherent modulation for certain number of symbols in the block till the channel estimate quality is good and operate noncoherently for the

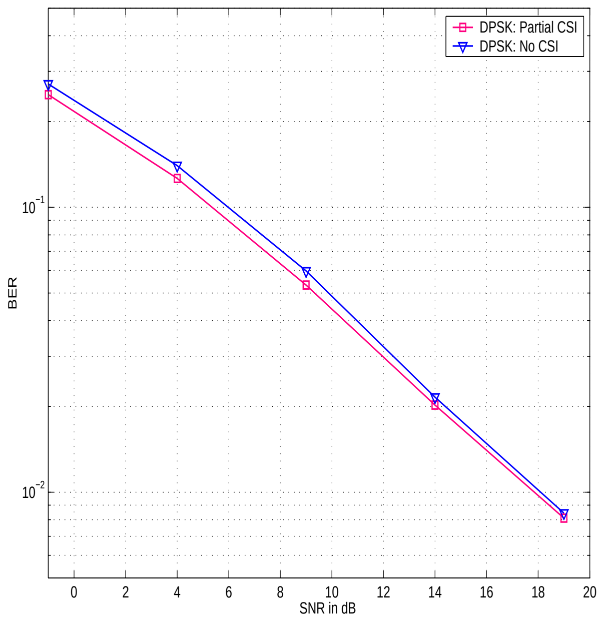

Fig. 8. Performance improvement in DPSK with partial CSI for a=0.999.

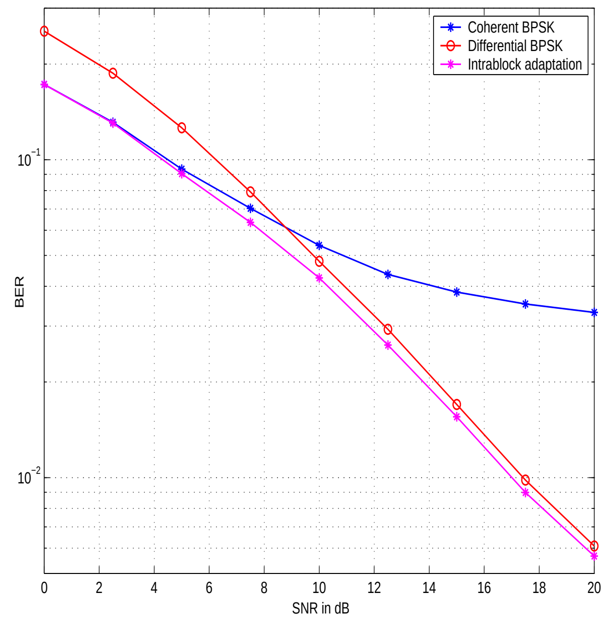

Fig. 9. Performance of the intra-block adaptation scheme at a=0.999 and N=100.

rest of the symbols in the block . We call this as an *intra-block adaptation*. From the BER plots in Fig. 1 and from the ML rule in (37), it is clear that an ideal modulation scheme should exhibit the performance of a coherent scheme at low SNR and low Doppler while noncoherent behavior is desired at high SNR or high Doppler. This is the basis for adaptation in *inter-block adaptation*.

A. Intra-Block Adaptation

With accurate channel estimate at the receiver, coherent BPSK is the best modulation and detection scheme. Ideally it is 3dB and 6dB superior to DPSK and NC-BFSK respectively. Therefore, if we can employ BPSK modulation in some part of the data transmission when the quality of the channel estimate is good, the overall performance will be better than with a wholly noncoherent operation. The symbol error probability of individual symbol positions with BPSK and DPSK are given by (10) and (16) respectively. It can be seen that for small values of k, BPSK performs better than DPSK regardless of a and SNR. Thus the transmission strategy is to send first N_t symbols with BPSK modulation and the remaining $(N-N_t)$ symbols with DPSK modulation.

$$\phi_k = \begin{cases} \theta_k & k \le N_t \\ \phi_{k-1}+\theta_k & k > N_t \end{cases} \tag{47}$$

The value of N_t is chosen such that

$$\overline{P}_e^{\text{BPSK}}(N_t) = \overline{P}_e^{\text{DPSK}}(N_t). \tag{48}$$

The average symbol error probability is then given by

$$\overline{P_e} = \frac{\sum_{i=1}^{N_t} \overline{P}_e^{\text{BPSK}}(i) + (N-N_t)\,\overline{P}_e^{\text{DPSK}}}{N} \tag{49}$$

Upon substituting (12) and (16), the average probability of error of the intra block adaptation scheme (49) becomes

$$\overline{P_e} = \frac{1}{2}\left(1 - \frac{a}{N}\left(\frac{1-a^{N_t}}{1-a}\right)\sqrt{\frac{E_s}{E_s+N_0}} - \frac{(N-N_t)\,a^2E_s}{N\,(1+a^2)\,(1+N_0)}\right) \tag{50}$$

If the average error probability alone is the metric to minimize, without considering the rate loss associated with FSK, adaptation strategy can include FSK modulation as well. In that case, the modulation scheme that has superior performance for that channel variation rate a should be selected. When the performance of NCFSK is superior to DPSK, the average probability of error for this adaptation is

$$\overline{P_e} = \frac{\sum_{i=1}^{N_t} \overline{P}_e^{\text{BPSK}}(i) + (N-N_t)\,\overline{P}_e^{\text{NCFSK}}}{N}. \tag{51}$$

Upon substituting (21), the above expression becomes

$$\overline{P_e} = \frac{N-N_t}{N}\left(\frac{N_0}{2N_0+E_s}\right) + \frac{N_t}{2N} - \frac{a}{2N}\left(\frac{1-a^{N_t}}{1-a}\right)\sqrt{\frac{E_s}{E_s+N_0}}. \tag{52}$$

Fig. 9 shows the performance of intra-block adaptive scheme versus the individual modulation schemes. A gain of about 1 dB is achieved for a wide range of SNR values.

B. Inter-Block Adaptation

Even though the modulation schemes like BPSK and DPSK are susceptible to error floors in a fast fading channel, they are optimal at low SNR when the noise power is comparable to the SNR loss caused due to channel decorrelation. Thus the knowledge of the received SNR is crucial in employing optimal modulation strategies. Therefore an adaptive scheme should also consider shadowing, which causes deviations in received SNR. A model to include shadowing is

$$h(k) = \sqrt{G}r(k).$$

where G is the local mean received power which varies slowly due to shadowing and $r(k)$ is the fast fade that follows the autoregressive model given by (1). We assume a lognormal distribution for the shadowing in which an entire block of N symbols experience a particular realization of lognormal shadowing and the shadowing parameter for the next block is independent of the previous block. Shadowing results in the average received power to slowly vary and can be tracked by the transmitter using the reverse transmission link. Fig. 10 shows the performance of inter-block adaptive scheme versus the individual modulation schemes for a standard deviation of 7 dB for the log-normal distribution. The performance gain is maximum near the cross over of the curves, as shadowing alters the order of the performance of the modulation schemes. The inter-block adaptive scheme in general can also include intra block adaptation.

VIII. CONCLUSION

Rapid channel variations caused by mobility, lead to loss in effective SNR for modulation schemes operating in the bandlimited regime of the capacity curve [2] resulting in an error floor. This is true even for differential schemes that do not require channel knowledge at the receiver. The performance loss due to mobility is lesser with orthogonal modulation schemes operating noncoherently, compared to differential or coherent schemes. We analyzed noncoherent FSK as a possible candidate for bandlimited systems with high mobility and found that its spectral inefficiency makes it unsuitable for operating at nominal values of Doppler and SNR. However FSK (and similar orthogonal modulation schemes like PPM that allow noncoherent detection) are the ideal modulation schemes for systems that need to operate under very large mobility as in some military applications.

Finally, as noncoherent schemes are increasingly being deployed in many wireless systems, there is a scope for significant performance improvement in these systems when the role of partial CSI is considered. The partially coherent detector that we derived for FSK is simple and at the same time provides significant performance improvement over both coherent and noncoherent FSK detection. We also showed that opportunistic adoption of various modulation schemes within a block or between blocks can result in substantial performance improvements.

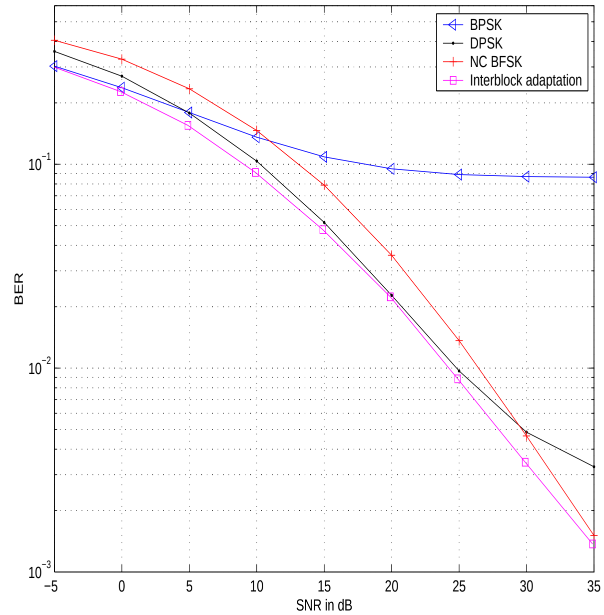

Fig. 10. Inter-block adaptation for a=0.999 and N=100.

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