Monetary Policy, Factor Substitution, and the Speed of Convergence

Anne Jurkat and Rainer Klump
(Goethe University Frankfurt)

Agenda

- Introduction
- The Intertemporal Monetary Growth Model
- Calibration and Sensitivity Analyses
  - Calibration of the Economy
  - Changes in the Parameters
  - Influence of the Normalization Point
- Conclusion

Introduction

We want to bring together three strands of discussion in macroeoconomic growth theory:

1. *(Optimal) Monetary growth theory*
2. *(Normalized) CES production functions*
3. *Speed of convergence analysis*

The most important benchmark papers are:

Turnovsky (JEDC 2002), Gokan (JEDC 2003), Klump/Saam (EL 2008)

Early ideas can already be found in Klump (1999):

„Keynes und die Neoklassiker: Verbindungen zwischen Keynesianischer Makroökonomik und Neoklassischer Wachstumstheorie“

The Intertemporal Monetary Growth Model

We let money enter the utility function (Sidrauski 1967) and work with a normalized CES production function (De LaGrandville 1989; Klump and De LaGrandville 2000)

$$u(c_t, m_t) = \frac{\left(c^{1-\theta} m^{\theta}\right)^{1-\eta} - 1}{1-\eta} \qquad (1)$$

$$f(k) = C(\sigma)\left[\alpha(\sigma) k^{\psi} - \left(1-\alpha(\sigma)\right)\right]^{\frac{1}{\psi}} \text{ with } \alpha(\sigma) = \frac{k_0^{1-\psi}}{k_0^{1-\psi} + \mu_0},$$

$$C(\sigma) = y_0 \left[\frac{k_0^{1-\psi} + \mu_0}{k_0 + \mu_0}\right]^{\frac{1}{\psi}}, \quad \mu_0 = \frac{F_L}{F_k} = \frac{1-\alpha}{\alpha} k_0^{1-\psi} \quad \text{and} \quad \psi = \frac{\sigma - 1}{\sigma} \qquad (2)$$

$$y = y_0[\pi_0 \left(\frac{k}{k_0}\right)^{\psi} + (1-\pi_0)]^{\frac{1}{\psi}}. \qquad (3)$$

[1]

$$\frac{dy}{d\sigma} = -\frac{1}{\sigma^2}\frac{1}{\psi^2} y \left\{ \pi \ln\left(\frac{\pi_0}{\pi}\right) + (1-\pi)\ln\left(\frac{1-\pi_0}{1-\pi}\right) \right\} > 0 \quad with \quad \pi = \frac{f'(k)k}{y}. \tag{4}$$

$$\dot{v} = f(k) - (\delta + n)k + z - (\Pi + n)m - c \tag{5}$$

The Hamiltonian of the intertemporal optimization problem is:

$$H = u(c,m) + \psi\left[f(k) - (\delta + n)k + z - (\Pi + n)m - c \right] \tag{6}$$

We obtain a system of three differential equations:

$$\frac{\dot{c}}{c} = \frac{(f'(k) - (n + \delta + \rho))}{(\eta(1-\theta) + \theta)} + \frac{\theta(1-\eta)}{(\eta(1-\theta) + \theta)}(\phi + f'(k) - n - \delta - \frac{c}{m}\frac{\theta}{1-\theta}) \tag{7}$$

$$\frac{\dot{k}}{k} = \frac{f(k)}{k} - (\delta + n) - \frac{c}{k} \tag{8}$$

$$\frac{\dot{m}}{m} = \phi - n - \Pi = \phi + f'(k) - n - \delta - \frac{c}{m}\frac{\theta}{1-\theta}. \tag{9}$$

We obtain the following steady state conditions:

$$f'\left(k^*\right) = \delta + \rho + \mathrm{n} \qquad (10)$$

$$c^* = f(k^*) - (\delta + n)k^* \qquad (11)$$

$$\phi - n = \Pi^* \qquad (12)$$

$$\frac{c^*}{m^*} = \frac{1-\theta}{\theta}(\rho + \phi) \qquad (13)$$

$$\frac{\partial \pi}{\partial \sigma} = \frac{1}{\sigma^2}\left(1-\pi\right)\pi \ln\left(\frac{k}{k_0}\right) > 0 \quad if \quad k > k_0 \ . \qquad (14)$$

$$\frac{\partial f'}{\partial \sigma} = \frac{y}{k}\frac{\partial \pi}{\partial \sigma} + \frac{\pi}{k}\frac{\partial y}{\partial \sigma} > 0 \quad if \ k > k_0 . \qquad (15)$$

In order to derive the speed of convergence we calculate the following Jacobian:

$$J = \begin{pmatrix} \frac{-\theta(1-\eta)(\rho+\phi)}{\theta+\eta(1-\theta)} & \frac{1+\theta(1-\eta)}{\theta+\eta(1-\theta)}k^* f''(k^*) & \frac{\theta(1-\eta)(\rho+\phi)}{\theta+\eta(1-\theta)} \\ -\frac{c^*}{k^*} & \rho & 0 \\ -(\rho+\phi) & k^* f''(k^*) & (\rho+\phi) \end{pmatrix} \quad (16)$$

$$\det J = \frac{(\phi+\rho)c^* f''(k^*)}{\theta+\eta(1-\theta)} < 0 \text{ if } f''(k^*) < 0 \text{ and trace } J>0. \quad (17)$$

We can derive the one negative eigenvalue from the following characteristic equation:

$$\mu = -\varepsilon^3 + \varepsilon^2\left(\rho + \frac{\eta(\rho+\phi)}{\theta+\eta(1-\theta)}\right) - \varepsilon\left(\frac{\eta(\rho+\phi)\rho}{\theta+\eta(1-\theta)} + c^* f''(k^*)\frac{(1+\theta(1-\eta))}{\theta+\eta(1-\theta)}\right) + c^* f''(k^*)\frac{(\rho+\phi)}{\theta+\eta(1-\theta)} \quad (18)$$

$$-\frac{\partial \mu}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \lambda}\frac{d\lambda}{d\phi}=\frac{\partial \mu}{\partial \phi} \tag{19}$$

with $\lambda=-\varepsilon$ where $\frac{d\varepsilon}{d\lambda}=-1$ and $\frac{d\mu}{d\varepsilon}<0$ [2] so that $\frac{d\lambda}{d\phi}=\frac{d\mu}{d\phi}$.

$$\frac{\partial \mu}{\partial \phi}\left[\theta+\eta(1-\theta)-\frac{\eta(\rho+\phi)}{\varepsilon}\right]=c^* f''(k^*)\left(\frac{(1-\eta)^2}{\theta+\eta(1-\theta)}\right)<0 \quad ^{3} \tag{20}$$

We can analyze the effect of a change in the e. o. s. on the transitional effect of monetary policy:

$$-\frac{\partial \mu}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \lambda}\frac{d\lambda}{d\phi d\sigma}=\frac{\partial \mu}{\partial \phi \partial \sigma}. \tag{21}$$

$$\frac{\partial \mu}{\partial \phi \sigma}\left[\theta+\eta(1-\theta)-\frac{\eta(\rho+\phi)}{\varepsilon}\right]=\left(\frac{dc^* f''(k^*)}{d\sigma}\right)\left(\frac{(1-\eta)^2}{\theta+\eta(1-\theta)}\right). \tag{22}$$

$$\begin{aligned}\frac{dc^*}{d\sigma}f''(k^*)+\frac{\partial f''(k^*)}{\partial \sigma}c^* &= \frac{dc^*}{d\sigma}f''(k^*)+(n+\delta+\rho)(1-\pi^*)\left(\frac{1}{\sigma^2}\right) \quad ^{4} \\ &+\frac{\partial \pi^*}{\partial \sigma}(n+\delta+\rho)\frac{1}{\sigma}>0 \quad \text{if } k^*>k_0\end{aligned} \tag{23}$$

Equation (23) together with (22) implies that $-\frac{d\lambda}{d\phi d\sigma}=\frac{\partial \mu}{\partial \phi \partial \sigma}>0$ since $\lambda=-\varepsilon$ where $\frac{\partial \varepsilon}{\partial \lambda}=-1$ and $\frac{\partial \mu}{\partial \varepsilon}<0$. This means that $\frac{d\lambda}{d\phi d\sigma}$ must be strictly negative.

Calibration of the Economy

For numerical simulations we calibrate the econony with parameters, if possible, close to the ones used by Turnovsky (2002) and Gokan (2003):

benchmark values:	point of normalization: $k_0 = y_0 = 1$; $\pi_0 = 1/3$
	aggregate elasticity of substitution: $\sigma = 0.8$
	key monetary policy parameter: $\phi = 0.03$
	weight of money in utility: $\theta = 0.3$
	elasticity of marginal utility: $\eta = 2.5$
other parameters:	rate of time preference: $\rho = 0.04$
	rate of depreciation: $\delta = 0.04$
	rate of population growth: $n = 0.01$.

Changes in the Parameters

• The speed of convergence will increase with the growth rate of the money supply

• This effect is the stronger the lower is the preference for money; however, the acceleration in the speed of convergence is stronger if the prererence for money is higher.

• The speed of convergence effect of expansive monetary policy decreases if the e.o.s. increases

• Even with a high baseline profit share we are not able to replicate a 2% speed of convergence with values of the e.o.s. below one.

Table 1: Speed of convergence with **low** baseline profit share

Speed of convergence in percentage	**Θ=0.8**		**Θ=0.3**	
	λ	half time	λ	half time
	$k_0=y_0=1$; $\mu_0=2$	**$\pi_0=1/3$**		
σ=1.2	$\pi^*=0.43$	$c^*/y^*=0.76$		
Φ=0.01	2.75	25.19	3.27	21.21
Φ=0.02	2.85	24.36	3.29	21.01
Φ=0.03	2.92	23.73	3.32	20.84
Φ=0.05	3.03	22.88	3.36	20.61
σ=1	$\pi^*=1/3$	$c^*/y^*=0.81$		
Φ=0.01	3.69	17.52	4.99	13.87
Φ=0.02	4.13	16.79	5.05	13.73
Φ=0.03	4.26	16.26	5.09	13.61
Φ=0.05	4.47	15.52	5.16	13.44
σ=0.8	$\pi^*=0.26$	$c^*/y^*=0.86$		
Φ=0.01	5.39	12.86	7.30	9.49
Φ=0.02	5.67	12.24	7.38	9.39
Φ=0.03	5.89	11.77	7.45	9.31
Φ=0.05	6.23	11.12	7.55	9.18
σ=0.6	$\pi^*=0.19$	$c^*/y^*=0.89$		
Φ=0.01	7.19	9.68	10.59	6.54
Φ=0.02	7.59	9.13	10.69	6.48
Φ=0.03	7.95	8.72	10.79	6.43
Φ=0.05	8.50	8.15	10.93	6.34
σ=0.2	$\pi^*=0.12$	$c^*/y^*=0.94$		
Φ=0.01	13.21	5.25	26.99	2.57
Φ=0.02	14.39	4.81	27.17	2.55
Φ=0.03	15.40	4.5	27.34	2.53
Φ=0.05	17.03	4.07	27.64	2.51

Table 2: Speed of convergence with **high** baseline profit share

Speed of convergence in percentage	Θ=0.8		Θ=0.3	
	λ	half time	λ	half time
	$k_0=y_0=1$; $\mu_0=1/2$	**$\pi_0=2/3$[1]**		
σ=1	$\pi^*=2/3$	$c^*/y^*=0.63$		
Φ=0.01	1.50	46.04	1.67	41.48
Φ=0.02	1.54	44.99	1.68	41.16
Φ=0.03	1.57	44.22	1.69	40.92
Φ=0.05	1.6	43.17	1.71	40.59
σ=0.8	$\pi^*=0.45$	$c^*/y^*=0.75$		
Φ=0.01	3.32	20.87	4.06	17.06
Φ=0.02	3.45	20.09	4.10	16.89
Φ=0.03	3.55	19.52	4.14	16.75
Φ=0.05	3.70	18.72	4.19	16.55
σ=0.6	$\pi^*=0.29$	$c^*/y^*=0.83$		
Φ=0.01	5.50	12.60	7.50	9.24
Φ=0.02	5.79	11.98	7.58	9.15
Φ=0.03	6.02	11.51	7.64	9.07
Φ=0.05	6.37	10.87	7.75	8..96
σ=0.2	$\pi^*=0.13$	$c^*/y^*=0.92$		
Φ=0.01	12.57	5.52	24.74	2.80
Φ=0.02	13.66	5.08	24.91	2.78
Φ=0.03	14.57	4.76	25.07	2.76
Φ=0.05	16.06	4.32	25.35	2.73

Influence of the Normalization Point

- Klump and Saam (2008) have shown that the *choice of the normalization point* has a major impact on the speed of convergence:

- Xue and Yip (2011) have identified *two different effects* – the efficiency and the distribution effect

- The distribution effect can be elimated by *normalizing in the steady state*

Table 3: Speed of convergence with various baseline points

Speed of convergence in percentage	Θ=0.3	
	λ	half time
	k_0=2, y_0=1; μ_0=2	**π_0=1/2**
σ=0.8	k^*=8.92	π^*=0.41
Φ=0.01	4.57	15.18
Φ=0.02	4.61	15.03
Φ=0.03	4.65	14.90
Φ=0.05	4.70	14.71
	k_0=5, y_0=1 ; μ_0=2	**π_0=5/7**
σ=0.8	k^*=16.07	π^*=0.65
Φ=0.01	2.08	33.26
Φ=0.02	2.10	32.98
Φ=0.03	2.12	32.77
Φ=0.05	2.14	32.26
	k_0=1, y_0=0.5; μ_0=2	**π_0=1/3**
σ=0.8	k^*=2.05	π^*=0.29
Φ=0.01	6.45	10.75
Φ=0.02	6.51	10.64
Φ=0.03	6.57	10.55
Φ=0.05	6.67	10.41

The higher the baseline point the lower is the speed of convergence

Table 4: Speed of convergence with baseline point in the steady state

Speed of convergence in percentage	Θ=0.3	
	λ	half time
baseline point in steady state of σ=0.8	**$k_0=k_{\sigma=0.8}$=4.409, y_0=1.547**	**π_0=0.26**
σ=1.2		
Φ=0.01	5.77	12.02
Φ=0.02	5.83	11.89
Φ=0.03	5.88	11.79
Φ=0.05	5.96	11.63
σ=0.8		
Φ=0.01	7.30	9.49
Φ=0.02	7.38	9.39
Φ=0.03	7.45	9.31
Φ=0.05	7.55	9.18
σ=0.2		
Φ=0.01	15.64	4.43
Φ=0.02	15.78	4.39
Φ=0.03	15.89	4.36
Φ=0.05	16.10	4.30

The impact of a change in the e.o.s. on the speed of convergence is largely due to the *distribution effect*

The impact of monetary policy is more channelled via the *efficiency effect*

Conclusion

- By using a *normalized CES production function* we are able to derive more realistic effects of monetary policy on the speed of convergence than Gokan (2002)
- Monetary policy is the more effective in speeding up convergence, the *lower ist the e.o.s*. and the *lower is the preference for money*
- The choice of the *normalization point* has a particular influence on the result as it determines the interplay between *distribution and efficiency effects* which can both be induced by changes in the e.o.s.

1 Appendix

Reformulation of the normalized CES production function

$$
\begin{aligned}
f(k) &= C(\sigma)\left[\alpha(\sigma)k^{\psi}-(1-\alpha(\sigma))\right]^{\frac{1}{\psi}} \\
&= y_0\left[\frac{k_0^{1-\psi}+\mu_0}{k_0+\mu_0}\right]^{\frac{1}{\psi}}\left[\frac{k_0^{1-\psi}}{k_0^{1-\psi}+\mu_0}k^{\psi}-\left(1-\frac{k_0^{1-\psi}}{k_0^{1-\psi}+\mu_0}\right)\right]^{\frac{1}{\psi}} \\
&= y_0\left[\frac{k_0^{1-\psi}}{k_0+\mu_0}k^{\psi}-\frac{k_0^{1-\psi}+\mu_0}{k_0+\mu_0}+\frac{k_0^{1-\psi}}{k_0+\mu_0}\right]^{\frac{1}{\psi}} \\
&= y_0\left[\frac{k_0}{k_0+\mu_0}\frac{k^{\psi}}{k_0^{\psi}}-\frac{\mu_0}{k_0+\mu_0}\right]^{\frac{1}{\psi}} \\
&= y_0\left[\pi_0\left(\frac{k}{k_0}\right)^{\psi}+(1-\pi_0)\right]^{\frac{1}{\psi}}
\end{aligned}
$$

Proof of Equation (4):

$$
y = y_0\left[\pi_0\left(\frac{k}{k_0}\right)^{\psi}+(1-\pi_0)\right]^{\frac{1}{\psi}}
$$

$$
\ln y = \ln y_0 + \frac{1}{\psi}\ln\left[\pi_0\left(\frac{k}{k_0}\right)^{\psi}+(1-\pi_0)\right]
$$

$$
\frac{1}{y}\frac{dy}{d\sigma} = -\frac{1}{\psi^2}\frac{1}{\sigma^2}\ln\left[\pi_0\left(\frac{k}{k_0}\right)^{\psi}+(1-\pi_0)\right]+\frac{1}{\psi}\frac{1}{\sigma^2}\frac{\pi_0\left(\frac{k}{k_0}\right)^{\psi}\ln\left(\frac{k}{k_0}\right)}{\pi_0\left(\frac{k}{k_0}\right)^{\psi}+(1-\pi_0)}
$$

$$
\frac{dy}{d\sigma} = -\frac{1}{\psi^2}\frac{1}{\sigma^2}y\left\{\ln\left(\pi_0\left(\frac{k}{k_0}\right)^{\psi}+(1-\pi_0)\right)-\frac{\psi\pi_0\left(\frac{k}{k_0}\right)^{\psi}\ln\left(\frac{k}{k_0}\right)}{\pi_0\left(\frac{k}{k_0}\right)^{\psi}+(1-\pi_0)}\right\}
$$

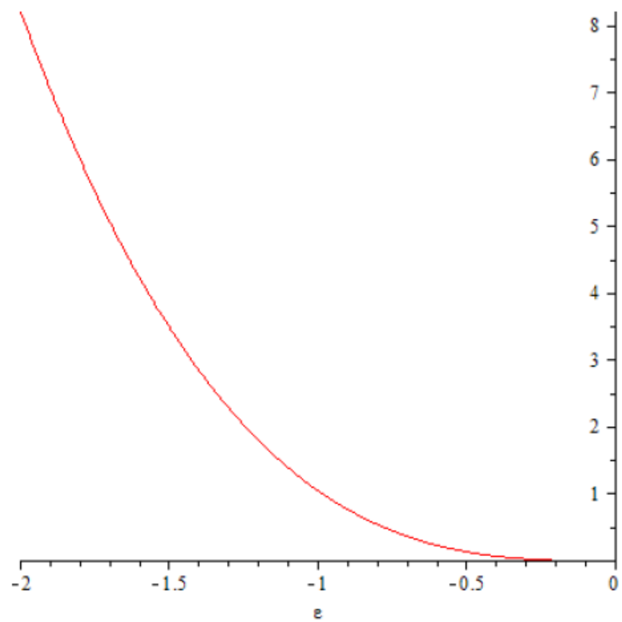
$$
\text{applying: } \left(\frac{k}{k_0}\right)^{\psi} = \frac{\pi(1-\pi_0)}{\pi_0(1-\pi)}
$$

$$
\frac{dy}{d\sigma} = -\frac{1}{\psi^2}\frac{1}{\sigma^2}y\left\{\pi\ln\left(\frac{\pi_0}{\pi}\right)+\ln\left(\frac{1-\pi_0}{1-\pi}\right)-\pi\ln\left(\frac{1-\pi_0}{1-\pi}\right)\right\}
$$

$$
\frac{dy}{d\sigma} = -\frac{1}{\psi^2}\frac{1}{\sigma^2}y\left\{\pi\ln\left(\frac{\pi_0}{\pi}\right)+(1-\pi)\ln\left(\frac{1-\pi_0}{1-\pi}\right)\right\}
$$

The characteristic equation is clearly negative for any negative root as can be seen from the plot below.

$\partial\mu / \partial\varepsilon < 0$ if ε<0



Derivation of equation (22)

$$f''(k^*) = -f'(k^*)(1-\psi)\left(\frac{\pi_0\left(\frac{k^*}{k_0}\right)^{\psi}}{\pi_0\left(\frac{k^*}{k_0}\right)^{\psi}+(1-\pi_0)}-1\right)\frac{1}{k}$$

$$= -f'(k^*)(1-\psi)(1-\pi) = -(n+\delta+\rho)(1-\pi)\frac{1}{\sigma}$$

$$f'(k^*) = (n + \delta + \rho)$$

$$\frac{dk^*}{d\sigma} f''(k^*) = -\frac{\partial f'(k^*)}{\partial \sigma}$$

$$\frac{dk^*}{d\sigma} = -\frac{\partial f'(k^*)}{\partial \sigma} \frac{1}{f''(k^*)} > 0 \quad \text{since} \quad f''(k^*) < 0 \quad and \quad \frac{\partial f'(k^*)}{\partial \sigma} > 0$$

$$c^* = f(k^*) - (n+\delta)k^*$$

$$\frac{dc^*}{d\sigma} = \frac{\partial f(k^*)}{\partial \sigma} + f'(k^*)\frac{dk^*}{d\sigma} - (n+\delta)\frac{dk^*}{d\sigma}$$

$$= \frac{\partial f(k^*)}{\partial \sigma} + \frac{dk^*}{d\sigma}(f'(k^*) - (n+\delta))$$

$$= \frac{\partial f(k^*)}{\partial \sigma} + \frac{dk^*}{d\sigma}\rho > 0$$

$$\pi = \frac{f'(k)k}{f(k)} = \pi_0 \left(\frac{k}{k_0}\right)^{\psi} [\pi_0 \left(\frac{k}{k_0}\right)^{\psi} + (1-\pi_0)]^{-1}$$

$$\frac{\partial \pi}{\partial \sigma} = \frac{1}{\sigma^2}(1-\pi)\pi \ln\left(\frac{k}{k_0}\right) > 0 \quad if \quad k > k_0$$

$$\frac{dc^*}{d\sigma} f''(k^*) + \frac{\partial f''(k^*)}{\partial \sigma} c^* = \frac{dc^*}{d\sigma} f''(k^*) + (n+\delta+\rho)(1-\pi^*)\left(\frac{1}{\sigma^2}\right)$$

$$+ \frac{\partial \pi^*}{\partial \sigma}(n+\delta+\rho)\frac{1}{\sigma} > 0 \quad if \quad k^* > k_0$$

Influence of elasticity of substitution on the speed of convergence

$$\frac{\partial \mu}{\partial \sigma} = -\varepsilon \frac{(1+\theta(1-\eta))}{\theta + \eta(1-\theta)} * \frac{\partial(c^* f''(k^*))}{\partial \sigma} + \frac{\partial(c^* f''(k^*))}{\partial \sigma} \frac{(\rho+\phi)}{\theta + \eta(1-\theta)} < 0 \quad if\, \varepsilon < 0$$

$$\frac{\partial(c^* f''(k^*))}{\partial \sigma} = \frac{dc^*}{d\sigma} f''(k^*) + \frac{\partial f''(k^*)}{\partial \sigma} c^* > 0$$