Moduli instability in warped compactification - 4D effective theory approach

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We consider a 5D BPS dilatonic two brane model which reduces to the Randall-Sundrum model or the Hořava-Witten theory for a particular choice of parameters. Recently new dynamical solutions were found by Chen *et al.*, which describe a moduli instability of the warped geometry. Using a 4D effective theory derived by solving the 5D equations of motion, based on the gradient expansion method, we show that the exact solution of Chen *et al.* can be reproduced within the 4D effective theory and we identify the origin of the moduli instability. We revisit the gradient expansion method with a new metric ansatz to clarify why the 4D effective theory solution can be lifted back to an exact 5D solution. Finally we argue against a recent claim that the 4D effective theory allows a much wider class of solutions than the 5D theory and provide a way to lift solutions in the 4D effective theory to 5D solutions perturbatively in terms of small velocities of the branes.

I. INTRODUCTION

The idea of using the degrees of freedom of extra spatial dimensions to unify gravity and the other interactions has attracted much interest. Especially, M-Theory has the potential to achieve this ultimate goal. Hořava and Witten showed that M-Theory compactified as $S^1/Z_2 \times M^{10}$ reduces to a known string theory ($E_8 \times E_8$ Heterotic String) [1]. From a low energy effective theory point of view, this model has a 5D spacetime (the bulk) with two 4D boundary hypersurfaces (the branes) [2]. The remaining six spatial dimensions are assumed to be compactified. All the Standard Model particles are confined to the branes while gravity can propagate in the bulk. As a consequence of the compactification of the six spatial dimensions, a 5D effective scalar field appears in the bulk, which describes the volume of the compactified 6D space.

A novel aspect of this model is that the fifth dimension is not homogeneous. The line element for a static solution is given in the form $ds_5^2 = h^\alpha(r)dr^2 + h^\beta(r)ds_4^2(x)$, where r is the fifth coordinate, α, β are constants and h is called the warp factor. This warped geometry is used by Randall and Sundrum [3] to address the hierarchy problem. They have also showed that it is possible to localize gravity around the brane with the warped geometry, providing in this way an alternative to compactification [4] (see Ref. [5] for a review).

Recently, a new class of dynamical solutions that describes an instability of the warped geometry has been found. Chen *et al.* noticed that it is possible to obtain a dynamical solution by replacing the constant modulus in the warp factor h by a linear function of the 4D coordinates [6]. This solution describes an instability of the model as the brane will hit the singularity in the bulk, where $h = 0$. This kind of solution exists also in the 10D type IIB supergravity [7].

In this paper, we study the moduli instability in a dilatonic two brane model, where the potentials for the scalar field on the brane and in the bulk obey the BPS condition [8]. For particular values of parameters we retrieve either the Hořava-Witten theory or the Randall-Sundrum model. We identify the origin of the moduli instability using a 4D effective theory derived in Ref. [9] (see Refs. [10] and [11] for different approaches). This effective theory is derived by solving the 5D equations of motion using the gradient expansion method [12], where we assume that the velocities of the branes are small compared with the curvature scale of the bulk determined by the warp factor.

Despite the fact that the 4D effective theory is based on a slow-motion approximation, we will show that the 5D exact solutions can be reproduced in the 4D effective theory. In order to understand the relation between 4D solutions in an effective theory and full 5D solutions, we revisit the gradient expansion method by employing a new metric ansatz. Using this metric ansatz, we can clearly see why the moduli instability solution in the 4D effective theory can be lifted to an exact 5D solution.

We also comment on a recent claim that the 4D effective theory allows a much wider class of solutions than the 5D theory [13]. We disagree with that conclusion and we show that it is based on the restricted form of the 5D metric ansatz used in Ref. [13]. Using our metric ansatz, we provide a way to lift solutions in the 4D effective theory to 5D solutions perturbatively in terms of small velocities of the branes.

The structure of the paper is as follows. In section II, the model used in this paper is described in detail. In section III, we identify the solution in the 4D effective theory that describes the moduli instability. In section IV, we revisit

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the gradient expansion method to derive the 4D effective theory. We propose a new metric ansatz which is useful to relate 4D solutions in the effective theory to 5D solutions. Using this formalism we explain why the 4D solution for the moduli instability can be lifted to an exact 5D solution. In section V, we comment on the arguments against the 4D effective theory. Section VI is devoted to conclusions.

II.   THE MODEL

Our model consists of a 5D spacetime (bulk) filled with a scalar field. The fifth dimension is a compact space S^1 with a Z_2 symmetry (i.e. identification $r \to -r$) and will be parameterized by the coordinate r, $-L \leq r \leq L$. The bulk action takes the form

$$S_{bulk} = \frac{1}{2\kappa_G^2} \int d^5x \sqrt{-g_5} \left[{}^5R - \nabla_M \varphi \nabla^M \varphi + V_{bulk}(\varphi) \right],$$ (1)

where κ_G denotes the 5D gravitational constant. Throughout this work latin indices can have values in $\{r, t, x, y, z\}$, while greek indices do not include the extra dimension coordinate r. The scalar field potential will be

$$V_{bulk}(\varphi) = -\left(b^2 - \frac{2}{3} \right) e^{-2\sqrt{2}b\varphi} \sigma^2,$$ (2)

where σ and b are the remaining parameters of this model. We can retrieve either the Randall-Sundrum model ($b = 0$) or the Hořava-Witten model ($b = 1$) according to the value of b. The orbifold S^1/Z_2 has two fixed points, at $r = 0$ and $r = L$, and we can put branes there. The branes' action is

$$S_{branes} = S_+ + S_-,$$ (3)

where S_+ and S_- are the positive and negative tension brane action, respectively. They are given by

$$S_\pm = \mp \frac{\sqrt{2}}{\kappa_G^2} \int d^4x \sqrt{-g_4} e^{-\sqrt{2}b\varphi} \sigma.$$ (4)

The action of our model is

$$S_{total} = S_{bulk} + S_{branes}.$$ (5)

III.   EXACT SOLUTIONS FOR MODULI INSTABILITY IN 4D EFFECTIVE THEORY

A.   The exact 5D solution

Exact static solutions for the equations of motion that result from (5) were obtained in [8]. Recently, an interesting dynamical solution was found by Chen *et al.* [6]. They noticed that if we add a linear function of time to the warp factor of the solution of [8], it would still be a solution of the equations of motion. Their exact 5D solution reads

$$dS_5^2 = \left(h\tau - \frac{|r|}{l} \right)^{\frac{6b^2-2}{3b^2+1}} dr^2 + \left(h\tau - \frac{|r|}{l} \right)^{\frac{2}{3b^2+1}} \eta_{\mu\nu} dx^\mu dx^\nu,$$ (6)

$$\varphi = \frac{3\sqrt{2}b}{3b^2+1} \ln \left(h\tau - \frac{|r|}{l} \right), \quad l = \frac{3}{3b^2+1} \frac{\sqrt{2}}{\sigma},$$ (7)

where h is an arbitrary constant.

From the above equations we can read the scale factor of the positive tension brane ($r = 0$) as

$$a_+^2(\tau) = (h\tau)^{\frac{2}{3b^2+1}},$$ (8)

and the scalar field as

$$\varphi(r = 0, \tau) = \frac{3\sqrt{2}b}{3b^2+1} \ln(h\tau).$$ (9)

Let us choose $h < 0$ and $\tau < 0$. In this case the proper distance between branes, the so-called radion, is decreasing according to

$$\mathcal{R} = \int_0^L \left(h\tau - \frac{r}{l} \right)^{\frac{3b^2-1}{3b^2+1}} dr = l\frac{3b^2+1}{6b^2}(h\tau)^{\frac{6b^2}{3b^2+1}} \left[1 - \left(1 - \frac{L/l}{h\tau} \right)^{\frac{6b^2}{3b^2+1}} \right]. \tag{10}$$

When $\tau \to -\infty$, $\mathcal{R} \to L\,(h\tau)^{\frac{3b^2-1}{3b^2+1}}$ and for $\tau = L/hl$, $\mathcal{R} = l(3b^2 + 1)\,(L/l)^{\frac{6b^2}{3b^2+1}}/6b^2$. Before the two branes collide, a curvature singularity will appear in the negative tension brane ($r = L$) at $\tau = L/hl$. This singularity will move towards the positive tension brane and will reach it at $\tau = 0$. This event represents the total annihilation of the spacetime.

It is useful to note that if we drop the modulus sign in (6), the bulk spacetime is a static black brane or black hole solution depending on b, see [6] for more details. If $b = 1$, there is a timelike curvature singularity at $h\tau = |r|/l$. From the static bulk point of view, the two branes are moving in this static bulk and the negative tension brane first hits the singularity. Even if the bulk spacetime is static, the existence of the branes, which gives the modulus sign for r, makes the spacetime truly time dependent.

Moduli instability is a serious problem for these types of model. In this work, we will try to understand it from a 4D effective theory viewpoint.

B. The exact solutions in 4D effective theory

In [9], Kobayashi and Koyama applied the gradient expansion method to solve perturbatively the 5D equations of motion resulting from (5). Their 0th order solution reads

$$dS_5^2 = d(t)^2 e^{2\sqrt{2}b\phi(t)} dr^2 + \mathcal{F}(r,t)^{\frac{1}{3b^2}} h_{\mu\nu}(t) dx^\mu dx^\nu, \tag{11}$$

$$\varphi(r,t) = \frac{1}{\sqrt{2}b} \ln \mathcal{F}(r,t) + \phi(t), \quad \mathcal{F}(r,t) = 1 - \sqrt{2}b^2 d(t)\sigma|r|. \tag{12}$$

The main result of the paper is a set of equations involving only four dimensional quantities that describe the dynamics of the unknown functions $d(t)$, $h_{\mu\nu}(t)$ and $\phi(t)$, appearing in Eqs. (11) and (12). It can be shown that their dynamical equations can be consistently deduced from the following action

$$S_{eff} = \frac{l}{2\kappa_G^2} \int d^4x \sqrt{-h} e^{\sqrt{2}b\phi} \left[\psi R(h) - \frac{3}{2(1+3b^2)} \frac{1}{1-\psi} \nabla_\alpha \psi \nabla^\alpha \psi - \psi \nabla_\alpha \phi \nabla^\alpha \phi \right], \tag{13}$$

where here ∇_α means the covariant derivative with respect to $h_{\mu\nu}$ and $\psi = 1 - (1-d)^{\frac{3b^2+1}{3b^2}}$. This is the action we obtain if we substitute Eqs. (11) and (12) into (5) and integrate over the extra dimension [10].

Performing the conformal transformation $h_{\mu\nu} = e^{-\sqrt{2}b\phi} f_{\mu\nu}/\psi$ and defining new scalar fields as

$$\psi = 1 - \tanh^2 \left[\sqrt{\frac{1+3b^2}{6}} \Psi \right], \tag{14}$$

$$\phi = \frac{\Theta}{\sqrt{1+3b^2}} + \lambda \ln \psi, \quad \lambda \equiv -\frac{3}{\sqrt{2}} \frac{b}{3b^2+1}, \tag{15}$$

we arrive at the action in the Einstein frame

$$S_E = \frac{l}{2\kappa_G^2} \int d^4x \sqrt{-f} \left[R(f) - \Theta_{|\alpha}\Theta^{|\alpha} - \Psi_{|\alpha}\Psi^{|\alpha} \right]. \tag{16}$$

It is clear that the moduli fields have no potential and this is the origin of the instability. We are interested in cosmological solutions so we choose $f_{\mu\nu}$ to be a flat FRW metric. The Einstein equations resulting from the action (16) are

$$\frac{a''}{a} = -\frac{1}{6} \left(\Psi'^2 + \Theta'^2 \right), \quad \left(\frac{a'}{a} \right)^2 = \frac{1}{6} \left(\Psi'^2 + \Theta'^2 \right). \tag{17}$$

and the equations of motion for the scalar fields are

$$\Psi'' + 2\frac{a'}{a}\Psi' = 0, \quad \Theta'' + 2\frac{a'}{a}\Theta' = 0, \tag{18}$$

where $a(\tau)$ is the FRW scale factor and the prime denotes derivative with respect to the conformal time τ. Equations (17,18) can be easily integrated to find

$$\Psi(\tau) = \sqrt{\frac{6}{3b^2+1}}\alpha \ln a^2 + \gamma, \quad \Theta(\tau) = \sqrt{1+3b^2}\beta \ln a^2 + \delta, \quad a^2(\tau) = \xi\tau + \zeta, \tag{19}$$

$$\alpha^2 + \frac{(1+3b^2)^2}{6}\beta^2 = 1 + 3b^2, \tag{20}$$

where α, β, γ, δ, ζ and ξ are integration constants. The solutions for the fields in the original frame can be obtained as

$$d(\tau) = 1 - \left[\frac{(\mu\tau+\nu)^{2\alpha}-1}{(\mu\tau+\nu)^{2\alpha}+1}\right]^{\frac{6b^2}{3b^2+1}}, \tag{21}$$

$$\phi(\tau) = \ln\left[\frac{(\mu\tau+\nu)^{2\alpha\lambda(b)+\beta}}{[(\mu\tau+\nu)^{2\alpha}+1]^{2\lambda(b)}}\right] + \varrho, \tag{22}$$

where $\lambda(b)$ is defined in Eq. (15) and integration constants μ, ν, ϱ are redefined from ξ, ζ, γ and δ. The radion is calculated as

$$\mathcal{R}(\tau) = \frac{1}{\sqrt{2}b^2\sigma}d(\tau)e^{\sqrt{2}b\phi(\tau)}, \tag{23}$$

and the square of the scale factor on the positive tension brane is

$$a_+^2(\tau) = \frac{e^{-\sqrt{2}b\phi(\tau)}}{\psi(\tau)}a^2(\tau). \tag{24}$$

We have found the remarkable fact that for a particular choice of the integration constants we can reproduce the exact solution found by Chen *et al.* described in the previous section. If we choose integration constants obeying the relations

$$\alpha = \frac{1}{2}, \quad \beta = -\lambda(b), \quad \mu = \frac{2hl}{L}, \quad \nu = -1, \tag{25}$$

we get the same 4D quantities (scale factor and scalar field on the positive tension brane) and radion as the Chen *et al.* solution, Eqs. (8, 9, 10).

For this solution, the scale factor in the Einstein frame is given by

$$a^2(\tau) = \frac{2L}{l}\left(h\tau - \frac{L}{2l}\right). \tag{26}$$

If we take $h < 0$, $\tau < 0$, this corresponds to a collapsing universe, due to the kinetic energy of the scalar fields. At $\tau = L/2hl < 0$, the universe in the Einstein frame reaches the Big-bang singularity. However, we should be careful to interpret this singularity. In fact, in the original 5D theory, $\tau = L/2hl$ does not correspond to any kind of singularity. In 5D theory, the negative tension brane hits the singularity at $\tau = L/hl$ and the positive tension brane hits the singularity at $\tau = 0$. In fact at $\tau = L/2hl$, $\psi = 0$ and so the conformal transformation becomes singular and the Einstein frame metric loses physical meaning.

IV. GRADIENT EXPANSION METHOD WITH A NEW METRIC ANSATZ

In the previous section, we found that the exact solution derived in [6] can be reproduced from the 0th order of the perturbative method. In this section, we will revisit the gradient expansion method which is used to derive the effective theory to clarify the reason why the exact solution derived in [6] can be reproduced within the 4D effective theory.

A. 5D Equations

In this subsection we study the 5D equations of motion. From the action (5) we derive the Einstein's equations

$$
{}^5G_{AB} = \nabla_A\varphi\nabla_B\varphi + \frac{1}{2}g_{AB}\left[-\nabla^C\varphi\nabla_C\varphi + V_{bulk}(\varphi)\right]
$$
$$
+\sqrt{2}\sigma\left[-\frac{\sqrt{g_4}}{\sqrt{g_5}}g_{\mu\nu}\delta^\mu_A\delta^\nu_B e^{-\sqrt{2}b\varphi}\delta(r) + \frac{\sqrt{g_4}}{\sqrt{g_5}}g_{\mu\nu}\delta^\mu_A\delta^\nu_B e^{-\sqrt{2}b\varphi}\delta(r-L)\right]. \tag{27}
$$

Assuming that the 5D line element has the form

$$
dS_5^2 = g_{AB}(X)dX^AdX^B = g_{rr}(x,r)dr^2 + g_{\mu\nu}(x,r)dx^\mu dx^\nu, \tag{28}
$$

(x dependence means dependence of all the other coordinates $\{x,y,z,t\}$, except the extra dimension r) we can extract the junction conditions for the metric tensor from Einstein's equation,

$$
\sqrt{g^{rr}}\left(K^\mu_\nu - \delta^\mu_\nu K\right)\Big|_{\substack{r=0^+ \\ r=L^-}} = \mp\frac{1}{\sqrt{2}}\sigma e^{-\sqrt{2}b\varphi}\delta^\mu_\nu\Big|_{\substack{r=0^+ \\ r=L^-}}, \tag{29}
$$

where the extrinsic curvature $K_{\mu\nu}$ is defined by $K_{\mu\nu} = -\frac{1}{2}\partial_r g_{\mu\nu}$. The scalar field equation of motion is

$$
\Box\varphi + \sqrt{2}b\left(b^2 - \frac{2}{3}\right)\sigma^2 e^{-2\sqrt{2}b\varphi} = -2b\sigma\left[\sqrt{g^{rr}}e^{-\sqrt{2}b\varphi}\delta(r) - \sqrt{g^{rr}}e^{-\sqrt{2}b\varphi}\delta(r-L)\right], \tag{30}
$$

and so the junction conditions for the scalar field are

$$
\sqrt{g^{rr}}\partial_r\varphi\Big|_{\substack{r=0^+ \\ r=L^-}} = \mp b\sigma e^{-\sqrt{2}b\varphi}\Big|_{\substack{r=0^+ \\ r=L^-}}. \tag{31}
$$

In order to proceed we shall assume that the 5D line element has further symmetries, described by

$$
dS_5^2 = e^{2\sqrt{2}b\tilde\varphi(x,r)}H^{\frac{6b^2-2}{3b^2+1}}(x,r)dr^2 + H^{\frac{2}{3b^2+1}}(x,r)\tilde g_{\mu\nu}dx^\mu dx^\nu, \quad H(x,r) = C(x) - \frac{r}{l}. \tag{32}
$$

In this section, we will assume that the position of the second brane is $r = L = l$. For the scalar field we will assume the form

$$
\varphi(x,r) = \frac{3\sqrt{2}b}{3b^2+1}\ln H + \tilde\varphi(x,r). \tag{33}
$$

This metric ansatz is inspired by the time dependent solution (6,7) of Chen *et al.* [6]. Their solution was found by replacing the modulus parameter in the static solution by a linear function of time. In the same manner, we introduce an x dependence in H through the modulus parameter $C(x)$ in a covariant way . We also introduce the function $\tilde\varphi$ for the scalar field moduli. In order to satisfy the junction conditions (31) we must have the exponential factor in the g_{rr} metric component. The tensor $\tilde g_{\mu\nu}$ is left completely general.

After some mathematical manipulations of the Einstein equations we obtain

$$
\frac{1}{l}e^{-2\sqrt{2}b\tilde\varphi}H^{\frac{-9b^2+1}{3b^2+1}}\left(\frac{3b^2-5}{3b^2+1}\tilde K^\mu_\nu - \frac{1}{3b^2+1}\delta^\mu_\nu\tilde K\right) + e^{-2\sqrt{2}b\tilde\varphi}H^{\frac{-6b^2+2}{3b^2+1}}\left(\tilde K^\mu_{\nu,r} - \tilde K\tilde K^\mu_\nu\right)
$$
$$
-\sqrt{2}be^{-2\sqrt{2}b\tilde\varphi}H^{\frac{-3b^2+1}{3b^2+1}}\tilde\varphi_{,r}\left(\frac{1/l}{3b^2+1}H^{\frac{-6b^2}{3b^2+1}}\delta^\mu_\nu + H^{\frac{-3b^2+1}{3b^2+1}}\tilde K^\mu_\nu\right) - H^{\frac{-3b^2-3}{3b^2+1}}\left(C^{|\mu}_{|\nu} + \frac{1}{3b^2+1}C^{|\alpha}_{|\alpha}\delta^\mu_\nu\right)
$$
$$
-H^{-\frac{2}{3b^2+1}}\left[\sqrt{2}b\tilde\varphi^{|\mu}_{|\nu} + (2b^2+1)\tilde\varphi^{|\mu}\tilde\varphi_{|\nu}\right] - \frac{\sqrt{2}b}{3b^2+1}H^{\frac{-3b^2-3}{3b^2+1}}\left[(3b^2+1)\left(\tilde\varphi^{|\mu}C_{|\nu} + \tilde\varphi_{|\nu}C^{|\mu}\right) + \tilde\varphi^{|\alpha}C_{|\alpha}\delta^\mu_\nu\right]
$$
$$
+H^{-\frac{2}{3b^2+1}}R^\mu_\nu(\tilde g) = 0, \tag{34}
$$

$$
\frac{1}{l}\frac{3b^2-3}{3b^2+1}e^{-2\sqrt{2}b\tilde\varphi}H^{\frac{-9b^2+1}{3b^2+1}}\tilde K + e^{-2\sqrt{2}b\tilde\varphi}H^{\frac{-6b^2+2}{3b^2+1}}\left(\tilde K_{,r} - \tilde K^{\alpha\beta}\tilde K_{\alpha\beta}\right)
$$
$$
+e^{-2\sqrt{2}b\tilde\varphi}H^{\frac{-6b^2+2}{3b^2+1}}\tilde\varphi_{,r}\left(\frac{1}{l}\frac{2\sqrt{2}b}{3b^2+1}H^{-1} - \sqrt{2}b\tilde K - \tilde\varphi_{,r}\right)
$$
$$
-H^{\frac{-3b^2-3}{3b^2+1}}\left(\frac{3b^2-1}{3b^2+1}C^{|\alpha}_{|\alpha} + \frac{6\sqrt{2}b^3}{3b^2+1}\tilde\varphi^{|\alpha}C_{|\alpha}\right) - \sqrt{2}bH^{-\frac{2}{3b^2+1}}\left(\tilde\varphi^{|\alpha}_{|\alpha} + \sqrt{2}b\tilde\varphi^{|\alpha}\tilde\varphi_{|\alpha}\right) = 0, \tag{35}
$$

$$\tilde{K}^{\alpha}_{\beta|\alpha} - \tilde{K}_{|\beta} + \sqrt{2}b\left(\tilde{\varphi}_{|\beta}\tilde{K} - \tilde{\varphi}_{|\alpha}\tilde{K}^{\alpha}_{\beta}\right) + H^{-1}\left(\frac{3b^2-2}{3b^2+1}C_{|\beta}\tilde{K} - \frac{3b^2-5}{3b^2+1}C_{|\alpha}\tilde{K}^{\alpha}_{\beta}\right)$$

$$= -\tilde{\varphi}_{,r}\left(\frac{3\sqrt{2}b}{3b^2+1}H^{-1}C_{|\beta} + \tilde{\varphi}_{|\beta}\right), \tag{36}$$

where $\tilde{K}_{\mu\nu}$ is defined like $\tilde{K}_{\mu\nu} = -\frac{1}{2}\partial_r \tilde{g}_{\mu\nu}$, $\tilde{K}$ is the trace of $\tilde{K}_{\mu\nu}$, $_{|\alpha}$ denotes covariant derivative with respect to $\tilde{g}_{\mu\nu}$ and $R_{\mu\nu}(\tilde{g})$ is the Ricci tensor of $\tilde{g}_{\mu\nu}$. Equation (30) transforms into

$$e^{-2\sqrt{2}b\tilde{\varphi}}H^{\frac{-3b^2+5}{3b^2+1}}\left[\tilde{\varphi}_{,rr} - \sqrt{2}b\left(\tilde{\varphi}_{,r}\right)^2 - \tilde{\varphi}_{,r}\tilde{K} + \frac{1}{l}\frac{1}{3b^2+1}H^{-1}\left(3\sqrt{2}b\tilde{K} + \left(9b^2-5\right)\tilde{\varphi}_{,r}\right)\right]$$

$$+H\tilde{\varphi}^{|\alpha}_{|\alpha} + \frac{3\sqrt{2}b}{3b^2+1}C^{|\alpha}_{|\alpha} + \sqrt{2}bH\tilde{\varphi}_{|\alpha}\tilde{\varphi}^{|\alpha} + \frac{9b^2+1}{3b^2+1}\tilde{\varphi}_{|\alpha}C^{|\alpha} = 0. \tag{37}$$

With this particular ansatz the junction conditions (29,31) are significantly simplified and read

$$\left[\tilde{K}^{\mu}_{\nu}\right]\Big|^{r=0^+}_{r=l^-} = 0, \tag{38}$$

$$\left[\tilde{\varphi}_{,r}\right]\Big|^{r=0^+}_{r=l^-} = 0. \tag{39}$$

It is impossible to solve these equations in general, so in the next section, we will solve them using the gradient expansion method up to first order in the perturbations.

B.   The gradient expansion

1.   The approximation

In order to use perturbation theory to solve a system of differential equations we need to identify the characteristic scale of the different terms involved in the equations and then see if there is a small parameter.

The derivatives along the extra dimension of the conformal metric $\tilde{g}_{\mu\nu}$ as well as the derivative of $\tilde{\varphi}$ are of order $1/l$. We assume that variations along the branes' coordinates are small in comparison with $1/l$. This implies that the radion changes slowly or that the speed of the branes is small. More precisely, our small parameters will be

$$l^2\partial^2_x(\dots) \ll 1 \quad \text{and} \quad \left(l\partial_x\dots\right)^2 \ll 1, \tag{40}$$

where $\dots$ represents the conformal metric functions, C or $\tilde{\varphi}$.

As in the usual perturbation method, we expand the unknown functions in a series

$$\tilde{g}_{\mu\nu}(r,x) = \overset{(0)}{\tilde{g}}_{\mu\nu}(r,x) + \overset{(1)}{\tilde{g}}_{\mu\nu}(r,x) + \cdots, \tag{41}$$

$$\tilde{\varphi}(r,x) = \overset{(0)}{\tilde{\varphi}}(r,x) + \overset{(1)}{\tilde{\varphi}}(r,x) + \cdots. \tag{42}$$

We impose the boundary conditions at the position of the positive tension brane

$$\overset{(n)}{\tilde{g}}_{\mu\nu}(r=0,x) = 0, \text{ for all } n > 0, \tag{43}$$

$$\overset{(n)}{\tilde{\varphi}}(r=0,x) = 0, \text{ for all } n > 0. \tag{44}$$

Other quantities are naturally expanded as

$$\tilde{K}_{\mu\nu} = \overset{(0)}{\tilde{K}}_{\mu\nu} + \overset{(1)}{\tilde{K}}_{\mu\nu} + \cdots. \tag{45}$$

2. 0th Order (Background geometry)

The 0th order system can be easily integrated with respect to the extra dimension coordinate r to get the particular solution

$$\overset{(0)}{\tilde{g}_{\mu\nu}}(r,x) = \overset{(0)}{\tilde{g}_{\mu\nu}}(x), \tag{46}$$

$$\overset{(0)}{\tilde{\varphi}}(r,x) = \overset{(0)}{\tilde{\varphi}}(x). \tag{47}$$

This solution clearly satisfies the 0th order junction conditions.

3. 1st Order

At 1st order the evolution equations are

$$\frac{1}{l}e^{-2\sqrt{2}b\overset{(0)}{\tilde{\varphi}}}H^{\frac{-9b^2+1}{3b^2+1}}\left(\frac{3b^2-5}{3b^2+1}\overset{(1)}{\tilde{K}^\mu_\nu}-\frac{1}{3b^2+1}\delta^\mu_\nu\overset{(1)}{\tilde{K}}\right)+e^{-2\sqrt{2}b\overset{(0)}{\tilde{\varphi}}}H^{\frac{-6b^2+2}{3b^2+1}}\overset{(1)}{\tilde{K}^\mu_{\nu,r}}-\frac{\sqrt{2}b}{l}\frac{1}{3b^2+1}e^{-2\sqrt{2}b\overset{(0)}{\tilde{\varphi}}}H^{\frac{-9b^2+1}{3b^2+1}}\overset{(1)}{\tilde{\varphi}_{,r}}\delta^\mu_\nu$$

$$-H^{\frac{-3b^2-3}{3b^2+1}}\left(C^{|\mu}_{|\nu}+\frac{1}{3b^2+1}C^{|\alpha}_{|\alpha}\delta^\mu_\nu\right)-H^{-\frac{2}{3b^2+1}}\left[\sqrt{2}b\,\overset{(0)}{\tilde{\varphi}^{|\mu}_{|\nu}}+(2b^2+1)\,\overset{(0)}{\tilde{\varphi}^{|\mu}}\overset{(0)}{\tilde{\varphi}_{|\nu}}\right]$$

$$-\frac{\sqrt{2}b}{3b^2+1}H^{\frac{-3b^2-3}{3b^2+1}}\left[(3b^2+1)\left(\overset{(0)}{\tilde{\varphi}^{|\mu}}C_{|\nu}+\overset{(0)}{\tilde{\varphi}_{|\nu}}C^{|\mu}\right)+\overset{(0)}{\tilde{\varphi}^{|\alpha}}C_{|\alpha}\delta^\mu_\nu\right]+H^{-\frac{2}{3b^2+1}}R^\mu_\nu\left(\overset{(0)}{\tilde{g}}\right)=0, \tag{48}$$

$$\frac{1}{l}\frac{3b^2-3}{3b^2+1}e^{-2\sqrt{2}b\overset{(0)}{\tilde{\varphi}}}H^{\frac{-9b^2+1}{3b^2+1}}\overset{(1)}{\tilde{K}}+e^{-2\sqrt{2}b\overset{(0)}{\tilde{\varphi}}}H^{\frac{-6b^2+2}{3b^2+1}}\overset{(1)}{\tilde{K}_{,r}}+\frac{1}{l}\frac{2\sqrt{2}b}{3b^2+1}e^{-2\sqrt{2}b\overset{(0)}{\tilde{\varphi}}}H^{\frac{-9b^2+1}{3b^2+1}}\overset{(1)}{\tilde{\varphi}_{,r}}$$

$$-H^{\frac{-3b^2-3}{3b^2+1}}\left(\frac{3b^2-1}{3b^2+1}C^{|\alpha}_{|\alpha}+\frac{6\sqrt{2}b^3}{3b^2+1}\overset{(1)}{\tilde{\varphi}^{|\alpha}}C_{|\alpha}\right)-\sqrt{2}bH^{-\frac{2}{3b^2+1}}\left(\overset{(0)}{\tilde{\varphi}^{|\alpha}_{|\alpha}}+\sqrt{2}b\,\overset{(0)}{\tilde{\varphi}^{|\alpha}}\overset{(0)}{\tilde{\varphi}_{|\alpha}}\right)=0, \tag{49}$$

$$e^{-2\sqrt{2}b\overset{(0)}{\tilde{\varphi}}}H^{\frac{-3b^2+5}{3b^2+1}}\left[\overset{(1)}{\tilde{\varphi}_{,rr}}+\frac{1}{l}\frac{1}{3b^2+1}H^{-1}\left(3\sqrt{2}b\,\overset{(1)}{\tilde{K}}+(9b^2-5)\overset{(1)}{\tilde{\varphi}_{,r}}\right)\right]$$

$$+H\overset{(0)}{\tilde{\varphi}^{|\alpha}_{|\alpha}}+\frac{3\sqrt{2}b}{3b^2+1}C^{|\alpha}_{|\alpha}+\sqrt{2}bH\overset{(0)}{\tilde{\varphi}_{|\alpha}}\overset{(0)}{\tilde{\varphi}^{|\alpha}}+\frac{9b^2+1}{3b^2+1}\overset{(0)}{\tilde{\varphi}_{|\alpha}}C^{|\alpha}=0. \tag{50}$$

The junction conditions at this order are

$$\left[\overset{(1)}{\tilde{K}^\mu_\nu}\right]\Bigg|_{\substack{r=0^+\\r=l^-}}=0, \tag{51}$$

$$\left[\overset{(1)}{\tilde{\varphi}_{,r}}\right]\Bigg|_{\substack{r=0^+\\r=l^-}}=0. \tag{52}$$

In the preceding equations all the indices are raised with the zeroth order metric. Combining the trace of equation (48) with equation (49) we obtain

$$\frac{1}{l}e^{-2\sqrt{2}b\overset{(0)}{\tilde{\varphi}}}\overset{(1)}{\tilde{K}}=-\left(C^{|\alpha}_{|\alpha}+\sqrt{2}bC_{|\alpha}\overset{(0)}{\tilde{\varphi}^{|\alpha}}\right)H^{\frac{6b^2-4}{3b^2+1}}+\frac{3b^2+1}{6}\left[R\left(\overset{(0)}{\tilde{g}}\right)-\overset{(0)}{\tilde{\varphi}_{|\alpha}}\overset{(0)}{\tilde{\varphi}^{|\alpha}}\right]H^{\frac{9b^2-3}{3b^2+1}}-\frac{\sqrt{2}b}{l}e^{-2\sqrt{2}b\overset{(0)}{\tilde{\varphi}}}\overset{(1)}{\tilde{\varphi}_{,r}}. \tag{53}$$

Imposing the junction conditions (51,52) we get

$$\tilde{R}\left(\overset{(0)}{\tilde{g}}\right) = \overset{(0)}{\tilde{\varphi}}_{|\alpha}\overset{(0)}{\tilde{\varphi}}^{|\alpha}, \tag{54}$$

and the equation of motion for the 4D effective scalar field

$$C_{|\alpha}^{|\alpha} + \sqrt{2}b C_{|\alpha}\,\overset{(0)}{\tilde{\varphi}}^{|\alpha} = 0. \tag{55}$$

Equation (53) now reads

$$\overset{(1)}{\tilde{K}} = -\sqrt{2}b\,\overset{(1)}{\tilde{\varphi}}_{,r}. \tag{56}$$

Using the decomposition of $\overset{(1)}{\tilde{K}^{\mu}_{\nu}}$ in

$$\overset{(1)}{\tilde{K}^{\mu}_{\nu}} = \overset{(1)}{\tilde{\Sigma}^{\mu}_{\nu}} + \frac{1}{4}\delta^{\mu}_{\nu}\,\overset{(1)}{\tilde{K}}, \tag{57}$$

equation (48) can be easily integrated to find

$$e^{-2\sqrt{2}b\,\overset{(0)}{\tilde{\varphi}}}\,H^{-\frac{3b^2-5}{3b^2+1}}\,\overset{(1)}{\tilde{\Sigma}^{\mu}_{\nu}} =$$
$$\left[\left(C^{|\mu}_{|\nu} + \sqrt{2}b\left(C_{|\nu}\,\overset{(0)}{\tilde{\varphi}}^{|\mu} + C^{|\mu}\,\overset{(0)}{\tilde{\varphi}}_{|\nu}\right)\right)r - \left(R^{\mu}_{\nu}\left(\overset{(0)}{\tilde{g}}\right) - \sqrt{2}b\,\overset{(0)}{\tilde{\varphi}}^{|\mu}_{|\nu} - (2b^2+1)\,\overset{(0)}{\tilde{\varphi}}_{|\nu}\overset{(0)}{\tilde{\varphi}}^{|\mu}\right)\left(Cr - \frac{r^2}{2l}\right)\right]_{traceless} + \chi^{\mu}_{\nu}(x), \tag{58}$$

where $\chi^{\mu}_{\nu}(x)$ is an integration constant and the subscript $[\ ]_{traceless}$ means the traceless part of the quantity between square brackets. In terms of $\tilde{\Sigma}^{\mu}_{\nu}$, the junction conditions (51) are

$$\left.\left[\overset{(1)}{\tilde{\Sigma}^{\mu}_{\nu}}\right]\right|_{r=0^+}^{r=l^-} = 0. \tag{59}$$

From the previous junction conditions (59) we can obtain the 4D effective equations of motion

$$\tilde{R}^{\mu}_{\nu}\left(\overset{(0)}{\tilde{g}}\right) = \left(C - \frac{1}{2}\right)^{-1}\left[C^{|\mu}_{|\nu} + \frac{1}{4}\delta^{\mu}_{\nu}C^{|\alpha}_{|\alpha} + \sqrt{2}b\left(C_{|\nu}\,\overset{(0)}{\tilde{\varphi}}^{|\mu} + C_{|\mu}\,\overset{(0)}{\tilde{\varphi}}^{|\nu}\right)\right] + \sqrt{2}b\left(\overset{(0)}{\tilde{\varphi}}^{|\mu}_{|\nu} - \frac{1}{4}\delta^{\mu}_{\nu}\overset{(0)}{\tilde{\varphi}}^{|\alpha}_{|\alpha}\right)$$
$$+ (2b^2+1)\,\overset{(0)}{\tilde{\varphi}}_{|\nu}\overset{(0)}{\tilde{\varphi}}^{|\mu} - \frac{b^2}{2}\delta^{\mu}_{\nu}\,\overset{(0)}{\tilde{\varphi}}_{|\alpha}\overset{(0)}{\tilde{\varphi}}^{|\alpha}, \tag{60}$$

and

$$\chi^{\mu}_{\nu}(x) = 0. \tag{61}$$

Combining equation (56) with the scalar field equation (50) and integrating it once with respect to the extra dimension, we get (after using the previous equations of motion to simplify the result)

$$e^{-2\sqrt{2}b\,\overset{(0)}{\tilde{\varphi}}}\,H^{\frac{-3b^2+5}{3b^2+1}}\,\overset{(1)}{\tilde{\varphi}}_{,r} = -\left(Cr - \frac{r^2}{2l}\right)\left(\overset{(0)}{\tilde{\varphi}}^{|\alpha}_{|\alpha} + \sqrt{2}b\,\overset{(0)}{\tilde{\varphi}}_{|\alpha}\overset{(0)}{\tilde{\varphi}}^{|\alpha}\right) - \overset{(0)}{\tilde{\varphi}}^{|\alpha}\,C_{|\alpha}r + \Xi(x), \tag{62}$$

where $\Xi(x)$ is just an integration constant.

The junction conditions (52) give

$$\Xi(x) = 0 \tag{63}$$

and the equation of motion for the second 4D effective scalar field

$$\overset{(0)}{\tilde{\varphi}}^{|\alpha}_{|\alpha} + \sqrt{2}b\,\overset{(0)}{\tilde{\varphi}}_{|\alpha}\overset{(0)}{\tilde{\varphi}}^{|\alpha} = -\left(C - \frac{1}{2}\right)^{-1}\overset{(0)}{\tilde{\varphi}}^{|\alpha}\,C_{|\alpha}. \tag{64}$$

C. The 4D Effective theory

The 4D effective equations of motion are summarized as

$$\tilde{R}^{\mu}_{\nu}\left(\overset{(0)}{\tilde{g}}\right) = \left(C-\frac{1}{2}\right)^{-1}\left[C^{|\mu}_{|\nu}+\frac{1}{4}\delta^{\mu}_{\nu}C^{|\alpha}_{|\alpha}+\sqrt{2}b\left(C_{|\nu}\overset{(0)}{\tilde{\varphi}^{|\mu}}+C_{|\mu}\overset{(0)}{\tilde{\varphi}^{|\nu}}\right)\right]+\sqrt{2}b\left(\overset{(0)}{\tilde{\varphi}^{|\mu}_{|\nu}}-\frac{1}{4}\delta^{\mu}_{\nu}\overset{(0)}{\tilde{\varphi}^{|\alpha}_{|\alpha}}\right)$$

$$+\left(2b^2+1\right)\overset{(0)}{\tilde{\varphi}_{|\nu}}\overset{(0)}{\tilde{\varphi}^{|\mu}}-\frac{b^2}{2}\delta^{\mu}_{\nu}\overset{(0)}{\tilde{\varphi}_{|\alpha}}\overset{(0)}{\tilde{\varphi}^{|\alpha}}, \tag{65}$$

$$C^{|\alpha}_{|\alpha}+\sqrt{2}bC_{|\alpha}\overset{(0)}{\tilde{\varphi}^{|\alpha}}=0, \tag{66}$$

$$\overset{(0)}{\tilde{\varphi}^{|\alpha}_{|\alpha}}+\sqrt{2}b\overset{(0)}{\tilde{\varphi}_{|\alpha}}\overset{(0)}{\tilde{\varphi}^{|\alpha}}=-\left(C-\frac{1}{2}\right)^{-1}\overset{(0)}{\tilde{\varphi}^{|\alpha}}C_{|\alpha}, \tag{67}$$

and they can be deduced from the following action

$$S_{eff}=\frac{l}{\kappa_G^2}\int d^4x\sqrt{-\overset{(0)}{\tilde{g}}}\left(C-\frac{1}{2}\right)e^{\sqrt{2}b\overset{(0)}{\tilde{\varphi}}}\left[R(\overset{(0)}{\tilde{g}})-\overset{(0)}{\tilde{\varphi}_{|\alpha}}\overset{(0)}{\tilde{\varphi}^{|\alpha}}\right], \tag{68}$$

where $_{|\alpha}$ denotes covariant derivative with respect to $\overset{(0)}{\tilde{g}_{\mu\nu}}$. We should note that this effective action can be derived by substituting in (5) the 5D solutions up to the first order and integrating it over the fifth dimension [11].

As a consistency check, we see that if we perform the conformal transformation

$$h_{\mu\nu}(x)=C^{\frac{2}{3b^2+1}}(x)\overset{(0)}{\tilde{g}_{\mu\nu}}(x), \tag{69}$$

the previous action reduces to (13) if the effective scalar fields of the two theories are related through

$$\psi=2C^{-2}\left(C-\frac{1}{2}\right), \tag{70}$$

$$\phi(x)=\frac{3\sqrt{2}b}{3b^2+1}\ln C+\overset{(0)}{\tilde{\varphi}}(x). \tag{71}$$

The check consists in seeing that these relations are exactly the ones required so that the two observables (the proper distance between branes and the scalar field on the positive tension brane) agree in both approaches.

D. The 5D exact solution

It is straightforward to find a cosmological solution of this 4D effective theory. For example we can easily find the following particular solution

$$\overset{(0)}{\tilde{g}_{\mu\nu}}(x)=\eta_{\mu\nu}, \quad C(x)=ht, \quad \overset{(0)}{\tilde{\varphi}}(x)=0, \tag{72}$$

where h is an integration constant.

Now we are ready to address the question why the above solution can be lifted to an exact 5D solution. Let us start by calculating the next order correction $^{(1)}\tilde{g}_{\mu\nu}$ and $^{(1)}\tilde{\varphi}$. Eq. (62) and the boundary conditions (44) give $^{(1)}\tilde{\varphi}=0$, if we take as 0th order solution Eqs. (72). We can construct $^{(1)}\tilde{K}_{\mu\nu}$ from Eqs. (56) and (58). For the 0th order solution (72) this gives $^{(1)}\tilde{K}_{\mu\nu}=0$. After imposing the boundary conditions (43), we obtain that the next order correction

vanishes, $^{(1)}\tilde{g}_{\mu\nu}(r, x) = 0$. For solution (72) it turns out that all the corrections vanish and the 0th order solution is an exact solution of the non-perturbed 5D Eqs. (34-39).

For other solutions of the 4D effective theory, higher order corrections will not vanish and therefore they should be taken into account in the reconstruction of the 5D metric. Using the gradient expansion method, we can reconstruct the 5D solution perturbatively. We should emphasize that the choice of the 0th order metric is quite important in order to reconstruct 5D solutions efficiently. Our metric ansatz has the advantage that it is possible to recover the exact solution of Chen *et al.* (72) at 0th order. Indeed, if we had started with an ansatz like Eqs. (11,12) we would need an infinite number of higher order terms to obtain the exact 5D solution.

V.   VALIDITY OF 4D EFFECTIVE THEORY

In this section we will make comments on a recent work by Kodama and Uzawa [13]. Let us start by briefly describing their arguments. After deriving the 4D effective theory for warped compactification of the 5D Hořava-Witten model (they also extend their analysis to 10D IIB supergravity and obtain the same conclusions), the authors show that the 4D effective theory allows a wider class of solutions than the fundamental higher dimensional theory. Therefore we should be careful in using this effective theory approach, because we may find 4D solutions that do not satisfy the equations of motion once lifted back to 5D.

The authors assume a metric ansatz of the form

$$dS_5^2 = h(x, r)dr^2 + h^{\frac{1}{2}}(x, r)\tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu, \tag{73}$$

where the warp factor has the form $h(x, r) = C(x) - r/L$. This corresponds to taking $\tilde{K}_{\mu\nu} = 0$, $\tilde{\varphi} = 0$ and $b = 1$ (for the Hořava-Witten case) in our work. Then Eq. (34) reduces to

$$-H(r, x)^{-\frac{3}{2}}\left(C(x)^{|\mu}_{|\nu} + \frac{1}{4}C(x)^{|\alpha}_{|\alpha}\delta^\mu_\nu\right) + H(r, x)^{-\frac{1}{2}}R^\mu_\nu(\tilde{g}(x)) = 0. \tag{74}$$

In order to satisfy this equation for all values of r, we should have

$$R_{\mu\nu}(\tilde{g}) = 0, \quad C_{|\mu\nu} = 0. \tag{75}$$

They obtain the 4D effective action, by integrating the fifth dimension, as

$$S_{eff} \propto \int d^4x \sqrt{-\tilde{g}}\left(C(x) - \frac{1}{2}\right)R(\tilde{g}), \tag{76}$$

which agrees with our effective action (68). As we have shown, this theory admits solutions with $R_{\mu\nu}(\tilde{g}) \neq 0$ (see Eq. (65)), which do not obey the constraint (75) obtained from the 5D equations of motion.

However, it is clear from our analysis that their metric ansatz is too restrictive. If we consider a more general metric as our metric ansatz, we see that the 5D Einstein equations contain more terms given by $\tilde{K}_{\mu\nu}$. With the inclusion of these new terms, the 5D equations do not necessarily imply (75). Of course, the non-vanishing $\tilde{K}_{\mu\nu}$ changes the metric (73) and one could argue that the resultant 4D effective action would be also changed. However, it is shown that even if we include the first order corrections $^{(1)}\tilde{g}_{\mu\nu}(x)$ to the metric, the resultant 4D effective action derived by integrating out the fifth dimension does not change [11]. Therefore, for 4D solutions that do not satisfy (75), we should include the corrections to the metric (73). We have provided this correction perturbatively. Using Eqs. (53) and (58), we can reconstruct the correction to the metric, $^{(1)}\tilde{g}_{\mu\nu}(x)$, which is necessary to satisfy the 5D equation of motion.

We should emphasize that the validity of the 4D effective theory is based on the conditions (40). If the 4D effective theory admits a solution that violates the conditions (40), then there is no guarantee that the 4D solution can be lifted up to the 5D solution consistently. We should check the validity of the 4D effective theory by calculating the higher order corrections to ensure that the higher order corrections can be neglected consistently.

VI.   CONCLUSION

In this paper, we studied the moduli instability in a two brane model with a bulk scalar field recently found by Chen *et al.*. This model can be viewed as a generalization of the Hořava-Witten theory and the Randall-Sundrum model. The scalar field potentials in the bulk and on the branes are tuned in order to satisfy the BPS condition.

We used a low energy effective theory, which is derived by assuming that variations along the brane coordinates of the metric are small compared with variations along the dimension perpendicular to the brane. The effective theory is a bi-scalar tensor theory where one of the scalar fields arises from the bulk scalar field (dilaton) and the other arises from the degree of freedom of the distance between branes (radion). In the Einstein frame, the theory consists of two massless scalar fields, and the lack of potentials for these moduli fields was shown to be responsible for the instability.

We found that the exact solution derived in [6] can be reproduced from the 0th order of the perturbative method, despite the fact that slow-motion approximations are used. We revisited the gradient expansion method which is used to derive the effective theory, in order to understand why the exact solution derived in [6] can be reproduced within the 4D effective theory. We proposed a new metric ansatz which is useful to see the relation between the solutions in the effective theory and the full solutions for 5D equation of motion. Using this metric ansatz, it is transparent why the moduli instability solution can be lifted to a full 5D solution. We have also shown that not all solutions in the 4D effective theory can be lifted to exact 5D solutions. For these solutions, the solutions in the effective theory receive higher order corrections in velocities of the branes and we need to find 5D solutions perturbatively.

Finally, we comment on the recent arguments against the 4D effective theory. Ref. [13] claims that the 4D effective theory allows a much wider class of solutions than the 5D theory. We argued that this conclusion comes from a too restricted metric ansatz used in Ref. [13]. Using a more general metric ansatz, we provided a way to reconstruct the full 5D solutions from the solutions in the 4D effective theory.

Our method can be applied to other warped compactifications such as 10D type IIB supergravity models. In fact, there have been debates on the validity of the metric ansatz commonly used to derive the 4D effective theory. Our 10D generalization of the 0th order metric ansatz agrees with that proposed in Ref. [14] and the method presented in this paper will provide a consistent way to reduce the 10D theory to the 4D effective theory based on this metric ansatz. This will be reported in a future publication [15].

Acknowledgments

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