

**Monetary Policy and Exchange Rate Volatility  
in a Small Open Economy**

by

Jordi Galí

and

Tommaso Monacelli

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## Motivation

- The new Keynesian model for the closed economy
    - equilibrium dynamics: simple three-equation representation
    - optimal monetary policy design: optimality of inflation targeting
  - How does the introduction of open economy elements affect that analysis and prescriptions?
  - What role should the exchange rate play in the design of policy? What is the optimal degree of exchange rate volatility?
- ⇒ main finding: equivalence result (for a benchmark model)

# A New Keynesian Model of a Small Open Economy

*Households*

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

subject to

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + E_t\{Q_{t,t+1} D_{t+1}\} \leq D_t + W_t N_t + T_t$$

$$C_t = \left[ (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

$$C_{H,t} = \left( \int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$C_{F,t} \equiv \left( \int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} \quad ; \quad C_{i,t} \equiv \left( \int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

## Firms

$$Y_t(i) = A_t N_t(i)$$

$$a_t = \rho a_{t-1} + u_t$$

## Law of One Price

$$p_{i,t}(j) = e_{i,t} + p_{i,t}^i(j)$$

$$\Rightarrow p_{F,t} = e_t + p_t^*$$

## Some Identities and Definitions

*Terms of Trade*

$$s_t \equiv p_{F,t} - p_{H,t}$$

*CPI*

$$\begin{aligned} p_t &\equiv (1 - \alpha) p_{H,t} + \alpha p_{F,t} \\ &= p_{H,t} + \alpha s_t \end{aligned}$$

*CPI Inflation vs. Domestic Inflation:*

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t$$

*Real Exchange Rate and the Terms of Trade*

$$\begin{aligned} q_t &\equiv (e_t + p_t^*) - p_t \\ &= s_t + (p_{H,t} - p_t) \\ &= (1 - \alpha) s_t \end{aligned}$$

*Optimal Intratemporal Allocation of Expenditures:*

- within each category:

$$\begin{aligned}c_{H,t}(i) &= -\varepsilon (p_{H,t}(i) - p_{H,t}) + c_{H,t} \\c_{F,t}(i) &= -\gamma (p_{F,t}(i) - p_{F,t}) + c_{F,t}\end{aligned}$$

- between categories:

$$\begin{aligned}c_{H,t} &= -\eta (p_{H,t} - p_t) + c_t \\c_{F,t} &= -\eta (p_{F,t} - p_t) + c_t\end{aligned}$$

*Other Optimality Conditions:*

$$w_t - p_t = \sigma c_t + \varphi n_t$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} (r_t - E_t\{\pi_{t+1}\} - \rho)$$

*International Risk Sharing (Complete Markets)*

$$\beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) = Q_{t,t+1}$$

$$\beta \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left( \frac{P_t^i}{P_{t+1}^i} \right) \left( \frac{\mathcal{E}_t^i}{\mathcal{E}_{t+1}^i} \right) = Q_{t,t+1}$$

Combining domestic and world foc's:

$$C_t = \vartheta_i C_t^i Q_{i,t}^{\frac{1}{\sigma}}$$

Log-linearizing (after  $\vartheta_i = 1$ ):

$$\begin{aligned} c_t &= c_t^* + \frac{1}{\sigma} q_t \\ &= c_t^* + \frac{1-\alpha}{\sigma} s_t \end{aligned}$$

## *Uncovered Interest Parity*

Complete markets:

$$E_t\{Q_{t,t+1} [R_t - R_t^i (\mathcal{E}_{i,t+1}/\mathcal{E}_{i,t})]\} = 0$$

Log-linearizing and aggregating over  $i$ :

$$r_t - r_t^* = E_t\{\Delta e_{t+1}\}$$

Combined with the definition of the terms of trade:

$$s_t = (r_t^* - E_t\{\pi_{t+1}^*\}) - (r_t - E_t\{\pi_{H,t+1}\}) + E_t\{s_{t+1}\}$$

Integrating forward, and using  $\lim_{T \rightarrow \infty} E_t\{s_T\} = 0$  :

$$s_t = E_t \left\{ \sum_{k=0}^{\infty} [(r_{t+k}^* - \pi_{t+k+1}^*) - (r_{t+k} - \pi_{H,t+k+1})] \right\}$$



## Equilibrium Dynamics in the SOE: A Canonical Representation

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa_\alpha \tilde{y}_t$$

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma_\alpha} (r_t - E_t\{\pi_{H,t+1}\} - \overline{r}r_t)$$

where

$$\begin{aligned}\tilde{y}_t &= y_t - \bar{y}_t \\ \bar{y}_t &= \Omega + \Gamma a_t + \alpha \Psi y_t^* \\ \overline{r}r_t &\equiv \rho - \sigma_\alpha \Gamma (1 - \rho_a) a_t + \alpha \sigma_\alpha (\Theta + \Psi) E_t\{\Delta y_{t+1}^*\}\end{aligned}$$

$$\kappa_\alpha \equiv \lambda (\sigma_\alpha + \varphi) \quad ; \quad \sigma_\alpha \equiv \frac{\sigma}{(1 - \alpha) + \alpha \omega} \quad ; \quad \Gamma \equiv \frac{1 + \varphi}{\sigma_\alpha + \varphi} \quad ; \quad \Psi \equiv - \frac{\Theta \sigma_\alpha}{\sigma_\alpha + \varphi}$$

## Aggregate Demand and Output Determination

*World Market Clearing (WMC)*

$$y_t^* = c_t^*$$

*Domestic Market Clearing (DMC)*

$$\begin{aligned} y_t &= c_t + \alpha \gamma s_t + \alpha \left( \eta - \frac{1}{\sigma} \right) q_t \\ &= c_t + \frac{\alpha \omega}{\sigma} s_t \end{aligned}$$

where  $\omega \equiv \sigma \gamma + (1 - \alpha)(\sigma \eta - 1)$ .

Combining DMC and WMC with IRS we obtain:

$$y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t \tag{1}$$

where  $\sigma_\alpha \equiv \frac{\sigma}{(1-\alpha)+\alpha\omega} > 0$ .

Finally, combining DMC with the Euler equation:

$$\begin{aligned}
y_t &= E_t\{y_{t+1}\} - \frac{1}{\sigma} (r_t - E_t\{\pi_{t+1}\} - \rho) - \frac{\alpha\omega}{\sigma} E_t\{\Delta s_{t+1}\} \\
&= E_t\{y_{t+1}\} - \frac{1}{\sigma} (r_t - E_t\{\pi_{H,t+1}\} - \rho) - \frac{\alpha\Theta}{\sigma} E_t\{\Delta s_{t+1}\} \\
&= E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha} (r_t - E_t\{\pi_{H,t+1}\} - \rho) + \alpha\Theta E_t\{\Delta y_{t+1}^*\}
\end{aligned}$$

where  $\Theta \equiv (\sigma\gamma - 1) + (1 - \alpha)(\sigma\eta - 1) = \omega - 1$  and  $\sigma_\alpha \equiv \frac{\sigma}{(1-\alpha)+\alpha\omega}$ .

Letting  $\tilde{y}_t = y_t - \bar{y}_t$ ,

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma_\alpha} (r_t - E_t\{\pi_{H,t+1}\} - \bar{r}r_t)$$

where  $\bar{r}r_t \equiv \rho + \sigma_\alpha(\Delta\bar{y}_{t+1} + \alpha\Theta E_t\{\Delta y_{t+1}^*\})$

# The New Keynesian Phillips Curve in the Small Open Economy

*Domestic Price Dynamics:*

$$p_{H,t} \equiv \theta p_{H,t-1} + (1 - \theta) \bar{p}_{H,t}$$

*Optimal Price Setting:*

$$\bar{p}_{H,t} = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{mc_{t+k} + p_{H,t}\}$$

*Domestic Inflation Dynamics*

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda \widehat{mc}_t \tag{2}$$

where  $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$

### *Marginal Cost and the Output Gap*

$$\begin{aligned} mc_t &= -\nu + (w_t - p_{H,t}) - a_t \\ &= -\nu + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\ &= -\nu + \sigma c_t + \varphi n_t + \alpha s_t - a_t \\ &= -\nu + \sigma y_t^* + \varphi y_t + s_t - (1 + \varphi) a_t \end{aligned}$$

Substituting for  $s_t$  using (1):

$$mc_t = -\nu + (\sigma_\alpha + \varphi) y_t + (\sigma - \sigma_\alpha) y_t^* - (1 + \varphi) a_t$$

Under flexible prices,

$$-\mu = -\nu + (\sigma_\alpha + \varphi) \bar{y}_t + (\sigma - \sigma_\alpha) y_t^* - (1 + \varphi) a_t$$

Thus,

$$\bar{y}_t = \Omega + \Gamma a_t + \alpha \Psi y_t^*$$

Also

$$\widehat{mc}_t = (\sigma_\alpha + \varphi) \tilde{y}_t$$

which combined with (2) yields:

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa_\alpha \tilde{y}_t$$

# Optimal Monetary Policy

*Background and Strategy*

*A Special Case*

$$\sigma = \eta = \gamma = 1$$

$$\Rightarrow C_t = Y_t^{1-\alpha}(Y_t^*)^\alpha \quad (3)$$

*Optimal Allocation:*

$$\max \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi}$$

subject to

$$\begin{aligned} C_t &= Y_t^{1-\alpha}(Y_t^*)^\alpha \\ &= (A_t N_t)^{1-\alpha}(Y_t^*)^\alpha \end{aligned}$$

*Optimality condition:*

$$N = (1 - \alpha)^{\frac{1}{1+\varphi}}$$

### *Flexible Price Equilibrium*

$$\begin{aligned} 1 - \frac{1}{\varepsilon} &= \overline{MC}_t \\ &= - \frac{(1 - \tau)}{A_t} \overline{S}_t^\alpha \frac{U_N(\overline{C}_t, \overline{N}_t)}{U_C(\overline{C}_t, \overline{N}_t)} \\ &= \frac{(1 - \tau)}{A_t} \frac{\overline{Y}_t}{\overline{C}_t} \overline{N}_t^\varphi \overline{C}_t \\ &= (1 - \tau) \overline{N}_t^{1+\varphi} \end{aligned}$$

### *Optimality of Flexible Price Equilibrium:*

$$(1 - \tau)(1 - \alpha) = 1 - \frac{1}{\varepsilon}$$

### *Implied Monetary Policy Objectives*

$$y_t = \overline{y}$$

$$\pi_{H,t} = 0$$

for all  $t$ .

## *Implementation*

$$r_t = \overline{r} \overline{r}_t + \phi_\pi \pi_{H,t} + \phi_x x_t$$

where  $\kappa_\alpha (\phi_\pi - 1) + (1 - \beta) \phi_x > 0$ .

## **Other Macroeconomic Implications**

### *Terms of Trade*

$$\begin{aligned} \overline{s}_t &= \sigma_\alpha (\overline{y}_t - y_t^*) \\ &= \sigma_\alpha \Omega + \sigma_\alpha \Gamma a_t - \sigma_\alpha \Phi y_t^* \end{aligned}$$

where  $\Phi \equiv \frac{\sigma + \varphi}{\sigma_\alpha + \varphi} > 0$ .

Special case

$$\overline{s}_t = a_t - y_t^*$$

### *Exchange Rate*

$$\overline{e}_t = \overline{s}_t - p_t^*$$

### *CPI*

$$\begin{aligned} \overline{p}_t &= \alpha (\overline{e}_t + p_t^*) \\ &= \alpha \overline{s}_t \end{aligned}$$



## Consequences of Suboptimal Policies

*Welfare Losses (special case)*

$$\mathbb{W} = - \frac{(1 - \alpha)}{2} \sum_{t=0}^{\infty} \beta^t \left[ \frac{\varepsilon}{\lambda} \pi_{H,t}^2 + (1 + \varphi) x_t^2 \right]$$

Taking unconditional expectations and letting  $\beta \rightarrow 1$ ,

$$\mathbb{V} = - \frac{(1 - \alpha)}{2} \left[ \frac{\varepsilon}{\lambda} \text{var}(\pi_{H,t}) + (1 + \varphi) \text{var}(x_t) \right]$$

## Three Simple Rules

*Domestic inflation-based Taylor rule* (DITR)

$$r_t = \rho + \phi_\pi \pi_{H,t}$$

*CPI inflation-based Taylor rule* (CITR):

$$r_t = \rho + \phi_\pi \pi_t$$

*Exchange rate peg* (PEG)

$$e_t = 0$$

TABLE 1

*Cyclical properties of alternative policy regimes*

	Optimal sd%	DI Taylor sd%	CPI Taylor sd%	Peg sd%
Output	0.95	0.68	0.72	0.86
Domestic inflation	0.00	0.27	0.27	0.36
CPI inflation	0.38	0.41	0.27	0.21
Nominal I. rate	0.32	0.41	0.41	0.21
Terms of trade	1.60	1.53	1.43	1.17
Nominal depr. rate	0.95	0.86	0.53	0.00

TABLE 2  
*Contribution to welfare losses*

	DI Taylor	CPI Taylor	Peg
Benchmark $\mu = 1.2, \varphi = 3$			
Var(domestic infl)	0.0157	0.0151	0.0268
Var(output gap)	0.0009	0.0019	0.0053
Total	0.0166	0.0170	0.0321
Low steady state mark-up $\mu = 1.1, \varphi = 3$			
Var(Domestic infl)	0.0287	0.0277	0.0491
Var(Output gap)	0.0009	0.0019	0.0053
Total	0.0297	0.0296	0.0544
Low elasticity of labour supply $\mu = 1.2, \varphi = 10$			
Var(Domestic infl)	0.0235	0.0240	0.0565
Var(Output gap)	0.0005	0.0020	0.0064
Total	0.0240	0.0261	0.0630
Low mark-up and elasticity of labour supply $\mu = 1.1, \varphi = 10$			
Var(Domestic infl)	0.0431	0.0441	0.1036
Var(Output gap)	0.0005	0.0020	0.0064
Total	0.0436	0.0461	0.1101

*Note:* Entries are percentage units of steady state consumption.

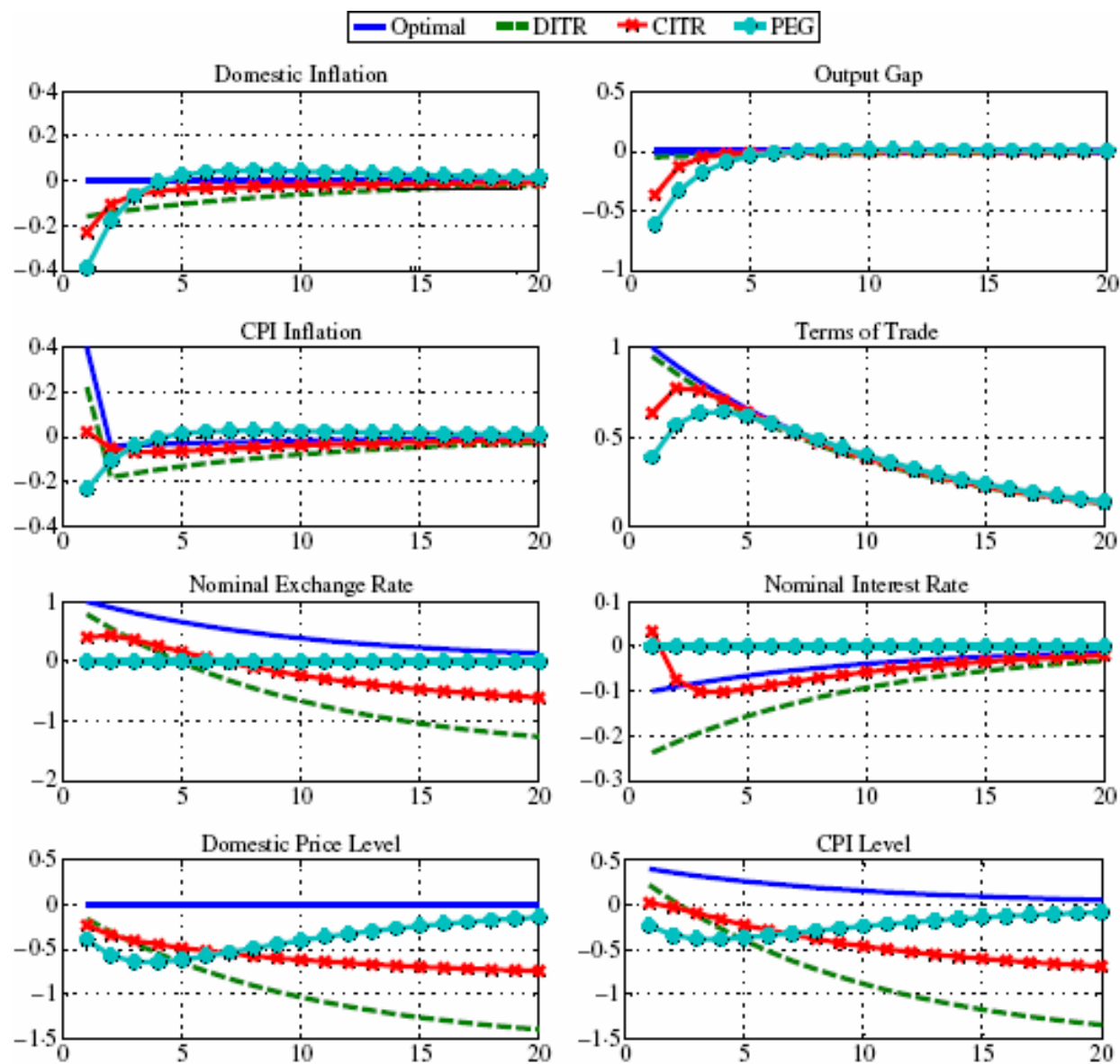


FIGURE 1  
Impulse responses to a domestic productivity shock under alternative policy rules

## Concluding Remarks

- small open economy version of the new Keynesian model
- under baseline assumptions (complete markets, full pass-through), equilibrium dynamics equivalent to the closed economy
- in a special (but not implausible) case: same optimal policy implications as in the closed economy (domestic inflation targeting).
- optimal policy associated with large fluctuations in nominal exchange rate.
- extensions:
  - sticky wages
  - limited pass-through
  - incomplete markets.
  - fiscal policy
  - optimal policy design in a monetary union