Moments, positive polynomials and their applications

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Roughly speaking, the Generalized Problem of Moments (GPM) is an infinite-dimensional linear optimization problem (i.e., an infinite dimensional linear program) on a convex set of measures with support on a given subset $\mathbf{K} \subset \mathbb{R}^n$. From a theoretical viewpoint, the GPM has developments and impact in various area of Mathematics like algebra, Fourier analysis, functional analysis, operator theory, probability and statistics, to cite a few. In addition, and despite its rather simple and short formulation, the GPM has a large number of important applications in various fields like optimization, probability, mathematical finance, optimal control, control and signal processing, chemistry, cristallography, tomography, quantum computing, etc.

In its full generality, the GPM is untractable numerically. However when $\mathbf{K}$ is a compact basic semi-algebraic set, and the functions involved are polynomials (and in some cases piecewise polynomials or rational functions), then the situation is much nicer. Indeed, one can define a systematic numerical scheme based on a hierarchy of semidefinite programs, which provides a monotone sequence that converges to the optimal value of the GPM. (A semidefinite program is a convex optimization problem which up to arbitrary fixed precision, can be solved in polynomial time.) Sometimes finite convergence may even occcur.

In the talk, we will present the semidefinite programming methodology to solve the GPM and describe in detail several applications of the GPM (notably in optimization, probability, optimal control and mathematical finance).

Références

[1] J.B. Lasserre, *Moments, Positive Polynomials and their Applications*, Imperial College Press, in press.

[2] J.B. Lasserre, *A Semidefinite programming approach to the generalized problem of moments*, Math. Prog. 112 **(2008)**, pp. 65–92.

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