

Monetary and Fiscal Policy with Sovereign Default*

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Abstract

How does the option to default on debt payments affect the conduct of public policy? To answer this question, this paper studies optimal monetary and fiscal policy without commitment in a model with nominal debt and endogenous sovereign default. When the government can default on its debt, public policy changes in the short and the long run relative to a setting without default option. The risk of default increases the volatility of interest rates, impeding the government's ability to smooth tax distortions across states. It also limits public debt accumulation and reduces the government's incentive to implement high inflation in the long run. Having the option to default on debt payments is furthermore shown to be negligible in terms of welfare.

Keywords: Monetary and Fiscal Policy, Sovereign Default, Lack of Commitment, Markov-Perfect Equilibrium

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1 Introduction

Suppose that a government faces high nominal debt payments that can only be refinanced at high interest rates. If it is not willing (or able) to raise primary surpluses to pay bond holders, there are essentially two options left: inflation and sovereign default. While default and inflation both can lower the real debt burden, there are several differences between these two policy options which make them imperfect substitutes. For example, a government can collect seigniorage when engineering inflation by issuing currency while a default does not generate additional tax revenues. Another difference is that default directly affects the return on government bonds whereas inflation impacts on the return on all nominal assets. Being a continuous variable, inflation can also be adjusted rather easily while the discrete default choice does not offer the same degree of flexibility. Given the distinct role of money and government bonds for the private sector, default and inflation may also distort economic activity through different channels.

The contribution of this paper is to study the implications of allowing a policy maker not only to use standard instruments of monetary and fiscal policy but also to choose outright sovereign default. In particular, it extends previous studies on optimal monetary and fiscal policy with nominal debt that focus on the case of lack of commitment but still assume that the policy maker is always committed to service debt (see e.g. Diaz-Gimenez et al., 2008, Martin, 2009, Niemann et al., 2013a). In the model studied here, a benevolent government jointly chooses monetary and fiscal policy under discretion to finance exogenous government spending in a representative agent cash-credit economy that is subject to productivity shocks.¹ More specifically, it sets a labor income tax rate, chooses the money growth rate, issues nominal non-state contingent bonds and decides on whether to repay its outstanding debt or not. The default decision is modeled as a binary choice (see Eaton and Gersovitz, 1981). Following the quantitative sovereign default literature,² a default is costly because it leads to a deadweight loss of resources that takes the form of a reduction in aggregate productivity and exclusion from financial markets for a random number of periods.

As is common in the literature on optimal monetary and fiscal policy, I consider a closed economy. This paper thus contributes to the study of domestic debt default which,

¹I assume that there is only one policy maker, referred to as the government, who is in charge of both, monetary and fiscal policy. Niemann (2011), Niemann et al. (2013b) and Martin (2014) study time-consistent public policy without sovereign default in models where a central bank and a fiscal authority interact. Röttger (2015) considers a model with independent monetary and a fiscal authorities that allows for sovereign default and political frictions.

²See e.g. Hamann (2004), Aguiar and Gopinath (2006) and Arellano (2008)).

despite being a historically recurring phenomenon with severe economic consequences, has not received a lot of attention in the sovereign default literature (see Reinhart and Rogoff, 2011). In a closed economy, a default does not redistribute resources from foreign lenders to domestic citizens. The government may still choose not to repay its debt to relax its budget constraint and reduce distortionary taxes. The model is calibrated to the Mexican economy which has experienced periods of high inflation and sovereign risk in the recent past. In addition, domestic nominal debt matters for the Mexican government. Du and Schreger (2014) document that Mexican nominal government debt held by domestic residents has roughly increased from 10% to 20% of GDP over the last decade, while the ratio of foreign currency debt to GDP has strongly declined.³

I study the Markov-perfect equilibrium of the public policy problem (see Klein et al., 2008). The government's decisions hence only depend on the payoff-relevant state of the economy which consists of aggregate productivity, the beginning-of-period public debt position and whether the government is in financial autarky or not. Since the government optimizes sequentially, it cannot commit to future policies and does not internalize how its current decisions affect household expectations in previous periods. However, the government is aware that (expected) future policy will depend on its borrowing decision because it will affect the incentive to reduce the real debt burden via default or inflation in the next period. With lack of commitment, the option to default thus matters for the government's response to adverse shocks by allowing it to adjust the real debt burden as well as by affecting the cost of borrowing and thus the attractiveness of debt as a shock absorber. However, only the first effect is internalized by the government.

Compared to an otherwise identical economy without default option (or equivalently an economy with prohibitively high costs of default) the availability of sovereign default results in lower average inflation. Since inflation does not reduce the real debt burden when a default takes place, it is lower when default is chosen instead of repayment. However, this direct effect of default on inflation is negligible at a plausible default frequency. The key mechanism that leads inflation to be lower when the default option is available is an indirect one. The attractiveness and hence the probability of default increases with public debt and decreases with aggregate productivity. With default risk, bond prices become more debt elastic in recessions than with only inflation risk and the marginal revenue from debt issuance decreases faster.⁴ Consequently, the government borrows less

³Between 2000 and 2009, the amount of Mexican nominal public debt held by foreign residents increased as well but only accounted for up to about 20% of total nominal debt.

⁴Even without sovereign risk, higher debt issuance leads to higher interest rates by increasing expected inflation.

which reduces its incentive to implement high inflation rates. Since lower average debt is associated with less inflation, less money is issued and seigniorage revenues decline. The government then has to adjust the primary surplus, leading to a higher labor tax rate in the long run. In the short run, the increased sensitivity of bond prices to productivity shocks and bond issuance that is induced by sovereign risk impedes the government's ability to smooth tax distortions across states. Relative to an economy without default option, tax and inflation rates are thus more volatile, amplifying the impact of productivity shocks on the economy. In times of high sovereign risk, debt issuance is costly and the government tries to avoid a default by reducing the real debt burden via inflation. A sovereign debt crisis thus is inflationary which is consistent with empirical evidence (see Reinhart and Rogoff, 2009).

From a welfare perspective, it is not obvious whether it is desirable to endow the government with the option to default when it cannot commit to future actions.⁵ As discussed above, the risk of default affects public policy in the short and the long run. With productivity shocks, the government would like to smooth tax distortions by running a budget deficit (surplus) during bad (good) times, following the logic of Barro (1979). Default risk makes debt issuance more expensive in recessions which leads to welfare losses due to more volatile public policy. The long-run implications of sovereign default might however outweigh these costs. As in Martin (2009) and Diaz-Gimenez et al. (2008), the government chooses positive average debt positions because of its lack of commitment and the presence of a liquidity constraint. By increasing the cost of borrowing in recessions, risk of default renders public debt accumulation less attractive, thus avoiding high debt levels and the implementation of high average inflation. A welfare exercise reveals that the counterfactual elimination of sovereign default leads to a negligible welfare loss. From a welfare perspective, lack of commitment to debt service hence is not important.

This paper builds on the literature on optimal Markov-perfect monetary and fiscal policy with nominal government debt. Martin (2009, 2011, 2013) extensively studies the short- and long-run properties of public debt and inflation when the government lacks commitment. I will discuss how his findings relate to mine in Section 3. Diaz-Gimenez et al. (2008) show how public policy and welfare depend on whether debt is indexed to inflation or not. Among other things, they find that without commitment welfare can be lower when debt is indexed. In a model with nominal rigidities, Niemann et al. (2013a)

⁵The same is true in the context of consumer default where there exists a trade-off when reducing the costs of filing for bankruptcy. On the one hand, indebted consumers receive the ability to make debt payments state contingent. On the other hand, this flexibility comes at the cost of higher borrowing costs that compensate lenders for the increased risk of default (see e.g. Livshits et al., 2007).

show that the presence of lack of commitment and nominal government debt leads to persistent inflation. Despite highlighting the role of lack of commitment for public policy, all of these studies maintain the assumption that debt is always repaid thus abstract from sovereign default. This work is also related to recent papers that study domestic debt default. In a model with incomplete markets and idiosyncratic income risk, D'Erasmus and Mendoza (2013) show that a sovereign default can occur in equilibrium as an optimal distributive policy. Pouzo and Presno (2014) extend the incomplete markets model of Aiyagari et al. (2002) by considering a policy maker who cannot commit to debt payments. Sosa-Padilla (2013) studies Markov-perfect fiscal policy in a model where a sovereign default triggers a banking crisis. All of these papers feature real economies and hence do not discuss monetary policy.

This paper also relates to the quantitative sovereign default literature that studies how risk of default affects business cycles in emerging economies.⁶ With this literature, it shares the assumption of the government's lack of commitment and the way sovereign default is modeled. Within this literature, the studies that are closest to this paper are Cuadra et al. (2010), Hur et al. (2014) and Du and Schreger (2015). Cuadra et al. (2010) study a production economy with endogenous fiscal policy but abstract from monetary policy and - as is common in the sovereign default literature - look at a small open economy that trades real bonds with foreign investors. Hur et al. (2014) consider an endowment economy with nominal debt, exogenous shocks to inflation and risk-averse investors. They find that the cyclical nature of inflation matters for public debt dynamics by affecting risk-premia and thus the cost of borrowing. Du and Schreger (2015) study a model of a small open economy where the government borrows in local currency from foreign investors and can reduce its real debt burden by using inflation. Since domestic entrepreneurs have liabilities denominated in foreign currency but earn revenues in local currency, inflation hurts firm balance sheets by depreciating the local currency.

In independent and contemporaneous work, Sunder-Plassmann (2014) also studies time-consistent public policy for a version of Martin (2009)'s cash-credit good economy that allows for a default decision as in Arellano (2008). However, there are several differences between our studies that make them complementary. First, as in Diaz-Gimenez et al. (2008), the focus of her paper is on comparing the long-run properties of a model economy with nominal government debt and an otherwise identical model economy with real government debt. She finds that real debt leads to higher average debt, more frequent default events and higher inflation than nominal debt, which is shown to be consistent with

⁶A recent summary of this literature can be found in Aguiar and Amador (2014).

cross-country evidence for selected emerging economies. By contrast, I focus on how the ability to default changes the conduct of monetary and fiscal policy in the short and the long run relative to an economy without default, using a model that can replicate short- and long-run properties of the Mexican economy. To model business cycle fluctuations, I study a model with productivity shocks, whereas she considers government expenditure shocks. Another difference between our two studies is that her model assumes an exogenous and constant labor tax rate, whereas I allow the government to choose the tax rate.

Recently, Aguiar et al. (2013) have also developed a model to jointly study inflation and sovereign default when a government cannot commit to future policy. However, their analysis differs from mine in several ways. First, their model features an endowment economy that is not subject to fundamental shocks and borrows from abroad. Second, the authors assume that the government experiences an ad-hoc utility cost of inflation. Third, in the spirit of Cole and Kehoe (2000), they exclusively focus on self-fulfilling sovereign defaults and characterize analytically how a government's "inflation credibility" - as measured by the weight on its disutility of inflation - affects an economy's vulnerability to sunspot-driven rollover crises.

The rest of the paper is organized as follows. Section 2 presents the model that is analyzed in Section 3. The welfare implications of sovereign default are discussed in Section 4. Section 5 concludes.

2 Model

The model extends the cash-credit economy studied in Martin (2009) by allowing for productivity shocks and sovereign default.⁷ Time is discrete, starts in $t = 0$ and goes on forever. The economy is populated by a unit mass continuum of homogeneous infinitely-lived households and a benevolent government. Taking government policies and prices as given, the households optimize in a competitive fashion. They supply labor n_t to produce the marketable good y_t , using a linear technology to be specified below. In addition, they choose consumption of a cash good c_{1t} and a credit good c_{2t} , and decide on money (\tilde{m}_{t+1}), nominal government bond (\tilde{b}_{t+1}) and nominal risk-free private bond (\tilde{b}_{t+1}^{rf}) holdings. The unit price of a government (private) bond is denoted as q_t (q_t^{rf}). The risk-free bonds are only traded by households and will thus be in zero net supply. While all assets are nominal

⁷The focus on productivity shocks allows me to study how the possibility of sovereign default affects the business cycle properties of a monetary economy.

and thus subject to inflation risk, only government bonds are subject to default risk. A role for money is introduced by tying consumption of c_{1t} to beginning-of-period money holdings via a cash-in-advance constraint (see Lucas and Stokey, 1983)

$$\tilde{m}_t \geq \tilde{p}_t c_{1t},$$

with \tilde{p}_t denoting the price of consumption in terms of \tilde{m}_t .

To finance constant government spending g and outstanding nominal debt payments \tilde{B}_t , the government chooses from a set of policies that includes the money growth rate μ_t , a linear labor income tax rate τ_t , the binary default decision $d_t \in \{0, 1\}$, and issuance of nominal non-state contingent one-period bonds \tilde{B}_{t+1} . A default occurs when $d_t = 1$ is chosen, while the government fully repays its obligations for $d_t = 0$. In the default case, the government is excluded from financial markets for a random number of periods (see Aguiar and Gopinath, 2006, or Arellano, 2008). It can thus neither borrow from nor lend to households during this time.

2.1 Private Sector

Before formulating the government decision problem, I will first discuss the behavior of the private sector.

2.1.1 Households

The households have preferences given by

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, n_t) \right],$$

with discount factor $\beta \in (0, 1)$ and period utility function $u : \mathbb{R}_+^3 \rightarrow \mathbb{R}$. The utility function is additively separable in all its arguments and satisfies $u_1, u_2, -u_n > 0$ and $u_{11}, u_{22}, u_{nn} < 0$ with u_x (u_{xx}) denoting the first (second) derivative of $u(\cdot)$ with respect to $x \in \{c_1, c_2, n\}$. Households have initial assets (b_0, b_0^{rf}, m_0) and take as given prices $\{\tilde{p}_t, q_t, q_t^{rf}\}_{t=0}^{\infty}$ and government policies $\{d_t, \mu_t, \tau_t, \tilde{B}_{t+1}\}_{t=0}^{\infty}$. The aggregate money stock evolves according to $\tilde{M}_{t+1} = (1 + \mu_t)\tilde{M}_t$. The labor productivity $\{a_t\}_{t=0}^{\infty}$ of the households is subject to random shocks and follows a stationary first-order Markov process with continuous support $\mathbb{A} \subseteq \mathbb{R}_+$ and transition function $f_a(a_{t+1}|a_t)$.

Households maximize their expected lifetime utility subject to the period budget constraint

$$(1 - \tau_t)\psi(a_t, d_t)n_t + \frac{\tilde{m}_t + (1 - d_t)\tilde{b}_t + \tilde{b}_t^{rf}}{\tilde{p}_t} \geq c_{1t} + c_{2t} + \frac{\tilde{m}_{t+1} + q_t\tilde{b}_{t+1} + q_t^{rf}\tilde{b}_{t+1}^{rf}}{\tilde{p}_t},$$

the cash-in-advance constraint

$$\frac{\tilde{m}_t}{\tilde{p}_t} \geq c_{1t},$$

as well as the No-Ponzi game constraint $\lim_{T \rightarrow \infty} \mathbb{E}_t \left[\left(q_T^{rf} \tilde{b}_{T+1}^{rf} + q_T \tilde{b}_{T+1} \right) \prod_{s=0}^{T-1} q_{t+s}^{rf} \right] \geq 0$. Households use their labor supply n_t to produce a marketable good according to the linear technology $y_t = \psi(a_t, d_t)n_t$. They take as given their effective labor productivity $\psi: \mathbb{R}_+ \times \{0, 1\} \rightarrow \mathbb{R}_+$ which depends on random productivity a_t and the government's default decision d_t . Effective productivity $\psi(\cdot)$ increases with exogenous productivity ($\partial \psi(a_t, d_t) / \partial a_t \geq 0$) and is negatively affected by a default ($\psi(a_t, 0) \geq \psi(a_t, 1)$).⁸

In the model, lack of commitment to public debt repayment will require costs of default that are internalized by the government to sustain positive levels of debt. As is common in the quantitative sovereign default literature (see e.g. Arellano, 2008 and Cuadra et al., 2010), there are two types of default costs. First, the government is excluded from the bond market in the default period. Conditional on being in autarky, the economy regains access to financial markets with constant probability θ in the subsequent period. Second, the economy experiences a direct resource loss governed by $\psi(\cdot)$. As in Cuadra et al. (2010) and Pouzo and Presno (2014), these costs capture in reduced form productivity losses that occur in periods of default (and financial autarky). Despite being arguably ad hoc, such a specification allows me not to take a stand on how exactly a sovereign default is propagated through the economy. While there is evidence for domestic output costs, there is still no consensus on which mechanism is the most relevant one (see Panizza et al., 2009). In addition, two recent papers show that models with endogenous default costs that arise due to private credit disruptions (Mendoza and Yue, 2012) or banking crises (Sosa-Padilla, 2013) deliver similar qualitative and quantitative results as those with exogenous default costs.

⁸It is straightforward to modify the model to include a representative firm that is owned by households and produces the homogeneous good y_t , using labor supplied by households at a real wage w_t . Due to linearity of the production function, the wage rate will equal effective productivity $\psi(a_t, d_t)$ and profits will be zero, such that the behavior of the economy will not change with such a firm sector.

The household optimality conditions are given by the first-order conditions

$$-\frac{u_n(n_t)}{u_2(c_{2t})} = (1 - \tau_t)\psi(a_t, d_t), \quad (1)$$

$$u_2(c_{2t}) = \beta \mathbb{E}_t \left[u_1(c_{1t+1}) \frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right], \quad (2)$$

$$u_2(c_{2t})q_t = \beta \mathbb{E}_t \left[(1 - d_{t+1}) u_2(c_{2t+1}) \frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right], \quad (3)$$

$$u_2(c_{2t})q_t^{rf} = \beta \mathbb{E}_t \left[u_2(c_{2t+1}) \frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right], \quad (4)$$

and the complementary slackness conditions

$$\lambda_t = u_1(c_{1t}) - u_2(c_{2t}) \geq 0, \tilde{m}_t / \tilde{p}_t - c_{1t} \geq 0, \lambda_t (\tilde{m}_t / \tilde{p}_t - c_{1t}) = 0,$$

with λ_t denoting the Kuhn-Tucker multiplier on the cash-in-advance constraint.⁹

Intuitively, the cash-in-advance constraint is binding whenever the marginal utility of cash-good consumption exceeds the marginal utility of credit-good consumption. The inequality

$$u_1(c_{1t}) - u_2(c_{2t}) \geq 0, \quad (5)$$

needs to hold in equilibrium to satisfy $\lambda_t \geq 0$. Equation (1) characterizes the optimal household labor supply decision which is distorted for non-zero tax rates $\tau_t \neq 0$. The conditions (2)-(4) are the Euler equations for money and bonds. Since nominal government bonds are defaultable, they have to compensate households for the risk of default (see condition (3)). However, all assets need to compensate for expected (gross) inflation $\tilde{p}_{t+1}/\tilde{p}_t$.

As in Martin (2009), I normalize all nominal variables by the beginning-of-period aggregate money stock \tilde{M}_t : $x_t \equiv \tilde{x}_t / \tilde{M}_t$ with $x \in \{B, b, b^{rf}, m, p\}$. This normalization renders the model stationary. It implies that the inflation rate in period t is given as

$$\pi_t \equiv \frac{p_t (1 + \mu_{t-1})}{p_{t-1}} - 1,$$

such that inflation equals money growth in the long run.

⁹In a household optimum, the household budget constraint and the No-Ponzi game constraint hold with equality.

After normalizing nominal variables, the Euler equations are now given as

$$u_2(c_{2t}) = \beta \mathbb{E}_t \left[u_1(c_{1t+1}) \frac{p_t}{p_{t+1}} \frac{1}{1 + \mu_t} \right], \quad (6)$$

$$u_2(c_{2t})q_t = \beta \mathbb{E}_t \left[(1 - d_{t+1}) u_2(c_{2t+1}) \frac{p_t}{p_{t+1}} \frac{1}{1 + \mu_t} \right], \quad (7)$$

$$u_2(c_{2t})q_t^{rf} = \beta \mathbb{E}_t \left[u_2(c_{2t+1}) \frac{p_t}{p_{t+1}} \frac{1}{1 + \mu_t} \right]. \quad (8)$$

2.1.2 Market Clearing

In equilibrium, goods and asset markets clear:

$$\begin{aligned} \psi(a_t, d_t)n_t &= c_{1t} + c_{2t} + g, \\ b_{t+1} &= B_{t+1}, \\ b_{t+1}^{rf} &= 0, \\ m_{t+1} &= 1. \end{aligned}$$

If real balances are high enough, i.e. $1/p_t \geq u_1^{-1}(u_2(c_{2t}))$ holds (see condition (5)), households equalize marginal utility across cash and credit goods. If not, households are cash constrained and the allocation of consumption is distorted. As in Martin (2009), in a monetary equilibrium, i.e. an equilibrium in which money is valued,

$$c_{1t} = 1/p_t,$$

needs to hold. Note that this still allows for an unconstrained consumption allocation if the cash-in-advance constraint is just binding, i.e. when p_t is such that $\lambda_t = 0$ and $c_{1,t} = 1/p = u_1^{-1}(u_2(c_{2t}))$.

Given the policy instruments $\{d_t, \mu_t, \tau_t, B_{t+1}\}_{t=0}^{\infty}$, the private sector equilibrium then is characterized by the conditions (1),(5),(6)-(8), the market clearing conditions above and binding cash-in-advance, household budget and No-Ponzi game constraints.

2.2 Government Problem

In this section, I formulate the decision problem of the benevolent government. When the government defaults, the economy suffers a productivity loss governed by $\psi(\cdot)$ and

it cannot access the bond market for a random number of periods.¹⁰ Conditional on the default decision, the government budget constraint is

$$g - \tau_t \psi(a_t, d_t) n_t = \begin{cases} \frac{\tilde{M}_{t+1} + q_t \tilde{B}_{t+1}}{\tilde{p}_t} - \frac{\tilde{M}_t + \tilde{B}_t}{\tilde{p}_t}, & \text{if } d_t = 0 \\ \frac{\tilde{M}_{t+1} - \tilde{M}_t}{\tilde{p}_t}, & \text{if } d_t = 1 \end{cases}$$

It can be written as

$$g - \tau_t \psi(a_t, d_t) n_t = \begin{cases} (1 + \mu_t) \frac{1 + q_t B_{t+1}}{p_t} - \frac{1 + B_t}{p_t}, & \text{if } d_t = 0 \\ \frac{\mu_t}{p_t}, & \text{if } d_t = 1 \end{cases}$$

by using $\tilde{M}_{t+1} = (1 + \mu_t) \tilde{M}_t$ and applying the normalization of nominal variables used before.¹¹ In the default (and autarky) case, the government has to finance public spending g with income tax revenues $\tau_t \psi(a_t, 1) n_t$ and seigniorage $\tau_t^m \equiv \mu_t / p_t$. When the government repays its debt, it additionally has to make debt payments but can access the bond market and issue debt.

The government is benevolent and sets its policy instruments to maximize the expected life-time utility of the households, anticipating the response of the private sector to its policies. However, it cannot commit itself to a state-contingent (Ramsey) policy plan for all current and future policies but optimizes from period to period instead. To analyze the public policy problem of the government, I restrict attention to stationary Markov-perfect equilibria (see Klein et al., 2008). In a Markov-perfect equilibrium, the optimal decisions of the government in any period will be characterized by time-invariant functions that only depend on the minimal payoff-relevant state of the economy in that respective period. In the model, this state consists of the beginning-of-period debt-to-money ratio B_t , labor productivity a_t and whether the government is in financial autarky or not. By requiring the government to only condition its decisions on the current payoff-relevant aggregate state, the Markov-perfect equilibrium concept rules out the possibility that the government is able to keep promises made in the past. This is because at the start of a period, the government does not care about the past and only considers its payoff in current and future periods.¹² By construction, the government thus is ensured to act in a time-consistent way.

¹⁰Note that households can still trade risk-free bonds among each other when the government is in financial autarky. However, since they are homogeneous and private bonds are in zero net supply, this is not going to affect the way public policy is conducted.

¹¹Note that the normalized aggregate money stock is constant and equal to one.

¹²The focus on Markov-perfect strategies also rules out the possibility of reputational considerations based on complex trigger strategies as in Chari and Kehoe (1990, 1993).

The Markov-perfect policy problem will be formulated recursively. In the remainder, I will thus adopt the notation of dynamic programming. Time indices are hence dropped and a prime is used to denote next period's variables. Given the aggregate state at the start of a period, the government takes as given the policy function $\mathcal{D}(B', a')$ that determines next period's default decision as well as the policy functions $\mathcal{X}^r(B', a')$ and $\mathcal{X}^d(a')$, with $\mathcal{X} \in \{\mathcal{C}_2, \mathcal{P}\}$, that determine consumption and the price index in the next period for the case of repayment (r) and default (d).¹³ Expectations of these variables enter the household optimality conditions (6) and (7) and thus matter for the allocation in the current period.¹⁴ Despite lacking the ability to commit to future policies, the government fully recognizes today that it affects (expected) future policies via its choice of B' , which in turn have an effect on the behavior of the private sector in the current period. In a stationary Markov-perfect equilibrium, the policy functions that govern future decisions then coincide with the policy functions that determine current public policy for all states.

As in Klein et al. (2008), one can interpret the formulation of the public policy problem as a Markov-perfect (Nash) game played between successive governments. Following this interpretation, in each period, a different government is in charge of choosing public policy, taking as given the policies set by the government in the next period. Each government then chooses its optimal strategies, taking as given the optimal responses of the government in the next period and the private sector in the current period.

In each period, the government anticipates how the private sector responds to its actions in the current period as given by the private sector equilibrium conditions.¹⁵ Using the household optimality conditions (1),(6)-(7), the binding cash-in-advance constraint and the aggregate resource constraint, the government budget constraint can be rewritten as

$$\beta \mathbb{E}_{a'|a} \left[\begin{array}{c} u_1 (\mathcal{P}^r(B', a'))^{-1} \frac{1 - \mathcal{D}(B', a')}{\mathcal{P}^r(B', a')} \\ + u_1 (\mathcal{P}^d(a'))^{-1} \frac{\mathcal{D}(B', a')}{\mathcal{P}^d(a')} \end{array} \right] + \beta \mathbb{E}_{a'|a} \left[u_2 (\mathcal{C}_2^r(B', a')) \frac{1 - \mathcal{D}(B', a')}{\mathcal{P}^r(B', a')} \right] B' \quad (9)$$

$$+ u_n(n)n + u_2(c_2)(c_2 - B/p) = 0,$$

¹³Remember that cash-good consumption c_1 is directly linked to the price index p via the cash-in-advance constraint.

¹⁴Households do not have an impact on the future government policies but form rational expectations about them based on the policy functions listed above.

¹⁵The government thus plays a Stackelberg game against the (passive) private sector in every period.

for the repayment case and as

$$\beta \mathbb{E}_{a'|a} \left[\theta \times \left\{ \begin{aligned} &u_1 (\mathcal{P}^r(0, a')^{-1}) \frac{1 - \mathcal{D}(0, a')}{\mathcal{P}^r(0, a')} \\ &+ u_1 (\mathcal{P}^d(a')^{-1}) \frac{\mathcal{D}(0, a')}{\mathcal{P}^d(a')} \end{aligned} \right\} + (1 - \theta) \times \frac{u_1(c_1^d(a'))}{\mathcal{P}^d(a')} \right] + u_n(n)n + u_2(c_2)c_2 = 0, \quad (10)$$

for the default (and autarky) case. This constraint can be seen as the period implementability constraint for the government.¹⁶

In addition to this constraint, the government has to respect the following two private sector equilibrium conditions:

$$0 = \psi(a, d)n - 1/p - c_2 - g, \quad (11)$$

$$0 \leq u_1(1/p) - u_2(c_2). \quad (12)$$

The household budget constraint is satisfied by Walras' Law, given the government budget constraint and the resource constraint. Let $\mathbb{B} \equiv [\underline{B}, \bar{B}]$ be the set of possible aggregate debt values with $-\infty < \underline{B} \leq 0$ and $0 < \bar{B} < \infty$. Conditional on entering a period with access to financial markets, the decision problem of the government is given by the following functional equation:

$$\mathcal{V}(B, a) = \max_{d \in \{0, 1\}} \left\{ (1 - d)\mathcal{V}^r(B, a) + d\mathcal{V}^d(a) \right\} \quad (13)$$

with the value of repayment given as

$$\mathcal{V}^r(B, a) = \max_{c_2, n, p, B' \in \mathbb{B}} u(1/p, c_2, n) + \beta \mathbb{E}_{a'|a} [\mathcal{V}(B', a')] \text{ s.t. (9), (11), (12),}$$

and the value of default as

$$\mathcal{V}^d(a) = \max_{c_2, n, p} u(1/p, c_2, n) + \beta \mathbb{E}_{a'|a} \left[\theta \mathcal{V}(0, a') + (1 - \theta) \mathcal{V}^d(a') \right] \text{ s.t. (10) - (12).}$$

The value $\mathcal{V}(\cdot)$ is the option value of default. As is standard in the sovereign default literature, the government is assumed to honor its obligations whenever it is indifferent between default and repayment. If the government starts a period in financial autarky, it solves the same problem as in the default case. In periods of default and autarky, the government will regain access to financial markets in the subsequent period with probability

¹⁶The derivation of the implementability constraint can be found in Appendix A.1.

θ . With probability $1 - \theta$, it will stay in financial autarky.¹⁷

2.3 Equilibrium

The Markov-perfect equilibrium is defined as follows:

Definition 1. A stationary Markov-perfect equilibrium is given by a set of value functions $\mathcal{V}(B, a)$, $\mathcal{V}^r(B, a)$, $\mathcal{V}^d(a)$, and policy functions $\mathcal{D}(B, a)$, $\mathcal{B}^r(B, a)$, $\mathcal{X}^r(B, a)$, $\mathcal{X}^d(a)$, with $\mathcal{X} \in \{\mathcal{C}_2, \mathcal{N}, \mathcal{P}\}$, such that for all $(B, a) \in \mathbb{B} \times \mathbb{A}$:

$$\mathcal{D}(B, a) = \arg \max_{d \in \{0,1\}} \left\{ (1-d)\mathcal{V}^r(B, a) + d\mathcal{V}^d(a) \right\},$$

$$\begin{aligned} \left\{ \mathcal{X}^r(B, a) \right\}_{\mathcal{X} \in \{\mathcal{C}_2, \mathcal{N}, \mathcal{P}, \mathcal{B}\}} &= \arg \max_{c_2, n, p, B' \in \mathbb{B}} u(1/p, c_2, n) + \beta \mathbb{E}_{a'|a} [\mathcal{V}(B', a')] \\ &\text{s.t. (9), (11), (12),} \end{aligned}$$

and

$$\begin{aligned} \left\{ \mathcal{X}^d(a) \right\}_{\mathcal{X} \in \{\mathcal{C}_2, \mathcal{N}, \mathcal{P}\}} &= \arg \max_{c_2, n, p} u(1/p, c_2, n) + \beta \mathbb{E}_{a'|a} \left[\theta \mathcal{V}(0, a') + (1-\theta) \mathcal{V}^d(a') \right] \\ &\text{s.t. (10) - (12),} \end{aligned}$$

as well as

$$\begin{aligned} \mathcal{V}(B, a) &= (1 - \mathcal{D}(B, a)) \times \mathcal{V}^r(B, a) + \mathcal{D}(B, a) \times \mathcal{V}^d(a), \\ \mathcal{V}^r(B, a) &= u\left(\mathcal{P}^r(B, a)^{-1}, \mathcal{C}_2^r(B, a), \mathcal{N}^r(B, a)\right) + \beta \mathbb{E}_{a'|a} [\mathcal{V}(\mathcal{B}^r(B, a), a')], \end{aligned}$$

and

$$\mathcal{V}^d(a) = u(\mathcal{P}^d(a)^{-1}, \mathcal{C}_2^d(a), \mathcal{N}^d(a)) + \beta \mathbb{E}_{a'|a} \left[\theta \mathcal{V}(0, a') + (1-\theta) \mathcal{V}^d(a') \right].$$

This definition highlights the stationarity of the policy problem since the functions that solve the decision problem of the government in a given period coincide with the policy functions that govern the optimal decisions of the government in future periods.¹⁸

¹⁷On average, the government thus spends $1/\theta$ periods in autarky after a default.

¹⁸The definition of the equilibrium is formulated following Martin (2009).

3 Model Analysis

In this section, the role of sovereign default for public policy is investigated. Because the model cannot be evaluated analytically due to the discrete default option, numerical methods are applied. Appendix A.3 contains details regarding the numerical computation of the equilibrium. The next section presents the model specification. A discussion of the public policy choices can be found in Section 3.2. Simulation results are presented in Section 3.3.

3.1 Model Specification

To explore the model properties by computational means, functional forms and parameters need to be chosen.

3.1.1 Functional Forms

Productivity follows a log-normal AR(1)-process,

$$a_t = a_{t-1}^{\rho} \exp(\sigma \varepsilon_t), \quad \varepsilon_t \stackrel{i.i.d.}{\sim} N(0, 1).$$

The household utility function is specified as

$$u(c_1, c_2, n) = \gamma_1 \frac{c_1^{1-\sigma_1} - 1}{1-\sigma_1} + \gamma_2 \frac{c_2^{1-\sigma_2} - 1}{1-\sigma_2} + (1 - \gamma_1 - \gamma_2) \frac{(1-n)^{1-\sigma_n} - 1}{1-\sigma_n},$$

with $\gamma_1, \gamma_2, \sigma_i > 0, i \in \{1, 2, n\}$ and $\gamma_1 + \gamma_2 < 1$.¹⁹

The resource costs of default are specified as in Cuadra et al. (2010):

$$\psi(a, d) = a - d \times \max\{0, a - \tilde{a}\}.$$

If a default takes place, effective productivity equals \tilde{a} when a exceeds \tilde{a} while there are no costs of default when productivity a is below the threshold \tilde{a} . This default cost specification implies that a default is more costly in booms than in recessions. In the quantitative sovereign default literature, it is well known that this feature is crucial for default to mostly take place in bad states and hence for countercyclical sovereign risk to emerge (see e.g. Aguiar and Amador, 2014). This property is consistent with empirical

¹⁹For $\sigma_i = 1, i \in \{1, 2, n\}$, household utility is logarithmic for the respective variable.

Parameter	Description	Value
β	Discount factor	0.9900
g	Government spending	0.0379
γ_1	Cash-good weight	0.0030
γ_2	Credit-good weight	0.3370
σ_1	Cash-good curvature	2.4300
σ_2	Credit-good curvature	1.0000
σ_n	Leisure curvature	2.0000
\tilde{a}	Default cost parameter	0.9900
θ	Probability of reentry	0.2000
σ	Std. dev. productivity shock	0.0169
ρ	Persistence of productivity	0.9000

Table 1: Parameter values

evidence (see Tomz and Wright, 2007) and also present in models with endogenous costs of default (see Mendoza and Yue, 2012, Sosa-Padilla, 2013).²⁰

3.1.2 Parameters

A model period corresponds to one quarter. The selected model parameters are listed in Table 1. They are either set to standard values or chosen to replicate certain short- or long-run properties of the Mexican economy.²¹ The productivity parameter ρ is set to 0.9 while σ is chosen to match the standard deviation of HP-filtered Mexican log real GDP. As is common in business cycle models, a discount factor of $\beta = 0.99$ is selected, implying an annual real risk-free rate of 4%. Based on World Bank data for 1980-2008, g is set to 0.0379 to match an average ratio of public spending to GDP of around 11%.²² The credit-good preference parameter σ_2 is normalized to 1. Targeting a cash-credit good ratio and an average working time of one third each, γ_1 is set to 0.003 and γ_2 to 0.337. For the inverse of the elasticity of leisure σ_l , a rather standard value of 2 is selected. The probability of reentry θ is set to 0.2, implying that financial autarky lasts for 5 quarters on average. This parameter value is in line with values considered in the quantitative sovereign default literature which range from 0.0385 (Chatterjee and Eyigungor, 2012) to

²⁰Allowing for default costs that enter the the aggregate resource constraint (or the government budget constraint) in a lump-sum way does not change the results of this paper as long as these losses are also relatively higher in good than in bad states, preserving countercyclical default incentives.

²¹The time series for real GDP and the GDP deflator are taken from Cuadra et al. (2010) and cover the time period from 1980:I to 2007:I. They are seasonally adjusted via EViews' multiplicative X-12 routine.

²²More specifically, I use annual data from the World Bank's World Development Indicators for general final government consumption as a share of GDP.

0.282 (Arellano, 2008).

The size of σ_1 is crucial for long-run debt and inflation as will be explained in the next section. To match the Mexican average annual inflation rate of 29.69%, σ_1 is set to 2.43.²³ The incentive to default critically depends on \tilde{a} . For Mexico, Reinhart (2010) documents that there have been domestic defaults in 1982 and between 1929 and 1938. Based on this observation, I set the default cost parameter to match an annual default frequency of 2%. The model is also solved and simulated with prohibitively high productivity costs of default which rule out equilibrium default. This benchmark economy yields the same results as a model without default option and will be referred to as "no-default economy". The model with default option will be referred to as "baseline economy".

3.2 Public Policy Decisions

The optimal policies for the economy with default can be seen in Figure 1. Debt is normalized by nominal output $Y \equiv py$, evaluated at the unconditional mean of productivity. The default decision is visualized using the default threshold $\hat{a}(B)$ which is the lowest productivity level that leads to repayment for given debt B : $\mathcal{V}^r(B, \hat{a}(B)) = \mathcal{V}^d(\hat{a}(B))$. The threshold separates the state space (B, a) into two areas: the default region ($a < \hat{a}(B)$), i.e. below the line) and the repayment region ($a \geq \hat{a}(B)$), i.e. on and above the line). As in the quantitative sovereign default literature, default becomes more attractive with higher debt and lower productivity (see Arellano, 2008).

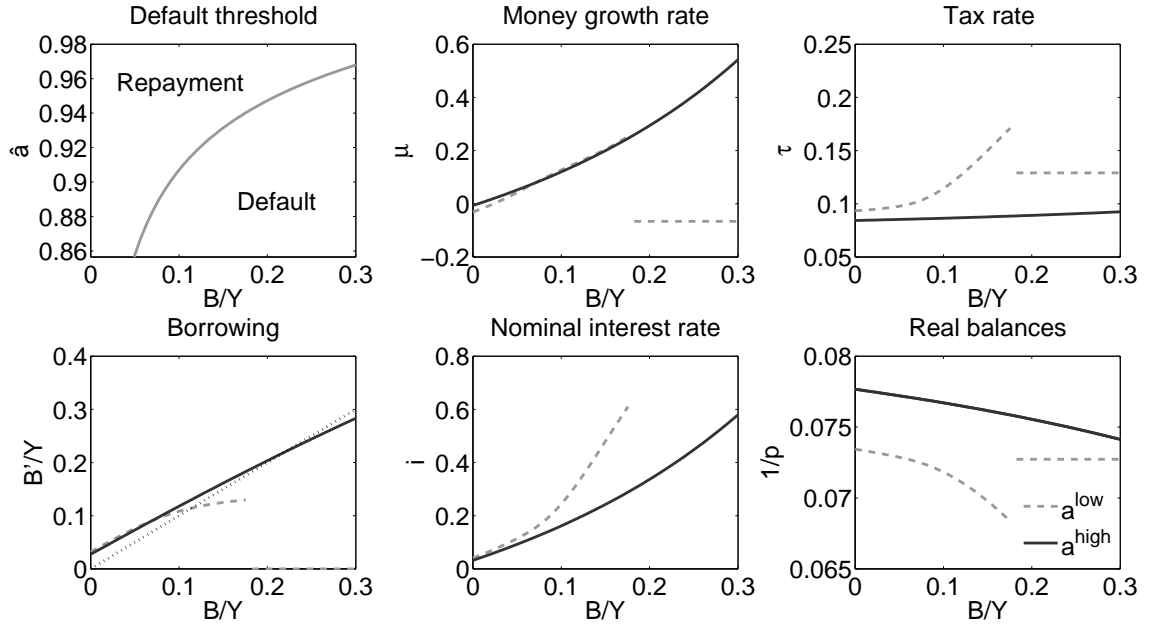
The remaining policies are presented for productivity levels 1.5 standard deviations below (dashed line) and above (solid line) the unconditional mean of productivity. The nominal interest rate is defined as $i = 1/q - 1$. Since the continuous policy decisions depend on the default decision, the objects displayed in Figure 1 exhibit kinks at states where default is optimally chosen.²⁴ In the default case, the policies also do not change with B anymore.

The optimal labor and inflation tax distortions reflect the government's financing needs. By relaxing the government's budget, a sovereign default allows to reduce labor taxation and increase real balances relative to full debt repayment. The income tax rate and the price index p both increase with B . An inflationary monetary policy becomes

²³The inflation rate is calculated based on the quarterly GDP deflator time series for Mexico provided by Cuadra et al. (2010). Using alternative measures such as the CPI also yields average inflation rates of around 30%.

²⁴The policy functions displayed in Figure 1 are given as $\mathcal{X}(B, a) = (1 - \mathcal{D}(B, a))\mathcal{X}^r(B, a) + \mathcal{D}(a)\mathcal{X}^d(a)$, with $\mathcal{X} \in \{\mathcal{B}, \mathcal{C}_2, \mathcal{N}, \mathcal{P}\}$. The remaining variables (μ, τ, i) are calculated by using these policy functions and the private sector equilibrium conditions (1),(6)-(7).

Figure 1: Default threshold and selected policy functions (baseline economy)



Notes: The policy functions are depicted for productivity values 1.5 standard deviations below (a^{low}) and above (a^{high}) the unconditional mean of productivity. The money growth rate and the nominal interest rate are displayed in annual terms.

particularly more attractive with higher debt because it lowers the real debt burden. This implies that default and inflation are substitutes since inflation as "partial default" becomes useless for $d = 1$. However, they are only imperfect substitutes due to the discrete nature of default.

The intertemporal policy trade-off can be illustrated via the generalized Euler equation

$$\beta \int_{\hat{a}(B')}^{\infty} (\xi' - \xi) \frac{u_2(c_2')}{p'} f_a(a'|a) da' = \xi \left(\frac{\partial R^b}{\partial B'} B' + \frac{\partial R^m}{\partial B'} \right), \quad (14)$$

where ξ denotes the multiplier on the implementability constraint (9), $R^b = ((1 + \mu) qu_2) / p$ average revenues from bond issuance and $R^m = ((1 + \mu) u_2) / p$ (gross) revenues from money creation.²⁵ With the model specification of Section 3.1.1, these revenues are given

²⁵The derivation of the Euler equation assumes differentiability of \mathcal{V}' , R^b and R^m with respect to debt (see Appendix A.2 for details). As is common in the sovereign default literature (see e.g. Cuadra and Sapriza, 2008, or Hatchondo et al., 2015), the generalized Euler equation is only presented here to highlight the intertemporal public policy trade-off in an intuitive way. The numerical algorithm that is used to solve the model is not based on this Euler equation and does not require differentiability to hold (see Appendix A.3 for details).

by the following functions of productivity a and borrowing B' :

$$R^b(B', a) = \beta \int_{\hat{a}(B')}^{\infty} \frac{\gamma_2 C_2^r(B', a')^{-\sigma_2}}{\mathcal{P}^r(B', a')} f_a(a'|a) da',$$

$$R^m(B', a) = \beta \times \left\{ \begin{array}{l} \int_0^{\hat{a}(B')} \gamma_1 \mathcal{P}^d(a')^{\sigma_1-1} f_a(a'|a) da' \\ + \int_{\hat{a}(B')}^{\infty} \gamma_1 \mathcal{P}^r(B', a')^{\sigma_1-1} f_a(a'|a) da' \end{array} \right\}.$$

Households dislike volatile consumption and leisure. In the presence of productivity shocks, the government can issue debt to accommodate these preferences and smooth tax distortions as measured by ξ across states (see the LHS of (14)). Its ability to do so is constrained by financial market incompleteness and lack of commitment. Since only nominal non-state contingent bonds are available, the government has an incentive to make real debt payments state contingent via inflation or default. However, because it cannot commit to a state-contingent repayment plan for the next period, public financing conditions will depend on the chosen debt position B' since it affects the risk of inflation and default. The derivatives on the RHS of (14) reflect this channel. The optimal debt policy then trades off the tax smoothing motive against the time-inconsistency problem, i.e. how current debt issuance affects revenues $R^b(B', a) \times B'$ and $R^m(B', a)$ by changing household expectations of inflation and default, which matter for household bond and money demand.²⁶

The impact of debt issuance on public revenues is visualized in Figure 2. It depicts the functions R^b and R^m for the no-default economy (first row) and the baseline economy with equilibrium default (second row). The no-default case has previously been studied by Martin (2009, 2011, 2013) and Diaz-Gimenez et al. (2008).

It is useful to first look at the case without default to understand how sovereign risk affects the debt policy.²⁷ The shape of R^b reflects the relation between inflation and beginning-of-period debt (see panel (a)). Due to its lack of commitment, the government optimizes from period to period and therefore does not internalize how its current policy choices affect outcomes in previous periods. More specifically, it does not internalize that its current actions have an impact on households' demand for money and bonds in the last period. Failing to recognize this impact, the government decides to erode the real value of beginning-of-period debt via inflation to relax its budget. Since the temptation to use inflation in this way increases with B , expected inflation becomes an

²⁶Note that revenues are weighted by the marginal utility of credit-good consumption $u_2(c_2)$ and expressed in real terms.

²⁷In this case, the default threshold is given as $\hat{a}(B) = 0$, i.e. default is prohibitively costly.

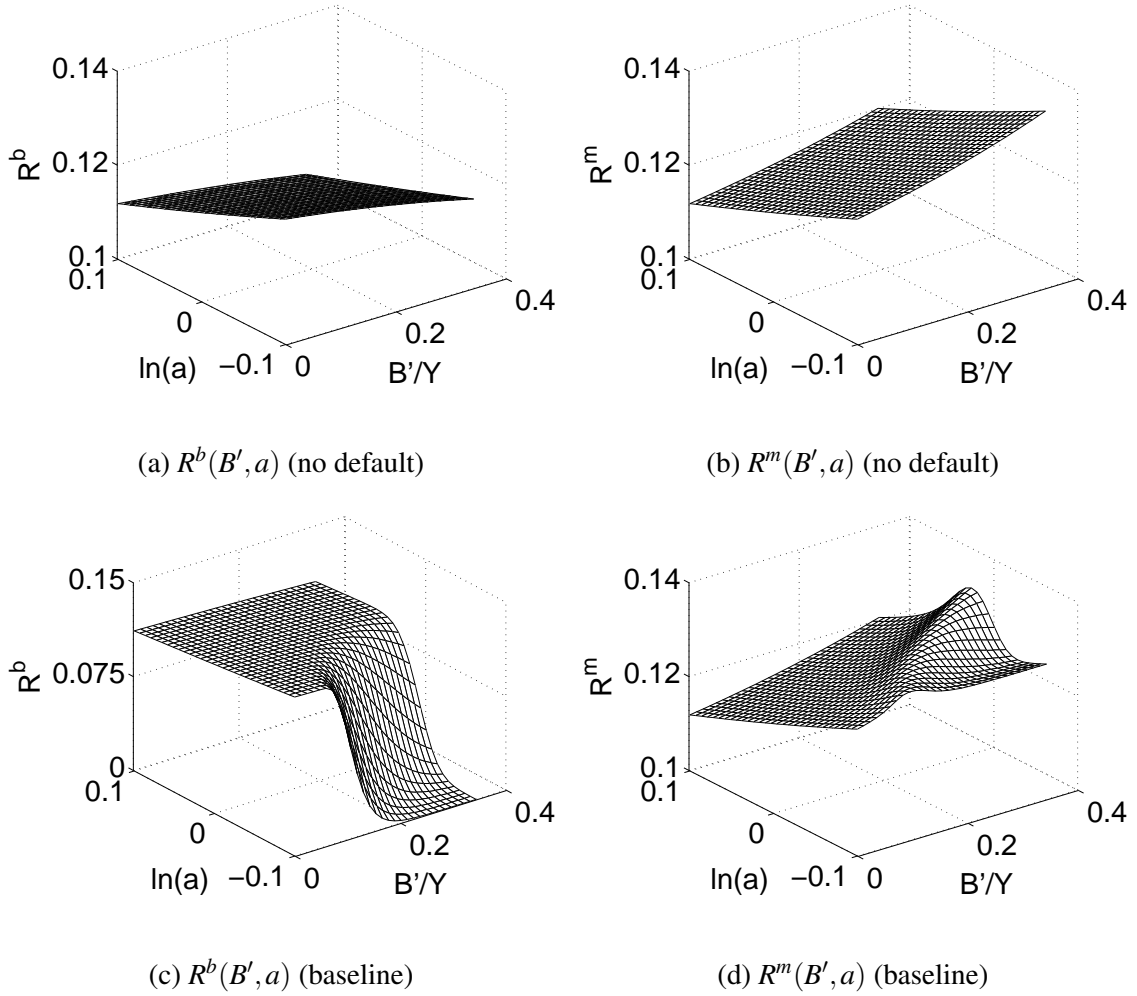
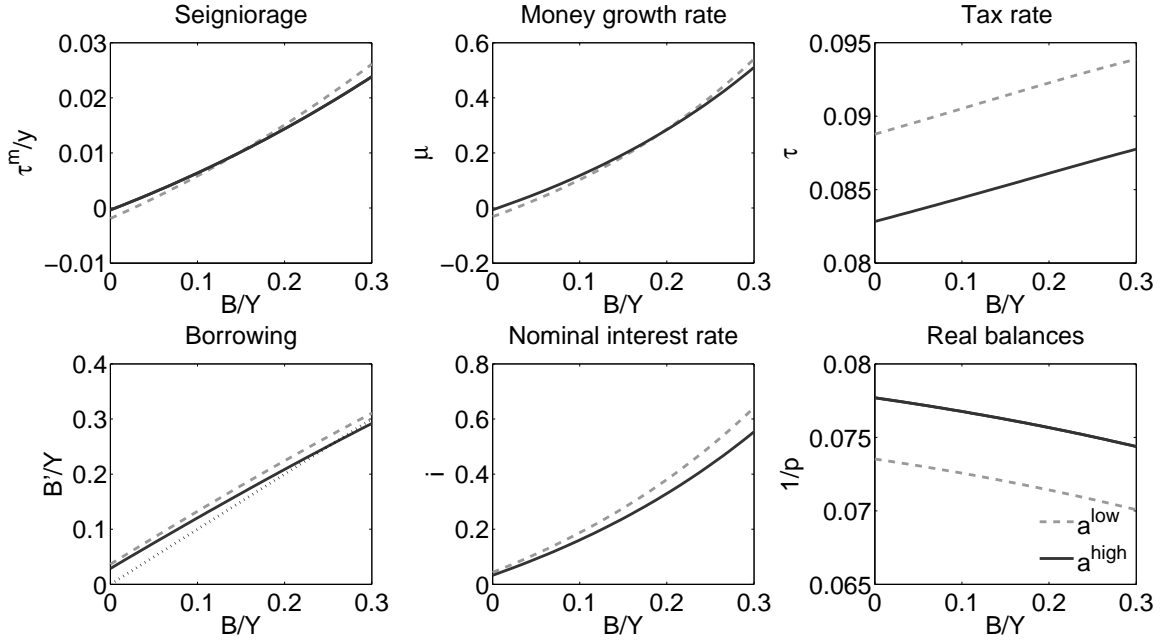


Figure 2: Average bond revenues $R^b(B', a)$ and money revenues $R^m(B', a)$ for the no-default economy and the baseline economy

increasing function of end-of-period debt and the price of nominal government bonds responds to B' in a negative way, causing borrowing to become more expensive when more debt is issued ($\partial R^b / \partial B' < 0$). The shape of R^m reflects the way public debt, inflation and household money demand are related (see panel (b)). Given that real balances $1/p$ are a decreasing function of B , the real payoff of money is expected to decrease with borrowing B' , reducing household money demand today. However, lower real balances $1/p$ also increase the marginal value of money $u_1(1/p)$, such that the demand for money increases with B' since households expect to be more cash-constrained in the subsequent period. Whether higher borrowing B' increases net household money demand depends on the size of the parameter σ_1 . For $\sigma_1 > 1$, it does and an increase in B' leads to higher money revenues ($\partial R^m / \partial B' > 0$). Since a household's valuation of money increases with

Figure 3: Selected policy functions (no-default economy)



Notes: The policy functions are depicted for productivity values 1.5 standard deviations below (a^{low}) and above (a^{high}) the unconditional mean of productivity. The money growth rate and the nominal interest rate are displayed in annual terms.

B' , the government can simply issue more currency to implement a particular price index and thus obtain more revenues from money issuance.

The long-run debt position is determined by the two effects described above. A positive sign for $\partial R^m / \partial B'$ is crucial for non-zero long-run debt.²⁸ For $\sigma_1 = 1$, money revenues do not respond to borrowing ($\partial R^m / \partial B' = 0$) which eliminates the incentive to borrow in the long run. With $\sigma_1 \in (0, 1)$, the government even has an incentive to accumulate assets ($B' < 0$) due to $\partial R^m / \partial B' < 0$.²⁹ These two cases are not further discussed here because they make default a redundant policy option. When there are productivity shocks, a positive response of money revenues to borrowing also matters for the government's ability to smooth tax distortions across states since, without this effect, only the negative bond price effect would be operative and make it more expensive to issue debt in low productivity states.

²⁸Looking at Markov-perfect public policy in a real economy setting with endogenous government spending and without default, Debortoli and Nunes (2013) show - for analytical and quantitative examples - that long-run debt only deviates from zero for a small range of parameter values.

²⁹For more details see Proposition 5 in Martin (2009) who shows these properties for a deterministic model without default option.

	Baseline	No default
<u>Mean</u>		
Default probability (annual)	0.0206	0
Debt-to-GDP	0.1866	0.3340
Tax rate	0.1043	0.0912
Seigniorage-to-GDP	0.0155	0.0296
Inflation rate (annual)	0.3011	0.6503
Nominal interest rate (annual)	0.3958	0.7175
<u>Standard deviation</u>		
Output	0.0236	0.0192
Tax rate	0.0124	0.0012
Inflation rate (annual)	0.1077	0.0544
Nominal interest rate (annual)	0.0782	0.0252
<u>Correlation with output</u>		
Tax rate	-0.8597	-0.9991
Inflation rate (annual)	-0.2990	-0.5599
Nominal interest rate (annual)	-0.4725	-0.9093

Table 2: Selected model statistics

When the government can default on its debt, sovereign risk changes the impact of borrowing on public revenues. Panel (c) displays R^b for the baseline economy with sovereign default. Because a default is more likely for higher amounts of debt, the bond price would respond to B' in a negative way even in the absence of inflation risk. As in Arellano (2008), the debt elasticity of interest rates is higher in bad (low productivity) than in good (high productivity) states, reflecting the default incentives of the government. Panel (d) shows how money revenues respond to B' when there is sovereign risk. The non-monotonic shape results from the optimal mix of default and inflation. Since default and inflation are substitutes, higher borrowing can lower expected inflation by increasing the probability of default. While making money as a store of value more valuable, this interaction also lowers the expected marginal value of money and hence R^m . Due to its adverse effect on bond and money revenues, sovereign risk thus ultimately makes debt issuance less attractive. The consequences for the long-run debt position can be illustrated via Figure 1 and Figure 3. By looking at the intersection between the (dotted) 45-degree line and the borrowing policies, one can already see without having simulated the model that average debt is going to be lower in the baseline model with default. The quantitative dimension of the model properties discussed so far is explored in the next section.

3.3 Simulation Results

Table 2 presents the averages of statistics calculated for 2500 simulated economies with 2000 periods each. The time series are filtered using the Hodrick-Prescott filter with a smoothing parameter of 1600. Output is in logs. All simulations are initialized with $S_0 = (0, \mathbb{E}[a])$ and the first 500 observations of each sample are discarded.

Average debt and inflation are lower with default option. More specifically, the average inflation rate in the no-default economy is more than twice as large as in the baseline economy with default. The possibility of default reduces average inflation through a direct and an indirect effect. When the government chooses to default, there is no incentive to use inflation to reduce the real debt burden anymore. As a result, inflation is lower in default (and autarky) periods than in periods of repayment. The role of this direct effect is however limited by the frequency of default and does not contribute much to the average inflation rate.³⁰ The indirect effect of default on inflation is related to how the risk of default affects the government's borrowing behavior. Default risk raises the cost of rolling over even low amounts of debt in recessions. This mechanism restricts the build up of large public debt positions which would make higher inflation more attractive. Less debt also implies that the tax base of the income tax increases relative to that of inflation. Hence, the benefit of raising inflation is lower, leading to a higher average labor tax rate in the baseline economy.³¹ While the accumulation of debt crucially depends on the government's ability to collect seigniorage (see the discussion in the previous section), the average seigniorage-to-GDP ratio is moderate and of plausible size.³² When the government has the option to default, borrowing is more expensive in recessions due to the increased risk of default. The average nominal interest rate however is higher in the no-default economy since it experiences more inflation on average.

The short-run implications of sovereign risk for public policy can be illustrated via Figure 4. For the baseline and the no-default economy, it displays impulse responses of selected model variables to a negative one-time productivity shock. The variables are expressed as absolute deviations from their values at the stationary state to which the economies converge when productivity is kept fixed at its long-run mean.³³ Since pro-

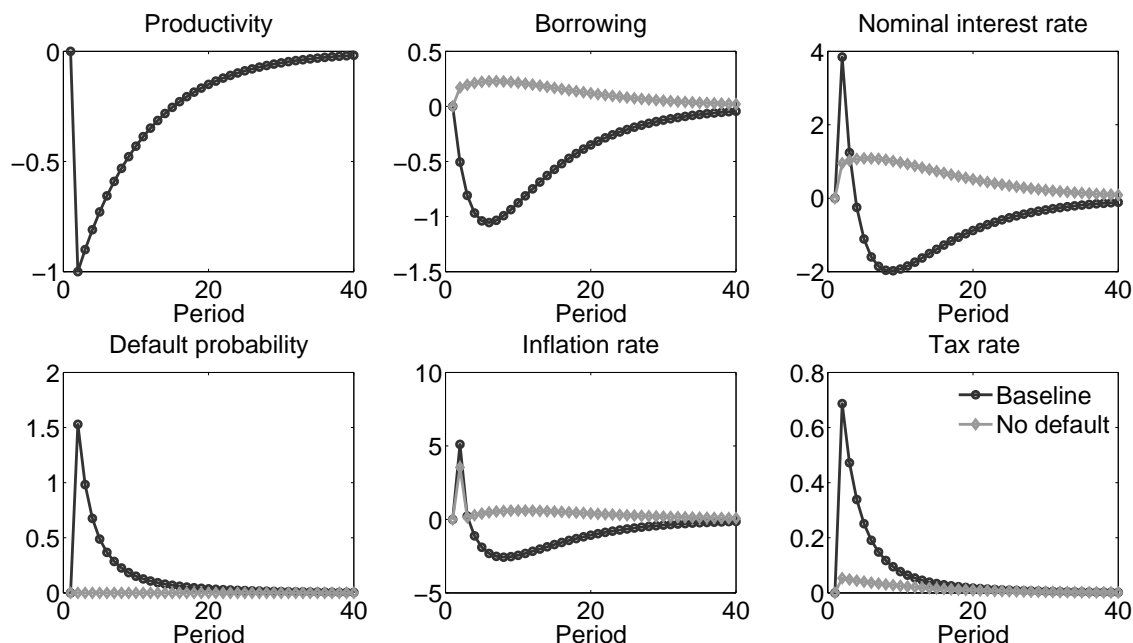
³⁰The average inflation rate for periods of repayment only is 31.08% and thus only slightly higher than the overall average inflation rate of 30.11% which includes periods of default and autarky as well.

³¹For Mexico, Ilzetzki (2011) calculates an average marginal income tax rate of 12.1% which is close to the average tax rate in the baseline model (10.43%).

³²Using the same definition of seigniorage as in the model, Aisen and Veiga (2008) calculate that average seigniorage is 2.2% of GDP for Mexico.

³³Of course, the two economies do not exhibit the same stationary state. The variables in such a stationary state are close to the average values listed in Table 2.

Figure 4: Impulse responses of selected model variables to a negative one-time productivity shock



Notes: All variables are expressed as absolute deviations (multiplied by 100) from their values at the stationary state to which the respective economy converges when productivity is kept fixed at its unconditional mean.

ductivity is persistent ($\rho > 0$), the negative shock immediately raises the risk of default as the incentive to default is more likely to be strong in the subsequent period. The high sensitivity of interest rates to changes in debt issuance forces the government to reduce its debt position in order to avoid an even larger decline of the bond price. As a result, the government has to resort to large increases in inflation and taxes to finance debt payments and government spending.³⁴ Consistent with empirical evidence (see Reinhart and Rogoff, 2009), a sovereign debt crisis thus is inflationary. As productivity reverts back to its mean and debt is reduced even further, expected inflation and sovereign risk both decline, leading the government to take advantage of the improved borrowing conditions and accumulate debt again. In the no-default economy, borrowing conditions do not deteriorate very much in response to the negative productivity shock. This property allows the government to effectively smooth tax distortions across states by issuing debt which avoids large increases in taxes and inflation. Because debt cannot be easily rolled over in the

³⁴This mechanism is related to the one studied by Cuadra et al. (2010) in a model of a small open economy with real government debt. The authors show that countercyclical default risk can rationalize the procyclical consumption taxation observed in emerging economies.

	Mexico (1980-2007)	Baseline	No default (recalibrated)
Standard deviation	0.2423	0.1077	0.0703
Correlation with output	-0.2734	-0.2990	-0.4585

Table 3: Cyclical properties of inflation

baseline model, the impact of productivity shocks on the economy is more pronounced and output volatility is 23% higher relative to the model without default.

Since one of the main contributions of this paper is to offer a joint analysis of inflation and sovereign default, it is interesting to compare the cyclical properties of inflation generated by the model with and without default to those observed in the data. Table 3 shows the results. In Mexico, inflation is very volatile and countercyclical. The baseline model with default can replicate these findings. To give the model without default a fair chance, it is recalibrated to match the average inflation rate and the volatility of output in Mexico.³⁵ While the recalibrated no-default economy yields countercyclical inflation, the baseline model predicts more volatile and less countercyclical inflation than the no-default model which is closer to what is observed empirically.

4 The Welfare Implications of Sovereign Default

This section discusses the welfare implications of sovereign default. With commitment, the option to default will not decrease welfare since the government would otherwise refrain from using it.³⁶ Without commitment, this is not necessarily the case anymore. Section 3 has shown that the default option has implications for public policy in the short and the long run. On the one hand, by increasing the sensitivity of bond prices with respect to debt and productivity, countercyclical risk of default entails short-run costs because the government loses some of its ability to smooth tax distortions across states. On the other hand, default risk might lead to welfare gains due to its impact on long-run debt. The model features a long-run borrowing motive that stems from the presence of two frictions, lack of commitment and a liquidity constraint (see the discussion in Section 3.2). The government acts in a time-consistent way and does not internalize the effect of its current choice of inflation on the borrowing behavior in previous periods. When

³⁵The changed model parameter values are $\gamma_1 = 0.015$, $\gamma_2 = 0.325$, $\sigma_1 = 1.73$ and $\sigma = 0.02$.

³⁶For a real small open economy with incomplete markets and costly sovereign default, Adam and Grill (2012) show that welfare can be increased when the Ramsey planner can commit to a state-contingent default plan.

household money demand and thus the value of money are increasing in the amount of issued debt, the government persistently chooses positive debt positions which then lead to high average inflation. By limiting public debt accumulation via more sensitive interest rates, the default option reduces average inflation and the misallocation of consumption compared to the no-default setting.

To evaluate whether the addition of the default option to the set of policy instruments is welfare enhancing, welfare measure Δ is calculated. It measures the percentage increase in credit-good consumption that households in the no-default economy need to be given in each period to achieve the same expected lifetime utility as in the baseline economy with default:

$$\mathbb{E}_0 \left[\sum_{t=0}^T \beta^t u(c_{1t}^D, c_{2t}^D, n_t^D) \right] = \mathbb{E}_0 \left[\sum_{t=0}^T \beta^t u(c_{1t}^N, c_{2t}^N(1 + \Delta), n_t^N) \right].$$

The sequences of consumption and labor supply in the economy with ($j = D$) and without default option ($j = N$) are denoted as $\{c_{1t}^j, c_{2t}^j, n_t^j\}_{t=0}^T$. Expected lifetime utility is calculated for both types of economies by averaging realized lifetime utility of 2500 samples with simulated time series of effective length $T = 1500$ each.

The calculated welfare measure is $\Delta = 0.00006$. For the no-default economy, credit-good consumption thus needs to be increased by only 0.006% in each period to equalize household welfare in both types of economies. Since these gains are of negligible size, one can argue that, from a welfare perspective, lack of commitment to repayment is not important.

5 Conclusion

To understand the implications of the option to default on debt payments for public policy, this paper has studied optimal monetary and fiscal policy without commitment in a cash-credit economy with nominal debt and endogenous government default. While a default allows the government to reduce inflation and distortionary labor taxation by relaxing its budget constraint, the default option mainly induces lower rates of inflation by constraining debt issuance via endogenous default risk premia. This mechanism reduces the average debt position and the government's incentive to implement high inflation in the long-run. Less debt also implies that the income tax becomes more attractive relative to inflation, resulting in a higher average labor tax rate. Taxes and inflation are more volatile when the default option is available because the government's ability to smooth

tax distortions across states is reduced by the presence of default risk. For the case of Mexico, a counterfactual exercise has demonstrated that the consequences of the option to default for welfare are negligible.

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A Appendix

A.1 Derivation of the Implementability Constraint

I will only derive the implementability constraint for the repayment case. The constraint for the default case is derived similarly. First, take the household optimality conditions (1),(6)-(7) and rewrite them (in recursive notation) as

$$\begin{aligned}\tau &= 1 + \frac{u_n(n)}{u_2(c_2)} \frac{1}{\psi(a,0)}, \\ \frac{1+\mu}{p} &= \beta \mathbb{E}_{a'|a} \left[\frac{u_1(1/p')}{u_2(c_2)} \frac{1}{p'} \right], \\ \frac{(1+\mu)q}{p} &= \beta \mathbb{E}_{a'|a} \left[\frac{u_2(c'_2)}{u_2(c_2)} \frac{1-d'}{p'} \right].\end{aligned}$$

After using these expressions to eliminate the terms on the LHS of these equations in the government budget constraint

$$g - \tau \psi(a,0)n + \frac{1+B}{p} = (1+\mu) \frac{1+qB'}{p},$$

one obtains

$$\begin{aligned}& g - \left(1 + \frac{u_n(n)}{u_2(c_2)} \frac{1}{\psi(a,0)} \right) \psi(a,0)n + 1/p + B/p \\ &= \beta \mathbb{E}_{a'|a} \left[\frac{u_1(1/p')}{u_2(c_2)} \frac{1}{p'} \right] + \beta \mathbb{E}_{a'|a} \left[\frac{u_2(c'_2)}{u_2(c_2)} \frac{1-d'}{p'} \right] B',\end{aligned}$$

or

$$\begin{aligned}& g - \psi(a,0)n - \frac{u_n(n)}{u_2(c_2)} + 1/p + B/p \\ &= \beta \mathbb{E}_{a'|a} \left[\frac{u_1(1/p')}{u_2(c_2)} \frac{1}{p'} \right] + \beta \mathbb{E}_{a'|a} \left[\frac{u_2(c'_2)}{u_2(c_2)} \frac{1-d'}{p'} \right] B'.\end{aligned}$$

Now, eliminate $\psi(a,0)n$ via the resource constraint $\psi(a,0)n = 1/p + c_2 + g$,

$$\begin{aligned}& g - (1/p + c_2 + g) - \frac{u_n(n)}{u_2(c_2)} + 1/p + B/p \\ &= \beta \mathbb{E}_{a'|a} \left[\frac{u_1(1/p')}{u_2(c_2)} \frac{1}{p'} \right] + \beta \mathbb{E}_{a'|a} \left[\frac{u_2(c'_2)}{u_2(c_2)} \frac{1-d'}{p'} \right] B'.\end{aligned}$$

After multiplying both sides of the equation with $u_2(c_2)$ and using the policy functions to replace next period's variables, one arrives at the implementability constraint

$$-u_n(n)n - u_2(c_2)(c_2 - B/p) \\ = \beta \mathbb{E}_{a'|a} \left[u_1(\mathcal{P}^r(B', a')^{-1}) \frac{1 - \mathcal{D}(B', a')}{\mathcal{P}^r(B', a')} + u_1(\mathcal{P}^d(a')^{-1}) \frac{\mathcal{D}(B', a')}{\mathcal{P}^d(a')} \right] + \beta \mathbb{E}_{a'|a} \left[u_2(C_2'(B', a')) \frac{1 - \mathcal{D}(B', a')}{\mathcal{P}^r(B', a')} \right] B'.$$

A.2 First-Order Conditions for the Policy Problem

Conditional on repayment, the necessary first-order condition for an interior debt choice B' is

$$0 = \xi \left(R^b + \frac{\partial R^b}{\partial B'} B' + \frac{\partial R^m}{\partial B'} \right) + \beta \int_{\hat{a}(B')}^{\infty} \frac{\partial \mathcal{V}^r(B', a')}{\partial B'} f_a(a'|a) da',$$

with ξ denoting the multiplier on the implementability constraint (9).³⁷

When combined with definition

$$R^b = \beta \int_{\hat{a}(B')}^{\infty} \frac{u_2(c_2')}{p'} f_a(a'|a) da',$$

and envelope condition

$$\frac{\partial \mathcal{V}^r(B, a)}{\partial B} = -\xi \frac{u_2(c_2)}{p},$$

the first-order condition yields the generalized Euler equation

$$\beta \int_{\hat{a}(B')}^{\infty} (\xi' - \xi) \frac{u_2(c_2')}{p'} f_a(a'|a) da' = \xi \left(\frac{\partial R^b}{\partial B'} B' + \frac{\partial R^m}{\partial B'} \right).$$

The first-order conditions for the price index p , credit consumption c_2 , and labor supply n are

$$0 = -p^{-2} [u_1(1/p) - \xi u_2(c_2) B - \phi + \vartheta u_{11}(1/p)], \quad (15)$$

$$0 = u_2(c_2) + \xi [u_{22}(c_2)(c_2 - B/p) + u_2(c_2)] - \phi - \vartheta u_{22}(c_2), \quad (16)$$

$$0 = u_n(n) + \xi [u_{nn}(n)n + u_n(n)] + \phi \Psi(a, 0), \quad (17)$$

where ξ , ϕ and ϑ are the multipliers related to the constraints (9), (11) and (12). In

³⁷The derivation of the generalized Euler equation follows Martin (2009) and Arellano and Ramaranayanan (2012).

addition to these three conditions, the complementary slackness conditions

$$\vartheta \geq 0, u_1(1/p) - u_2(c_2) \geq 0, \vartheta \times [u_1(1/p) - u_2(c_2)] = 0, \quad (18)$$

need to be satisfied as well.

A.3 Numerical Solution

The task of the numerical solution algorithm is to find the policy and value functions $\mathcal{X}^r(B, a)$, $\mathcal{X} \in \{\mathcal{B}, \mathcal{C}_2, \mathcal{N}, \mathcal{P}\}$, and $\mathcal{X}^d(a)$, $\mathcal{X} \in \{\mathcal{C}_2, \mathcal{N}, \mathcal{P}\}$. Following Hatchondo et al. (2010), I approximate these functions on discrete grids for debt and productivity, and use cubic spline interpolation to allow for off-grid values of B and a . The solution algorithm involves the following steps:

1. Construct discrete grids for debt $[\underline{B}, \bar{B}]$ and productivity $[\underline{a}, \bar{a}]$.
2. Choose initial values for the policy and value functions $\mathcal{X}_{start}^r(B, a)$ and $\mathcal{X}_{start}^d(a)$, $\mathcal{X} \in \{\mathcal{C}_2, \mathcal{N}, \mathcal{P}, \mathcal{V}\}$, at all grid point combinations.
3. Set $\mathcal{X}_{next}^j = \mathcal{X}_{start}^j$, $j \in \{r, d\}$ and fix an error tolerance ε .
4. For each discrete grid point combination $(B, a) \in [\underline{B}, \bar{B}] \times [\underline{a}, \bar{a}]$, find the optimal policies $\mathcal{X}_{new}^r(B, a)$, $\mathcal{X} \in \{\mathcal{B}, \mathcal{C}_2, \mathcal{N}, \mathcal{P}\}$, and the associated value of repayment $\mathcal{V}_{new}^r(B, a)$. For each productivity value $a \in [\underline{a}, \bar{a}]$, compute the optimal policies $\mathcal{X}_{new}^d(a)$, $\mathcal{X} \in \{\mathcal{C}_2, \mathcal{N}, \mathcal{P}\}$, and the value of default $\mathcal{V}_{new}^d(a)$.
5. If $|\mathcal{X}_{new}^r(B, a) - \mathcal{X}_{next}^r(B, a)| < \varepsilon$ and $|\mathcal{X}_{new}^d(a) - \mathcal{X}_{next}^d(a)| < \varepsilon$, $\mathcal{X} \in \{\mathcal{C}_2, \mathcal{N}, \mathcal{P}, \mathcal{V}\}$, for all grid point combinations, go to step 6, else set $\mathcal{X}_{next}^j = \mathcal{X}_{new}^j$, $j \in \{r, d\}$ and repeat step 4.
6. Use $\mathcal{X}_{new}^j(\cdot)$, $j \in \{r, d\}$, as approximations of the respective equilibrium objects in the infinite-horizon economy.

The grid points are distributed evenly. Since the asymmetric default cost specification leads to a kink at $a = \bar{a}$ in $\mathcal{X}^d(a)$, $\mathcal{X} \in \{\mathcal{C}_2, \mathcal{N}, \mathcal{P}\}$, I partition the productivity grid for the default case as in Hatchondo et al. (2010) to account for this discontinuity.

As is known in the literature (see e.g. Krusell and Smith, 2003, or Martin, 2009), there might be multiple Markov-perfect equilibria in models with infinitely-lived agents. In particular, there could be equilibria with discontinuous policy functions which do not arise

in the infinite-horizon limit of a finite-horizon model version. To avoid such equilibria, I follow Hatchondo et al. (2010) and solve for the infinite-horizon limit of a finite-horizon model version.³⁸ In practice, this means that I compute the value and policy functions for the final period problem where no borrowing takes place and use these objects as initial values \mathcal{X}_{start}^j , $j \in \{r, d\}$, for step 2.

For a given state $(B, a) \in [\underline{B}, \overline{B}] \times [\underline{a}, \overline{a}]$, the objective function of the government is the sum of two parts, the period utility function $u(1/p, c_2, n)$ and (in the repayment case) the continuation value $\beta \mathbb{E}_{a'|a} [\mathcal{V}_{next}(B', a')]$, with $\mathcal{V}_{next}(B, a) = \max \{ \mathcal{V}_{next}^r(B, a), \mathcal{V}_{next}^d(a) \}$. The optimal policies for step 4 are then computed as follows. I use a sub-routine that calculates the optimal static policies c_2 , n , and p for given debt and productivity values $(B, a) \in [\underline{B}, \overline{B}] \times [\underline{a}, \overline{a}]$ and an arbitrary, i.e. possibly off-grid, borrowing value \hat{B}' . More specifically, I use a non-linear equation solver to find the variables that satisfy the static optimality conditions (9),(11),(12),(15)-(18).³⁹ Since these conditions involve the complementary slackness conditions (18), I follow Brumm and Grill (2014) and use the trick by Garcia and Zangwill (1981) to transform the set of optimality conditions into a system of equations.⁴⁰ Using the static policy sub-routine, (c_2, n, p) and thus period utility $u(1/p, c_2, n)$ can be expressed as functions of (B, a, \hat{B}') .⁴¹ As a result, given $(B, a) \in [\underline{B}, \overline{B}] \times [\underline{a}, \overline{a}]$, the whole government objective can be expressed as a function of \hat{B}' .

For each discrete grid point combination $(B, a) \in [\underline{B}, \overline{B}] \times [\underline{a}, \overline{a}]$, the optimal debt policy $\mathcal{B}_{new}^r(B, a)$ then is computed via a global non-linear optimizer, calling the static policy routine to calculate the objective function for each candidate debt value \hat{B}' . More specifically, for each (B, a) , I first perform a grid search over a pre-defined grid for \hat{B}' and then use the solution as an initial guess for the Nelder-Mead algorithm.⁴² The optimal policies $\mathcal{X}^r(B, a)$, $\mathcal{X} \in \{C_2, \mathcal{N}, \mathcal{P}\}$ then are computed by using the static policy routine for the optimal borrowing value $\mathcal{B}_{new}^r(B, a)$. The algorithm iterates on the policy and

³⁸Martin (2009) also solves for the infinite-horizon limit. As pointed out by him, using a Svensson (1985)-type beginning-of-period cash-in-advance constraint in a finite-horizon model requires a terminal money value for a monetary equilibrium to exist. Otherwise, households will not be willing to invest in money in the final period and by backward induction not in any of the previous periods. The impact of the final-period value of money vanishes over time and does not affect the final results.

³⁹In the default case, condition (9) is replaced by condition (10)

⁴⁰Alternatively, one can also use a non-linear constrained optimizer to compute the optimal static policies for each combination (B, a, \hat{B}') . Using a sequential quadratic programming algorithm (see e.g. Nocedal and Wright, 1999 for details), I found this approach to be both slower and less accurate.

⁴¹The routine is also used to obtain the optimal policies in the final period, where $\hat{B}' = 0$ holds.

⁴²Instead of the Nelder-Mead algorithm, I also solved for the optimal debt policy using Golden section search and a mesh adaptive direct search algorithm (as implemented by the optimizing routine NOMAD provided by the OPTI Toolbox), which did not affect the results but slowed down the algorithm considerably.

value functions until the maximum absolute difference between value and policy functions obtained in two subsequent iterations is below $\varepsilon = 10^{-5}$ for each combination $(B, a) \in [\underline{B}, \bar{B}] \times [\underline{a}, \bar{a}]$.⁴³

To evaluate value and policy functions at debt and productivity states that are off-grid, cubic spline interpolation is used.⁴⁴ To approximate expected values in an accurate way, one needs to account for the default threshold. This can be seen by looking at the expected option value of default:

$$\mathbb{E}_{a'|a} [\mathcal{V}_{next}(B', a')] = \int_0^{\hat{a}(B')} \mathcal{V}_{next}^d(a') f_a(a'|a) da' + \int_{\hat{a}(B')}^{\infty} \mathcal{V}_{next}^r(B', a') f_a(a'|a) da'.$$

Gauss-Legendre quadrature nodes and weights are used to approximate the integrals above. The default threshold $\hat{a}(B)$ satisfies $\mathcal{V}_{next}^r(B, \hat{a}(B)) - \mathcal{V}_{next}^d(\hat{a}(B)) = 0$ and is computed via bisection method.

⁴³Using a tighter convergence criterion did not affect the results.

⁴⁴Hatchondo et al. (2010) show that allowing for a continuous state space is crucial for accurate solutions of models with equilibrium default.