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MONEY AND ASSET PRICES IN A CASH-IN-ADVANCE ECONOMY

by

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## 1. Introduction

This paper is a study of the demand for money, of the determination of the price level and nominal and real interest rates, and to some extent of asset prices in general, in a monetary economy; in particular how these variables are affected by changes in money supply and in income. We examine Friedman's (1969) optimum monetary policy and some frequently discussed parity conditions, namely the Fisher relation and the premium on nominal bonds relative to indexed bonds. We also discuss the cash-in-advance approach versus money-in-the-utility-function.

We derive the demand for money by treating money symmetrically with other assets. Ordinary assets have a value and are held because they give a return - dividends - in the future. In complete analogy, we can think of money having a value and being held because it gives a return - liquidity services - in the future. Once these liquidity services have been specified, the price of money can be determined by an asset-pricing equation, as the price of other assets; only the liquidity services replace the dividends in the equation. This idea is not new; here we follow for instance the works of Dixit and Goldman (1970), Kouri (1977), Fama and Farber (1979), Hodrick (1981), LeRoy (1982a, b), Jones (1983) and Stulz (1983). They all derive a demand for money by considering money as an asset that pays

liquidity services rather than dividends. This literature has some shortcomings, though. First, with the exception of LeRoy (1982a, b) and Jones (1983), it is not fully general equilibrium, in the sense that the stochastic processes of some or all prices and interest rates are exogenously given, and not functions of more fundamental stochastic processes, of money supply and output, say. Second, by putting real balances directly into the utility functions, they assume that real balances give direct utility and the liquidity services of money are simply given by the direct marginal utility of real balances. (Fama and Farber (1979) and Stulz (1983) assume that consumption goods and real balances jointly produce consumption services, which in turn give direct utility. This is essentially the same assumption.)

The present paper attempts to improve upon this literature in both these respects. First, by following the analysis of the two seminal papers of Lucas (1978, 1982) and using his general equilibrium asset-pricing model of a pure-exchange economy, we can indeed specify a stochastic steady state where all prices and interest rates are endogenous functions of exogenous stochastic processes of money supply and output.<sup>1</sup> Second, instead of postulating that real balances give direct utility, we derive the demand for money via a cash-in-advance constraint, the liquidity constraint associated with Clower (1967) and thoroughly discussed by Kohn (1981). This way the liquidity services of money are endogenously determined by the value of relaxing the liquidity constraint - the shadow price of the liquidity constraint. However, when introducing the cash-in-advance constraint, we avoid a common pitfall of the cash-in-advance literature, namely that the elaborate structure results in the trivial quantity equation with a unitary income

velocity of money. We do this by modifying Lucas' (1982) cash-in-advance model in a crucial way.

In Lucas' model consumers enter each period with a portfolio of assets, but (in equilibrium) without any money. In the beginning of each period they learn the current state of the economy after which they trade assets and money. After this they buy consumption goods. Consumption goods must be paid for with cash, which gives rise to the liquidity constraint. With this set-up, since consumers acquire cash after they know the current state and their planned current consumption, they end up acquiring exactly the amount of cash they need to buy their consumption goods. The result is that equilibrium cash balances acquired within each period are exactly equal to the nominal value of output. As in most other cash-in-advance models, for instance Wilson (1979), Helpman (1981), Persson (1982), and Kouri (1983), the demand for money is then given by the simple quantity equation, since monetary velocity is driven up to its technological maximum and is identically equal to unity. There is then only a transactions demand for money, money is not a store of value, and an asset-pricing equation for money cannot be derived.

The important difference between our model and Lucas' is that in our model consumers must decide upon their cash holdings before they know the current state, and hence before they know their consumption.<sup>2</sup> This gives rise to a combined transactions, precautionary and store-of-value demand for money. Money is held for the future liquidity services it provides, and the value of these liquidity services is the value of relaxing the future liquidity constraint. Under these circumstances an asset-pricing equation for money can indeed be derived. In particular, with uncertainty about future consumption the simple quantity equation does not hold, yielding a richer - and more reasonable - demand for money.

Previous papers with cash-in-advance models with a variable non-unitary monetary velocity include Goldman (1974), Lucas (1980), Stockman (1980), Helpman and Razin (1982), Krugman, Persson and Svensson (1983), and Obstfeld and Stockman (1983). These papers derive a combined transactions, precautionary and store-of-value money demand, but they do not specify a full stochastic stationary general equilibrium with several assets, and they do not exploit the symmetry between the asset-pricing equations for money and other assets.<sup>3</sup>

In summary, relative to Dixit and Goldman (1970), Kouri (1977), Fama and Farber (1979), Jones (1983) and Stulz (1983), we have a full general equilibrium solution rather than partial equilibrium (again with the exception of Jones), and a better microfoundation of money. Relative to Lucas (1982) and most of the cash-in-advance literature we have a more reasonable demand for money with variable velocity.

In Section 2 we present the model and derive the equilibrium equations. We see that the existence of a binding liquidity constraint drives a wedge between the marginal utility of real wealth and the marginal utility of consumption, the latter exceeding the former by the marginal utility of real balances. This expresses the circumstance that wealth is less liquid than cash.

In Section 3 we examine the demand for real balances as a function of temporary and permanent disturbances in monetary expansion and income. We note that the definition of real balances is subject to choice. Since the concept of real balances is only an intermediate step to determine and interpret the equilibrium effects on the price level and nominal and real interest rates, the choice can be made

according to convenience only. We then show, for instance, that the income velocity of money and the price level are increasing in monetary expansion. They are increasing or decreasing in income depending upon whether risk aversion (intertemporal substitutability in consumption) is low or high (high or low), respectively. The demand for real balances and the price level are more sensitive to permanent changes in monetary expansion than to temporary. A temporary increase in monetary expansion increases monetary velocity and the price level, decreases the real interest rates, but leaves the nominal interest rate constant. Hence, monetary expansion may have a direct influence on the demand for real balances, over and above any influence via the nominal interest rate. A permanent increase in monetary expansion increases monetary velocity and the price level more, increases the nominal interest rate, and has an ambiguous effect on the real rate of interest.

Next, in Section 4 we apply the model to study Friedman's (1969) optimum monetary policy which minimizes the distortion relative to a barter economy. We show that the optimum (net) monetary expansion should be counter- or pro-cyclical depending upon whether the degree of risk aversion (intertemporal substitution) in consumption is low or high), respectively. The optimum monetary policy does not involve stabilizing nominal income, nominal prices, inflation or money growth.

In Section 5 we look at the Fisher relation and the premium on nominal bonds relative to indexed bonds. In contrast to previous partial equilibrium literature, for instance Fama and Farber (1979), we can specify the effects on these parity conditions of the parameters of the fundamental stochastic processes of monetary expansion and income, and it turns out that these effects differ considerably from

results of previous general equilibrium approaches involving the quantity theory, for instance Kouri (1983). We find, for instance, that a high variance of monetary expansion and a negative covariance between income and monetary expansion contribute to the difference between the nominal and real interest rate exceeding the expected rate of inflation, whereas the variance of income has no effect. The premium on nominal bonds is higher with higher variance of income, whereas the covariance between monetary expansion and income has an ambiguous effect. Increased variance in monetary expansion decreases the premium.

The issue of liquidity constraints versus money-in-the-utility function is discussed in Section 6. It has been argued that the two approaches are essentially identical (see Fischer's (1983b) comment on Lucas and Stokey (1983)). We argue that they are not.

Some conclusions, limitations of the analysis, and directions for further research are presented in Section 7. Appendix 1 compares in some detail with Lucas (1978, 1982), and Appendix 2 deals with some technical matters.

## 2. The Model

We consider a closed monetary economy, with stochastic endowments/output/income  $y_t$  in period  $t$ ,  $t = \dots, -1, 0, 1, \dots$ , of a single non-storable good. The supply  $\bar{M}_t$  of money in period  $t$  is random and is given by

$$(2.1) \quad \bar{M}_{t+1} = \omega_t \bar{M}_t,$$

where  $\omega_t$ , (the gross rate of) monetary expansion, is stochastic. Let  $s_t = (y_t, \omega_t)$  be the state in period  $t$ . It follows a Markov process with the probability of  $s_{t+1}$  given by the distribution

function  $F(s_{t+1}|s_t)$ . That is, the probability distribution of  $s_{t+1}$  depends on the realization of  $s_t$ .

The economy has a representative consumer with preferences

$$(2.2) \quad E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_{\tau}), \quad 0 < \beta < 1,$$

in period  $t$ , where  $u(c)$  is a concave utility function of consumption  $c$  and  $E_t$  is the expectations operator conditional upon information available at  $t$ . We shall first look at the decision problem of the consumer. We assume that there exist traded claims to future endowments of goods, and that the consumer receives net transfers of money in each period.<sup>4</sup>

The timing of events is crucial: The consumer enters period  $t$  with predetermined holdings  $M_t$  of money and a predetermined share  $z_t$  of claims to endowments of goods. He/she learns the current state  $s_t$  and then has the opportunity to purchase goods with money, at a price  $P(s_t, \bar{M}_t)$ .<sup>5</sup> His/her purchases  $c_t$  must obey the liquidity constraint

$$(2.3) \quad P(s_t, \bar{M}_t) c_t \leq M_t.$$

After the goods market is closed, at the end of period  $t$  he/she receives his/her share of cash from the sale of the endowments,  $P(s_t, \bar{M}_t) y_t z_t$ , as well as the period's lump sum money transfers,  $(\omega_t - 1) \bar{M}_t$ . This is a positive net transfer if money supply expands ( $\omega_t > 1$ ) and a positive net tax if money supply contracts ( $\omega_t < 1$ ). Note that the consumer by assumption receives the money transfer after the close of the consumption goods markets - hence the money transfer cannot be used to buy consumption goods within the same period. Put differently, money transfers - when regarded as assets -



are by assumption not more liquid than other assets, in that they do not pay their "dividend" until after the consumption market is closed. When regarded as liabilities, taxes are paid at the same time as cash income is received - the consumer does not have to hold cash as a precaution for random tax payments.

Finally, at the end of period  $t$ , money and shares are traded. The consumer then faces the budget constraint

$$(2.4) \quad M_{t+1} + Q(s_t, \bar{M}_t)z_{t+1} \leq [M_t - P(s_t, \bar{M}_t)c_t] \\ + [Q(s_t, \bar{M}_t) + P(s_t, \bar{M}_t)y_t]z_t + (\omega_t - 1)\bar{M}_t.$$

Here  $M_{t+1}$  and  $z_{t+1}$  are new holdings of money and shares, to be carried into period  $t+1$ .  $Q(s_t, \bar{M}_t)$  is the share price in terms of money. The first term on the right-hand side is cash not spent on consumption, and the second term is the gross return on shares.

We now introduce the (real) price of money (that is, its purchasing power in terms of goods),  $\pi_t = 1/P_t$ , and the real price of shares,  $q_t = Q_t/P_t$ . For simplicity we let primed variables denote variables at time  $t+1$  at state  $s_{t+1}$  and money stock  $\bar{M}_{t+1}$ , where as non-primed variables denote variables in period  $t$  at state  $s_t$  and money stock  $\bar{M}_t$ . Then the budget constraint (2.4) and the liquidity constraint (2.3) can be written

$$(2.5a) \quad c + \pi M' + qz' \leq \pi M + (q + y)z + \pi(\omega - 1)\bar{M} \equiv w \quad \text{and}$$

$$(2.5b) \quad c \leq \pi M.$$

Here the right-hand side of (2.5a) is identified with total (real) wealth in period  $t$ . Wealth in period  $t+1$  will hence be given by

$$(2.5c) \quad w' = \pi' M' + (q' + y') z' + \pi' (\omega' - 1) \bar{M}'.$$

(Note that here the variables  $M'$  and  $z'$  are choice variables at period  $t$  and hence, in equilibrium, not functions of  $s'$  and  $\bar{M}'$  but of  $s$  and  $\bar{M}$ .)

The consumer will maximize (2.2) subject to (2.3) and (2.4).

In an equilibrium we will have

$$(2.6) \quad \begin{aligned} c_t &= y_t, \\ M_t &= \bar{M}_t, \\ M_{t+1} &= \bar{M}_{t+1} = \omega_t \bar{M}_t \quad \text{and} \\ z_t &= 1. \end{aligned}$$

That is, the goods, money and share markets clear at each date.

Note that  $\bar{M}_t$  in (2.6) and (2.1) is the beginning-of-period money stock in period  $t$ , and  $\bar{M}_{t+1}$  is the end-of-period money stock in period  $t$  (and the beginning-of-period money stock in period  $t+1$ ).

Since by assumption the period  $t$  money transfer is paid at the end of the period, that money transfer is included in the end-of-period money stock but not in the beginning-of-period money stock.

We assume the existence of a unique stochastic stationary rational-expectations equilibrium.<sup>6</sup> In such an equilibrium, the probability distributions of the endogenous variables are independent of  $t$ . Then the solution to maximizing (2.2) subject to (2.3) and (2.4) gives the value function  $v(w, M, s, \bar{M})$  defined implicitly as the maximum of

$$(2.7) \quad u(c) + \beta \int v(w', M', s', \bar{M}') dF(s' | s)$$

over  $c$ ,  $M'$  and  $z'$ , subject to (2.5). We let  $\lambda$  and  $\mu$  be the Lagrange multipliers associated with the constraints (2.5a) and (2.5b), respectively. We have, by standard properties, that the partials of the value function fulfill

$$(2.8) \quad v_w = \lambda \quad \text{and} \\ v_M = \mu\pi.$$

Combining (2.6) and (2.8) with the first-order conditions for maximizing (2.7) subject to (2.5) we can finally write the equations that define an equilibrium in period  $t$ , for each  $s$  and  $\bar{M}$ , as,

$$(2.9a) \quad y \leq \pi\bar{M} \quad [\mu \geq 0],$$

$$(2.9b) \quad \lambda + \mu = u_c(y),$$

$$(2.9c) \quad \lambda\pi = \beta E[(\lambda' + \mu')\pi'] \quad \text{and}$$

$$(2.9d) \quad \lambda q = \beta E[\lambda'(q' + y')].$$

Here non-primed variables are functions of  $s$  and  $\bar{M}$ , and  $Ef'$  denotes  $\int f' dF(s' | s)$  (all expectations throughout this section are conditional upon  $s$ ). Equation (2.9a), the liquidity constraint, says that real balances  $\pi\bar{M}$  equal or exceed output  $y$ . By (2.8) the multiplier  $\mu$  is the marginal utility of real balances ( $\mu\pi$  is the marginal utility of (nominal) money). The expression  $[\mu \geq 0]$  denotes the usual complementary slackness condition: if the liquidity constraint is not binding, the marginal utility of real balances is zero; and if the marginal utility of real balances is positive, the liquidity constraint is binding. By (2.8)  $\lambda$  is the marginal utility of wealth, and

by (2.9b) the sum of marginal utility of wealth and of real balances equals the marginal utility of consumption. Put differently, the existence of a binding liquidity constraint drives a wedge between the marginal utility of wealth and the marginal utility of consumption, since wealth cannot instantaneously buy consumption.

We also note that  $\mu$  is the liquidity component only of the marginal utility of real balances, what we shall call the liquidity services of real balances. Real balances are also part of wealth, and the total marginal utility of real balances is  $\lambda + \mu = u_c(y)$ , the sum of the marginal utility of wealth and the liquidity services, which equals marginal utility of consumption.

Expression (2.9d) is the standard capital-asset-pricing equation for the value of a claim to future output. The price of a claim to future output is equal to the discounted expected next period's marginal utility of wealth times the gross return on the claim, divided by the present marginal utility of wealth. The gross return is the sum of next-period's value of the claim  $q'$ , the "indirect" return, and next period's "direct" return  $y'$ . Expression (2.9d) can be interpreted as the equality between the price of the claim,  $q$ , and the marginal rate of substitution of the claim for current wealth,  $\beta E[\lambda'(q' + y')]/\lambda$ . Equation (2.9d) can be solved forward in the usual way, disregarding bubble solutions,<sup>7</sup> to give

$$(2.10) \quad q_t = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} E[\lambda_{\tau} y_{\tau} | s_t] / \lambda_t.$$

That is, the price of the claim equals the discounted sum of all future periods' expected marginal utility of its direct return (conditional upon current information), divided by the present marginal utility of wealth.

Equations (2.9d) and (2.10) are completely standard capital-asset-pricing equations, except in one way. The marginal utility of wealth,  $\lambda$ , is here not always equal to the marginal utility of consumption. As we have noted the existence of a binding liquidity constraint drives a wedge between the marginal utility of wealth and that of consumption, and the marginal utility of wealth is then less than the marginal utility of consumption, the difference being the marginal utility of real balances. Clearly the existence of the liquidity constraint will affect the pricing of the claim.

It is of interest here that equation (2.9c) also can be interpreted as a capital-asset-pricing equation: The real price of a unit of money is equal to the discounted expected next period's marginal utility of the gross return of money, divided by the present marginal utility of wealth. The marginal utility of the gross return is equal to the sum of the marginal utility of wealth times the sum of next period's value of money ( $\lambda'\pi'$ ) (the indirect return) and next period's marginal utility of money ( $\mu'\pi'$ ) (the direct return). That is, the price of money is equal to the marginal rate of substitution of money for current wealth,  $\beta E[u_c(y')\pi']/\lambda$ . Equation (2.9c) can of course also be solved forward to give

$$(2.11) \quad \pi_t = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} E[\mu_{\tau} \pi_{\tau} | s_t] / \lambda_t = \\ = \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} E[v_{M\tau} | s_t] / \lambda_t.$$

That is, the real price a unit of money is the discounted sum of all future periods' expected marginal utility of money, divided by the present marginal utility of wealth.

Hence, money is priced in complete symmetry with other assets, once its direct return has been appropriately defined. The direct return to money in this case is simply the value of the liquidity services provided by money, measured by the value of relaxing the liquidity constraint, which value is given by  $v_M = \mu\pi$ .<sup>8</sup>

In the present framework, as in Lucas (1982), the price of any arbitrary asset can be determined, if only its direct return as a function of the state is specified. Let us this way compute nominal and real interest rates. This can be done, even though in equilibrium the consumer's holdings of the bonds to be considered are zero.

Let the nominal interest rate,  $i$ , be defined from the nominal present value of a nominal bond that pays a sure unit of money in the next period. More precisely, the bond is bought on the asset markets at the end of period  $t$ , and it pays one unit of money at the end of period  $t+1$ . The present value at the end of period  $t$  of this bond, measured in goods, is  $\beta E[\lambda'\pi']/\lambda$ . The present value measured in money is  $\beta E[\lambda'\pi']/\lambda\pi$ . Hence, the nominal rate of interest is given by

$$(2.12) \quad 1/(1+i) = \beta E[\lambda'\pi']/\lambda\pi.$$

Using (2.9c) and rearranging, we have

$$(2.13) \quad i = E[\mu'\pi']/E[\lambda'\pi'].$$

Hence, the nominal interest rate is the expected utility of the liquidity services of money, over the expected utility of wealth of money. Put differently, the nominal interest rate is the ratio between the expected marginal utility of the direct return of money and the expected utility of the indirect return on money. We can understand this relation as follows: Money and bonds have the same indirect return. For one unit of money held as cash at the end of period  $t$ , and one unit of money invested in a nominal bond at the end of period  $t$ , both "pay" one unit of money at the end of period  $t+1$  as indirect return. The expected utility of that indirect return is  $\beta E[\lambda'\pi']$ . But one unit of money also gives liquidity services, the expected utility of which is  $\beta E[\mu'\pi']$ ; whereas the bond pays interest, the expected utility of which is  $i\beta E[\lambda'\pi']$ . In equilibrium, the expected utility of the liquidity services of one unit of money must equal the expected utility of the interest on one unit of money invested as a nominal bond. This gives rise to (2.13).

We see from (2.13) that the nominal interest rate cannot be negative, and it is strictly positive when the expected next period's marginal utility of liquidity services is positive, that is when the liquidity constraint is binding in at least some state with positive probability.

The real interest rate,  $\rho$ , is defined from the real present value of an indexed bond that can be bought at the end of period  $t$  and that pays one sure unit of real wealth at the end of period  $t+1$ . It is given by

$$(2.14) \quad 1/(1 + \rho) = \beta E[\lambda']/\lambda, \text{ hence}$$

$$(2.15) \quad \rho = \lambda/\beta E[\lambda'] - 1.$$

The real interest rate thus defined is the present marginal utility of wealth over the discounted expected marginal utility of the next period's wealth, minus 1. It is related to the usual expected intertemporal marginal rate of substitution, although not between consumption in the current and next period, but between wealth in the current and next period.

Note that by assumption this indexed bond does not pay out physical units of the consumption good at the end of the next period. If it did, the real interest would be given by  $1/(1+\rho) = \beta E[u_c(y')]/\lambda$ , where the marginal utility of consumption  $u_c(y')$  replaces the marginal utility of wealth. The behaviour of that interest rate can of course also be examined. However, since we are considering a monetary economy, we restrict the analysis to an indexed bond that pays cash to an amount equivalent to one unit of real wealth, that is, paying  $1/\pi'$  units of money, at the end of the next period.

### 3. Real balances and interest rates

In this section we shall examine in some detail how the demand for real balances and how the price level and nominal and real interest rates are determined as functions of temporary and permanent disturbances in the rate of expansion of money supply and in output. We shall therefore solve the system (2.9a-c) for different assumptions about the distribution function  $F(s_{t+1}|s_t)$ . (We note in passing that (2.9 a-c) can be solved independently of (2.9d)).

Let us first introduce the notation

$$(3.1) \quad m_t = \pi_t \bar{M}_t$$



for real balances in period  $t$ , that is the goods value of the money stock. It is easy to show that in a stochastic steady state, real balances will depend on the state  $s$  only and not on the money stock. The same is the case for the variables  $\lambda$  and  $\mu$ , once  $m_t/\bar{M}_t$  has been substituted for  $\pi_t$ . Then, the equilibrium equations (2.9a-c) can be written explicitly, for each  $s = (y, \omega)$  as

$$(3.2a) \quad y \leq m(s) \quad [\mu(s) \geq 0] ,$$

$$(3.2b) \quad \lambda(s) + \mu(s) = u_c(y) \quad \text{and}$$

$$(3.2c) \quad \lambda(s)m(s) = \beta E[u_c(y')m(s')|s]/\omega ,$$

where  $E[f(s')|s]$  denotes  $\int f(s')dF(s'|s)$ .<sup>9</sup>

We shall solve this system and find the functions  $m(s)$ ,  $\lambda(s)$  and  $\mu(s)$ . These functions can then be substituted into (3.1) to give

$$(3.3) \quad \pi(s, \bar{M}) = m(s)/\bar{M} \quad \text{and} \quad P(s, \bar{M}) = 1/\pi(s, \bar{M}) = \bar{M}/m(s),$$

and then into (2.13) and (2.15) to give the nominal and real interest rates  $i(s)$  and  $\rho(s)$ . The price of money will depend on both the state and the stock of money, whereas the other variables will depend on the state only. The pricing function (3.3) for money can be interpreted as a demand-price function for money. Hence, we are indeed about to derive a proper demand function for money. The equilibrium real price of money, and hence the nominal price level, are then determined by the interaction between the demand for and supply of money, in the usual way.

Note that  $m_t$  in (3.1) is beginning-of-period real balances, the goods value of the money stock in the beginning of the period. We can also consider end-of-period real balances, defined as  $\tilde{m}_t = \pi_t \bar{M}_{t+1}$ , say. Of course, the two definitions of real balances are related by  $\tilde{m}_t = \omega_t m_t$ . The purpose of choosing a concept of real balances is to help understand the determinants of the demand for money and the price level. Which concept of real balances we choose to work with is a matter of convenience. We choose beginning-of-period real balances, since they have the advantage of varying directly with the real price of money, that is inversely with the price level (recall that  $\bar{M}$ , the beginning-of-period money stock, is predetermined).

#### Temporary Disturbances

We start by examining the demand for money as a function of temporary disturbances in income  $y$  and (the gross rate of) monetary expansion  $\omega$ . We do this by assuming that the probability distribution of  $s_{t+1} = (y_{t+1}, \omega_{t+1})$  is independent of the realization of  $s_t$ ; that is,  $s_{t+1}$  is serially uncorrelated with  $s_t$ , and the probability distribution fulfills

$$(3.4) \quad F(s' | s) = F(s') .$$

Then, for a given probability distribution, we can interpret the current realization of the state as a temporary disturbance, in the sense that it does not change the probability distribution of future states. Put differently, there is no information content in current realizations. A permanent disturbance, on the other hand, will affect future probability distributions, as further specified below. But first we shall deal separately with the case of temporary disturbances.

If states are serially uncorrelated, it follows that the term which is expected total marginal utility of real balances,  $E[u_c(y')m(s')|s]$ , on the right-hand side of (3.2c), is a constant independent of  $s$ , and we can write (3.2) as

$$(3.5a) \quad y \leq m(s) \quad [\mu(s) \geq 0] ,$$

$$(3.5b) \quad \lambda(s) + \mu(s) = u_c(y) \quad \text{and}$$

$$(3.5c) \quad \lambda(s)m(s) = \beta A/\omega , \quad \text{where}$$

$$(3.5d) \quad A = E[u_c(y')m(s')] .$$

To characterize the solution, we note that there will generally be two regions in  $(y, \omega)$ -space, one in which the liquidity constraint is not binding ( $m > y$  and  $\mu = 0$ ), and one in which it is binding ( $m = y$  and  $\mu > 0$ ). The border line between the two regions is given by the set of  $(y, \omega)$  fulfilling  $\mu(s) = 0$  and  $m(s) = y$ , which by (3.5 a-c) gives

$$(3.6) \quad \omega = \beta A/u_c(y)y \equiv \tilde{\omega}(y) .$$

When  $(y, \omega)$  fulfill  $\omega < \tilde{\omega}(y)$  the liquidity constraint will not bind, whereas it will be binding for  $\omega \geq \tilde{\omega}(y)$ .<sup>10</sup> Intuitively, we can understand this by looking at equation (3.2c), written as

$$(3.7) \quad \lambda(s)m(s) = (\beta/\omega)E[u_c(y')m(s')] = (\beta/\omega)A .$$

A high rate of monetary expansion acts like decreasing the "effective" discount factor  $(\beta/\omega)$ . Thus, the expected utility of future real balances is in effect discounted more and the real balances today tend to fall. Hence, for sufficiently large monetary expansion real balances will fall to hit the liquidity constraint. Put differently, in the

beginning of the current period a high current realization of monetary expansion means that more money will be distributed at the end of the period, and that the future value of money,  $\pi' = m'/\bar{M}'$  will be lower in all future states since  $\bar{M}' = \omega\bar{M}$  is higher. Therefore it will be less attractive to hold money and more attractive to spend money on consumption, which will bid up the money price of consumption goods in the beginning of the current period. This lowers the current value of money and current real balances.

The explicit solution in the region below the border line, when  $\omega < \tilde{\omega}(y)$ , is

$$(3.8a) \quad \begin{aligned} m(s) &= \beta A / u_c(y) \omega > y, \\ \lambda(s) &= u_c(y) \quad \text{and} \\ \mu(s) &= 0; \end{aligned}$$

and the solution above the border line, when  $\omega \geq \tilde{\omega}(y)$ , is

$$(3.8b) \quad \begin{aligned} m(s) &= y, \\ \lambda(s) &= \beta A / y \omega \quad \text{and} \\ \mu(s) &= u_c(y) - \beta A / y \omega \geq 0. \end{aligned}$$

For small  $\omega$ , when  $m > y$ , real balances by (3.8a) fall with increasing  $\omega$ , to eventually hit the liquidity constraint. Then real balances cannot fall further, and instead the marginal utility of real balances  $\mu$  increases with  $\omega$ , by (3.8b).

The elasticity of the border line function  $\tilde{\omega}(y)$  is

$$(3.9) \quad \tilde{\omega} / \epsilon y = - \epsilon u_c / \epsilon y - 1 = r(y) - 1,$$

where  $r(y) = - \epsilon u_c / \epsilon y \equiv - y u_{cc} / u_c$  is the Arrow-Pratt measure of relative (consumption) risk aversion. (As is well known, with an additive inter-

temporal utility function (2.2) the degree of risk aversion is inversely related to the degree of intertemporal substitution in consumption.) Hence the slope of the border line between the region depends on whether the relative risk aversion is greater or smaller than unity. Let us for simplicity consider the case with constant relative (consumption) risk aversion, that is,

$$(3.10) \quad u(c) = \begin{cases} c^{1-r}/(1-r) & \text{for } r \neq 1 \text{ and} \\ \log c & \text{for } r = 1. \end{cases}$$

Then the equation for the border line is

$$(3.11) \quad \omega = \tilde{\omega}(y) \equiv \beta A y^{r-1}.$$

The solution is then, for  $\omega < \tilde{\omega}(y)$ ,

$$(3.12a) \quad \begin{aligned} m(s) &= \beta A y^r / \omega > y, \\ \lambda(s) &= y^{-r} \text{ and} \\ \mu(s) &= 0 ; \end{aligned}$$

and, for  $\omega \geq \tilde{\omega}(y)$ ,

$$(3.12b) \quad \begin{aligned} m(s) &= y, \\ \lambda(s) &= \beta A / y \omega \text{ and} \\ \mu(s) &= y^{-r} - \beta A / y \omega \geq 0. \end{aligned}$$

We illustrate the solution in  $(y, \omega)$ -space in Figure 1, by plotting the border line, the marginal utility of real balances  $r$ , and the income velocity of money,  $y/m$ . The latter is

$$(3.13) \quad y/m = \begin{cases} \omega y^{1-r} / \beta A & \text{for } \omega < \beta A y^{r-1} \text{ and} \\ 1 & \text{for } \omega \geq \beta A y^{r-1}. \end{cases}$$

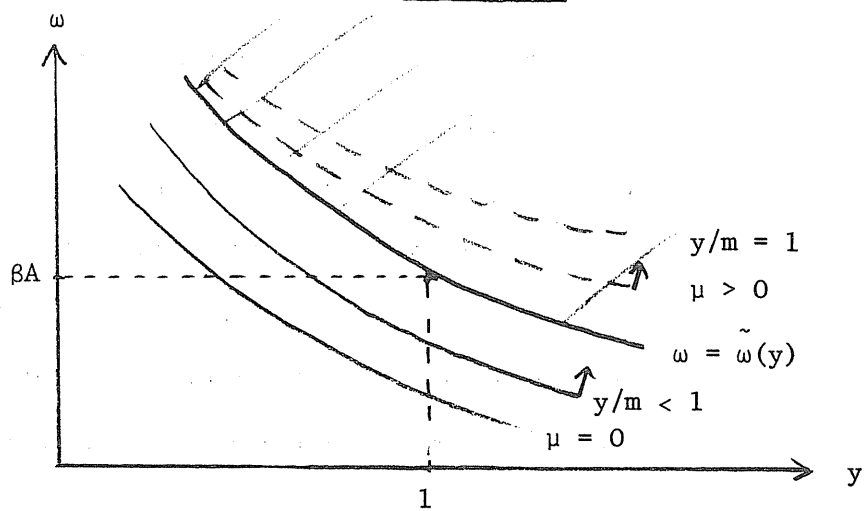
Panel (a)-(c) in Figure 1 corresponds to the three cases  $0 < m < 1$ ,  $r = 1$  and  $r > 1$ , respectively. The border line has a negative, zero, and positive slope in the three cases, respectively. (Panel (c) is drawn for  $r = 2$ . If  $r < 2$ , the border line is concave downward; if  $r > 2$  it is convex downward.) Below the border line, velocity is rising in monetary expansion up to its maximum - unity -, and above the border line velocity equals unity. The marginal utility of real balances is zero below the border line, and it is rising in monetary expansion above the border line. The iso-value curves for monetary velocity and marginal utility of real balances are plotted.<sup>11</sup>

We see in Figure 1 that below the border line velocity is increasing or decreasing in income depending upon whether the degree of risk aversion is less than or larger than unity, respectively. To understand this result, we recall from (3.8a) that below the border line, that is when the liquidity constraint is not binding, marginal utility of wealth equals the marginal utility of consumption. Assume that the degree of risk aversion is less than unity. This means that marginal utility of consumption and of wealth decreases less than proportionately to an increase in income. By (3.5c) real balances vary inversely with the marginal utility of wealth. Hence, real balances increase less than proportionately to income, and indeed velocity increases. Similarly, with a degree of risk aversion larger than unity, marginal utility of wealth and real balances vary more than proportionately to income, and velocity falls with income.

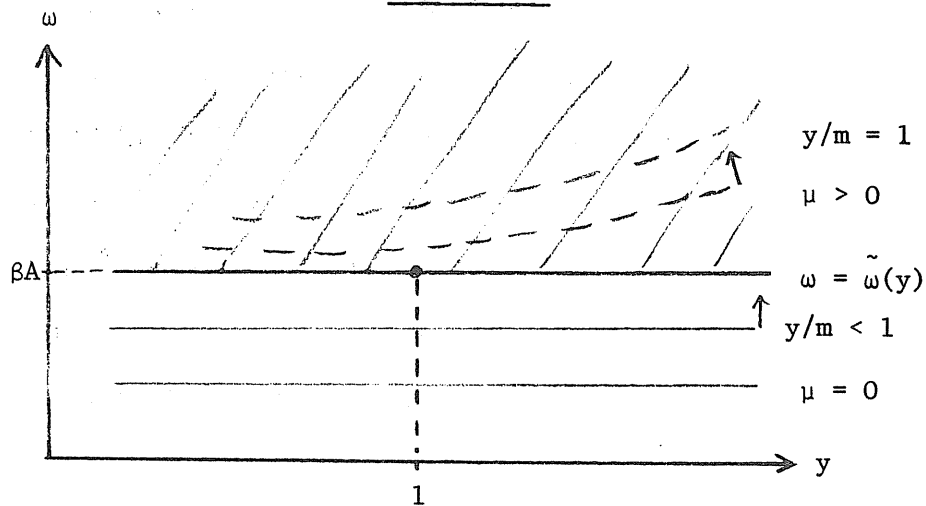
Let us again emphasize that the monetary velocity  $y/m$  is defined using beginning-of-period real balances. We can as well

Figure 1

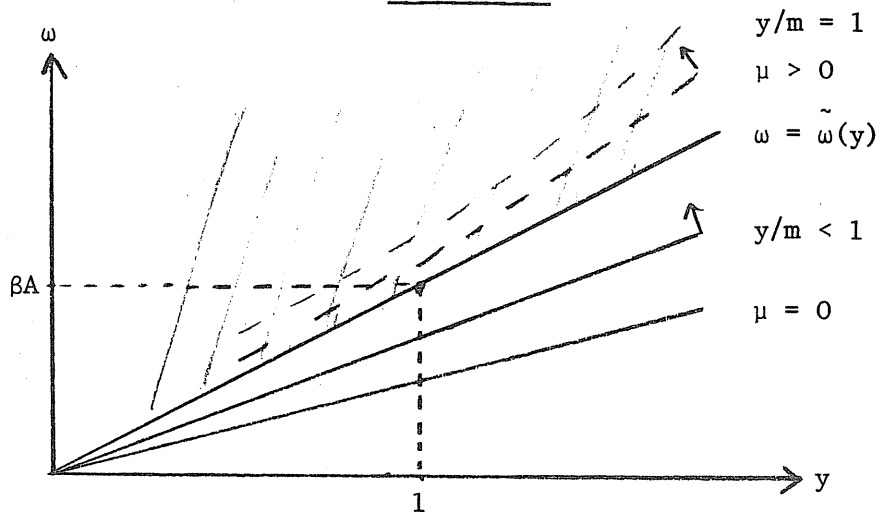
(a)  $r < 1$



(b)  $r = 1$



(c)  $r = 2$



look at end-of-period monetary velocity  $y/\bar{m} = y/\omega m$ . As mentioned, the advantage with using beginning-of-period real balances is the direct relation to the price level as given by (3.3). The advantage with defining monetary velocity as  $y/m$  is the ease with which the solution can be illustrated and interpreted.

Thus, the price level  $P(s, \bar{M})$  by (3.3) varies inversely with real balances  $m(s)$ . Past inflation  $P/P_{-1}$  of course varies with the price level  $P$ . Future inflation fulfills

$$(3.14) \quad P'/P = \pi/\pi' = \omega m(s)/m(s'),$$

where we use  $\pi' = m'/\bar{M}' = m'/\omega \bar{M}$ . Hence future inflation varies, for given next period state  $s'$ , directly with end-of-period real balances  $\bar{m} = \omega m$ . The same is the case with expected inflation,

$$(3.15) \quad E[P'/P] = E[\pi/\pi'] = \omega m E[1/m'],$$

since the term  $E[1/m']$  is constant. We see from (3.12a) that when the liquidity constraint is not binding, expected inflation varies with  $y^r$  and is hence independent of monetary expansion and increasing in income. When the liquidity constraint is binding, it is increasing in both monetary expansion and income. That expected inflation is increasing in income is natural: A temporary increase in income temporarily lowers the current price level, which increases inflation. That a temporary increase in monetary expansion increases expected inflation when the liquidity constraint binds also seems intuitive: Next period's price level increases across the board, whereas current real balances and the current price level are independent of current monetary expansion. Why is expected inflation independent of monetary expansion when the liquidity constraint is not binding? The reason is that current real balances fall, and hence



the current price level rises, in proportion to monetary expansion. Hence the current price level rises in proportion with next period's price level, and expected inflation remains unaffected.

Let us also see how nominal and real interest rates vary with temporary disturbances in monetary expansion and income. By substituting  $\pi' = m'/\bar{M}' = m'/\omega\bar{M}$  in (2.13) and simplifying, we get that the nominal interest rate fulfills

$$(3.16) \quad i(s) = i = E[\mu'm']/E[\lambda'm'],$$

which is constant and independent of current monetary expansion and income. That the nominal interest rate is independent of, rather than increasing in, a temporary monetary disturbance may appear somewhat surprising. How can we understand this result? One way is to recall from the discussion of (2.13) that the nominal interest rate compensates for the absence of liquidity services of nominal bonds. The relation between next period's liquidity services ( $\mu'$ ) and next period's marginal utility of wealth ( $\lambda'$ ) depends on next period's state only, and not on the current monetary expansion. A higher current monetary expansion lowers the value of money  $\pi'$  in all states, but it does not affect the relative attractiveness of nominal bonds and money.<sup>12</sup>

What about the real interest rate, given by  $\rho = \lambda/\beta E[\lambda'] - 1$  in (2.15)? The term  $E[\lambda']$ , the expected marginal utility of wealth, is independent of current monetary expansion and income, and the real interest rate varies with current marginal utility of wealth. By (3.12a) it follows that below the border line it varies with the marginal utility of consumption and is hence independent of monetary expansion and decreasing in income. Above the border line,

that is when the liquidity constraint is binding, it is decreasing in both monetary expansion and income.

Summing up, for a temporary increase in monetary expansion, real balances fall and the price level rises when the liquidity constraint is not binding, expected inflation rises when the liquidity constraint binds, the nominal interest rate remains constant, and the real interest rate falls when the liquidity constraint binds. This means that a temporary disturbance in monetary expansion has a direct effect on real balances and the price level, an effect that is not captured by any change in the nominal interest rate. We note that a proper demand function for money may need current monetary expansion as an independent argument, separately from current income and the nominal interest rate.<sup>13</sup>

#### Permanent Disturbances

Next, we shall examine the same endogenous variables as functions of permanent disturbances in monetary expansion and income. We model this as follows: Let income and monetary expansion each be given by the product of two stochastic processes,

$$(3.17) \quad y_t = \bar{y}_t \eta_t \quad \text{and} \\ \omega_t = \bar{\omega}_t \theta_t.$$

Here  $\eta_t$  and  $\theta_t$  are serially uncorrelated, and correspond to temporary disturbances, whereas  $\bar{y}_t$  and  $\bar{\omega}_t$  are positively serially correlated and correspond to permanent disturbances. Let  $s_t = (\eta_t, \theta_t, \bar{y}_t, \bar{\omega}_t)$  denote the state in period  $t$ . The probability distribution of  $s' = s_{t+1}$  hence depends on the realization of  $s = s_t$  according to

$$(3.18) \quad F(s'|s) = F(\eta', \theta', \bar{y}', \bar{\omega}' | \bar{y}, \bar{\omega}) .$$

We choose the particularly simple case

$$(3.19) \quad \bar{y}_{t+1} = (\bar{y}_t)^a \xi_{t+1}, \quad 0 < a < 1, \quad \text{and}$$

$$\bar{\omega}_{t+1} = (\bar{\omega}_t)^b \zeta_{t+1}, \quad 0 < b < 1,$$

with constant "autocorrelation" coefficients  $a$  and  $b$ , and with  $\xi_{t+1}$  and  $\zeta_{t+1}$  serially uncorrelated and jointly distributed with  $\eta_{t+1}$  and  $\theta_{t+1}$  according to the distribution function  $G(\eta', \theta', \xi', \zeta')$ . The constants  $a$  and  $b$  must be less than unity in absolute value, to ensure the existence of a stationary equilibrium. We restrict the discussion here to positive coefficients and hence to positive serial correlation. The distribution function (3.18) fulfills

$$(3.20) \quad F(\eta', \theta', \bar{y}', \bar{\omega}' | \bar{y}, \bar{\omega}) = G(\eta', \theta', \bar{y}'/\bar{y}^a, \bar{\omega}'/\bar{\omega}^b).$$

Since the probability distribution of next period's state depends on the current permanent disturbances  $\bar{y}$  and  $\bar{\omega}$ , the previous constant  $A$  in (3.5d), the expected total marginal utility of next period's real balances, is now a function of the current permanent disturbances, given by

$$(3.21) \quad A(\bar{y}, \bar{\omega}) = E[u_c(y')m(s') | \bar{y}, \bar{\omega}].$$

Let us consider the case with constant relative risk aversion, (3.10). Again the solution is characterised by two regions: in the region  $\omega < \beta A(\bar{y}, \bar{\omega})/u_c(y)y$ , the solution is

$$(3.22a) \quad m(s) = \beta A(\bar{y}, \bar{\omega})y^r/\omega > y,$$

$$\lambda(s) = y^{-r} \quad \text{and}$$

$$\mu(s) = 0;$$

and, for  $\omega \geq \beta A(\bar{y}, \bar{\omega})/u_c(y)y$ , the solution is

$$(3.22b) \quad \begin{aligned} m(s) &= y, \\ \lambda(s) &= \beta A(\bar{y}, \bar{\omega})/y\omega \quad \text{and} \\ \mu(s) &= y^{-r} - \beta A(\bar{y}, \bar{\omega})/y\omega \geq 0. \end{aligned}$$

This solution is the same as (3.12), except that  $A$  is now a function of  $\bar{y}$  and  $\bar{\omega}$ .

When the consumer learns the state  $s = (\eta, \theta, \bar{y}, \bar{\omega})$  in the beginning of the current period, he/she hence is assumed to observe both the temporary and permanent components of current income and monetary expansion. For given permanent disturbances  $\bar{y}$  and  $\bar{\omega}$ , the previous analysis of temporary disturbances applies directly to the effects of changes in the temporary components of  $\eta$  and  $\theta$ , and Figure 1 applies, if only  $\eta$  and  $\theta$  are substituted for  $y$  and  $\omega$  on the two axes.

Let us then look at how the solution looks for changes in the permanent disturbances, for constant temporary disturbances. The equation for the border line in  $(\bar{y}, \bar{\omega})$ -space, for constant  $\eta$  and  $\theta$ , is

$$(3.23) \quad \bar{\omega}\theta = \beta A(\bar{y}, \bar{\omega})/u_c(\bar{y}\eta)\bar{y}\eta = \beta A(\bar{y}, \bar{\omega})(\bar{y}\eta)^{r-1}.$$

Taking logarithmic differentials, we get

$$(3.24) \quad \frac{\hat{\bar{\omega}}}{\bar{\omega}} = (\epsilon A/\epsilon \bar{y} + r - 1)/(1 - \epsilon A/\epsilon \bar{\omega}).$$

It is shown in Appendix 2 that the elasticities of expected next period's total marginal utility of real balances fulfill

$$(3.25) \quad \begin{aligned} \epsilon A/\epsilon \bar{y} &= (1 - r)\alpha(\bar{y}, \bar{\omega}) \quad \text{and} \\ \epsilon A/\epsilon \bar{\omega} &< 0, \end{aligned}$$

where the function  $\alpha(\bar{y}, \bar{\omega})$  fulfills  $0 < \alpha(\bar{y}, \bar{\omega}) < 1$ . Substituting (3.25) in (3.24) gives

$$(3.26) \quad \hat{\bar{\omega}}/\hat{\bar{y}} = [(1 - \alpha)/(1 - \epsilon A/\epsilon \bar{\omega})](r - 1).$$

The bracketed coefficient is positive and less than one. Comparing with (3.9), we conclude that the border line (3.23) in  $(\bar{y}, \bar{\omega})$ -space has the same sign of its slope as the border line in (3.11) in  $(y, \omega)$ -space, except that it is flatter. That is, the border line (3.23) is less negatively sloped than the border line (3.11) when  $r < 1$ , and the border line (3.23) is less positively sloped than the border line (3.11) when  $r > 1$ .

Given this, it follows that the dependence of monetary velocity on permanent disturbances in monetary expansion and income can be illustrated exactly as in Figure 1, except that the border line and the iso-value curves for monetary velocity and marginal utility of real balances are flatter. In particular, it follows that, below the border line (when the liquidity constraint is not binding), and relative to temporary disturbances, monetary velocity is more sensitive to permanent disturbances in monetary expansion, and less sensitive to permanent disturbances in income. This is intuitive: looking at (3.7), we see that a temporary increase in monetary expansion decreases the "effective" discount factor  $(\beta/\omega)$  and hence depresses current real balances. A permanent increase in monetary expansion in addition decreases next period's real balances, hence decreasing expected total utility of next period's real balances  $(A(\bar{y}, \bar{\omega}) = E[u_c(y')m(s')|\bar{y}, \bar{\omega}])$ , and thus depressing current real balances even further, making them more sensitive to permanent disturbances in monetary expansion.

On the other hand, a current temporary increase in income decreases the marginal utility of wealth ( $\lambda$ ), requiring real balances to increase. A permanent increase in income in addition decreases next period's marginal utility of income, hence depresses the expected total marginal utility of next period's real balances, thus requiring less of a rise in real balances, making them less sensitive to permanent disturbances in income.

Thus, the price level and past inflation are more sensitive to permanent monetary expansion than temporary, and less sensitive to permanent income changes than to temporary.

Expected inflation is given by

$$(3.27) \quad E[P'/P | \bar{y}, \bar{\omega}] = \omega m(s') E[1/m(s') | \bar{y}, \bar{\omega}].$$

As shown in Appendix 2 it appears that expected inflation can be either decreasing or increasing in permanent monetary expansion. In Appendix 2, conditions are stated under which the unconditional mean of the elasticity  $\epsilon E[P'/P | \bar{y}, \bar{\omega}] / \epsilon \bar{\omega}$  is positive. I have not been able to show that those conditions hold in general.

The nominal interest rate can be written

$$(3.28a) \quad i(\bar{y}, \bar{\omega}) = A(\bar{y}, \bar{\omega}) / \beta C(\bar{y}, \bar{\omega}) - 1, \quad \text{where}$$

$$(3.28b) \quad C(\bar{y}, \bar{\omega}) = E[\lambda(s') m(s') | \bar{y}, \bar{\omega}].$$

In Appendix 2 it is shown that the elasticity of  $C(\bar{y}, \bar{\omega})$  with respect to permanent monetary expansion fulfills

$$(3.29) \quad \epsilon C / \epsilon \bar{\omega} \leq \epsilon A / \epsilon \bar{\omega} \leq 0.$$

Hence

$$(3.30) \quad \epsilon(1 + i) / \epsilon \bar{\omega} = \epsilon A / \epsilon \bar{\omega} - \epsilon C / \epsilon \bar{\omega} \geq 0,$$

and the nominal interest rate is increasing in permanent monetary expansion. Intuitively, recalling (2.13), a permanent increase in monetary expansion will decrease next period's expected marginal utility of wealth and hence increase next period's expected marginal utility of real balances, making money relatively more attractive than bonds. To compensate for that, the nominal interest rate must rise.

The real interest rate is given by

$$(3.31a) \quad \rho(s) = \lambda(s)/\beta D(\bar{y}, \bar{\omega}) - 1, \quad \text{where}$$

$$(3.31b) \quad D(\bar{y}, \bar{\omega}) = E[\lambda(s') | \bar{y}, \bar{\omega}] .$$

In Appendix 2 it is shown that  $\epsilon D/\epsilon \bar{\omega} < 0$ . Below the border line  $\lambda(s) = u_c(y)$  and  $\epsilon \lambda/\epsilon \bar{\omega} = 0$ , whereas above the border line  $\lambda(s) = \beta A(\bar{y}, \bar{\omega})/y\omega$  and  $\epsilon \lambda/\epsilon \bar{\omega} = \epsilon A/\epsilon \bar{\omega} - 1 < 0$ . It follows that below the border line

$$(3.32a) \quad \epsilon(1 + \rho)/\epsilon \bar{\omega} = - \epsilon D/\epsilon \bar{\omega} > 0 ,$$

whereas above the border line,

$$(3.32b) \quad \epsilon(1 + \rho)/\epsilon \bar{\omega} = \epsilon A/\epsilon \bar{\omega} - 1 - \epsilon D/\epsilon \bar{\omega} \gtrless 0 .$$

The real interest rate is increasing in permanent monetary expansion below the border line, whereas it seems that it can be either decreasing or increasing above the border line. A permanent increase in monetary expansion, in contrast to a temporary increase, decreases next period's expected marginal utility of wealth, giving rise to this more ambiguous response. In Appendix 2, conditions are stated under which the unconditional mean of the elasticity  $\epsilon(1+\rho(\bar{y}, \bar{\omega}))/\epsilon \bar{\omega}$  is negative.

We summarize our findings on the demand for money by noting that monetary velocity and the price level are increasing in monetary expansion, and they are more sensitive to permanent disturbances in monetary expansion than to temporary ones. Expected inflation is constant (when the liquidity constraint does not bind) or increasing in temporary monetary expansion, whereas it is either increasing or decreasing in permanent monetary disturbances. The nominal interest rate is independent of temporary disturbances, and increases in permanent monetary expansion. The real rate of interest is constant (when the liquidity constraint does not bind) or decreasing (when the liquidity constraint binds) in temporary monetary disturbances. It is increasing (when the liquidity constraint does not bind) or either increasing or decreasing (when the liquidity constraint binds) in permanent monetary disturbances.

Monetary velocity is decreasing or increasing for temporary increases in income, depending upon whether relative risk aversion is below or above unity. Monetary velocity is less sensitive to permanent increases in income than to temporary. The elasticities with respect to permanent income disturbances are reported in Appendix 2.

The results on elasticities of the crucial variables with respect to temporary and permanent disturbances in income are summarized in Table 1.



Table 1

|            | m > y  |          | m = y  |          | m > y                   |   | m = y                |                |
|------------|--------|----------|--------|----------|-------------------------|---|----------------------|----------------|
|            | $\eta$ | $\theta$ | $\eta$ | $\theta$ | $\bar{y}$               | $\bar{\omega}$                              | $\bar{y}$            | $\bar{\omega}$ |
| m          | r      | -1       | 1      | 0        | $\alpha + (1-\alpha)r$  | $\varepsilon A/\varepsilon\bar{\omega} - 1$ | 1                    | 0              |
| y/m        | 1-r    | 1        | 0      | 0        | $(1-r)(1-\alpha)$       | $1 - \varepsilon A/\varepsilon\bar{\omega}$ | 0                    | 0              |
| P          | -r     | 1        | -1     | 0        | $-\alpha - (1-\alpha)r$ | $1 - \varepsilon A/\varepsilon\bar{\omega}$ | -1                   | 0              |
| P'/P       | r      | 0        | 1      | 1        | $\alpha + (1-\alpha)r$  | $\varepsilon A/\varepsilon\bar{\omega}$     | 1                    | 1              |
| E[P'/P]    | r      | 0        | 1      | 1        | $\pm ?$                 | $\pm ?$                                     | $\pm ?$              | $\pm ?$        |
| 1 + i      | 0      | 0        | 0      | 0        | $(1-r)(\alpha-\psi)$    | +   | $(1-r)(\alpha-\psi)$ | +              |
| 1 + $\rho$ | -r     | 0        | -1     | -1       | $\pm ?$                 | +   | $\pm ?$              | $\pm ?$        |

Note:  $0 < r < \infty$ ,  $\varepsilon A/\varepsilon\bar{\omega} \leq 0$ ,  $0 \leq \alpha$ ,  $\psi < 1$ .

4. Optimum monetary policy

What is optimum monetary policy here? First, we note that in this one-consumer pure-exchange economy, monetary expansion has no (general-equilibrium) effect on consumption and on utility, since consumption in equilibrium is equal to output. (Here we of course assume that the probability distribution of output is independent of previous realizations of monetary expansion.) However, monetary expansion affects the relative prices of assets and the real interest rate, since the marginal utility of wealth ( $\lambda(s)$ ), which enters the asset pricing equation, depends on monetary expansion. In that sense, monetary policy has real effects in the model. Presumably, in a more complicated model, say with heterogeneous consumers, those effects on relative prices would affect the allocation of consumption, and monetary policy would then have real effects also on consumption and welfare.

As we emphasized in Section 2, the existence of a binding liquidity constraint drives a wedge between marginal utility of wealth and marginal utility of consumption ( $u_c(y)$ ), the former falling short of the latter by the marginal utility of real balances ( $\mu(s)$ ). Indeed, the marginal utility of real balances can taken to be a measure of the distortion relative to a barter economy that the monetary institutional structure causes in our monetary economy.<sup>14</sup>

It follows that the optimum monetary policy in this case is a stochastic process for monetary expansion that results in zero marginal utility of real balances in all states,

$$(4.1) \quad \mu(s) = 0, \text{ all } s.$$

Such a policy makes marginal utility of wealth equal marginal utility of consumption and results in the same asset prices as in a barter economy. In a more complicated model with real effects of monetary policy on consumption and welfare, the policy fulfilling (4.1) would give the same equilibrium as the corresponding barter economy. This policy is of course Friedman's (1969) optimum monetary policy, and it will by (2.13) correspond to a zero interest rate,

$$(4.2) \quad i(s) = 0, \text{ all } s.$$

Let us consider the case when output is serially uncorrelated, that is when there are no permanent income disturbances. When also monetary expansion is serially uncorrelated, the solution is given by (3.8). Substitution (4.1) in (3.8), we get

$$(4.3) \quad m(s) \geq y \quad \text{and}$$

$$\lambda(s) = u_c(y) = \beta A / y\omega, \quad \text{where}$$

$$A = E[u_c(y')m(s')].$$

Let us restrict ourselves to the solution where the liquidity constraint is just binding, which gives

$$(4.4) \quad \omega = \beta A / u_c(y)y \quad \text{where}$$

$$A = E[u_c(y')y'].$$

Indeed the optimum monetary policy is to choose monetary expansion conditional upon output such that it fulfills (4.4). Put differently, monetary expansion should be chosen so that the state  $s = (y, \omega)$  is always on the border line  $\omega = \tilde{\omega}(y)$  in Figure 1.

For the deterministic case, when  $y$  is constant, (4.4) collapses to

$$(4.5) \quad \omega = \beta,$$

the constant optimum rate of contraction of money supply ( $\beta < 1$ ). Under uncertainty, the elasticity of monetary expansion with respect to output is, by differentiation of (4.4),

$$(4.6) \quad \varepsilon\omega/\varepsilon y = r(y) - 1,$$

the degree of relative risk aversion minus one. Hence, the optimum monetary policy is in general not a constant monetary contraction, and whether monetary policy is procyclical or countercyclical depends on whether the degree of relative risk aversion is larger or smaller than unity. For the case of a unitary degree of risk aversion, the optimum monetary policy is (4.5), else it implies a stochastic monetary expansion.

It is easy to see that the optimum monetary policy does not stabilize nominal income or the nominal money stock. It does not stabilize beginning-of-period or end-of-period real balances. Nor does it stabilize the rate of inflation or money growth.<sup>15</sup>

We note in passing that the average monetary expansion is, for constant relative risk aversion, given by the second-order approximation

$$(4.7) \quad E[\omega] = \beta E[y^{1-r}] E[y^{r-1}] = \\ \beta \{1 + (r-1)^2 [\sigma_y^2 / (Ey)^2] [1 + r(2-r) \sigma_y^2 / (2Ey)^2]\},$$

where  $Ey$  and  $\sigma_y^2$  are the mean and variance of income, respectively.

Clearly, there are parameters such that  $E[\omega] > \beta$  and even  $E[\omega] > 1$ ,

that is, the optimum monetary policy can under uncertainty imply that on average there is net monetary expansion rather than contraction.

Also, let us note that what we call monetary policy here cannot in this model be separated from fiscal policy, since it involves transfers/taxes. Considering open market operations as distinct from fiscal policy requires more structure of the model and is beyond the scope of this paper.

Finally, let us note that since the consumer by assumption observes the current state directly, the problem of inferring the state from observation of endogenous variables (cf. Lucas (1972, 1983)) does not arise.

#### 5. The Fisher Relation and the Premium on Nominal Bonds

We shall also mention some frequently discussed parity conditions (see for instance Roll and Solnik (1979) and Kouri (1983)). Let us first look at the Fisher relation, between nominal and real interest rates and the expected rate of inflation. By (2.12) and (2.14) we can write the relation between nominal and real interest rates as

$$(5.1) \quad (1 + i)/(1 + \rho) = 1/E\{\lambda'(\pi'/\pi)/E[\lambda']\}.$$

The right-hand side is one over the probability and marginal-utility-of-wealth weighted (gross) real rate of appreciation of money. According to the simple Fisher relation it should equal the expected (gross) rate of inflation

$$(5.2) \quad E[P'/P] = E[\pi/\pi'] ,$$

but as noted by many authors the simple Fisher relation does not hold under uncertainty. The right-hand side can be rewritten to give

$$(5.3) \quad (1+i)/(1+\rho) = 1/\{E[\pi'/\pi] + \text{Cov}[\lambda', \pi'/\pi]/E[\lambda']\}.$$

Hence we see that the sign of  $\text{Cov}[\lambda', \pi'/\pi]$ , the covariance between the marginal utility of wealth and the rate of appreciation of money, is crucial to whether the left-hand side of (2.18) exceeds or falls short of  $1/E[\pi'/\pi]$ , one over the expected real rate of appreciation of money.

We note that real wealth risk-neutrality, meaning that marginal utility of wealth  $\lambda'$  is independent of the state  $s'$ , would imply that the covariance between the marginal utility of wealth and the real rate of appreciation of money is zero, and hence that the right-hand side of (5.3) equals one over the expected real rate of appreciation of money. We also note in passing that the distinction between marginal utility of wealth and marginal utility of consumption makes it necessary to distinguish wealth risk behavior from consumption risk behavior. Put differently, the concavity of the value function in wealth is relevant, in addition to the concavity of the direct utility function in consumption.

To express the Fisher relation explicitly in terms of the rate of inflation, we can rewrite the right-hand side of (5.3) to get

$$(5.4) \quad (1+i)/(1+\rho) = E[P'/P] + \text{Cov}[\lambda'\pi', P'/P]/E[\lambda'\pi'].$$

Hence, whether the left-hand side of (5.4) exceeds or falls short of the expected rate of inflation depends on  $\text{Cov}[\lambda'\pi', P'/P]$ ,

the covariance between the marginal utility of nominal wealth ( $\lambda'\pi'$ ) and the rate of inflation.

Here we note that nominal wealth risk-neutrality, meaning that the marginal utility of nominal wealth  $\lambda'\pi'$  is independent of the state  $s'$ , would imply that the right-hand side is indeed equal to the expected rate of inflation.

Another frequently discussed parity relation is that between the expected real return on nominal bonds and the real rate of interest on indexed bonds. More precisely, the issue is whether there is a risk premium on nominal bonds relative to indexed bonds.

Let us define the expected real return on nominal bonds as

$$(5.5) \quad R = (1 + i)E[\pi'/\pi] - 1.$$

We can understand (5.5) as follows: One unit of money invested in nominal bonds at the end of period  $t$  pays  $1+i$  units of money at the end of period  $t+1$ . The real value of this is, ex post at  $t+1$ ,  $(1+i)\pi'$ . The real value of one unit of money at the end of period  $t$  is  $1/\pi$ . Hence, the real ex post rate of return is  $(1+i)\pi'/\pi - 1$ . The expected real rate of return is then given by (5.5).

We say that there is a risk premium on nominal bonds if the expected real rate of return on nominal bonds exceeds the real interest rate on a real bond. Using (5.5) and (2.14) we can write<sup>16</sup>

$$(5.6) \quad R - \rho = - (1 + \rho) \text{Cov}[\lambda', \pi'/\pi]/E[\lambda'\pi'/\pi].$$

Hence, whether or not there is a risk premium on nominal bonds depends on the sign of the covariance between the marginal utility of wealth and the real rate of appreciation of money, the same covariance that appears in the Fisher relation (5.3). Of course, the issue of a risk premium on nominal bonds is connected with the Fisher relation: If there is a negative correlation between marginal utility of wealth and the real appreciation of money, nominal bonds become a less attractive asset for a risk averse consumer than an indexed bond, which requires a higher equilibrium nominal rate of interest relative to the real rate of interest.

These relations have been discussed in previous literature. What is new here, is following two circumstances. First, the marginal utility of wealth is not identical to the marginal utility of consumption, the difference being the marginal utility of real balances. This marginal utility of real balances is not postulated by having real balances in the utility function, but carefully derived as the endogenous shadow price of a liquidity constraint. Second, the variables, in particular the prices and interest rates, that enter these relations are the outcome of a general equilibrium, where only monetary supply and output are exogenous. Previous literature has taken at least some prices and interest rates as given, and is hence partial equilibrium, with the exception of Kouri (1983), which however relies on the quantity equation.

Since the variables in the relation above are endogenous functions of the exogenous stochastic income and monetary expansion, we can go behind the relations and see how properties of income



and monetary expansion affect the Fisher relation and the premium on nominal bonds. Let us limit the discussion to the case when there is no serial correlation in income and monetary expansion, that is, when the solution (3.8) applies. We note that then marginal utility of real wealth ( $\lambda(s)$ ) and marginal utility of nominal wealth ( $\lambda(s)\pi(s) = \lambda(s)m(s)/\bar{M}$ ) are both functions of the state  $s$ . Hence, what we have called real and nominal risk neutrality does not occur here.

Let us first look at the covariance between marginal utility of wealth and the real rate of appreciation of money,  $\text{Cov}[\lambda', \pi'/\pi]$ , which is crucial to the Fisher relation (5.3) and to the premium on nominal bonds (5.6). Since  $\pi'/\pi = m'/\omega m$ , we have

$$(5.7) \quad \text{Cov}[\lambda', \pi'/\pi] = \text{Cov}[\lambda', m']/\omega m,$$

and we can look at the covariance between marginal utility of wealth and real balances. Here  $\lambda' = \lambda(y', \omega')$  and  $m' = m(y', \omega')$  are both functions of income and monetary expansion. In general the covariance between  $\lambda'$  and  $m'$  will depend on the first-order partials of these functions together with the variances and covariances of  $y'$  and  $\omega'$ . It will also depend on the second order partials and the third- and fourth-order moments of  $y'$  and  $\omega'$ .<sup>17</sup> Let us restrict the discussion to first-order partials and second-order moments only. Then, from (3.8) we see that  $\lambda'$  is decreasing in  $y'$  and  $m'$  is increasing in  $y'$ , in both regions. Hence, a large variance in  $y'$  contributes to a large negative covariance between  $\lambda'$  and  $\pi'$ . In region  $m' > y'$ ,  $\lambda'$  is independent of  $\omega'$  and  $m'$  is decreasing in  $\omega'$ . In region  $m' = y'$ ,  $\lambda'$  is decreasing in  $\omega'$  and  $m'$  is constant. It follows that increased covariance between  $y'$  and  $\omega'$  has an ambiguous influence on

(5.7). Increased variance of  $\omega'$  contributes to a positive effect on (5.7).<sup>18</sup>

We conclude that increased variance in income contributes to the ratio  $(1 + i)/(1 + \rho)$  being larger than one over the expected real rate of appreciation of money ( $1/E[\pi'/\pi]$ ), and to a premium on nominal bonds relative to indexed bonds. Increased variance in monetary expansion has the opposite effect. The covariance between income and monetary expansion has an ambiguous effect.

With regard to the covariance between marginal utility of nominal wealth and the rate of inflation, we have

$$(5.8) \quad \text{Cov}[\lambda'\pi', P'/P] = \beta A \text{Cov}[1/\omega', 1/m'],$$

by (3.8). Again we disregard the effect of third- and fourth-order moments.  $1/\omega'$  is obviously decreasing in  $\omega'$ . So is  $m'$ , in the region  $m' > y'$ . Hence, increased variance in  $\omega'$  contributes to a positive covariance between marginal utility of nominal wealth and inflation.  $m'$  is increasing in  $y'$ . Thus, negative covariance between  $y'$  and  $\omega'$  contributes to positive covariance between  $\lambda'\pi'$  and  $P'/P$ . Variance in  $y'$  has no effect on (5.8).

In summary, negative covariance of income and monetary expansion, and high variance of monetary expansion both contribute to make the ratio  $(1 + i)/(1 + \rho)$  exceed expected inflation ( $E[P'/P]$ ), whereas the variance in income has no effect. We summarize these results in Table 2.

Let us also look at the premium on bonds in Lucas' (1982) monetary model. Lucas does not himself examine this parity condition, but it is not difficult to do that in his model. As shown in Appendix 1, in Lucas' model marginal utility of wealth equals

Table 2

|  | Var[y'] | Cov[y', ω'] | Var[ω'] |
|--|---------|-------------|---------|
| $\frac{(1+i)}{(1+\rho)} - \frac{1}{E[\pi'/\pi]}$ | +       | ?           | -       |
| $\frac{1+i}{1+\rho} - E[P'/P]$                   | 0       | -           | +       |
| R - ρ  | +       | ?           | -       |

marginal utility of consumption. Also, the quantity equation holds. Finally, monetary transfers are distributed in the beginning of each period and included in the beginning-of-period money supply. Hence, in summary

$$(5.8) \quad \lambda = U_c(y), \quad \pi \bar{M} = y \quad \text{and} \quad \bar{M}' = \omega' \bar{M}.$$

Then

$$(5.9) \quad \pi'/\pi = (y'/\bar{M}')/(y/\bar{M}) = y'/\omega'y,$$

and the covariance crucial to the premium on bonds can be written

$$(5.10) \quad \text{Cov}[\lambda', \pi'/\pi] = \text{Cov}[\bar{u}_c(y'), y'/\omega']/y.$$

Since  $u_c$  is decreasing in  $y$ , increased variance in  $y'$  contributes to a negative covariance in (5.10). Since  $y'/\omega'$  is decreasing in  $\omega'$  and increasing in  $y'$ , it follows that a negative covariance between  $y'$  and  $\omega'$  contributes to a negative (5.10). Hence, variance of  $y'$  as well as a negative correlation between  $y'$  and  $\omega'$  contribute towards a premium on nominal bonds. In Lucas' model the correlation between  $y'$  and  $\omega'$  does not have an ambiguous effect on the premium

on nominal bonds, in contrast to our case. This is due to the simplification caused by an always binding liquidity constraint. In particular, variance in monetary expansion has no effect in Lucas' model, whereas in our case it decreases the premium on nominal bonds.

In a somewhat different model, Kouri (1983) (see also the discussion by Fischer (1983a)), derives the result that the premium on bonds depends positively on the covariance between income and the price of money, in our notation  $\text{Cov}[y', \pi']$ . This is the same result as what we have derived above for the Lucas model. Since Kouri assumes the quantity equation, it follows also in his model that increased variance in income and increased positive covariance between income and monetary expansion contribute to a premium on nominal bonds, but variance in monetary expansion has no effect.

#### 6. Money in the utility function

It has been argued (see Fischer's (1983b) comment on Lucas and Stokey (1983)) that imposing a cash-in-advance constraint is in effect no different from postulating that money gives direct utility. Let us discuss this somewhat. The previous literature - referred to in the second paragraph of the Introduction - on money and asset pricing has in general assumed that real balances give direct instantaneous utility. This literature has mostly used the continuous-time stochastic differential equation approach. We follow Dixit and Goldman (1970) and LeRoy (1982a, b) in using discrete time, to facilitate comparison with our previous analysis. Hence, in our notation, the utility function is

$$(6.1) \quad E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U(c_{\tau}, m_{\tau})$$

where  $m_{\tau} = \pi_{\tau} M_{\tau}$  is real balances in period  $\tau$ . Assume that in the beginning of the period, after learning the current state, the consumer with wealth  $w$  can trade consumption, money and claims to output according to the budget constraint

$$(6.2a) \quad c + \pi M + qz' \leq w.$$

Wealth next period is

$$(6.2b) \quad w' = \pi' M + (q' + y')z' + \pi'(\omega' - 1)\bar{M}.$$

It is also assumed that the money transfer is distributed in the beginning of the period and hence included in the beginning-of-period money supply, that is,  $\bar{M} = \omega \bar{M}_{-1}$  and  $\bar{M}' = \omega' \bar{M}$ . In a stochastic steady state, maximizing (6.1) subject to (6.2) gives rise to the value function  $V(w, s)$  defined as the maximum of

$$(6.3) \quad U(c, \pi M) + \beta E[V(w', s')]$$

over  $c, M, z'$ , subject to (6.2). Note that this value function does not depend separately on money. The first-order conditions will be

$$(6.4a) \quad \lambda = U_c,$$

$$(6.4b) \quad \lambda \pi = \beta E[\lambda' \pi'] + U_m \pi \quad \text{and}$$

$$(6.4c) \quad \lambda q = \beta E[\lambda'(q' + y')],$$

where  $\lambda$  is the Lagrange multiplier of the budget constraint, and hence fulfills

$$(6.4d) \quad \lambda = V_w.$$

With the nominal interest rate given by

$$(6.5) \quad 1/(1+i) = \beta E[\lambda' \pi'] / \lambda \pi,$$

substituting (6.5) in (6.4b) gives

$$(6.6) \quad i = U_m(c, m) / [U_c(c, m) - U_m(c, m)],$$

the familiar condition that the nominal interest rate is directly related to the marginal rate of substitution of consumption for real balances. Solving for real balances in (6.6), using that consumption equals income in equilibrium, we get

$$(6.7) \quad m = f(y, i),$$

that is real balances are simply a function of consumption and the nominal interest rate only. If the utility function  $U(c, m)$  is homothetic, we get

$$(6.8) \quad m = g(i)y,$$

that is, the income velocity of money - the reciprocal of  $g(i)$  - is independent of income and an increasing function of the nominal interest rate.

This simple demand function for real balances is clearly not identical to the demand for real balances we have discussed and illustrated in Figure 1. In particular, (6.6) implies that the demand for real balances is independent of the consumer's degree of risk aversion/intertemporal substitution in consumption.

However, the simple demand function does not result if we change the timing of transactions. Let us presume that the consumer enters the current period with total wealth  $w$ , part of which is money  $M$ . Assume that the direct utility in the current period is derived from this initial stock of money, and that the consumer acquires the new stock of money  $M'$  at the end of the period, when also the current period's monetary transfer occurs. The new stock of money gives direct utility in the next period.<sup>19</sup> This timing is more similar to the timing in our previous analysis. Then, let us write the budget constraint and the next period's wealth as

$$(6.9a) \quad c + \pi M' + qz' \leq w \quad \text{and}$$

$$(6.9b) \quad w' = \pi' M' + (q' + y')z' + \pi'(\omega' - 1)\bar{M}'.$$

Now the value function  $V(w, M, s)$  will be a function of money separately, and it will be given by the maximum of

$$(6.10) \quad U(c, \pi M) + \beta E[V(w', M', s')]$$

over  $c, M', z'$ , subject to (6.9). The first-order conditions are

$$(6.11a) \quad \lambda = U_c,$$

$$(6.11b) \quad \mu = U_m,$$

$$(6.11c) \quad \lambda \pi = \beta E[(\lambda' + \mu')\pi'] \quad \text{and}$$

$$(6.11d) \quad \lambda q = \beta E[\lambda'(q' + y')],$$

where we use  $V_M = \pi U_m$  and identify  $\mu$  with  $U_m$ . Here (6.11c) and

(6.11d) are identical to (2.9c) and (2.9d). Then the nominal interest rate fulfills (2.13), and it may appear that the setting is essentially the same as in the case with a cash-in-advance constraint.

However, (6.11a) is not identical to (2.9b). With money in the utility function, marginal utility of wealth is always equal to marginal utility of consumption, whereas with a binding liquidity constraint it falls short of marginal utility of consumption by the marginal utility of real balances. Hence, the two approaches are not formally equivalent.

There is, though, a more fundamental argument why the two approaches are different.<sup>20</sup> The similarity appears of course because the indirect value function  $v(w, M, s)$  defined in (2.7) has money as a separate argument, and because money gives indirect utility. If money gives indirect utility, mustn't there be a direct utility function with money as a separate argument that somehow results in the same indirect value function? The previous paragraph explains why this conjecture is false. But suppose it were true, and that a direct utility function always existed. Then we should not forget that we have defined the value function here only for stationary conditions, that is when all exogenous and endogenous variables have stationary distributions. Under non-stationary conditions, the value function will be time-dependent. Then, any direct utility function with money as an argument would - if it were to exist - presumably also be time-dependent. This makes such a direct utility function much less useful.

Moreover, and perhaps more importantly, the value function  $v(w, M, s)$  depends on the "financial system" - for instance the



existing markets and the nature and timing of transactions. Hence, any corresponding direct utility function with money as an argument would in turn depend on the financial system, making it a rather elusive construction. For instance, it is impossible to use the same direct utility function to discuss different financial systems.<sup>21</sup>

This last point is nicely exemplified by the discussion of the optimum monetary policy in Section 4. There we could easily compare the monetary economy with the corresponding barter economy, hence indeed compare two financial systems. Clearly, one cannot do the same thing with a utility function of real balances, and let such a utility function apply to the barter economy.

It seems one can safely conclude that the two approaches are indeed different.

#### 7. Conclusions, Limitations and Possible Extensions

As far as I know, this paper is the first that derives a demand for money in a general-equilibrium asset-pricing framework with stochastic output and monetary expansion, where the liquidity services of money are endogenously determined via a cash-in-advance constraint. This framework makes it possible to express the demand for real balances, and the equilibrium price level, rate of inflation, and nominal and real interest rates, as functions of temporary and permanent disturbances in output and monetary expansion. In particular we are able to sign most of the effects on the endogenous variables.

What do we learn from this? From a more methodological point of view we see that the cash-in-advance approach can relatively easily be extended to a general-equilibrium asset-pricing framework, and need not be restricted to give a unitary income velocity of money. The cash-in-advance approach then is a convenient way to model the circumstance that cash is more liquid than non-money wealth. Although for some problems it may make little difference whether money is introduced via a cash-in-advance constraint or directly in the utility function, we argue that the two approaches are essentially different, and that the cash-in-advance approach offers some distinct advantages. Not only does it model more specifically the transactions role of money, it does not need ad hoc assumptions about cross partials of the utility function - frequent in the money-in-the-utility-function literature, it conveniently represents the difference in liquidity between cash and other wealth, and in particular it allows the comparison of different financial systems. This is illustrated in our discussion in Section 4 of the optimum monetary policy, where we easily can compare our monetary economy with a barter one.

More specifically, it is certainly of independent interest to be able to derive and sign the effects on the different endogenous prices and interest rates of disturbances in output and monetary expansion. This opens up the possibility of a systematic study of the much-discussed correlation between inflation and rates of return on different assets. This is clearly a suitable area for future application of this model. It seems obvious, for instance, that the various correlations between endogenous inflation and rates of return will depend crucially on the nature of the exogenous

disturbances, that is whether in output or in monetary expansion, and temporary or permanent, say.

Another obvious extension is to international finance issues, as in Lucas (1982). Some international issues are discussed in Svensson (1983).

With regard to the demand for money, it seems that one can conclude that it is in general not enough to specify a demand for real balances as a function of the nominal interest rate and income, but that temporary and permanent disturbances in income and monetary expansion probably should enter as separate arguments. The choice of definition of real balances should be a matter of convenience and ease of interpretation, but should in general affect the precise specification of the demand function.

From the discussion of the optimum monetary policy we see that the latter is a relatively sophisticated construction. In our model, whether it is pro- or countercyclical depends on parameters of preferences. In particular it is not at all in agreement with the usual simple rules of thumb, like a fixed rate of monetary growth as in Friedman (1969). In this paper, however, the discussion of optimum monetary policy completely avoids the problem of inferring the state of the economy by observing endogenous variables, since the state is assumed directly known by consumers. Also, the optimum monetary policy is here inseparable from fiscal policy, since we can interpret the monetary transfers as being government expenditure in the form of transfers which give rise to a budget deficit financed by money creation. Extending the model to a richer menu of monetary and fiscal policy, by introducing government expenditure on goods, outstanding government debt, and proportional

taxation (for instance as in Lucas and Stokey (1983)), seems a suitable area for future research. It seems that the method of modelling temporary and permanent disturbances in income and monetary expansion we employ here could be applied to temporary and permanent shifts between bond and money financing of government expenditures, say.

The results on the Fisher relation and the premium on nominal bonds show the possibilities in a general equilibrium model to relate these relations between endogenous variables to properties of the exogenous stochastic processes. This is not specific to the cash-in-advance approach, of course. What it contributes is the distinction between marginal utility of wealth and of consumption, and the insight that the relevant attitude to risk is with respect to wealth rather than to consumption. Also, the variability of velocity gives other results than the quantity equation, in that the variance of the rate of monetary expansion matters.

The general limitations of this kind of general equilibrium analysis with a pure exchange economy are discussed by Lucas (1982) and are now well known. In particular the assumption of identical consumers and absence of physical investment, and the reliance on a "perfectly pooled" stationary equilibrium, are serious restrictions. They imply that portfolios are never revised, and consumption and utility are in equilibrium independent of monetary expansion. Relative prices are affected by monetary policy, though, and the idea is of course that in a richer model, say with heterogeneous consumers, there would be real effects on consumption and welfare of monetary policy. The cash-in-advance approach has so far the additional limitation of treating the transactions period as

exogenous. Also, only goods transactions are assumed to absorb cash. If also asset transactions absorb cash, the demand for real balances is modified. This is another area for future research.<sup>22</sup>

Footnotes

\* A preliminary version of this paper, with a different title and including some international aspects, was presented at the Workshop in International Economics at Tel-Aviv University, July 11-13, 1983. I am grateful for comments by participants of the Workshop, especially Elhanan Helpman, and for comments by participants of an IIES seminar. Torsten Persson has contributed many thorough comments on the present version. Remaining errors and obscurities are my own.

1. A preliminary paper by Danthine and Donaldson (1983) extends Lucas' (1978) barter model to include real balances in the utility function. Only endowment shocks are dealt with, though.
2. Lucas mentions the possibility of this modification in the concluding section of Lucas (1982).
3. Lucas (1980) considers an economy where money is the only asset. Helpman and Razin (1982) and Krugman, Persson and Svensson (1983) consider a two-period general equilibrium with bonds and money, but with very asymmetric periods.

The cash-in-advance tradition represents one way of several to provide a somewhat better microfoundation of money (than just plugging money into the utility function), by assuming that transactions costs are zero with money and infinite without money. Most of this tradition seems too simplified in that the trivial quantity equation emerges as determining the price level. The present paper, as Krugman, Persson and Svensson (1983), can be seen as an attempt to improve on the cash-in-advance models.

Another tradition is the overlapping-generations one, like Wallace (1980), which by many seems inadequate in that money is given a role only as a store of value, and can in equilibrium not be rate-of-return dominated by other assets. The Baumol-Tobin tradition has previously been partial equilibrium only. Some recent work by Jovanovic (1982), Grossman and Weiss (1982) and Rotemberg (1982) provide very interesting general equilibrium extensions of it.

4. As emphasized by Lucas (1978, 1982) there is a certain arbitrariness in the number and kinds of assets assumed traded in these models, as long as there exist enough many assets to ensure existence of a stochastic steady state equilibrium.
5. We let prices be a function of the current state and money stock, but not of the current date, which presumes the stochastic stationary equilibrium that is to be defined below.
6. The methods to be used in proving existence of such an equilibrium are demonstrated in Lucas (1978, 1982), where existence is proved for Lucas' barter and monetary models.
7. See for instance Blanchard and Watson (1982).
8. We should emphasize that the pricing equations (2.9c) - (2.11) do not require the existence of a stochastic steady state. Then the variables in each period  $t$  depend on  $t$  in addition to  $s_t$  and  $\bar{M}_t$ .
9. In transforming (2.9c) into (3.2c) we use  $\pi = m/\bar{M}$  and  $\pi' = m'/M' = m'/\omega\bar{M}$ , since  $\bar{M}' = \omega\bar{M}$ . This gives rise to the  $\omega$  in equation (3.2c).

10. This can be shown as follows: Suppose  $\omega = \tilde{\omega}(y)$ , where  $m = y$ ,  $\mu = 0$  and  $\lambda = u_c$ . (i) Let  $\omega$  increase for constant  $y$ . Suppose  $\mu$  remains equal to zero and hence  $\lambda$  remains constant equal to  $u_c$ . Then, by (3.5c)  $m$  must fall. But then  $m < y$  which is impossible. Hence  $\mu$  must increase and be positive for  $\omega > \tilde{\omega}(y)$ . (ii) Let instead  $\omega$  decrease below  $\tilde{\omega}(y)$ , for constant  $y$ . Assume  $m = y$ . By (3.5c)  $\lambda$  must rise. But by  $\mu \geq 0$ ,  $\lambda \leq u_c(y)$  which is a contradiction. Hence  $m > y$  for  $\omega < \tilde{\omega}(y)$ .
11. The iso-value curves for monetary velocity fulfill the equation  $\omega = \beta A y^{r-1} (y/m)$  for constant  $y/m$ , and they are of the same form as the equation for the border line. The iso-value curves for marginal utility of real balances fulfill the equation  $\omega = \beta A y^{r-1} / (1 - \mu y^r)$  for constant  $\mu$ .
12. Hence, it is not always true that random money implies a random nominal interest rate, as is presumed in Jones (1983).
13. Note that this is true also for the demand for end-of-period real balances, although in opposite regions in  $(y, \omega)$  space: For  $m > y$ ,  $\tilde{m} = \beta A y^r$ ; whereas for  $m = y$ ,  $\tilde{m} = \omega y$ .
14. On this point, for the perfect foresight case, see for instance Grandmont and Younés (1975).
15. Nominal income in period  $t$  here equals the beginning-of-period money stock,  $y/\pi = y/(m/\bar{m}) = \bar{M}$ . Beginning-of-period real balances  $m$  are  $y$ , end-of-period real balances  $\tilde{m} = \omega m$  are  $\beta A / u_c(y)$ . The rate of inflation is  $P'/P = \pi/\pi' = \omega m / m' = \omega y / y' = \beta A / u_c(b) y'$ .



16. We get  $(1 + R)/(1 + \rho) = E\lambda'E\pi'/E[\lambda'\pi']$ , from which  $R - \rho =$   
 $= (1 + \rho)\{E\lambda'E\pi' - E[\lambda'\pi']\}/E[\lambda'\pi'] = -(1 + \rho) \text{Cov}[\lambda', \pi'/\pi]/E[\lambda'\pi'/\pi]$ .
17. Let the vector  $x$  be stochastic and let  $f(x)$  and  $g(x)$  be two real-valued functions. Then  $\text{Cov}[f(x), g(x)]$  is given by the formidable expression
- $$\begin{aligned} & f_x' E[(x - Ex)(x - Ex)'] g_x + f_x' E[(x - Ex)(x - Ex)'] g_{xx}(x - Ex) / 2 + \\ & + g_x' E[(x - Ex)(x - Ex)'] f_{xx}(x - Ex) / 2 + \\ & + E[(x - Ex)'] f_{xx}(x - Ex)(x - Ex)' g_{xx}(x - Ex) / 4, \end{aligned}$$
- where  $f_x$  and  $f_{xx}$  denote the gradient and Hessian of  $f(x)$  (evaluated at  $Ex$ ), respectively, etc., where all non-primed vectors are column vectors, and where "' denotes transpose.
18. Consider the linear approximations  $\lambda' = E\lambda' + a(y' - Ey') + b(\omega' - E\omega')$  and  $m' = Em' + c(y' - Ey') + d(\omega' - E\omega')$ . Then it is easy to show that  $\text{Cov}[\lambda', m'] = ac \text{Var}[y'] + (ad + bc) \text{Cov}[y', \omega'] + bd \text{Var}[\omega']$ .
19. This set-up is used in Danthine and Donaldson (1983).
20. I owe this point to Elhanan Helpman.
21. The specific example discussed by Lucas and Stokey (1983) and Fischer (1983) is where two goods ("cash goods" and "credit goods") are consumed in quantities  $c_1$  and  $c_2$ , giving utility  $U(c_1, c_2)$  (we suppress leisure). Cash goods require cash in advance,  $P c_1 \leq M$ . Lucas and Stokey then define the function  $V(c_1, c_1 + c_2) \equiv U(c_1, c_2)$  and then write  $V(M/P, c_1 + c_2)$  as the appropriate direct utility function

with real balance. Their function obviously relies on the assumption that cash good and credit goods are perfect substitutes in production; in which case  $P_1 = P_2 = P$ , and  $c_1 + c_2 = c$  can be used as an index of real consumption. If cash goods and credit goods are not perfect substitutes in consumption, one can of course define the function  $V^*(m, c_1, c_2) \equiv U(\min(m, c_1), c_2)$ , making  $V^*(M/P_1, c_1, c_2)$  the appropriate function with money. Even so, these two direct utility functions with money thus derived are meaningful only due to the assumption that cash balances for current purchases of cash goods are decided upon and acquired after the current state is known, which means that no excess cash balances are held, and that the cash-in-advance constraint is always binding. Hence, they indeed depend upon the "financial system".

22. The possibility of part of the transactions demand for money origination in asset trade seems not to have received much attention in the literature. The effects on exchange rate dynamics of such a transactions demand from asset trade in a cash-in-advance economy are examined by Helpman and Razin (1983).

Appendix 1: Comparison with Lucas (1978, 1982)

Lucas (1978) considers a barter economy. Then the budget constraint and the definition of wealth (in our notation) are

$$(A.1) \quad c + qz' \leq w \quad \text{and} \\ w' = (q' + y')z'.$$

In a stochastic steady state, the equations describing the pricing of a claim to future output is

$$(A.2a) \quad \lambda = u_c(y) \quad \text{and} \\ (A.2b) \quad \lambda q = \beta E[\lambda'(q' + y')],$$

which gives rise to the standard capital-asset-pricing equation

$$(A.3) \quad q = \beta E[u_c(y')(q' + y')]/u_c(y).$$

The real rate of interest is

$$(A.4) \quad \rho = u_c(y)/\beta E[u_c(y')] - 1.$$

It is clear that when  $\mu(s)$  is zero for all  $s$ , (2.9b) and (2.9d) simplifies to (A.2a-b), since then marginal utility of wealth equals marginal utility of consumption. In other cases, (2.9d) differs from (A.2b), marginal utility of wealth differs from marginal utility of consumption, and the price of the claim to output and the real rate of interest in our monetary economy differ from the price and the real interest rate in the barter economy.

Lucas (1982) considers a monetary economy. He actually examines a two-good two-country two-currency world. Here we are only interested in the corresponding closed-economy one-good version of Lucas' model.

Our timing of events is different from Lucas'. In his model, the consumer enters period  $t$  with shares  $z$  and possibly with money (although we shall see that in equilibrium no money is carried over). The consumer learns the state, receives his/her share of the cash from the sales of last period's endowment, receives money transfers, and trades cash and shares. After these transactions are completed, the consumer buys goods and pays with cash, and the period ends.

The period  $t+1$  money supply is defined including the period  $t+1$  transfer (since the transfer occurs in the beginning of the period),

$$(A.5) \quad \bar{M}_{t+1} = \omega_{t+1} \bar{M}_t.$$

The present period's budget constraint can be written (in our notation)

$$(A.6a) \quad \pi M + qz' = w,$$

where  $w$  is wealth in the beginning of period  $t$ . The liquidity constraint is

$$(A.6b) \quad c \leq \pi M,$$

and wealth at the beginning of next period is

$$(A.6c) \quad w' = \pi'(M - c/\pi) + (q' + (y/\pi)\pi')z' + \pi'(\omega' - 1)\bar{M}.$$

The first term on the right-hand side of (A.6c) is the real value of money not spent in period  $t$  and carried over into period  $t+1$ . In equilibrium this term will be zero, since no money will be carried over by the consumer, for the following reason: Period

t cash holdings are determined after the state is known. Then the consumer chooses to hold cash exactly equal to his planned consumption, and the liquidity constraint (A.6b) is always binding. (This is if nominal interest rates are positive). In equilibrium consumption equals output, and hence real balances will, in contrast to our case, always equal output,

$$(A.7) \quad y = \pi \bar{M}.$$

The second term on the right-hand side of (A.6c) is the total return on the claim to future output. Here  $(y/\pi)$  is cash from the sale of present output, and  $(y/\pi)\pi'$  is the real value of it in next period. Now since by (A.7)  $y/\pi = \bar{M}$ ,  $\pi' = y'/\bar{M}'$  and  $\bar{M}' = \omega'\bar{M}$ , this term can be written  $y'/\omega'$ . The claim to future output is really a claim to cash, and the real value of that cash is diluted by monetary expansion next period. This does not occur in our case, since with our timing cash from the sale of output is distributed within the same period.

With this the budget constraint and next period's wealth in Lucas' model can be written

$$(A.8a) \quad c + qz' = w \quad \text{and}$$

$$(A.8b) \quad w' = (q' + y'/\omega')z' + \pi'(\omega' - 1)\bar{M}.$$

It follows that the equations describing the equilibrium, in addition to (A.7), are

$$(A.9a) \quad \lambda = u_c(y) \quad \text{and}$$

$$(A.9b) \quad \lambda q = \beta E[\lambda'(q' + y'/\omega')],$$

where  $\lambda$  is the multiplier of (A.8a) and hence the marginal utility of wealth. Since shares and cash are traded in the beginning of the period when the state is known, and the cash then is used to buy the planned consumption, there is no wedge between the marginal utility of wealth and the marginal utility of consumption, in contrast to our case.

In Lucas' case, the pricing of the claim to output in the monetary economy is different from the barter economy equation (A.3) because the real value of the direct return is diluted by monetary expansion (the term  $y'/\omega'$ ). In our case the monetary economy equation (2.9d) differs from the barter one by the wedge between marginal utility of consumption and marginal utility of wealth.

The pricing of money in our case differs fundamentally from that of Lucas. In our case, money is held to provide future liquidity services, it is a store of value and part of wealth, and it is priced by a capital-asset-pricing equation in complete symmetry with other assets, once its direct return - its liquidity services - have been specified. In Lucas' case, money is not held by the consumer between periods, it is not a store of value, and it is not part of wealth. It is acquired and disposed of by the consumer within the period, only. Its real price is given directly by the quantity equation (A.7). There is no marginal utility of money and a capital-asset-pricing equation can of course not be derived. Hence, expectations of future states have no effect on the current value of money. (This is not to say that expectations of future monetary policy have no real current effects; they do influence the price of claims to output.)

The nominal and real rates of interest in Lucas' model are

$$(A.10) \quad i = u_c(y)y / [\beta E(u_c(y')y' / \omega')] - 1 \quad \text{and}$$

$$(A.11) \quad \rho = u_c(y) / [\beta E(u_c(y'))] - 1.$$

The real interest rate in Lucas' model is equal to that of the barter model, since marginal utility of wealth equals marginal utility of consumption. Equation (A.10) is derived by using the equality between marginal utility of wealth and marginal utility of consumption, as well as the quantity equation (A.7), and the nominal interest rate in Lucas' model will be different from that of our model.

We should note here that throughout the paper we define the real interest rate via the real present value of a sure unit of real wealth next period, that is, as in (2.14). Lucas (1982, p. 346) defines the real interest rate as (in our notation, and for one good rather than two)

$$(A.12) \quad 1/(1 + \tilde{\rho}) = \beta E[u_c(y')y'] / u_c(y)y,$$

(With two goods, the outputs of which are  $x$  and  $y$ , the definition is precisely  $1/(1 + \tilde{\rho}) = \beta E[u_c(x', y')x' + u_y(x', y')y'] / [u_x(x, y)x + u_y(x, y)]$ .) Clearly, this definition of the real interest rate is different from (A.11) except in the deterministic case when  $y'$  is constant. Lucas' real interest rate equals the nominal interest rate (A.10) when money supply is deterministic and constant, that is  $\omega' = 1$ , and this circumstance is used as a rationale for the definition (A.12).

In our model, the Lucas' definition would be

$$(A.13) \quad 1/(1 + \tilde{\rho}) = \beta E[\lambda'y'] / \lambda y.$$

Comparing with (2.12) we see that even if money supply is deterministic and constant ( $\omega' = 1$ ), the nominal rate of interest would not equal the Lucas' definition, unless the quantity equation (A.7) holds. But in our model the quantity equation need not hold even if money supply is deterministic, as long as output varies.

Hence, for our purposes the definition according to (2.14) appears more convenient. Indeed, with the definition (2.14), the nominal and real rates of interest coincide if the price level,  $P' = 1/\pi'$ , is deterministic and constant, that is, if the rate of inflation is constant and equal to zero. A zero rate of inflation does not follow from zero monetary expansion, since the value of money is affected by output as well. Zero inflation, rather than zero monetary growth, seems more intuitive as the situation under which nominal interest rates and an appropriately defined real interest rate should coincide. (In a many-good model, the appropriately defined real interest rate would be defined from a consumer price index such that nominal and real interest rates coincide when the CPI is deterministic and constant. This requires intertemporally weakly homothetically separable preferences, as discussed in, for instance, Svensson and Razin (1983).)

Appendix 2. The Elasticities of  $A(\bar{y}, \bar{\omega})$ ,  $(1 + i(\bar{y}, \bar{\omega}))$ ,  $(1 + \rho(\bar{y}, \bar{\omega}))$  and  $E[P'/P|\bar{y}, \bar{\omega}]$ .

In computing the elasticities of  $A(\bar{y}, \bar{\omega})$ , we shall use the following general result: Let the distribution function  $F(x'|x)$  fulfill

$$(A.14) \quad F(x'|x) = G(x'/x^a), \quad 0 < a < 1,$$

that is,  $x'$  fulfills



$$(A.15) \quad x' = x^a \xi'$$

where  $\xi'$  is distributed according to  $G(\xi')$ . For a given function  $f(x)$ , let the function  $g(x)$  be defined by

$$(A.16) \quad g(x) = \int f(x') dF(x' | x)$$

for each  $x$ . Then the elasticity of the function  $g$  is given by

$$(A.16) \quad \varepsilon g(x) / \varepsilon x = a \int [f(x') / g(x)] (\varepsilon f(x') / \varepsilon x) dF(x' | x).$$

That is, if  $g(x)$  is the mean of  $f(x')$  conditional upon  $x$ , then the elasticity of  $g(x)$  is a weighted mean of the elasticity of  $f(x')$ , the weights being  $f(x')/g(x)$ .

To show this, we first rewrite (A.16) as

$$(A.17) \quad g(x) = \int f(x^a \xi') dG(\xi')$$

by substituting  $\xi' = x' / x^a$ . Then the derivative of  $g(x)$  is

$$(A.18) \quad g_x(x) = \int f_x(x^a \xi') a x^{a-1} \xi' dG(\xi') = (a/x) \int x' f_x(x') dF(x' | x)$$

and (A.16) follows.

The function  $A(\bar{y}, \bar{\omega})$  is defined by

$$(A.19) \quad A(\bar{y}, \bar{\omega}) = \int u_c(y') m(s') dF(s' | \bar{y}, \bar{\omega}).$$

Hence, by (A.16) the elasticities of  $A(\bar{y}, \bar{\omega})$  are the weighted means of the elasticities of the integrand. In the region  $m' > y'$  real balances are given by (3.22a), hence the elasticities are

$$(A.20a) \quad \varepsilon m / \varepsilon \bar{y} = \varepsilon A / \varepsilon \bar{y} + r \quad \text{and} \quad \varepsilon m / \varepsilon \bar{\omega} = \varepsilon A / \varepsilon \bar{\omega} - 1.$$

In the region  $m' = y'$ , the elasticities are

$$(A.20b) \quad \varepsilon m / \varepsilon \bar{y} = 1 \quad \text{and} \quad \varepsilon m / \varepsilon \bar{\omega} = 0.$$

Hence, the elasticity  $\epsilon A/\epsilon \bar{y}$  is given by

$$(a.21) \quad \begin{aligned} \epsilon A(\bar{y}, \bar{\omega})/\epsilon \bar{y} = & a[\int u_c(y')m(s')(\epsilon A(\bar{y}, \bar{\omega})/\epsilon \bar{y})dF(s'|\bar{y}, \bar{\omega}, m' > y') + \\ & + \int u_c(y')m(s')(1-r)dF(s'|\bar{y}, \bar{\omega}, m' = y')] / A(\bar{y}, \bar{\omega}). \end{aligned}$$

This is a functional equation, the solution to which gives the function  $\epsilon A(\bar{y}, \bar{\omega})/\epsilon \bar{y}$ . Define the mapping  $T(f)$  that maps a function  $f$  into a function  $T(f)$  by

$$(A.22) \quad \begin{aligned} T(f)(\bar{y}, \bar{\omega}) = & a[\int u_c m' f(\bar{y}', \bar{\omega}')dF(s'|\bar{y}, \bar{\omega}, m' > y') + \\ & + (1-r)\int u_c m' dF(s'|\bar{y}, \bar{\omega}, m' = y')] / A(\bar{y}, \bar{\omega}). \end{aligned}$$

This mapping is a contraction mapping (see Luenberger (1969)).

(With sup norm it is easy to show that  $\|T(f^1) - T(f^2)\| \leq \alpha \|f^1 - f^2\|$  for  $0 \leq \alpha < 1$  and any two functions  $f^1$  and  $f^2$ ). Hence, by the Contraction Mapping Theorem, if it maps functions of a closed subset of a Banach space into the same subset, it has a unique fixpoint (that is, a function  $f^0$  that fulfills  $f^0 = T(f^0)$ ) in that subset. That fixpoint is in our case the elasticity  $\epsilon A/\epsilon \bar{\omega}$ . Consider now the contraction mapping  $T(f)/(1-r)$ , that is the right-hand side of (A.22) divided by  $(1-r)$ . It is straight forward to show that it maps positive functions fulfilling  $f(\bar{y}, \bar{\omega}) \leq 1$ , all  $\bar{y}, \bar{\omega}$ , into the set of positive functions fulfilling the same condition with strict inequality. Hence, we can conclude that it has a fixpoint  $\alpha(\bar{y}, \bar{\omega})$  fulfilling  $0 \leq \alpha(\bar{y}, \bar{\omega}) < 1$ . But then  $T(f)$  has a fixpoint equal to  $(1-r)\alpha(\bar{y}, \bar{\omega})$ . Hence,

$$(A.23) \quad \epsilon A(\bar{y}, \bar{\omega})/\epsilon \bar{\omega} = (1-r)\alpha(\bar{y}, \bar{\omega}), \quad 0 \leq \alpha(\bar{y}, \bar{\omega}) < 1.$$

The elasticity  $\epsilon A/\epsilon \bar{\omega}$  is given by

$$(A.24) \quad \begin{aligned} \epsilon A(\bar{y}, \bar{\omega})/\epsilon \bar{\omega} = & b[\int u_c(y')m(s')(\epsilon A(\bar{y}', \bar{\omega}')/\epsilon \bar{\omega})dF(s'|\bar{y}, \bar{\omega}, m' > y') \\ & - \int u_c(y')m(s')dF(s'|\bar{y}, \bar{\omega}, m' > y')] / A(\bar{y}, \bar{\omega}). \end{aligned}$$

The right-hand side of (A.24) also defines a contraction mapping, and it is straightforward to show that it maps negative functions into the space of negative functions. Hence we conclude that

$$(A.25) \quad \epsilon A(\bar{y}, \bar{\omega}) / \epsilon \bar{\omega} \leq 0.$$

The nominal interest rate fulfills

$$(A.26a) \quad 1 + i(\bar{y}, \bar{\omega}) = A(\bar{y}, \bar{\omega}) / \beta C(\bar{y}, \bar{\omega}), \text{ where}$$

$$(A.26b) \quad C(\bar{y}, \bar{\omega}) = \int \lambda(s') m(s') dF(s' | \bar{y}, \bar{\omega}) = \int (A(\bar{y}', \bar{\omega}') / \bar{\omega}' \theta') dF(s' | \bar{y}, \bar{\omega}).$$

Hence the elasticity  $\epsilon C / \epsilon \bar{y}$  is

$$(A.27a) \quad \begin{aligned} \epsilon C / \epsilon \bar{y} &= a \int (A' / \omega') (\epsilon A' / \epsilon \bar{y}) dF(s' | \bar{y}, \bar{\omega}) / C(\bar{y}, \bar{\omega}) = \\ &= \psi(\bar{y}, \bar{\omega}) (1 - r), \text{ where} \end{aligned}$$

$$(A.27b) \quad 0 \leq \psi(\bar{y}, \bar{\omega}) < 1,$$

and where we have used (A.23). Hence,

$$(A.28) \quad \epsilon(1 + i) / \epsilon \bar{y} = (\alpha(\bar{y}, \bar{\omega}) - \psi(\bar{y}, \bar{\omega})) (1 - r) \geq 0.$$

and it seems that (A.28) can be of any sign. We cannot say whether  $\psi(\bar{y}, \bar{\omega})$  is smaller or larger than  $\alpha(\bar{y}, \bar{\omega})$ . However, we may rewrite

(A.27a) as

$$(A.29a) \quad \begin{aligned} \epsilon C / \epsilon \bar{y} &= a E[(A' / \omega') (\epsilon A' / \epsilon \bar{y}) | \bar{y}, \bar{\omega}] / C(\bar{y}, \bar{\omega}) = \\ &= a \{E[\epsilon A' / \epsilon \bar{y} | \bar{y}, \bar{\omega}] + \text{Cov}[A' / \omega', \epsilon A' / \epsilon \bar{y} | \bar{y}, \bar{\omega}] / C(\bar{y}, \bar{\omega})\} \\ &= a(1-r) \{E[\alpha' | \bar{y}, \bar{\omega}] + \text{Cov}[A' / \omega', \alpha' | \bar{y}, \bar{\omega}] / C(\bar{y}, \bar{\omega})\}. \end{aligned}$$

Hence, if it would be the case that

$$(A.29b) \quad \text{Cov}[A' / \omega', \alpha' | \bar{y}, \bar{\omega}] \leq 0,$$

we would have

$$(A.29c) \quad (\varepsilon C / \varepsilon \bar{y}) / (1-r) \leq a E[\alpha' | \bar{y}, \bar{\omega}] .$$

Then it would follow that the unconditional mean of (A.28) would obey

$$(A.30) \quad (1-a) E[\alpha(\bar{y}, \bar{\omega})] \leq E[\varepsilon(1+i) / \varepsilon \bar{y}] / (1-r) \leq E[\alpha(\bar{y}, \bar{\omega})] .$$

If  $\alpha(\bar{y}, \bar{\omega})$  is constant, (A.29b) and hence (A.30) holds. In general, it seems that (A.29b) may or may not hold.

The elasticity  $\varepsilon C / \varepsilon \bar{\omega}$  is

$$(A.31) \quad \varepsilon C / \varepsilon \bar{\omega} = b \int (A' / \omega') (\varepsilon A' / \varepsilon \bar{\omega} - 1) dF(s' | \bar{y}, \bar{\omega}) / C(\bar{y}, \bar{\omega}) .$$

Comparing with (A.24) and noting that

$$(A.32) \quad \begin{aligned} u_c' m' &= \beta A' / \omega' \quad \text{for } m' > y', \quad \text{and} \\ C &= \int \lambda' m' dF \leq A = \int u_c' m' dF, \quad \text{since } \lambda' \leq u_c', \end{aligned}$$

we can conclude

$$(A.33) \quad \varepsilon C / \varepsilon \bar{\omega} \leq \varepsilon A / \varepsilon \bar{\omega} < 0 .$$

Hence,

$$(A.34) \quad \varepsilon(1+i) / \varepsilon \bar{\omega} = \varepsilon A / \varepsilon \bar{\omega} - \varepsilon C / \varepsilon \bar{\omega} \geq 0 .$$

The real interest rate is

$$(A.35a) \quad 1 + \rho(\bar{y}, \bar{\omega}) = \lambda(s) / \beta D(\bar{y}, \bar{\omega}) \quad \text{where}$$

$$(A.35b) \quad D(\bar{y}, \bar{\omega}) = \int \lambda(s') dF(s' | \bar{y}, \bar{\omega}) .$$

Hence the elasticities of  $D(\bar{y}, \bar{\omega})$  are

$$(A.36a) \quad \varepsilon D / \varepsilon \bar{y} = a \int \lambda' (\varepsilon \lambda' / \varepsilon \bar{y}) dF / D(\bar{y}, \bar{\omega}) \quad \text{and}$$

$$(A.36b) \quad \epsilon D/\epsilon \bar{\omega} = b \int \lambda' (\epsilon \lambda' / \epsilon \bar{\omega}) dF/D(\bar{y}, \bar{\omega}).$$

For  $m' > y'$  we have

$$(A.37a) \quad \epsilon \lambda / \epsilon \bar{y} = -r \quad \text{and} \quad \epsilon \lambda / \epsilon \bar{\omega} = 0,$$

and for  $m' = y'$  we have

$$(A.37b) \quad \epsilon \lambda / \epsilon \bar{y} = \epsilon A / \epsilon \bar{y} - 1 \quad \text{and} \quad \epsilon \lambda / \epsilon \bar{\omega} = \epsilon A / \epsilon \bar{\omega} - 1.$$

Hence,

$$(A.38) \quad \begin{aligned} \epsilon D / \epsilon \bar{y} &= a [\int \lambda' (-r) dF(s' | \bar{y}, \bar{\omega}, m' > y') + \\ &+ \int \lambda' (\epsilon A' / \epsilon \bar{y} - 1) dF(s' | \bar{y}, \bar{\omega}, m' = y')] / D(\bar{y}, \bar{\omega}) < 0, \text{ and} \\ \epsilon D / \epsilon \bar{\omega} &= a \int \lambda' (\epsilon A' / \epsilon \bar{\omega} - 1) dF(s' | \bar{y}, \bar{\omega}, m' = y') / D(\bar{y}, \bar{\omega}) < 0. \end{aligned}$$

It seems that we cannot say anything definite about  $\epsilon(1+\rho)/\epsilon \bar{y} = \epsilon \lambda / \epsilon \bar{y} - \epsilon D / \epsilon \bar{y}$ . However, we can rewrite (A.36a) as

$$(A.39a) \quad \epsilon D / \epsilon \bar{y} = a \{ E[\epsilon \lambda' / \epsilon \bar{y} | \bar{y}, \bar{\omega}] + \text{Cov}[\lambda', \epsilon \lambda' / \epsilon \bar{y} | \bar{y}, \bar{\omega}] / D(\bar{y}, \bar{\omega}) \}.$$

Hence, if

$$(A.39b) \quad \text{Cov}[\lambda', \epsilon \lambda' / \epsilon \bar{y} | \bar{y}, \bar{\omega}] / E[\epsilon \lambda' / \epsilon \bar{y} | \bar{y}, \bar{\omega}] \leq 0,$$

we can conclude that the unconditional mean fulfills

$$(A.39c) \quad 1-a \leq E[\epsilon(1+\rho)/\epsilon \bar{y}] / E[\epsilon \lambda' / \epsilon \bar{y} | \bar{y}, \bar{\omega}] \leq 1.$$

With regard to  $\epsilon(1+\rho)/\epsilon \bar{\omega}$  we have

$$(A.40) \quad \epsilon(1+\rho)/\epsilon \bar{\omega} = \begin{cases} -\epsilon D / \epsilon \bar{\omega} > 0 & \text{for } m' > y', \text{ and} \\ \epsilon A / \epsilon \bar{\omega} - 1 - \epsilon D / \epsilon \bar{\omega} \geq 0 & \text{for } m' = y'. \end{cases}$$

By rewriting (A.36b) we have (A.41a)

$$(A.41a) \quad \varepsilon D/\varepsilon \bar{\omega} = b\{E[\varepsilon \lambda'/\varepsilon \bar{\omega} | \bar{y}, \bar{\omega}] + \text{Cov}[\lambda', \varepsilon \lambda'/\varepsilon \bar{\omega} | \bar{y}, \bar{\omega}]/D(\bar{y}, \bar{\omega})\}.$$

If

$$(A.41b) \quad \text{Cov}[\lambda', \varepsilon \lambda'/\varepsilon \bar{\omega}] \geq 0, \text{ we have}$$

$$(A.41c) \quad |\varepsilon D/\varepsilon \bar{\omega}| \leq b |E[\varepsilon \lambda'/\varepsilon \bar{\omega} | \bar{y}, \bar{\omega}]|,$$

and it follows that the unconditional mean fulfills

$$(A.42) \quad E[\varepsilon \lambda'/\varepsilon \bar{\omega}] \leq E[\varepsilon(1+\rho)/\varepsilon \bar{\omega}] \leq (1-b)E[\varepsilon \lambda'/\varepsilon \bar{\omega}] \leq 0.$$

With respect to expected inflation we have

$$(A.43) \quad \begin{aligned} \Pi(\bar{y}, \bar{\omega}) &= E[\mathcal{P}'/P | \bar{y}, \bar{\omega}] = \omega m(s) H(\bar{y}, \bar{\omega}) \text{ where} \\ H(\bar{y}, \bar{\omega}) &= \int (1/m(s')) dF(s' | \bar{y}, \bar{\omega}). \end{aligned}$$

For  $m' > y'$ ,  $\omega m = \beta A/u_c$  and

$$(A.44a) \quad \varepsilon(\omega m)/\varepsilon \bar{y} = \varepsilon A/\varepsilon \bar{y} + r = \alpha + r(1-\alpha) > 0, \text{ and}$$

$$\varepsilon(\omega m)/\varepsilon \bar{\omega} = \varepsilon A/\varepsilon \bar{\omega} < 0;$$

for  $m' = y'$ ,  $\omega m = \omega y$ , and

$$(A.44b) \quad \varepsilon(\omega m)/\varepsilon \bar{y} = \varepsilon(\omega m)/\varepsilon \bar{\omega} = 1.$$

With regard to  $H(\bar{y}, \bar{\omega})$ , by (A.20) we have

$$(A.45a) \quad \begin{aligned} \varepsilon H/\varepsilon \bar{y} &= a \int (1/m') (-\varepsilon m'/\varepsilon \bar{y}) dF/H = \\ &= -a \left[ \int (1/m') [\alpha' + r(1-\alpha')] dF(s' | \bar{y}, \bar{\omega}, m' > y') \right. \\ &\quad \left. - \int (1/m') dF(s' | \bar{y}, \bar{\omega}, m' = y') \right] / H < 0, \text{ and} \end{aligned}$$

$$(A.45b) \quad \begin{aligned} \epsilon_H/\epsilon_{\bar{\omega}} &= b \int (1/m') (-\epsilon_{m'}/\epsilon_{\bar{\omega}}) dF/H = \\ &- b \int (1/m') (\epsilon_{A'}/\epsilon_{\bar{\omega}} - 1) dF(s'|\bar{y}, \bar{\omega}, m' > y')/H \geq 0. \end{aligned}$$

It seems that the elasticities of expected inflation with respect to  $\bar{y}$  and  $\bar{\omega}$ ,

$$(A.46a) \quad \epsilon_{\Pi}/\epsilon_{\bar{y}} = \epsilon_{m'}/\epsilon_{\bar{y}} + \epsilon_H/\epsilon_{\bar{y}} \quad \text{and}$$

$$(A.46b) \quad \epsilon_{\Pi}/\epsilon_{\bar{\omega}} = 1 + \epsilon_{m'}/\epsilon_{\bar{\omega}} + \epsilon_H/\epsilon_{\bar{\omega}},$$

can be of either sign.

Rewriting (A.45) we get

$$(A.47a) \quad \epsilon_H/\epsilon_{\bar{y}} = -a \{E[\epsilon_{m'}/\epsilon_{\bar{y}}|\bar{y}, \bar{\omega}] + \text{Cov}[1/m', \epsilon_{m'}/\epsilon_{\bar{y}}|\bar{y}, \bar{\omega}]/H(\bar{y}, \bar{\omega})\}.$$

and

$$(A.47b) \quad \epsilon_H/\epsilon_{\bar{\omega}} = -b \{E[\epsilon_{m'}/\epsilon_{\bar{\omega}}|\bar{y}, \bar{\omega}] + \text{Cov}[1/m', \epsilon_{m'}/\epsilon_{\bar{\omega}}|\bar{y}, \bar{\omega}]/H(\bar{y}, \bar{\omega})\}.$$

If

$$(A.48a) \quad \text{Cov}[1/m', \epsilon_{m'}/\epsilon_{\bar{y}}|\bar{y}, \bar{\omega}] \leq 0$$

the unconditional mean fulfills

$$(A.48b) \quad 0 \leq (1-a)E[\epsilon_{m'}/\epsilon_{\bar{y}}] \leq E[\epsilon_{\Pi}/\epsilon_{\bar{y}}] \leq E[\epsilon_{m'}/\epsilon_{\bar{y}}]$$

and if

$$(A.49a) \quad \text{Cov}[1/m', \epsilon_{m'}/\epsilon_{\bar{\omega}}|\bar{y}, \bar{\omega}] \leq 0$$

we will have

$$(A.49b) \quad 1 + (1-b)E[\epsilon_{m'}/\epsilon_{\bar{\omega}}] \leq E[\epsilon_{\Pi}/\epsilon_{\bar{\omega}}] \leq 1 + E[\epsilon_{m'}/\epsilon_{\bar{\omega}}].$$

If it can be established that

$$(A.49c) \quad E[\epsilon_{m'}/\epsilon_{\bar{\omega}}] > -1/(1-b),$$

it follows that the unconditional mean of the elasticity  $\epsilon_{\Pi}/\epsilon_{\bar{\omega}}$  is positive.

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