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Money and Monetary Policy in Dynamic Stochastic General Equilibrium Models*

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ABSTRACT

We compare two methods of motivating money in New Keynesian DSGE Models: Money-in-the-utility function and cash-in-advance constraint, as well as two ways of modelling monetary policy: interest rate feedback rule and money growth rules. As an aid to model selection, we use a new econometric measure of the distance between model and data variance-covariance matrices. The proposed measure is useful in distinguishing between alternative general equilibrium models. We find that the models closed by an estimated interest rate feedback rule imply counter-cyclical policy and inflation rates, which is at odds with the data. This problem is not a feature of models closed by an estimated money growth rule. Drawing on our econometric analysis, we argue that the cash-in-advance model, closed by a money growth rule, comes closest to the data.

JEL Classification: C13, E32, E52.
Keywords: Intertemporal macroeconomics; role of money; monetary policy; model selection; moment matching.

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1 Introduction

How should one model money and monetary policy in quantitative analyses of the business cycle? Within the broad paradigm of the New Neoclassical Synthesis (NNS) models (characterised by monopolistic competition, price and wage stickiness and sometimes by endogenous capital accumulation), there are conflicting approaches. Following Woodford (2003), many economists, such as Canzoneri *et al.* (2004), do not model money explicitly, or introduce real money balances into an additively separable utility function, and instead incorporate an interest rate feedback rule with coefficients estimated using actual data. Others, *e.g.*, Yun (1996) and Danthine and Kurmann (2004), include estimated money growth processes into their analyses and model the demand for money explicitly (in their case as a cash-in-advance constraint). Interestingly, the literature is not very explicit regarding the differences between or benefits of modelling monetary policy through an interest rate feedback rule as opposed to a money growth rule, or indeed different ways of modelling the role of money in these models.

In this paper, we argue that the way we model the demand for money and monetary policy matters for the model's ability to match salient features of the business cycle. Based on US data, we examine two popular ways to model the role of money: (i) By postulating that real money balances enter the representative agent's utility function directly, and (ii) by postulating that the representative agent requires previously accumulated real money balances to purchase goods. Similarly, we analyse the differences between modelling monetary policy through an interest rate feedback rule compared to a money growth rule.

Comparing across variants of our models in this way raises some important issues. How does one carry out such a comparison? Often, researchers 'eyeball' observable characteristics of simulated data, such as impulse response functions and unconditional second moments. The problem with this type of heuristic approach is that it may well allow the researcher to claim that model 'A' is better in terms of explaining the data along a certain dimension than model 'B', but along an other, equally important, dimension model 'B' might just be better than model 'A', making a meaningful ranking of different variants of the model difficult.

We address this problem by introducing a new distance measure that allows us to formalise the comparison of second order moments of the data with those for various competing models. While the applicability of the proposed econometric methodology goes beyond the issues addressed in this paper, the models presented here, as well as the questions we seek to address, serve to illustrate our new distance measure between competing models and the data.

There has been substantial research on formal empirical validation of dynamic macroeconomic models (see Canova and Ortega (2000) for a review). Based on the chosen application as well as the nature of sampling variability, the literature suggests the use of multiple alternative metrics for model validation. However, most of the available methods have serious limitations – they often assume an arbitrary error structure, are computation intensive and are usually based on tests for model fit rather than on methodology appropriate for model selection[1]. We address each of these issues by proposing a new measure of distance between moments of selected state variables in the data and under the model. In other words, we develop methodologies for model selection for comparing different estimated or calibrated DSGE models. The idea is to compare moments from

[1] See Fuhrer (1997) and Canova and Ortega (2000) for further discussion.

simulated data on the competing models with actual macroeconomic data, in our case data for the United States.

We find that so long as we model monetary policy with an estimated Taylor-type interest rate feedback rule, the NNS model generates, in contrast to the data, counter-cyclical inflation and policy rates, regardless of how we model money. This is a substantial drawback to this way of modelling money as it is often motivated by appeals to 'realism'; that is, the interest rate is the instrument of monetary policy and hence modelling monetary policy in this way is often argued to be a natural choice. Under an estimated money growth rule, however, both inflation and policy rates become pro-cyclical. The magnitude of the correlation is however only correct when the cash-in-advance constraint is combined with a money growth rule; as a result, this model comes closest to the data.

The remainder of this paper is structured as follows: Section 2 sets out our basic model, the different approaches to modelling the role of money and monetary policy and our proposed calibration of structural parameters and shock processes. Section 3 introduces our econometric methodology. Section 4 compares a set of key second moments generated by the models to those observed in United States data. Section 5 analyses impulse responses to monetary and real shocks. In section 6, we use our econometric technique for model selection by ranking the different models according to their ability to match data. In section 7 we discuss the economic intuition behind the ranking of our models. Section 8 concludes.

2 The basic modelling framework

Our emphasis is on formally comparing, as opposed to modelling, different ways of introducing money and modelling monetary policy. Therefore, we take as our starting point a canonical new Keynesian DSGE model with monopolistic competition and nominal rigidities in the goods and labour markets, along the lines of Erceg *et al.* (2000), but where firms produce differentiated goods using labour as well as capital services. For recent examples of this kind of model in the literature, see *inter alia* Danthine and Kurmann (2004), Canzoneri *et al.* (2004) and Nolan and Thoenissen (2005).

We start this section by discussing those aspects of the model that are not affected by the way we model the role of money and that are therefore common to all versions of the basic model analysed in this paper.

2.1 Representative firm: factor demands

Firms are monopolistic competitors who produce their distinctive goods according to the following constant returns technology:

$$Y_t(i) = F(A_t, K_t, N_t(i)) \equiv A_t {K_t}^{s_K} N_t(i)^{1-s_K}. \quad (1)$$

A_t is the Solow residual, K_t is the capital stock which is predetermined in period t, and $s_K < 1$. Firms contract labour and capital in economy-wide competitive markets. The optimal demand for capital and labour is given by (2) and (3) respectively

$$\rho_t = mc_t \partial F_t(i) / \partial K_t \tag{2}$$

$$w_t = mc_t \partial F_t(i) / \partial N_t(i). \tag{3}$$

ρ_t denotes the economy-wide rental rate for capital and mc_t denotes real marginal cost. Capital accumulation is described by

$$K_{t+1} = (1-\delta)K_t + \phi\left(\frac{x_t(i)}{K_t}\right)K_t. \tag{4}$$

The initial capital stock, K_{-1}, is given and assumed equal across all firms. $\phi(\cdot)$ is strictly concave. In addition to (4) optimal capital accumulation is described by (5) and (6),

$$\mu_t = \lambda_t \phi'\left(\frac{x_t(i)}{K_t}\right) \tag{5}$$

$$\beta E_t \mu_{t+1}\rho_{t+1} + \beta E_t \lambda_{t+1}\left[(1-\delta) - \phi'\left(\frac{x_{t+1}(i)}{K_{t+1}}\right)\frac{x_{t+1}(i)}{K_{t+1}} + \phi\left(\frac{x_{t+1}(i)}{K_{t+1}}\right)\right] = \lambda_t. \tag{6}$$

Equation (5) recognises the utility foregone, μ_t measures the marginal utility of consumption, from investment at date t, taking into account the adjustment costs noted above. Equation (6) captures the dynamic properties of this trade-off; a higher capital stock next period, *ceteris paribus*, enables higher consumption next period, taking into account depreciation between this period and next, and the discounted impact of next period adjustment costs. At the economy-wide level, we have the following constraint which states that all output is either consumed or invested:

$$Y_t = C_t + x_t \tag{7}$$

2.2 Representative firm: price setting

In all the variants of the New Keynesian models that we analyse, prices are sticky in a time dependent manner. The firm will reprice in accordance with the framework suggested by Calvo (1983). That is, if the firm reprices in period t it faces the probability α^k of having to charge the same price in period $t+k$. The criterion facing a firm presented with the opportunity to reprice is given by

$$\max \sum_{k=0}^{\infty} (\alpha\beta)^k E_t \left\{ \frac{\mu_{t+k}}{\mu_t} \left[\frac{p_t(i)}{P_{t+k}} \left(\frac{p_t(i)}{P_{t+k}}\right)^{-\theta} Y_{t+k}^d - mc_{t+k}\left(\frac{p_t(i)}{P_{t+k}}\right)^{-\theta} Y_{t+k}^d \right] \right\}, \tag{8}$$

where the terms in marginal utility ensure that the price set is what would have been chosen by any individual in the economy had they been in charge of price-setting. The optimal price is given by

$$p_t'(i) = \frac{\theta \sum_{k=0}^{\infty} (\alpha\beta)^k E_t \left\{ \mu_{t+k} mc_{t+k} P_{t+k}^{\theta} Y_{t+k}^d \right\}}{\theta - 1 \sum_{k=0}^{\infty} (\alpha\beta)^k E_t \left\{ \mu_{t+k} P_{t+k}^{\theta-1} Y_{t+k}^d \right\}}. \tag{9}$$

In the presence of economy-wide factor markets any producer given the chance to reprice will chose this value. As a result the price-level in our models evolves in the following way:

$$P_t = \left[(1-\alpha) p_t'^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{10}$$

2.3 Labour markets

We follow the work of Erceg *et al.* (2000) by assuming that labour is supplied by 'household unions' acting non-competitively. Household unions combine individual households' labour supply according to:

$$N_t = \left[\int_0^1 N_t(i)^{\frac{\theta_w - 1}{\theta_w}} di \right]^{\frac{\theta_w}{\theta_w - 1}}.$$

If we denote by W the price index for labour inputs and by $W(i)$ the nominal wage of worker i, then total labour demand for household i's labour is:

$$N_t(i) = \left[\frac{W_t(i)}{W_t} \right]^{-\theta w} N_t.$$

The household union takes into account the labour demand curve when setting wages. Given the monopolistically competitive structure of the labour market, if household unions have the chance to set wages every period, they will set it as a mark-up over the marginal rate of substitution of leisure for consumption. In addition to this monopolistic distortion, we also allow for the partial adjustment of wages using the same Calvo-type contract model as for price setters. This yields the following maximization problem:

$$\max \sum_{k=0}^{\infty} (\alpha^w \beta)^k E_t \left\{ \frac{\mu_{t+k}}{\mu_t} \left[\frac{W_t(i)}{P_{t+k}} \left[\frac{W_t(i)}{W_{t+k}} \right]^{-\theta w} N_{t+k} - mrs_{t+k} \left[\frac{W_t(i)}{W_{t+k}} \right]^{-\theta w} N_{t+k} \right] \right\} \tag{11}$$

where mrs is the marginal rate of substitution of leisure for consumption.

2.4 Representative agent: Money in the utility function

The first way we introduce money is to assume that there are a large number of agents in the economy who evaluate their utility in accordance with the following utility function:

$$E_t \left\{ U(C_t, \frac{M_t}{P_t}, N_t) \right\} \equiv E_t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} (m_t)^{1-b} - \Psi \frac{N_t^{1+\eta}}{1+\eta} \right\} \tag{12}$$

E_t denotes the expectations operator at time t, β is the discount factor, C is consumption, M is the nominal money stock, P is the price-level, m is the stock of real money balances, and N is labour supply. Consumption is defined over a basket of goods of measure one and indexed by i in the manner of Spence-Dixit-Stiglitz

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \tag{13}$$

where the optimal price level is

$$P_t = \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{14}$$

The demand for each good is given by

$$c_t^d(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t^d, \tag{15}$$

where Y_t^d denotes aggregate demand. Agents face a time constraint each period (normalised to unity) such that leisure, L_t, is given by

$$L_t = 1 - N_t. \tag{16}$$

Agents also face the following flow budget constraint:

$$C_t + E_t\{Q_{t,t+1} d_{t+1} \frac{P_{t+1}}{P_t}\} + m_t = d_t + m_{t-1} \frac{P_{t-1}}{P_t} + w_t N_t + \Pi_t + \tau_t \tag{17}$$

Here d_{t+1} denotes the real value at date $t+1$ of the asset portfolio held at the end of period t. $Q_{t,T}$ is the stochastic discount factor between period t and T, and

$$\frac{1}{1+i_t} = E_t \{Q_{t,t+1}\} \tag{18}$$

denotes the nominal interest rate on a risk-less one-period bond. w_t denotes the real wage in period t, and Π_t is the real value of income from the corporate sector remitted to the individual (e.g., think of rental income from the capital stock along with a proportionate share in any final profits). In addition to the standard boundary conditions, the necessary conditions for an optimum include:

$$U_{C_t}(.) = \mu_t \tag{19}$$

$$\mu_t = (1+i_t)\beta \mathrm{E} \mu_{t+1} \frac{P_t}{P_{t+1}} \tag{20}$$

$$\frac{U_{M_t}(.)}{U_{C_t}(.)} = \frac{i}{1+i_t} \tag{21}$$

where μ is the Lagrange multiplier associated with both the consumer's and the firm's optimisation problem.

2.5 Representative agent: Cash-in-advance constraint

Next, we introduce money into our model by assuming that agents face a cash-in-advance (CIA) constraint. In this version of our model timing is important. In each period agents must first go to the money market and obtain cash needed for transactions in both the consumption and capital goods markets. When the money market closes the goods markets open. This specification follows Chari and Kehoe (1999).

Agents obtain utility from consumption and disutility from labour:

$$E_t\left\{U(C_t,N_t)\right\} \equiv E_t\left\{\frac{C_t^{1-\sigma}}{1-\sigma}-\Psi\frac{N_t^{1+\eta}}{1+\eta}\right\}, \tag{22}$$

and face the standard budget constraint:

$$C_t+E_t\{Q_{t,t+1}d_{t+1}\frac{P_{t+1}}{P_t}\}+m_t=d_t+m_{t-1}\frac{P_{t-1}}{P_t}+w_tN_t+\Pi_t+\tau_t. \tag{23}$$

In addition, they also face a cash-in-advance constraint on consumption and investment

$$m_t=C_t+x_t, \tag{24}$$

which reflects our timing convention that goods and money market transactions occur sequentially in the same period. The necessary conditions for an optimum are:

$$U_{C_t}(.)=\mu_t+\varphi_t; \tag{25}$$

$$\mu_t=(1+i_t)\beta E_t\mu_{t+1}\frac{P_t}{P_{t+1}}; \tag{26}$$

$$\varphi_t=\mu_t\left(\frac{i_t}{1+i_t}\right); \tag{27}$$

$$m_t=C_t+x_t. \tag{28}$$

Since the cash-in-advance constraint applies to both consumption and investment, the first order conditions of the representative firm are affected in the following way:

$$\mu_t+\varphi_t=\lambda_t\phi'\left(\frac{x_t(i)}{K_t(i)}\right) \tag{29}$$

where φ_t is the Lagrange multiplier on the cash-in-advance-constraint. The remaining equations are unchanged.

2.6 Different ways to model monetary policy

We consider two popular ways of modelling monetary policy. First we consider that monetary policy is modelled in terms of an interest rate feedback rule. The monetary authority sets the nominal interest rate according to current economic conditions. A common example of this specification is the Taylor rule, under which the nominal interest rate reacts to current inflation and the output gap. Specifically, we assume a Taylor rule with an additional interest rate smoothing term. The log-linearised monetary policy reaction function takes the following form:

$$i_t = \phi_i i_{t-1} + \phi_\pi \pi_t + \phi_{\bar{y}}(y_t - \bar{y}_t) + \varepsilon_{i,t}. \tag{30}$$

where $\varepsilon_{i,t}$ is assumed to be an iid shock. This approach is followed by *inter alia*, Canzoneri *et al.* (2004) and Nolan and Thoenissen (2005). An alternative specification, used for example by Danthine and Kurmann (2004), Kollmann (2005) and Wang and Wen (2006) assumes that the monetary authority exogenously sets the growth rate of money, $g_{M,t}$, such that supply of real money balance evolves according to:

$$m_t = (1 + g_{M,t})m_{t-1}\frac{P_{t-1}}{P_t}. \tag{31}$$

The real money growth rate, $g_{M,t}$, follows an AR(1) process. The seigniorage from this activity is redistributed in a lump sum fashion to the consumer yielding real money transfers of:

$$\tau = g_{M,t}m_{t-1}\frac{P_{t-1}}{P_t}. \tag{32}$$

2.7 Solving the models

We solve the models by taking a log-linear approximation around the deterministic steady state. The stochastic system of linear difference equations is then solved using the solution algorithm of King and Watson (1998, 2000).

2.8 Calibration

Our calibration aims to capture the salient features of the US business cycle. Specifically, we assume a discount factor of 0.984, which yields an annualised steady-state rate of interest of 6.5%. We assume that utility in each of our models is defined according to (12) or (22). The intertemporal elasticity of consumption is taken as $\sigma = 1$ and that of the labour supply, $\eta = 1$. In the MIU model, we set b, the inverse of the interest elasticity of money demand to 2.56 in line with Chari *et al.* (2000). We assume an elasticity of substitution between individual varieties, $\theta = 7.67$, which yields a steady state mark-up over unit costs of 15%, a value commonly used in the literature[2]. We follow Erceg *et al.* (2000) in setting the elasticity of substitution between varieties of labour to 4.03, which yields a mark-up over the marginal rate of substitution between consumption and leisure of 33%. We assume the probability that a firm can not change prices in a given period to be $\alpha = 0.5$, which implies that each firm receives a signal to adjust prices on average every 2 quarters. This lies between the commonly used value of $\alpha = 0.67$ (every 3 quarters) as suggested by Canzoneri *et al.* (2004) and the much lower values in Bils and Klenow (2004) who suggest a value of around 0.3 suggesting firms change prices on average every 1.4 quarters. We assume that unions re-optimise wages on average once every 4 quarters. On the production side of the model,

[2] See, for example, Rotemberg and Woodford (1997).

we assume an annualised depreciation rate of the capital stock of 10% and a share of capital in production of 25%. The adjustment cost parameter, ϵ_K, is chosen so as to match to the data the relative volatility of investment to GDP generated by the calibrated models.[3]

Table 1: Parameters of the models

Parameter	Estimate	Parameter	Estimate
β	0.984	θ_w	4.03
θ	7.67	δ	0.025
$\bar{N}$	0.67	ϵ_K	calibrated
$\bar{L}$	0.33	s_K	0.25
σ	1	α	0.5
η	1	α_w	0.75

2.8.1 Driving processes

There are two types of shocks hitting our model economies: (a) shocks to total factor productivity, and (b) 'monetary policy shocks', defined either as shocks to the interest rate rule or shocks to the money growth rate. We wanted to focus on the post Volcker-disinflation era as we think linearised models stand the best chance of matching the data in this relatively stable economic period. However, related studies such as Canzoneri *et al.* (2004) and others, suggest that measured TFP over such a relatively short sample may be subject unduly to cyclical factors, and we agree. Hence, we opt to estimate TFP over a longer sample period, whilst estimating our monetary policy rules (and shocks) over the post-Volcker period. Estimating the monetary policy shock over the whole sample period would have compounded our difficulties as we would have run up against issues such as nominal regime shifts, as documented by Gavin and Kydland (1999).

We measure total factor productivity by the Solow residual, which is estimated using quarterly US data from 1960 $q1$ through 2003 $q4$. We estimate the following relationship:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_{A,t}, \tag{33}$$

where $\ln A$ the log of the linearly detrended Solow residual[4]. The estimated coefficient ρ and the standard error of the equation are shown in Table 2, where we also report the t-statistic and the p value for the Breusch-Godfrey serial correlation LM test (upto 4 lags).

Table 2: Estimated Solow residual

ρ_A	$\sigma_{\varepsilon A}$
0.94343	8.424×10^{-3}
t-stat (39.76)	B-G test p value (0.231)

[3]We define capital adjustment costs as $\phi(x/k)$ where x is investment and k is the capital stock. The capital adjustment cost parameter is defined as: $\frac{\phi''(x/k)(x/k)}{\phi'(x/k)} = \epsilon_K$.

[4]In the appendix we provide details of how we constructed our Solow Residual.

To estimate a monetary policy feedback rule, we choose a shorter sample period from 1981 $q4$ to 2003 $q4$. We estimate the following Taylor-type rule, equation (30) above, using ordinary least squares:

$$i_t = c + \phi_i i_{t-1} + \phi_\pi \pi_t + \phi_{\bar{y}}(y_t - \bar{y}_t) + \varepsilon_{i,t}. \tag{34}$$

An alternative strategy would have been to estimate a richer rule and impose the estimated rule on to the calibrated model. In our theoretical model, we define the output gap $(y_t - \bar{y}_t)$ as the difference between actual output and 'natural' output, where we calculate natural output under the assumption that prices and wages are flexible, are expected to remain flexible and have been flexible in the past. This measure of the output gap does not have a direct empirical counterpart. Instead, we use the Congressional Budget Office measure of potential output, which no doubt involves considerable measurement error, but is in line with the literature, e.g. Canzoneri *et al.* (2004). We interpret departures from the systematic part of the policy rule, $\varepsilon_{i,t}$ as monetary policy shocks. Our results are reported in Table 3. The t-statistics are placed in parentheses and reported below the estimates of the regression coefficients; similarly in Table 4. Our estimates suggest quite strong

Table 3: Estimated Taylor rule coefficients

c	ϕ_i	$\frac{\phi_\pi}{(1-\phi_i)}$	$\frac{\phi_{\bar{y}}}{(1-\phi_i)}$	$\sigma_{\varepsilon i}$
0.000	0.901	1.358	0.199	1.635×10^{-3}
t-stat (0.60)	(33.39)	(3.08)	(1.720)	B-G test p value (0.091)

interest rate smoothing, and importantly the long-run coefficients of interest rate on inflation in excess of unity. Our estimate of the long-run coefficient on the output gap is statistically significant at the 5% level for a one-sided test, but only at the 10% level for the usual two-sided alternative. This may, in part reflect our imprecise measure of the output gap. Our estimated coefficients are similar to those reported in Canzoneri *et al.* (2004), who estimate their Taylor rule over a similar sample period.

We measure the growth rate of the money supply by the log difference of between period t and period $t-1$ per capita US M1 money stock (seasonally adjusted):

$$\ln(1 + g_{M_t}) = [\ln(M1_t) - \ln(pop_t)] - [\ln(M1_{t-1}) - \ln(pop_{t-1})] \tag{35}$$

and estimate the shock to money growth process at quarterly frequency from 1981 $q4$ through 2003 $q4$ as:

$$g_{M_t} = c + \rho_g g_{M_{t-1}} + \varepsilon_{g,t} \tag{36}$$

Table 4: Estimated money growth rule coefficients

c	ρ_g	$\sigma_{\varepsilon M}$
0.003	0.736	9.708×10^{-3}
t-stat (2.09)	(10.17)	B-G test p value (0.555)

Our point estimate of ρ_g, $\sigma_{\varepsilon i}$ and $\sigma_{\varepsilon M}$ are similar to those reported in Danthine and Kurmann (2004) and Yun (1996). As in the above mentioned studies, the aim of the estimation exercise in this section is two-fold:

(i) to obtain estimated values for the AR(1) coefficients of the Solow residual (ρ_A), and similarly for the money growth coefficient (ρ_g) and coefficients of the interest rate feedback rule (ϕ_i, ϕ_π and $\phi_{\bar{y}}$), and

(ii) to obtain estimates of residual variances for our estimated equations ($\sigma^2_{\varepsilon A}$, $\sigma^2_{\varepsilon i}$ and $\sigma^2_{\varepsilon M}$); white noise shocks. These estimated variances drive our model simulations.

For all three equations, we confirm that our estimated coefficients are significantly different from zero and that we are not able to reject the null-hypothesis of no serial correlation (up to 4 lags), using the Breusch-Godfrey serial correlation LM test.

3 Econometric Methodology

As discussed earlier, our main objective is to compare across variants of our models with regard to their ability to replicate observable business cycle characteristics of the data. In this section, we discuss econometric methods that facilitate such comparison and ranking of the various NNS models. This includes a new measure that formalises the common practice of eyeballing unconditional second moments of the data and those generated by simulations of a model. In addition, our proposed approach also allows us to calibrate unknown parameters of the considered models.

As emphasised in Pagan (1994) (see also Canova and Ortega, 2000), fit of calibrated general equilibrium models is typically judged by selecting a metric and comparing outcomes of the (simulated) model relative to a set of 'stylised facts' observable from actual data. Typically, the stylised facts are either moments of selected state variables and their functions (variances, covariances and correlations) or impulse responses (or parameters of a VAR, or spectra/ cross-spectra of variables).

Beginning with Watson (1993), there has been considerable research on formal empirical validation of dynamic macroeconomic models based on the degree of conformity in selected moments of the state variables, or second-order comparison over selected range of frequencies.[5] The literature suggests the use of multiple alternative metrics for model validation, depending primarily on the chosen application and the nature of sampling variation. However, most of the methods suggested in the literature have serious limitations, they often assume an arbitrary error structure, are computation intensive or are based on testing model fit as opposed to selecting an appropriate model.

We propose a frequentist measure of the fit of a model based on moment comparison that addresses each of these issues. First, our approach for dealing with the rank-deficient case (i.e., where there are fewer shocks in the model than in the data) does not require us to include additional noise into the underlying structural model. Second, the method is extremely simple to implement and involves much less computations than many other methods proposed in the literature. Finally, our method specifically allows model selection between parsimonious, and therefore essentially 'false' models. In the following subsections, we first describe our proposed methodology, followed

[5] In addition to Watson (1993), several papers in a special issue of *Journal of Applied Econometrics* (namely, Canova (1994), Nason and Cogley (1994) and Fève and Langot (1994)); Cogley and Nason (1995); Diebold *et al.* (1998); and Ireland (2004) are related to our work. See Canova and Ortega (2000) for a review.

by discussion of distinguishing and attractive features of our approach, and finally some comments on the use and interpretation of our distance measure.

3.1 A frequentist distance measure based on moments

Based on the random data VCM and a model VCM, which is fixed for each combination of parameter values and model, we propose a new measure of the distance between the two. This measure is based on Nagao's (1973) test (N-test) for the equality of the two covariance matrices. However, since most NNS models are driven by only a limited number of shocks and predetermined state variables, the model VCM is usually rank deficient. Therefore, the N-test cannot be directly applied to our situation. We modify Nagao's test for rank-deficient situations, by restricting to a full rank lower dimensional subspace of the state variables.

Ledoit and Wolf (2002) have recently proposed a modification of Nagao's test for the rank-deficient case. They show consistency of their test statistic when the number of variables is larger than the sample size, and derive asymptotic distribution under the assumption that both sample size and the number of variables increases to infinity asymptotically. However, since our DGP is different in that our population VCM is rank-deficient by design and not by inadequate sample size, the test is not applicable to our situation except under a restrictive asymptotic setup.

Instead, we consider an approach that is similar to Watson (1993) and Rotemberg and Woodford (1997), in its consideration that the model VCM would be comparable to the data VCM only when additional sources of variation are added. However, instead of quantifying the amount of error necessary to bring the two into alignment, we proceed in exactly the opposite direction. We project both the full-rank data VCM and the reduced rank model VCM onto the full-rank lower-dimensional subspace spanned by the model VCM. Nagao's N-test can then be implemented on this space to test the equality of the two transformed full-rank covariance matrices.

We begin by solving the models using the solution algorithm of King and Watson (1998, 2000), and estimate moments in the frequency domain. The asymptotic matrix of filtered (Hodrick-Prescott in our case) second moments of the state variables estimated using the King and Watson (1998, 2000) algorithm constitute the model VCM for each of the candidate general equilibrium models. Being driven by a small number of shocks, the models will almost certainly be rank-deficient, in the sense that the model VCMs will have a rank less than the number of state variables.

We denote by $[\Sigma_0]_{m\times m}$ the full-rank data VCM estimated using data for n sample periods; $\rho(\Sigma_0) = p$ (the number of state variables considered), where $\rho(.)$ denotes rank of a square matrix. Let $[\Sigma_{M_1}]_{m\times m}$, $[\Sigma_{M_2}]_{m\times m}$, $[\Sigma_{M_3}]_{m\times m}$, ... denote estimated asymptotic VCMs of competing models $M_1, M_2,\ M_3, \ldots$[6]. Since all our competing models are parsimonious, the model VCMs are rank deficient ($m_j = \rho(\Sigma_{M_j}) \leq \rho(\Sigma_0) = p$). In the following, we consider a single model VCM, Σ_M, having rank $m(< p)$ and describe a measure of the distance between this model VCM and the data VCM, Σ_0.

We explicitly take the view that each of our competing models is possibly "false" – hence, all the simulated model VCMs may be statistically different from actual data, except in very small

[6] In principle, these alternative models include all possible combination of parameter values for each of the general equilibrium models under consideration.

samples. In other words, if we conduct a statistical test for the hypotheses

$$H_0 : \Sigma_0 = \Sigma_M \quad \text{versus} \quad H_1 : \Sigma_0 \neq \Sigma_M, \tag{37}$$

we expect to reject the null hypothesis that the the data VCM is the same as the model VCM, at least for large enough data. Nevertheless, we argue that the p-values associated with the test statistics can be used as a measure of the distance between data and alternative competing models. Below we describe the methods in further detail: first, Nagao's N-test in the full rank case, followed by our extension for the rank-deficient situation, and finally discuss how the test can be used for model selection and calibration.

3.1.1 Σ_M is full-rank

As described above, this situation is not likely to hold for most of our models. However, discussion of this case facilitates treatment of the rank-deficient case discussed later. Here we can use a whole battery of tests developed in the statistics literature over 40 years of active research from the 1930s to the 1970s. One of the most popular of these tests is the one proposed by Nagao (1973), which can be used to compare data and model VCMs by using the Cholesky decomposition of Σ_0, as follows:

$$\begin{aligned} \Sigma_M &= P_M'.P_M, \\ \Sigma_0^{(M)} &= P_M'^{-1}.\Sigma_0.P_M^{-1} \\ I &= P_M'^{-1}.\Sigma_M.P_M^{-1} \end{aligned} \tag{38}$$

where I is the identity matrix, and testing $H_0 : \Sigma_0 = \Sigma_M$ is equivalent to testing $H_0 : \Sigma_0^{(M)} = I$ against the omnibus alternative.

In this full-rank case, Nagao's N-test statistic is given by:

$$\begin{aligned} N &\sim \chi^2 \left(m \left(m+1\right)/2\right) \text{ under } H_0, \text{ where} \\ N &= \frac{n_0}{2}.tr\left(\Sigma_0^{(M)} - I\right)^2, \end{aligned} \tag{39}$$

and $tr(.)$ denotes trace of a square matrix.

The N-test can be directly used for model selection and calibration if the number of state variables included for study is not larger than the number of shocks and predetermined variables included in the model. However, none of the NNS models considered in this paper is full rank. Below, we discuss a simple extension of Nagao's test to the rank-deficient setup.

3.1.2 Σ_M is rank deficient ($m = \rho(\Sigma_M) < \rho(\Sigma_0) = p$)

This situation represents a vast majority of models, and is justified by the fact that most models are parsimonious and intended to be abstractions of reality. The model here is clearly an abstraction driven by only a limited number of shocks and predetermined variables. In fact, this abstraction can also represent reality to a high degree, in the sense that often only a small number of drivers can explain a substantial part of the variation in actual data on a larger number of state variables.

In other words, if we were to compute principal components of the data VCM, we may indeed find that a limited number of leading eigenvalues (and their corresponding eigenvectors) account for most of the variation, the remaining eigenvalues are small in comparison.

Note that the rank of the model VCM is given by the sum of number of shocks in the model and the free predetermined variables. Since many of our models are populated by a small number of shocks, we explicitly consider the situation when the data VCM is full-rank, but the model VCM is not. Therefore, we assume an asymptotic setup where, under H_0, the estimated full-rank data VCM, $\widehat{\Sigma}_{0,n}$, asymptotically converges to the rank-deficient matrix Σ_M.

Under this setup, we consider the testing problem $H_0 : \Sigma_0 = \Sigma_M$ versus $H_1 : \Sigma_0 \neq \Sigma_M$,where $\Sigma_M = m < p$. We have sampling variation in the data but not in the model. In other words, Σ_M is assumed to be known, and in practice will be given by the asymptotic model VCM of the DSGE model for a given set of parameter values.[7]

Following Srivastava (2003), we write the singular value decomposition of Σ_M as:

$$\begin{aligned}\Sigma_M &= \left[\begin{array}{ccc} \Gamma^{/}_{1_{p\times m}} & : & \Gamma^{/}_{2_{p\times(p-m)}} \end{array}\right] . \left[\begin{array}{cc} \Lambda_{m\times m} & 0_{m\times(p-m)} \\ 0_{(p-m)\times m} & 0_{(p-m)\times(p-m)} \end{array}\right] . \left[\begin{array}{c} \Gamma_{1_{m\times p}} \\ .. \\ \Gamma_{2_{(m-p)\times p}} \end{array}\right] \\ &= \Gamma^{/}_1 . \Lambda . \Gamma_1,\end{aligned}$$

where $\Lambda = diag\,(\lambda_1, \ldots, \lambda_m)$ is the diagonal matrix containing the m positive eigenvalues $\lambda_1 \geq \ldots \geq \lambda_m$ of Σ_M, and the rows of Γ_1 contain the corresponding eigenvectors. Hence, under the stated null hypothesis (37), we have:

$$\begin{aligned}\left[\begin{array}{cc} I_{m\times m} & 0 \\ 0 & 0 \end{array}\right] &= \left[\begin{array}{cc} \Lambda^{-1/2}_{m\times m} & 0 \\ 0 & 0 \end{array}\right] . \left[\begin{array}{c} \Gamma_{1_{m\times p}} \\ .. \\ \Gamma_{2_{(m-p)\times p}} \end{array}\right] . \Sigma_M . \\ & \quad \left[\begin{array}{ccc} \Gamma^{/}_{1_{p\times m}} & : & \Gamma^{/}_{2_{p\times(p-m)}} \end{array}\right] . \left[\begin{array}{cc} \Lambda^{-1/2}_{m\times m} & 0 \\ 0 & 0 \end{array}\right] \\ &= \left[\begin{array}{c} \left(\Lambda^{-1/2} . \Gamma_1\right)_{m\times p} \\ 0_{(p-m)\times p} \end{array}\right] . \Sigma_M . \left[\begin{array}{cc} \left(\Gamma^{/}_1 . \Lambda^{-1/2}\right)_{p\times m} & 0_{p\times(p-m)} \end{array}\right] \\ &= \left[\begin{array}{c} \left(\Lambda^{-1/2} . \Gamma_1\right)_{m\times p} \\ 0_{(p-m)\times p} \end{array}\right] . \Sigma_0 . \left[\begin{array}{cc} \left(\Gamma^{/}_1 . \Lambda^{-1/2}\right)_{p\times m} & 0_{p\times(p-m)} \end{array}\right] \qquad (40) \\ &= \left[\begin{array}{cc} \left(\Lambda^{-1/2} . \Gamma_1 . \Sigma_0 . \Gamma^{/}_1 . \Lambda^{-1/2}\right)_{m\times m} & 0 \\ 0 & 0 \end{array}\right] .\end{aligned}$$

Hence, the null hypothesis can be restated as:

$$H_0 : \Sigma^{(M)}_0 = I_{m\times m}, \qquad (41)$$

[7]Note that: while the asymptotic model VCM does not have sampling variability, the data VCM is a random matrix.

where

$$\Sigma_0^{(M)} = \left(\Lambda^{-1/2}.\Gamma_1.\Sigma_0.\Gamma_1^{/}.\Lambda^{-1/2}\right)_{m\times m}$$

can be estimated by

$$\widehat{\Sigma}_0^{(M)} = \Lambda^{-1/2}.\Gamma_1.\widehat{\Sigma}_{0,n}.\Gamma_1^{/}.\Lambda^{-1/2},$$

using the singular value decomposition of the computed rank-deficient VCM Σ_M.

We now consider a modification of Nagao's test based on projection of both data and model VCM to a lower dimensional subspace where each of the two VCMs is full rank. For the test $H_0 : \Sigma_0^{(M)} = I_{m\times m}$ against the omnibus alternative $H_1 : \Sigma_0^{(M)} \neq I_{m\times m}$, the modified Nagao test statistic (N_{new}) on this lower dimensional full-rank subspace is given by

$$\begin{aligned} N_{new} &= \frac{1}{m}.tr\left(\widehat{\Sigma}_0^{(M)} - I_m\right)^2, \qquad (42) \\ &\frac{nm}{2}.N_{new} \sim \chi^2\left(\frac{m(m+1)}{2}\right) \qquad \text{under } H_0 \end{aligned}$$

We formally discuss our assumptions regarding the asymptotics, the data generating process, and moment conditions on eigenvalues in the appendix to this paper. Based on these assumptions, we show the statistical properties of our method. The modified Nagao's test (N_{new}) is consistent under the given assumptions, and has a simple asymptotic distribution.

3.2 Features of the proposed approach

Our proposed methodology has several important features, many of which are quite distinct from existing methods in the literature. First, the method is extremely simple to implement and involves much less computations than many other methods proposed in the literature.

Second, our approach for the rank-deficient case is less vulnerable to the Lucas critique. An important feature of general equilibrium models is that these models have typically fewer shocks than variables. Watson (1993) proposed comparison of moments by augmenting the state variables with just enough stochastic error to enable matching the moments of actual data (see also Rotemberg and Woodford, 1997). The size of the error provides a measure of model fit. This approach has a serious limitation in that one can augment a structural model that is dynamically deficient with an arbitrary error structure so as to exactly replicate the dynamic structure in the data. Therefore, the error processes so identified cannot be considered "structural" in any meaningful sense, and the model becomes vulnerable to a Lucas critique of its errors (Fuhrer, 1997). Instead of augmenting the model with additional error, we take the approach of projecting the data VCM onto a lower dimensional subspace spanned by the shocks and free predetermined variables driving the model.

Third, in contrast to the existing literature, we consider model selection and calibration rather than explicitly testing the fit of "false" general equilibrium models. We take the view that calibrated general equilibrium models are essentially "false", in the sense that they are simplified or improper approximations of the true DGP of the actual data, see for example Kydland and Prescott (1991). In essense, our model does not involve formally testing the fit of "false" models. We only use the p-values to aid model selection.

Fourth, we consider an approach that uses sampling variability of actual data to provide a measure of the distance between model and the data, holding the model VCM fixed[8]. This approach is explicitly based on the context of dynamic general equilibrium macroeconomic models, where given specific calibrated or estimated values for the parameters, the model can be simulated for as many periods of time as desired. Thus, for given parameter values, the asymptotic VCM of the state variables obtained from such simulation has no sampling variability. On the other hand, the data VCM is based on a data for a finite sample period. In most applications, this period would be from 1960 or later to the most recent period for which data are available. Thus, there is substantial sampling variation in the data VCM, while the model VCM can be considered fixed for a given combination of parameter values. By computing distances for distinct combinations of possible parameter values across all the competing models, we can ignore the uncertainty regarding calibration or estimation of parameters, while taking account of sampling variability in the actual data.

Fifth, as advocated in Diebold et al. (1998), our methodology allows a frequency domain approach to compare second-order properties only over a selected range of frequencies. In the application considered in this paper, we filter the data to rank competing models only over the business cycle frequency. Sixth, in contrast to existing methods that are often difficult to apply with calibrated models (not estimated using a likelihood-based approach), or when the models are non-nested (Watson, 1993; Canova and Ortega, 2000), our approach facilitates evaluation of non-nested and calibrated models.

Finally, unlike model evaluation methods based on impulse responses, we focus on second order moments. Hence, our approach does not take into explicit consideration the dynamics of competing models. This has implications for model evaluation using our methodology. However, by including lags of selected state variables in the VCM, we can actually evaluate dynamic features of the models in a limited sense. We view this aspect as a feature of our approach rather than a limitation; we will discuss more about this issue later in the context of modeling money and monetary policy in the US. In summary, our proposed methodology adequately addresses important limitations of most other methods in the existing literature.

3.3 Interpretation of the distance measure

Since we take the view that all candidate models are "false", we do not use the test statistics to conduct a formal test of the hypothesis that the data and model VCMs are the same. As discussed earlier, if we were to conduct such a test, the null hypothesis of equality would almost always be rejected, since the model is only a crude approximation to the true DGP. However, we can still use the p-values of the test statistic, or in fact even the value of the test statistic itself (after adjusting for degrees of freedom), as a measure of distance between the data and the model VCMs.

For example, suppose we wish to compare between two competing NNS models. We compute the asymptotic VCMs for the two models and compute their respective distances from the data VCM. The model with the lower distance can, in this sense, be considered as being a better representation of the data, even though both models may be too simplistic to be considered as a

[8]See Canova and Ortega (2000) for a discussion of other possible approaches: (a) informal approaches which ignore both sampling variability in the data and uncertainty regarding model parameters, (b) methods that consider uncertainty in model but not sampling variability in the data, and (c) approaches that account for both sampling variability in the data and model uncertainty.

true representation of the DGP of the actual data. Similarly, suppose we have a candidate model for which a parameter needs to be calibrated[9]. We can simulate the model under different potential values of the parameter, and find the parameter value for which the distance between data and model VCM is the minimum. This value can be considered a credible calibrated value for the parameter[10].

4 Second moments

Having set up the models and discussed our econometric methodology, we now analyse the second moments generated by the model economies for the different models and under the different monetary policy frameworks. The second column of table 5 reports selected unconditional second moments from quarterly US data for the period 1960 $q1$ to 2003 $q4$. Both the data and the model output are logged and Hodrick-Prescott filtered.[11] Figures in bold type face indicate a cross-correlation with the wrong sign. The appendix describes in detail our data sources and construction of data.

Table 5: Data and model economies: 1960:1 - 2003:4

	Data	*Interest rate rules*		*Money growth rules*	
		MIU	CIA	MIU	CIA
ϵ_K		-5.37	-4.43	-5.04	-4.92
σ_{GDP}	1.57	1.61	2.06	4.19	1.65
$\sigma_{\text{variable}}/\sigma_{\text{GDP}}$					
Consumption	0.79	0.81	0.81	0.81	0.81
Investment	3.18	3.18	3.18	3.18	3.18
Hours	0.92	0.86	0.88	1.28	1.13
Real wage	0.48	0.50	0.36	0.25	0.54
Inflation	0.31	0.27	0.18	0.27	0.47
Interest rate	0.25	0.10	0.07	0.01	0.38
Corr($_{\text{variable, GDP}}$)					
Consumption	0.86	0.99	0.99	0.99	0.99
Investment	0.89	0.99	0.99	0.99	0.99
Hours	0.88	0.74	0.88	0.96	0.76
Real wage	0.26	0.63	0.58	**-0.31**	0.28
Inflation	0.37	**-0.12**	**-0.12**	0.83	0.39
Policy rate	0.36	**-0.95**	**-0.89**	0.81	0.51
Autocorrelations					
GDP	0.85	0.69	0.67	0.62	0.90
Consumption	0.87	0.71	0.67	0.63	0.89
Investment	0.90	0.68	0.66	0.60	0.90
Inflation	0.30	0.29	0.27	0.43	0.56

[9] For example, if there are no available data to estimate this parameter. Canova and Ortega (2000) discuss the case of designing tax policy for a newly formed economy.

[10] Christiano *et al.* (2005) take a similar approach for calibration based on the difference between the estimated impulse response functions and the analogous objects in the candidate models.

[11] The time series for the policy rate is only H-P filtered, not logged.

Models closed with interest rate rules. Columns 3 and 4 contain the second moments of the MIU and CIA models. In each case we set the capital adjustment cost parameter, ϵ_K in such a way that the model generates a filtered series for investment that is 3.18 times as volatile as the filtered series for GDP. The MIU and the CIA models come close to matching the relative volatility of consumption and the absolute standard deviation of GDP. Under an interest rate rule, both models do a reasonable job at matching the volatility of hours, real wages and inflation relative to the volatility of output.

However, both models generate filtered time series for policy rates that are less volatile than the data. Cross-correlations between our variables of interest and GDP have the correct sign for consumption, investment, hours, and real wages but the correct magnitude only for the first three variables. Both models generate counter-cyclical time series for inflation and the policy rate, whereas the data suggest a pro-cyclical relationship for each. Both models under-predict the persistence of GDP, consumption and investment and match the persistence of inflation. Related work by Nolan and Thoenissen (2005) suggests that the negative cross-correlation between inflation and GDP may be attributed to the presence of sticky wages, and that the counter-cyclicality of both the policy rate and inflation is robust to the introduction of firm-specific capital and/or to changes in the Calvo parameter on prices.

A striking feature is how similar the second moments are for both models in the case of an interest rate feedback rule. This suggests that the differences in the first-order conditions that arise due to the non-separability of consumption implied by the CIA model do not play a significant role in the dynamics of the model. We now turn to the models under the assumption that monetary policy is characterised by a money growth rule.

Models closed with money growth rules. In this case, the MIU model generates a filtered time series for GDP that is almost three time as volatile as the data, unlike the CIA model which matches the standard deviation of GDP quite closely. Compared to the models under an interest rate rule, the relative standard deviation of hours worked is too high (more volatile than the data) and that of the real wage too low (for the MIU model). The MIU model come close to matching the relative volatility of inflation. Interestingly, the CIA model, when combined with a money growth rate, now predicts a more correct magnitude of the relative volatility of the policy rate. The models under a money growth rule overcome the problem of counter-cyclical inflation and nominal interest rates, but the MIU model yields a counter-cyclical real wage, which is in contrast to the data. Under a money growth rule, only the CIA model generates cross-correlations that have the correct sign for all variables analysed. Canzoneri *et al.* (2004) argue that the counter-cyclicality of inflation and the policy rate suggests a missing or incorrectly specified demand shock. Our analysis suggests an important role for the way money and monetary policy are modelled.

The MIU model generates even less persistence of GDP and its components under a money growth rule than under an interest rate rule and are therefore even further away from the data. The CIA model generates more persistence under a money growth rule than under an interest rule and comes close to matching the first order autocorrelation coefficient of GDP and its components, but somewhat overestimates the persistence of inflation.

Overall, the CIA model comes closer to the data under a money growth rule than under an interest rate rule, and appears to have the best fit overall. Under a money growth rule, the MIU model appears to be further away from the data than under an interest rate rule in terms of overall volatility, but comes closer to the data in terms of cross-correlations with GDP. Our distance measure, the modified Nagao test derived in the previous section will allow us to rank our models according to how close their VCMs are to the VCM of the data. Based on the above descriptive discussion of moments, we therefore *a priori* expect our proposed methods to favour the CIA model under a money growth rule.

However, before we rank our models using the modified Nagao test, the next section analyses and compares the short-run dynamics of the models using impulse response functions. The main objective is to explore what additional insights regarding model evaluation might be gained by analysing dynamic features of the competing models.

5 Impulse responses

In this section, we use impulse response functions to analyse how our calibrated models react to monetary policy as well as technology shocks. The calibration of each model, in particular the calibration of the capital adjustment cost parameter, corresponds to the calibration used to generate the moments summarised in table 5. Throughout, the response of a variable in the MIU model is denoted by a solid and the response of a variable in the CIA model by a dashed line.

Monetary shocks in models with an interest rate feedback rule. We know from table 5 that under an interest rate rule the both models produce very similar second moments. This result is borne out by the different panels of Figure 1. Apart from the demand for real money balances, the dynamics of output, consumption, investment, employment, real wage, marginal cost, the policy rate and inflation are very similar across model specifications. In response to an unexpected cut in the policy rate, output and its components rise (slightly more under CIA). Contrary to VAR evidence, the response of these three variables does not, however, have a 'hump' shape such that the greatest effect of a monetary easing occurs some quarters after the shock. Marginal cost and inflation also increase in response to monetary easing. Because prices adjust more frequently than nominal wages, the real wage falls initially. Employment, which is demand determined, rises along with the increase in output and the fall in the real wage. The policy rate reacts endogenously to the interest rate shock. In response to the rise in inflation and a positive output gap, the policy maker raises interest rates following the shock, but not by enough to stop the policy rate from falling relative to its initial equilibrium value.

Technology shocks in models with an interest rate feedback rule. As earlier, both models display similar dynamics in response to an increase in total factor productivity when monetary policy is modelled via an interest rate feedback rule. In Figure 2, output and its components rise in response to a technology shock. They rise by more in the CIA than in the MIU model. Employment and the real wage also increase - at least after the initial shock. The 'hump' shape of these response functions reflects, to some degree, the autocorrelation of the productivity innovation. A technology shock lowers the marginal costs and thus inflation. The policy maker responds to the fall in inflation by lowering the nominal interest rate.

The conclusion we can draw from our impulse response analysis thus far is that in terms of short-run dynamics, the way we introduce money into our models appears not to be crucial as long as we model monetary policy via an interest rate feedback rule. Next, we analyse our models under the assumption that monetary policy is modelled by a money growth rule.

Monetary shocks in models with a money growth rule. Figure 3 shows that an increase in the growth rate of the money supply has significantly different dynamic implications for the CIA model than for the MIU model. Unlike the CIA model, the dynamics of the MIU model is very similar to its counterpart when closed with an interest rate rule. Importantly, in the CIA model the response of output, consumption, investment and employment is 'hump' shaped. This finding is familiar from Danthine and Kurmann (2004) and more recently from Wang and Wen (2006) who analyse similar models. Following a money injection in the MIU model, output, consumption and investment are not 'hump' shaped. Agents fully raise aggregate demand in the first period of the shock in anticipation of future increases in money growth. Under a cash-in-advance constraint, the initial increase in aggregate demand is constrained to the current increase in money. Consumers and firms have to wait for future money injections to fully adjust to the monetary shock. Our impulse responses suggest that it takes the CIA model several quarters for the maximum effect of a monetary easing to impact on output and its components as well as on employment. The model does not succeed at generating a liquidity effect, whereby the nominal interest rate falls in response to a rise in the money growth rate. Instead, the nominal interest rate rises in response to a monetary expansion. A partial explanation of this response can be found in the path of consumption which is rising for the first few periods following the monetary shock. We know from the Euler equation that such path of consumption requires a rise in the real interest rate. Since inflation rises after a monetary injection, the nominal interest rate must also rise to generate a rise in the real interest rate.
Technology shocks in models with a money growth rule. Figure 4 shows impulse response functions with respect to a 1% technology shock when monetary policy is modelled by a money growth rule. Unlike for monetary shocks, the CIA model does not display qualitatively different dynamics than the alternative models following a technology innovation. Because agents need to accumulate real money balances for consumption and investment purposes, output and its components in the CIA model appears to be less volatile but more persistent than in the MIU model. Employment initially declines in both models. Where the MIU and CIA models differ most is the response of the policy rate. Whereas the interest rate in the MIU model barely reacts to a technology shock, the interest rate in the CIA model declines on impact and rises above its steady state value along the adjustment path.

[Figures 1 - 4 about here]

The main conclusion we can draw from our analysis of impulse response functions is that the CIA model, when combined with a money growth rule can generate 'hump' shaped responses in output and its components following a monetary shock. We find that this 'hump' shape arises only in the combination of CIA and money growth rules. Taken individually, neither feature is able to generate such a response.

Summarising our results so far: we have analysed the second moments as well as the impulse response functions of two models across two monetary policy specifications. We have seen that under an interest rate feedback rule, it is difficult to distinguish between the two models. We also found that in a money growth rule specification, the CIA model had certain desirable features not shared by the MIU model. Each model and specification appears to have advantages as well as disadvantages that makes a ranking based on the current criteria difficult. In the following section, we use our proposed distance measure to compare a set of second moments of the model (capturing the variables we discussed in table 5) with those of the data. Importantly, our method allows us to compare these non-nested models with the data and with one another.

6 Taking the models to the data

In this section, we apply our modification to the Nagao test (N_{new}) to measure distance between data and model VCMs for our different candidate models. The state variables that we compare across all models and specifications are: logged and Hodrick-Prescott filtered series for consumption, federal funds rate[12], investment, inflation, hours worked, GDP, and real wages. Our results are presented in Table 6.

Table 6: Test Statistics - Baseline Models

	Interest rate rules		*Money growth rules*	
	MIU	CIA	MIU	CIA
N_{new}	1.95e^8 (4)	5.81e^6 (3)	3.95e^6 (2)	4.38e^2 (1)

We can rank the models by the value of the modified Nagao test statistic adjusted for the degrees of freedom.[13] Across all models and policy specifications, the cash-in-advance model in conjunction with a money growth rule yields the lowest test statistics for the modified Nagao tests. As far as this distance measures is concerned it is the specification whose filtered VCM comes closes to filtered VCM of the data. Our distance measures also suggest that it is difficult to distinguish between alternative ways of introducing money when monetary policy is characterised by an interest rate feedback rule - both models have high adjusted test statistics. This suggests that any effects of an inflation tax associated with the cash-in-advance models may be rather minimal. When monetary policy is modelled via a money growth rule, however, there are large differences between models, with only the CIA model yielding satisfactory results.

As a caveat, we should add that currently this ranking is based only on the contemporaneous VCMs of models and data. As of now, our distance measure does not consider the models' ability to match the dynamic features in the data. However, as discussed earlier, this can be incorporated within the framework of the proposed methodology by including state variables at selected leads and lags. Since the conclusions of our analysis of impulse responses were largely consistent with

[12] Interest rate series are H-P filtered but not logged.

[13] The figures in parentheses in the Table represent relative ordering of the different models in terms of the distance measure. Ideally, one would want to use the p-values from the modified Nagao test statistic to rank the models in terms of the closeness of their VCMs with that from the data. However, since we intend to use the test to conduct model selection among "false" models, we also expect the null hypothesis (that the data VCM is the same as the model VCM) to be rejected in each case. In other words, we expect the p-value to be close to unity. This indeed turns out to be the case, as the high values of the modified Nagao test statistics show.

the inferences drawn from second order moments, we have kept this issue outside the purview of the current application.

7 Discussion

At the end of section 4, based on our descriptive analysis of the models' second moments, we concluded that the CIA model in combination with a money growth rule appeared to come closest to the data. Confidence in our distance measure is enhanced by the fact that ranking of models based on the modified Nagao test supports our earlier intuition.

How can we interpret this ranking? As emphasized earlier, our distance measure is about comparing competing models, and not so much about model fit. In other words, we accept that most of the models we consider are quite stylised and therefore essentially "false" models. Therefore, we cannot conclude from this ranking that the 'winning' model is an accurate description of the data. The only thing we can say with confidence is that the 'winning' model comes closer to the data in terms of our selected second moments than the competing models.

Why is the CIA model combined with a money growth rate better than the competing models? To answer this question, we make use of both the descriptive statistics (second moments) and the impulse responses. Table 5 suggests that a key difference between our models lies in the behaviour of the policy rate and inflation. These variables are pro-cyclical in the data and in models closed with a money growth rule, but counter-cyclical in models closed with a Taylor rule. Our impulse responses suggest that the key qualitative difference between our models lies not in their response to TFP shocks but in how they respond to monetary policy shocks.

Comparing figures 1 and 3, we find that a monetary easing is more inflationary under a money growth rule than under an interest rule, this raises the covariance between GDP and inflation. We also note that the nominal interest rate behaves differently following an interest rate shock than it does following a money growth shock. In the latter case, consumption (and income) are rising in the first few periods following the shock. From the Euler equation, we know that rising real consumption implies a rising real interest rate. Since inflation increases in response to a rise in the growth rate of the money supply, the nominal interest rate has to rise in response to a monetary easing to generate a rising real interest rate. This explains why the CIA model closed with a money growth rule has larger covariances between GDP and inflation and GDP and the policy rate than the models closed with an interest rule.

The natural complementary question in this context is: why do the models closed with a Taylor rule fail to match the data? There are a multitude of possible answers. For one, we could ask if our representation of the Taylor rule is appropriate. In our model, the monetary authority sets interest rates in response to current levels of inflation and the output gap as well as last period's interest rate. Taylor (1999) does not, for instance, include a lagged dependent variable, whereas Clarida *et al.* (2000) estimate a rule in which the Fed sets the current interest rate in response to forecasts of inflation and the output gap based on the previous period's information.

A further issue concerns the measurement and modelling of the output gap. It is unclear that the concept of the output gap used in the model corresponds well with our chosen empirical counterpart. An alternative way of modelling the output gap, as suggested by Canzoneri *et al.* (2004) is to take the deviation of output form its steady state level. Even if the output gap is measured correctly, it is not clear that a simple Taylor rule can accurately capture the systematic component of monetary policy. This matters because we interpret the deviations from this rule as monetary policy shocks, when in fact the residuals could simply arise because the policy maker is reacting to other indicators. An interesting extension of our current work would be to estimate the best possible monetary policy reaction function and impose this function onto the theoretical model and analyse to what extent the model improves.

A further issue for our models is the monetary transmission mechanism. Whereas we have kept the structure of our models deliberately simple to help us focus on our distance measure, Christiano *et al.* (2005) put forward a state-of-the-art NNS model that more or less accurately captures the US transmission mechanism of monetary policy (based on fitting the impulse responses of the model to monetary policy shocks to that of a VAR). In their model, they include features such as habit persistence and investment (as opposed to capital) adjustment costs.

Based on the above discussion, we would conjecture that by introducing lags into consumption and investment equations, the modified Nagao distance between model and data can be reduced. Further, while we include only contemporaneous values of the state variables in the VCM and use this VCM for model selection, one could also potentially include leads of selected variables. This could, in principle, incorporate dynamic considerations into our methodology and provide further inference regarding model evaluation.

In summary, as the current application demonstrates, the distance measure proposed in the paper is useful for evaluating alternative theoretical models. While the methodology is primarily aimed at model selection based on moments, our approach can also contribute to better understanding of dynamic features of competing models. Further, with the modifications implied by the previous discussion, it is possible in principle to do a better job at evaluating theoretical models. In other words, we can use our methodology to select a better theoretical model based on observable characteristics of simulated model scenarios

8 Conclusions

In this paper, we ask the questions: Does it matter how we model the role of money in the New Neoclassical Synthesis (NNS) model characterised by monopolistic competition, price and wage stickiness as well as endogenous capital accumulation? And does it matter how we model monetary policy? It is well understood that the NNS model when closed by an interest rate feedback rule generates a counter-cyclical policy and inflation rate, whereas the data suggest that the two series are in fact pro-cyclical. One aim of our paper is to investigate if this shortcoming of the NNS model is related to the way we model the role of money and how we model monetary policy. Our findings suggest that the counter-cyclical policy and inflation rates are common to the money-in-the-utility as well as the cash-in-advance versions of our basic NNS model as long as monetary policy is modelled via an interest rate feedback rule. On the other hand, when monetary policy is modelled via a money growth rate, we find that our NNS model generates pro-cyclical policy and inflation rates. We also observe that in response to an unexpected increase in the money growth rate the cash-in-advance model generates 'hump' shaped impulse responses for output, consumption and investment.

To answer the question as to which of the models and specifications comes closest to the data, we develop and employ a new econometric method that allows us to compare and rank nested and non-nested models against each other and against the data. The proposed distance measure compares the variance-covariance matrix of the model economy with that generated by the data. Our results suggest a clear and consistent ranking of models. The cash-in-advance model in conjunction with the money growth rule matches the data moments most closely, whereas the money-in-the-utility model closed by an estimated interest rate feedback rule is the model furthest removed from the data.

In future work, we plan to examine how we can use our distance measure to calibrate, and indeed estimate key structural parameters of the model. In addition, we plan to extend our distance measure to take account of the lead and lag structure of the data generating process.

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A Assumptions and statistical results

Let us now formally state our assumptions regarding the asymptotics, the data generating process, and the moment conditions on the eigenvalues.

Assumption 1: *(Sample size asymptotics). p and m* are fixed for all sample sizes, there is only large sample size n-asymptotics.

Assumption 2: *(Data generating process). For each sample size n, X_n is a $n \times p$ dimensional matrix of n observations on a system of p random state variables which are jointly normally distributed with mean μ_n and covariance matrix $\Sigma_{0,n}$. Hence, $Y_n = X_n.\Gamma_1^{/}.\Lambda^{-1/2}$ is a $n \times m$ dimensional matrix of n observations, $\left(Y_n^{(i)}\right)_{1\times m}$, $(i = 1, \ldots, n)$, on a system of m linear transformations of the random state variables X_n that are jointly normally distributed with a $m \times m$ covariance matrix*

$\Sigma_{0,n}^{(M)} = \Lambda^{-1/2}.\Gamma_1.\Sigma_{0,n}.\Gamma_1^{/}.\Lambda^{-1/2}$. *This covariance matrix can be consistently estimated by:*

$$\widehat{\Sigma}_{0,n}^{(M)} = \frac{1}{n}\sum_{i=1}^{n}\left(Y_n^{(i)} - \overline{Y}_n\right)^{/}.\left(Y_n^{(i)} - \overline{Y}_n\right). \tag{43}$$

Assumption 3: *(Moment assumptions on eigenvalues). Let $\lambda_{1,n} \geq \ldots \geq \lambda_{m,n} > 0$ denote the ordered positive eigenvalues of the covariance matrix $\Sigma_{0,n}^{(M)}$. We assume that the first two central moments of these eigenvalues, $\alpha = \frac{1}{m}\sum_{i=1}^{m}\lambda_{i,n}$ and $\delta^2 = \frac{1}{m}\sum_{i=1}^{m}(\lambda_{i,n}-\alpha)^2$ are independent of the sample size n, and that $\alpha > 0$. We further assume that, as $n \to \infty$, both the third and fourth moments of these eigenvalues converge to finite limits: $\lim_{n\to\infty}\frac{1}{m}\sum_{i=1}^{m}\lambda_{i,n}^{j} < \infty$, $(j = 3, 4)$.*

Assumption 1 sets out the large sample size asymptotics. The number of sources of variation is fixed and does not change with sample size. We will show that the modified Nagao's test (N_{new}) is consistent in this setup, and will derive its asymptotic distribution.

The specification of the data generating process (**Assumption 2**) is standard. The covariance matrix $\Sigma_{0,n}^{(M)}$ is estimated using the matrix of averages of squares and products, but on the transformed variables Y_n, instead of the original variables X_n. This approach has the advantage that we consider a matrix having the Wishart distribution, rather than explicitly working with a singular Wishart matrix.

The assumptions on the moments of the eigenvalues (**Assumption 3**) are weaker than standard assumptions in the literature. These are also the most critical assumptions for development of the asymptotic properties of our test statistic. The assumption that the first and second moments are independent of the sample size is essentially a scaling argument. What this says is that the covariance matrices should be transformed to orthogonal standardised variables before applying the asymptotic arguments. The assumptions trivially hold under the null hypothesis (Equation 41) since all the eigenvalues of the identity matrix are equal to unity. In fact, the null hypothesis $H_0 : \Sigma_0^{(M)} = I_{m\times m}$ can be equivalently stated in terms of the first two moments, as:

$$\alpha = 1 \text{ and } \delta^2 = 0. \tag{44}$$

The conditions on existence of limits for the third and fourth moments of the eigenvalues also holds under the null hypothesis, since $\frac{1}{m}\sum_{i=1}^{m}\lambda_{i,n}^{j} = 1$ for all $j = 1, 2, \ldots$. In the literature, it is standard to assume finite limits for all moments. However, as argued by Ledoit and Wolf (2002), this stronger assumption is not really necessary for the purpose of consistency and asymptotic distribution of the test statistic (see also Bai *et al.*, 1988).

Under **Assumptions 1–3**, we have the following asymptotic results.

Proposition 1 *(Weak Law of Large Numbers). Under Assumptions 1, 2 and 3,*

$$\frac{1}{m}.tr\left(\widehat{\Sigma}_{0,n}^{(M)}\right) \xrightarrow{P} \alpha$$
$$\frac{1}{m}.tr\left(\widehat{\Sigma}_{0,n}^{(M)}\right)^2 \xrightarrow{P} (1+c)\alpha^2 + \delta^2$$

where $\xrightarrow{P}$ *denotes convergence in probability.*
Proof. *The proof was given by Yin and Krishnaiah (1983), while considering spectral densities of the product of two independent matrices, the second of which is a Wishart matrix. They assume limits of all moments of the eigenvalues; however, only limits upto the fourth moment are required in their proof.*[14] ■

Proposition 2 *(Asymptotic Distribution). Under Assumptions 1, 2 and 3 (first and second moments only) and* $H_0 : \alpha = 1 \quad and \quad \delta^2 = 0,$

$$n \times \begin{bmatrix} \frac{1}{m}.tr\left(\widehat{\Sigma}_{0,n}^{(M)}\right) - 1 \\ \frac{1}{m}.tr\left(\widehat{\Sigma}_{0,n}^{(M)}\right)^2 - \frac{n+m+1}{n} \end{bmatrix}$$
$$\xrightarrow{D} N_2\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \frac{2}{c} & 4\left(1+\frac{1}{c}\right) \\ 4\left(1+\frac{1}{c}\right) & 4\left(\frac{2}{c}+5+2c\right) \end{bmatrix}\right),$$

where $\xrightarrow{P}$ *denotes convergence in law as* $n \to \infty$ *and* N_2 *denotes the bivariate normal distribution.*
Proof. *The Proof follows along the lines of Ledoit and Wolf (2002, Proposition 2), using the asymptotic distribution results in Jonsson (1982) and the moments derived in John (1972).* ■

Proposition 3 *(Consistency of modified Nagao's test). The modified Nagao test,* N_{new}*, is n-consistent (consistent as* $n \to \infty$ *with m fixed).*
Proof. *Note that testing* $H_0 : \alpha = 1 \quad and \quad \delta^2 = 0$ *against the omnibus alternative is equivalent to testing* $H_0 : (\alpha-1)^2 + \delta^2 = 0$ *versus* $H_1 : (\alpha-1)^2 + \delta^2 > 0$.
As n goes to infinity with m fixed, $\widehat{\Sigma}_{0,n}^{(M)} \xrightarrow{P} \Sigma_0^{(M)}$*, and therefore* $N_{new} = \frac{1}{m}.tr\left(\widehat{\Sigma}_{0,n}^{(M)} - I_m\right)^2 \xrightarrow{P} \frac{1}{m}.tr\left(\Sigma_0^{(M)} - I_m\right)^2$*. Hence the test based on* N_{new} *is n-consistent.* ■

Proposition 4 *(Asymptotic distribution of modified Nagao' s test statistic under* H_0*). As n goes to infinity with m fixed,*

$$\frac{nm}{2}.N_{new} \xrightarrow{D} \chi^2\left[m(m+1)/2\right],$$

where $\chi^2[d]$ *denotes the Chi-square distribution with d degrees of freedom.*
Proof. *Proof follows directly from Proposition 2.* ■

Note that consistency and asymptotic distribution are proved in Nagao (1973) without Assumption 3 on the moments of eigenvalues. However, the proof is lengthy and quite involved. If we make this assumption, however, the understanding of the results become much more intuitive.

B The data

Our data are of quarterly frequency and come from two main sources: the *US Department of Commerce: Bureau of Economic Analysis* (BEA) and *US Department of Labor: Bureau of Labor Statistics* (BLS) and span the sample period 1960:1 to 2003:4.

[14] The proof of the result is quite involved – an intuitive exposition of the proof is presented in Bai (1999).

1. GDP referred to in table 5 is real GDP per capita from BEA's NIPA table 7.1. 'Selected Per Capita Product and Income Series in Current and Chained Dollars', seasonally adjusted. The series was logged and H-P filtered.

2. Consumption referred to in table 5 is total consumption expenditures deflated by the relevant GDP deflator, both from BEA's NIPA tables 2.3.5 and 1.1.9.

3. Investment referred to in table 5 is real fixed investment per capita from BEA's NIPA table 5.3.3. Real Private Fixed Investment by Type. Population is from NIPA table 7.1.

4. Hours referred to in table 5 is per capita hours worked in non-farm businesses, from BLS, series code PRS85006033. Population is from NIPA table 7.1.

5. Real wage referred to in table 5 is real hourly compensation from BLS, series code PRS85006153.

6. Inflation referred to in table 5 is defined as $\pi = \log(P_t/P_{t-1})$, where P is consumer price index for all urban consumers, from BLS series CUSR0000SA0.

7. Wage inflation referred to in table 5 is constructed using nominal hourly compensation from BLS, series code PRS85006103 W_t. $\omega = \log(W_t/W_{t-1})$.

8. Interest rate referred to in table 5 is the effective US federal funds rate.

9. Potential output used to construct the output gap measure in our estimated Taylor rule is taken from the *Congressional Budget Office* measure of potential output.

10. The Solow residual is constructed as follows:

$$A_t = ynfb_t - s_k \log(K_t) - (1 - s_k)\log(N_t)$$

    where $ynfb$ is the log of real GDP in the non-farm business sector, series PRS85006043 from BLS. N_t is aggregate hours worked, as above, but not deflated by the population. K is real non-residential fixed assets, constructed following Stock and Watson (1999).

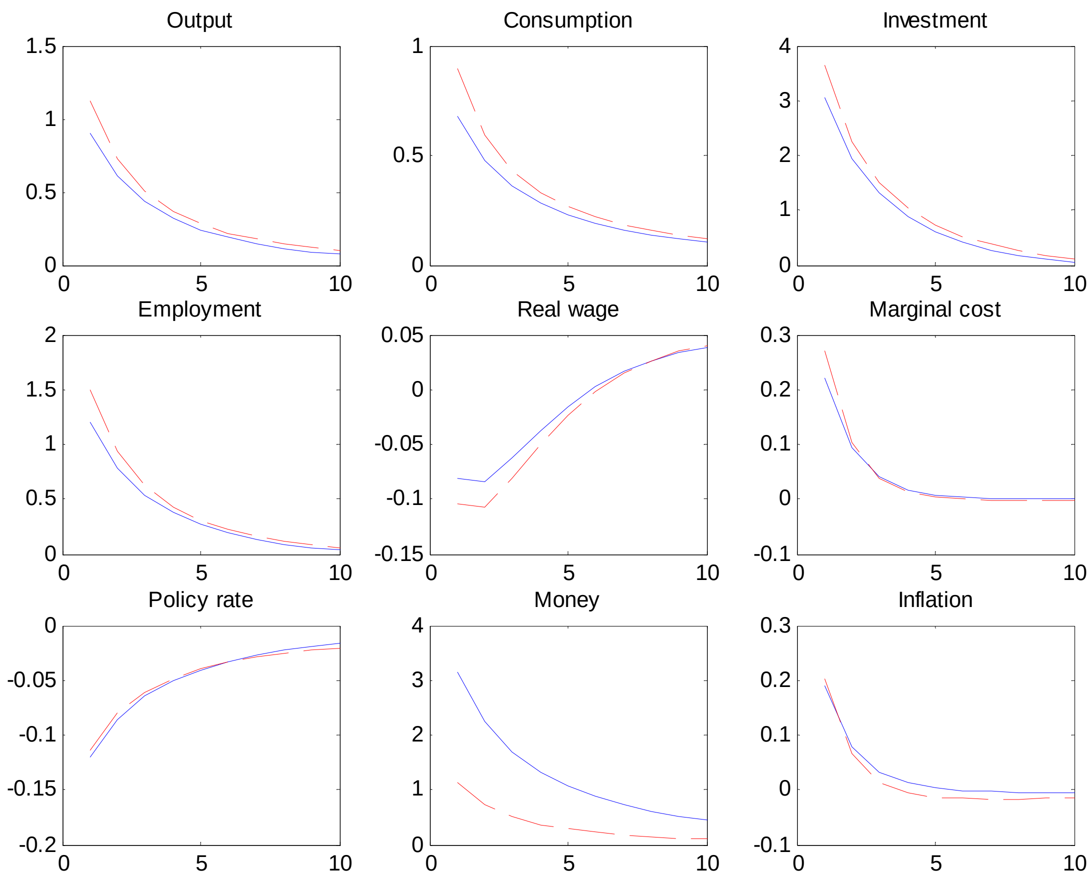

Figure 1: Impulse response functions with respect to a -1% interest rate shock in the MIU model (solid) and CIA model (dashed).

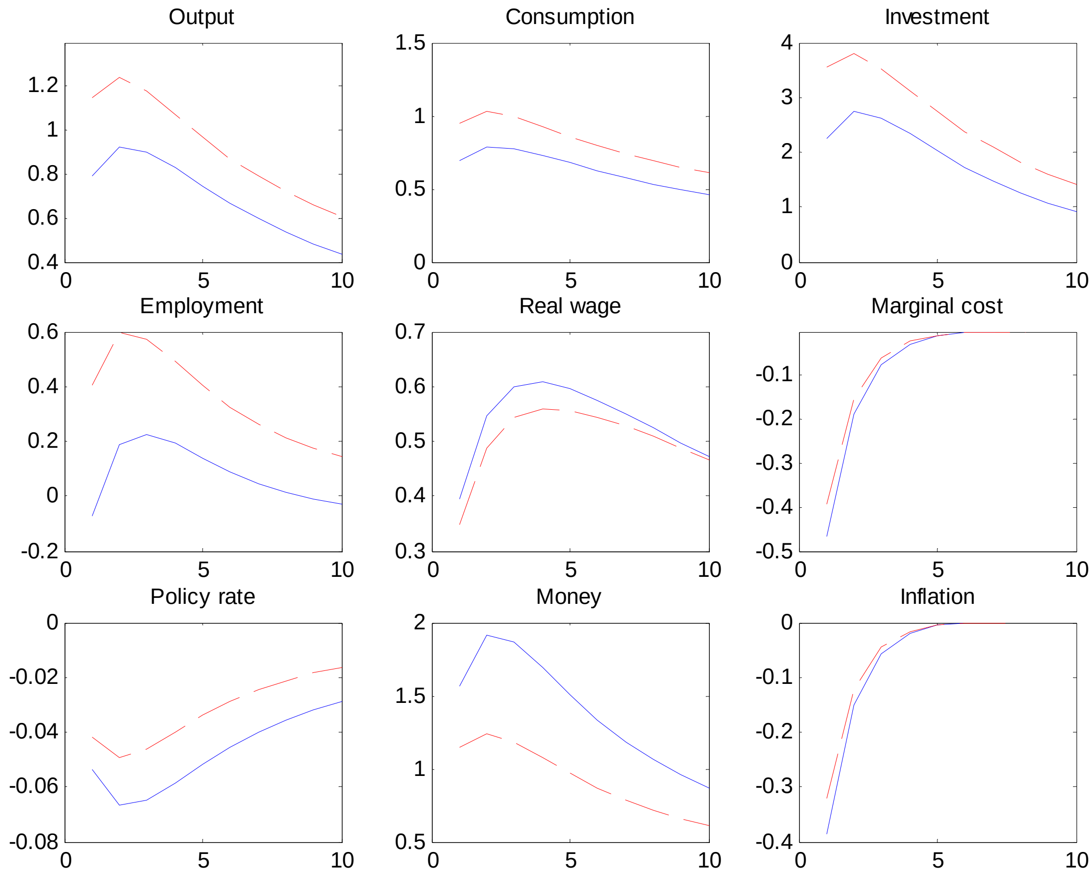

Figure 2: Impulse response functions with respect to a 1% increase in TFP in the MIU model (solid) and CIA model (dashed) when closed with an interest rate feedback rule.

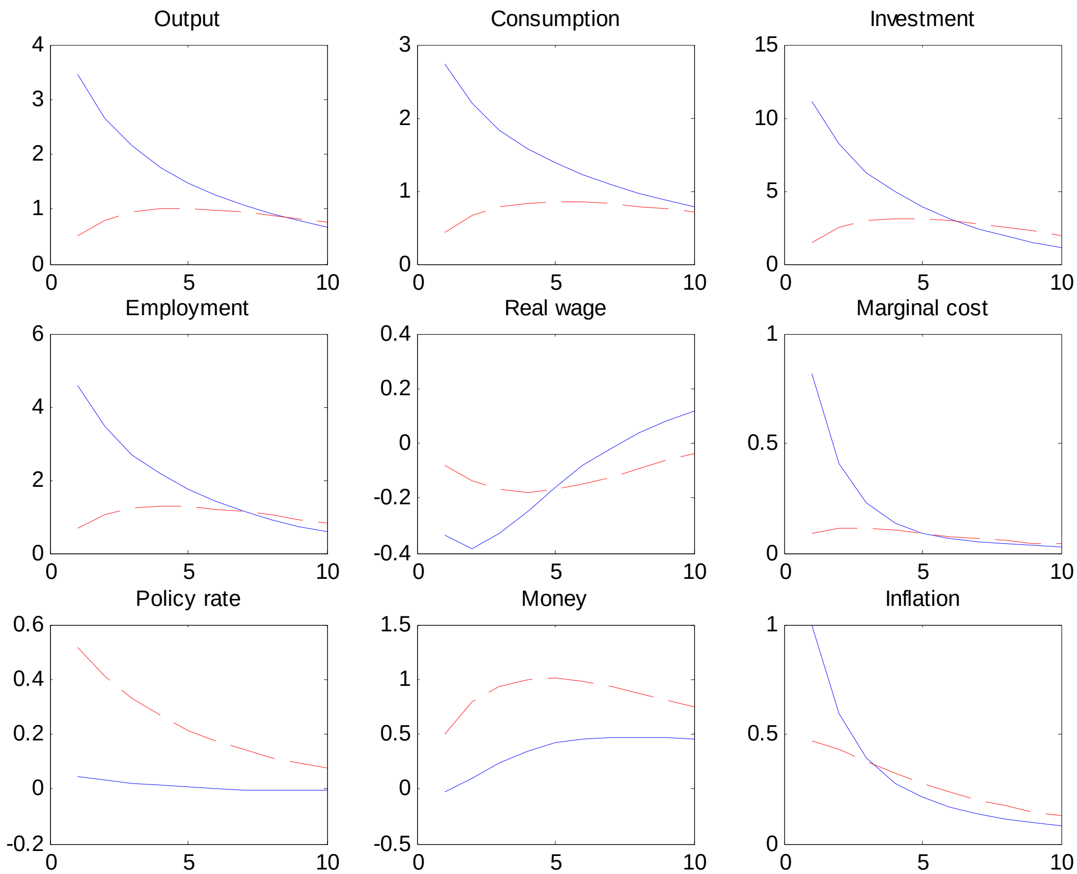

Figure 3: Impulse response functions with respect to a 1% increase in the growth rate of the nominal money supply for the MIU model (solid) and the CIA model (dashed).

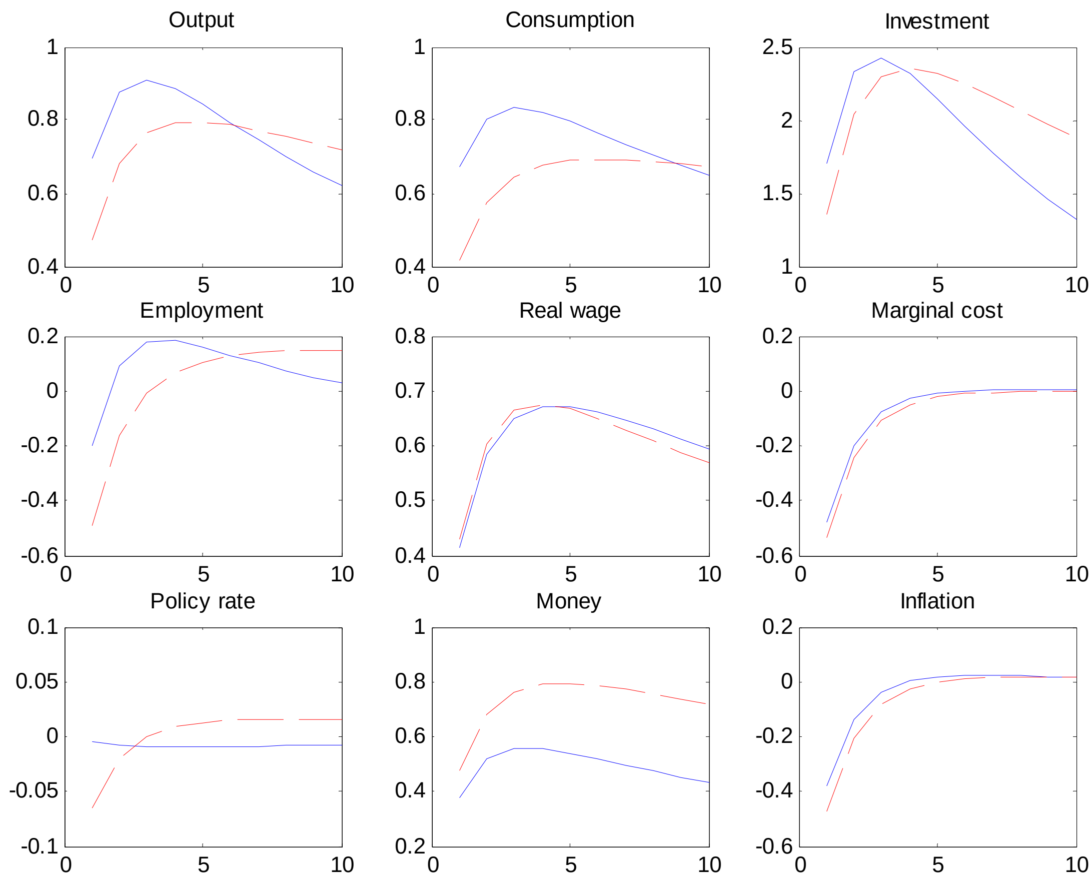

Figure 4: Impulse response functions with respect to a 1% increase in TFP in the MIU model (solid) and CIA model (dashed) when closed with a money growth rule.

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