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Money, Debt, and Economic Activity

by

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Money, Debt, and Economic Activity

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The paper develops an alternative to the standard *IS-LM* framework. There are two asset markets and three prices—the prices of real assets, financial assets, and output. Costs of adjustment and information prevent output prices and output from adjusting instantaneously. Both the size of deficits and the method of financing affect output and prices. Some principal implications are derived. Several of these are also demonstrated, using a graph to show the interaction of asset markets, output markets, and the financing of the budget deficit. Some main implications of standard analysis are rejected. The basis for several “monetarist” conclusions is shown.

Three deficiencies of the standard paradigm of macroeconomics, represented either by Hicks's (1937) classic restatement and adumbration of Keynes's *General Theory* (1936) or by the Metzler-Patinkin (1951, 1965) model, seem most important. First, several variables of interest are either omitted or combined. Bonds and real capital are treated as a single asset. There is only one relative price—the rate of interest. There is no way to analyze the substitutions between money, bonds, real capital, and current consumables set off by changes in monetary or fiscal policies or by autonomous changes in the productivity of capital. The only simultaneous solution for the price level and real output is the full-employment solution; elsewhere, real output is determined only if the price level is constant.

Second, the theory has not been successfully confirmed. Although evidence has been obtained about the properties of particular equations,

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none of the multiequation econometric models inspired by the standard paradigm provides a reliable explanation of prices, output, and interest rates. The many failures to obtain reliable evidence suggest that there is a gap between the standard theory and the events the theory seeks to explain.

Third, standard macrotheory has not been extended or modified to incorporate some main developments in monetary theory and price theory of the past two decades. One such development is the work on portfolio balance (Tobin 1969) that introduces relative prices into the analysis of asset demands. In this analysis, "money" is a substitute for existing real assets and not solely a substitute for "bonds." Once this work is extended to include the market for output, we are able to analyze the relations between money, other assets, and output and the interrelation of asset prices, output prices, and interest rates. Below, we make the extension.

A second development offers an explanation of the problem of persistent unemployment. Keynes (1936, chap. 2) discarded price theory as an explanation of persistent or "involuntary" unemployment, and instead assumed that money wages are rigid downward. Many researchers (Alchian and Allen 1967; Phelps 1968; Lucas and Rapping 1969) have now developed alternatives to Keynes's analysis. In this paper we explore one such approach based on an analysis of the credit market and its interaction with the rest of the economy.

The credit market also serves as a main link between the government and the private sector. The government issues either base money or bonds to finance a deficit and uses a surplus to retire money or debt from the public. A third recent development (Christ 1968; Silber 1970) included in our analysis is explicit consideration of the effects on assets and output of financing the government's budget.

The introduction of these new elements modifies the standard paradigm and increases its complexity. The power and relevance of the analysis increase also, and, we believe, justify the extension. Stating some main implications of our analysis shows that the changes introduce a number of differences. Seven are listed here.

1. The interest elasticities of the expenditure (or *IS*) and the demand for money (or *LM*) functions are neither necessary nor sufficient for determining the relative responses to fiscal and monetary policies.
 2. The real balance effect is neither a necessary nor sufficient condition for a positive response of output to changes in money or the monetary base.
 3. The dominant wealth effect induced by monetary (and some fiscal) policies is a change in the price of existing real assets relative to the price of new production (the price level).
-

4. A constant, maintained budget deficit financed by issuing debt raises market interest rates and the price of real capital.

5. A budget deficit (surplus) raises (lowers) the price level but does not induce sustained inflation (deflation). The change in prices is distributed through time. The length of time and the size of the total price change increase with the proportion of the deficit (surplus) financed by issuing (retiring) debt. The rate of price change per unit time increases with the proportion of the deficit financed by issuing (withdrawing) base money.

6. The length of the lag of output behind changes in money varies. The length depends on the relative size of accelerations and decelerations of money and the interaction of monetary and fiscal impulses.

7. The effects of money on output and prices do not depend on the exogeneity or endogeneity of money.

Many of these differences in implication result from our separation of the markets for money and credit, the latter defined as total bank-earning assets. If we were willing to assume that changes in the stock of government securities held by the public have no effect on wealth—by any of a number of devices that make the government debt and real capital perfect substitutes—our model could be pressed into the standard, *IS-LM* framework. Doing so, however, misses an essential point. In our view, the standard framework is an inadequate explanation of short-term changes in the economy. The present paper develops an alternative.

In the following section we present a model relating stocks and flows, the government's budget, and the financing of the budget. To bring out some principal differences between our model and the standard *IS-LM* paradigm, we reduce the scope of our model by omitting the labor market, tax parameters, anticipated prices, and other variables that we have included elsewhere (Brunner and Meltzer 1972). We then show that the interaction between the asset and output markets removes disequilibrium in the output market by changing prices and output. Finally, we consider the effects of the budget deficit and changes in the deficit on the markets for assets and output and show the conditions for full stock-flow equilibrium. A diagram in the familiar i, y plane helps to show how our model differs from the *IS-LM* model.

The Wealth and Price Adjustment Process

In some earlier work we developed and extended a model of the wealth-adjustment process (Brunner and Meltzer 1963, 1968, 1972; Brunner 1971) that relates money to relative and absolute prices, output, wealth, and financial assets. The response of output and prices to money and other variables summarizes the interaction on the markets for money, output,

and credit or bank-earning assets. This section presents a condensed version that omits the labor market, the determination of real and money wages, the interaction between output and labor markets, and the role of price anticipations. There is no growth in capital and labor force. A symbol dictionary (Appendix) defines the principal variables.

We assume, throughout, that costs of acquiring information and adjustment are smaller for the assets we consider than for output. Consequently, asset markets are cleared by suitable adjustment of asset prices within the time units relevant for our analysis. Output prices do not adjust rapidly enough to maintain equilibrium on the output market. One reason is that the adjustment of output and prices involves a production process that is slower and more costly than the adjustment process on the markets for money and credit. A second reason, that we do not pursue here, is that differences in costs of acquiring information give rise to differences in the prices anticipated by buyers and sellers.¹ Third, producers delay the adjustment of output and the labor force by allowing inventories to change. Once changes in expenditure are regarded as systematic, not random events, the adjustment of output and prices accelerates.

Three equations describe the interaction of real expenditure, real output and prices on the output market. Equation (1) describes the adjustment of real output of the private sector, y , to a discrepancy between aggregate real (private) output and aggregate real expenditure, $d + g$. Total expenditure is the sum of private (d) and government (g) expenditures on the output market. Equation (2) defines real private expenditure, and equation (3) describes the price level, p . Together, equations (1) and (3) constitute the supply side of the output market, and equation (2) plus real government purchases constitute the demand side:

$$\frac{d}{dt} \log y = h[\log(d + g) - \log y]; \quad (1)$$

$$d = d(i - \pi, p, P, W_n, W_h, e) \quad d_1, d_2 < 0; \quad d_3, d_4, d_5, d_6 > 0; \quad (2)$$

$$p = p(y, K, y^*) \quad p_1, p_3 > 0; \quad p_2 < 0. \quad (3)$$

The remaining variables in the three equations are defined as follows: i is an index of nominal (or market) rates of interest; π , the anticipated rate of inflation; P , the price of existing real capital; W_n and W_h , the values of nonhuman and human wealth; e , the anticipated return on real capital per unit of real capital; K , the stock of existing real capital; and y^* , anticipated real output.

¹ Producer's and purchaser's anticipations often differ, and both sets of anticipations differ, at times, from the anticipated rate of inflation reflected by the prevailing market rate of interest. In an explanation of very short-term changes, one might wish to distinguish also between actual, anticipated, and perceived prices and similarly for wages, if the labor market is included.

The (partial) elasticity of p with respect to y measures the response of p to short-run changes in current output, holding expected output, y^* , constant. We postulate that the response of p to short-run changes is less than the long-run response (Alchian and Allen 1967; Lucas and Rapping 1969; Phelps 1968); $\epsilon(p, y) < \epsilon(p, y^*)$. The size of the short-run response rises with the rate of capacity utilization, y^*/K , and is higher in periods of sustained expansion than in periods of prolonged contraction. The short-run response of y^* to y depends on the factors determining producer's anticipations. It is sufficient for present purposes to assume that $\epsilon(y^*, y)$ does not exceed unity.

Three of the variables just introduced— W_n , W_h , and e —require additional description. Equations (4), (5), and (6) define these variables and introduce the financing of the government budget into the analysis. The presence of y^* in equation (6) makes the anticipated yield on real capital depend on the factors affecting producers' short- and long-run anticipations. Changes in anticipations and particularly sudden, sharp, or autonomous changes in producers' market anticipations were emphasized by Keynes (1936) and Wicksell (1935) as a cause of cycles:

$$W_n = PK + (1 + \omega)B + v(i)S \quad v_1 < 0; \quad (4)$$

$$W_h = W_h(y^*, p) \quad W_{h_1}, W_{h_2} > 0; \quad (5)$$

$$e = \frac{n(y^*)}{K}. \quad (6)$$

The additional variables introduced in these equations are ω , the ratio of the banking system's net worth to the monetary base;² B , the monetary base; S , the outstanding stock of government debt (at face value); and v , the price per dollar of S . Our formulation of W_h and n assumes the prevalence of some stable distribution of real income.

The descriptions of the markets for credit and money follow our previously published work (1966, 1968, 1972) and are presented here with little elaboration. The equilibrium condition for the credit market, equation (7a), equates the banks' desired portfolio (aB) to the stock of earning assets offered to banks (σ).³ This equation proximately determines the

² Real capital, K , does not include capital invested in the monetary system. The capital invested in the monetary system and the value of the banks' monopoly contribute to the positive value of ω . Discussions following Pesek and Saving (1967) have suggested different methods of calculating the net worth of the banking system but agree that the value of a banking monopoly is part of net wealth. We treat ω as a constant.

³ We assume throughout that the markets for money and credit adjust more quickly than the output market. The reason is that, in general, costs of acquiring information and costs of adjusting are substantially smaller for assets than for new production. The adjustment of output and prices, the h and p functions of our analysis, require more analytic foundation in terms of costs of information and readjustment than we have provided here.

equilibrium value of the nominal rate, i . Equations (7b) and (7c) express the asset multiplier, a , and the stock of assets offered to banks, σ , as functions of the main arguments of our model:

$$aB = \sigma; \quad (7a)$$

$$a = a(i, i_t, P, W_n, W_h, e) \quad a_1 \dots a_5 > 0; a_6 < 0; \quad (7b)$$

$$\sigma = \sigma(i - \pi, P, p, W_n, W_h, e, S) \quad \sigma_1, \sigma_2 < 0; \sigma_3, \sigma_6, \sigma_7 > 0. \quad (7c)$$

The only variable introduced but not yet defined is i_t in (7b), the interest rate on deposits. The σ equation is obtained by appropriate aggregation of the specialized credit markets for mortgages, corporate bonds, etc. Our hypothesis separates aggregative and allocative effects of portfolio allocations and assigns so little aggregative significance to the allocation of credit between submarkets that we dismiss the allocative effect on macrovariables. However, the stock of government securities is not eliminated in the aggregation; S enters (7c) with partial derivative equal to unity.

The credit market allocates the stock of government securities between banks and nonbanks and permits an individual wealth owner to adjust the composition of his wealth by borrowing or repaying loans and by buying or selling securities. Since the credit market assumes the role of (proximately) determining market interest rates most often assigned to the demand and supply equations for money, we assign to the money equation the task of (proximately) determining P , the price of existing capital and, implicitly, the net rate of return on real capital.

Equation (8a), the equilibrium condition for money, is satisfied when the desired nominal stock of money held by the public, L , just equals mB , the nominal stock supplied by banks. The latter is the product of a money multiplier, m , and the monetary base. Equations (8b) and (8c) explain the money multiplier and L in terms of the arguments previously introduced:

$$mB = L; \quad (8a)$$

$$m = m(i, e, i_t, P, W_n, W_h) \quad m_1, m_2 > 0; m_3, \dots, m_6 < 0; \quad (8b)$$

$$L = L(i, e, P, W_n, W_h, p) \quad L_1, L_2 < 0; L_3, \dots, L_6 > 0. \quad (8c)$$

Several of the arguments of the credit and money equations have different effects on the two markets. The sign of the wealth elasticity of σ is ambiguous and much smaller than the (positive) wealth elasticity of the L function. Also, the elasticities of the σ function with respect to P and e are opposite to the signs of the elasticities of the L function. Several of the signs of (8b) are opposite to the signs of (7b).

Stability of the system requires that the interest elasticity of the credit-market equations exceed the interest elasticity of the money equations. The Hotelling conditions imply that this condition holds, and recent empirical studies either support or are consistent with the condition (see Brunner and Meltzer 1968; Zwick 1971).

The last set of equations introduces the government's monetary and

fiscal operations and the financing of the government's budget. In our analysis, government is capable of altering real expenditure for output of the private sector, g , and the outstanding stocks of debt and base money. Tax rates are held constant.

Current tax collections, t , (9a) depend on real income and the price level. The monetary base, equation (9b), has two components, the central bank's holding of government securities, B_1 , and all other source components, B_2 . In the United States, B_2 consists mainly of international reserves; in other countries, special advances from the central bank to the government are often included also. Changes in B_1 and changes in S occur whenever the central bank finances a budget deficit or surplus and/or engages in open-market operations. Equations (9c) and (9d) introduce μ and v to separate open-market operations from deficit finance:

$$t = t(y, p) \quad t_1, t_2 > 0; \quad (9a)$$

$$B = B_1 + B_2; \quad (9b)$$

$$dB_1 = \mu(G - t) + v; \quad (9c)$$

$$dS = (1 - \mu)(G - t) - v; \quad (9d)$$

$$G = pg + GI(i, S) \quad GI_1, GI_2 > 0. \quad (9e)$$

The portion of a deficit, $G - t$, financed by issuing base money or the portion of a surplus used to retire outstanding base money is denoted μ ; v is the change, dB_1 , resulting from central bank purchases or sales that are independent of the government's budget deficit or surplus—"pure" open-market operations. Pure open-market purchases or sales do not occur when a budget deficit or surplus is financed by changing both B_1 and S .

Three examples of open-market purchases (or sales) bring out the way in which μ and v help to distinguish between surplus or deficit finance and pure open-market operations. First, $\mu = 1$ and $G - t > 0$; B_1 may rise by more than the current deficit. This occurs whenever increases in the base to finance the entire deficit are supplemented by pure open-market purchases; $dB_1 > -dS$ and $v = dB_1 - (G - t) > 0$. Second, $\mu = 0$ and $G - t < 0$. A budget surplus is used to retire outstanding debt. A positive value of v again describes a "pure" open-market purchase, and $-dS > dB_1 > 0$ describes the change in the composition of the public's assets. Third, if the budget is balanced, $G - t = 0$; any change in B_1 and S is a pure open-market operation; the budget imposes no constraint on v .

Total budget expenditure, G , is the sum of the government's expenditure, at current prices, on output supplied by the private sector, pg , plus interest payments on the debt, GI . To simplify the analysis, we depart from convention and treat g rather than pg as exogenous. Main results are unaffected; GI , interest payments on outstanding debt, depend on i and S with positive derivatives.⁴

⁴ The government's budget has been simplified by ignoring purchases of labor services and identifying the deficit with conventional accounting measures. Our pro-

Interaction of Prices, Stocks, and Flows on the Money Credit and Output Markets

This section develops some major properties of the system with the aid of a two-panelled diagram. One panel describes the output market and shows the relations between prices and output given by equations (2) and (3). The other panel shows the determination of partial equilibrium values of P and i obtained from equations (7) and (8). To bring out some main properties of the system, we start from a position of disequilibrium on the output market and trace the adjustment of p and y implied by the dynamics of the output market. We show that the adjustment of the output market variables disturbs the equilibria of the asset markets and changes the values of P and i that clear these markets. The changes in P and i , in turn, shift the expenditure ($d + g$) curve inducing additional changes in expenditure, output, and the price level. Throughout this section, we neglect the effect on the budget deficit of changes in prices and output.

The size and direction of the changes in p , y , P , and i and the comparative responses to these changes determine whether the adjustment process converges to an equilibrium and the speed of convergence. The slopes and the determinants of the shifts of each curve are written as elasticities. The expression $\epsilon(x, z)$ denotes the elasticity of x with respect to z .

We begin in the output market. The negatively sloped line in panel 1 is the aggregate expenditure function, $d + g$. The slope of this line is given by the elasticity $\epsilon(p, y|d + g)$ and is derived from the expenditure function $\epsilon(p, y|d + g)$

$$= \frac{1 - (1 - \gamma)[\epsilon(d, W_h)\epsilon(W_h, y^*) + \epsilon(d, e)\epsilon(e, y^*)]\epsilon(y^*, y)}{(1 - \gamma)\epsilon(d, p)}$$

The slope depends on four factors: (1) the ratio of the government's expenditure on output to total expenditure, $\gamma = g/d + g$; (2) the responses of wealth and the yield on real capital to changes in expected output; (3) the response of expected output to actual output, $\epsilon(y^*, y)$; and (4) $\epsilon(d, p)$, the price elasticity of real private expenditure. Each of the first three items is positive; the fourth is negative, so the denominator is negative.

The sign of the numerator depends on $\epsilon(y^*, y)$. In the short run, producers do not adjust production schedules to every change in expenditure, so $\epsilon(y^*, y)$ is less than unity. The numerator is, therefore, positive, and $\epsilon(p, y|d + g)$ is negative. In the long run, $\epsilon(y^*, y) = 1$, $\epsilon(p, y|d + g)$ falls, making the $d + g$ line in panel 1 flatter and possibly upward sloping.

cedure omits loans and repayments between the government sector and the private sector. Such transactions change the cash-flow deficit relative to the national income deficit and induce changes in relative prices and wealth and adjustments on asset and output markets. These operations are not adequately represented here. Other government activities, for example the activities of regulatory bodies, are also neglected.

The position of the $d + g$ line depends on real government expenditure, g , the stock of real capital, K , the monetary base, B , the stock of government securities, S , the anticipated rate of inflation, π , and of particular interest here, on the interest rate, i , and the asset price of real capital, P .

Changes in P and i shift $d + g$ vertically. The size of each shift depends on the ratio of two elasticities:

$$\epsilon(p, P|d + g) = -\frac{\bar{\epsilon}(d, P)}{\epsilon(d, p)} > 0; \epsilon(p, i|d + g) = -\frac{\bar{\epsilon}(d, i)}{\epsilon(d, p)} < 0.$$

The bar on $\bar{\epsilon}(d, P)$, $\bar{\epsilon}(d, i)$, or other elasticities indicates a *total* elasticity; $\bar{\epsilon}(d, P)$ is the sum of two components, a partial elasticity and an induced change in wealth:

$$\bar{\epsilon}(d, P) = \epsilon(d, P) + \epsilon(d, W_n) \frac{PK}{W_n} > 0.$$

If the d -function is homogeneous of zero degree in all nominal values, $\epsilon(p, P|d + g)$ is positive and less than one. The elasticity with respect to i has similar form:

$$\bar{\epsilon}(d, i) = \epsilon(d, i) + \epsilon(d, W_n) \epsilon(v, i) \frac{vS}{W_n} < 0.$$

Equiproportionate changes in P and i almost certainly shift the $d + g$ curve in the same direction as the change in P . The reason is that $\epsilon(p, P) > -\epsilon(p, i)$ by an amount that depends on two factors. One, $\epsilon(v, i)$, is less than unity as long as the maturity of the debt is finite. (The debt is not entirely perpetuities). The other is the size of PK/W_n relative to vS/W_n ; PK is considerably larger than vS .

The short-run slope of the aggregate supply function, s , depends on $\epsilon(p, y)$ of the price-setting function, equation (3). The long-run slope depends on $\epsilon(p, y) + \epsilon(p, y^*)$ and is steeper than the short-run slope. The position of the aggregate supply curve depends on y^* and K as shown in equation (3). Increases in y^* reduce s (for given p), and increases in K raise s .

Starting from any set of initial conditions, equations (1)–(3) determine aggregate expenditure, aggregate output, and a price level. Every combination is not an equilibrium position for the output market. Expenditure may exceed or fall short of output at the prevailing price level. The curves in panel 1 show an initial position of disequilibrium. Output is y_0 . The suppliers' behavior associates a price level, p_0 , with this output. At this price aggregate expenditure is y_1 , and excess demand is $y_1 - y_0$. The suppliers' behavior, described by equation (1), implies that output increases by dy/y proportional to $y_1 - y_0$. This adjustment is a movement to the right along the supply curve in panel 1. Output price rises to p_1 , and excess demand falls.

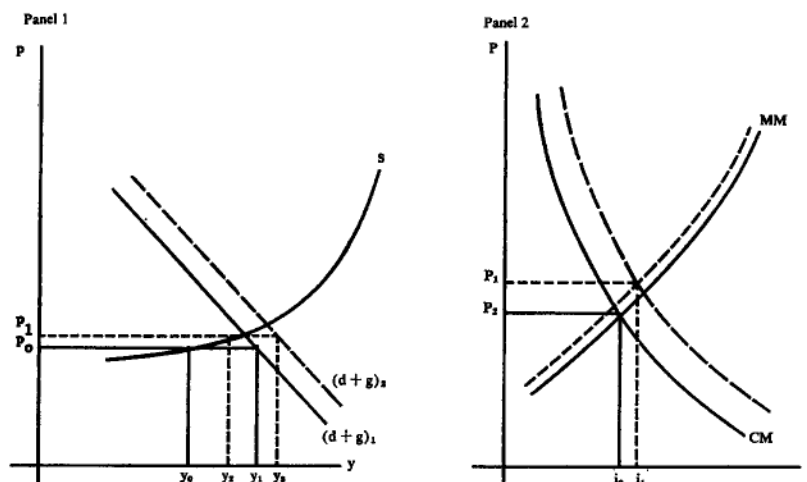


FIG. 1

The system does not converge to the intersection of s and $(d + g)_1$. The increases in p and y disturb the asset-market equilibrium and change tax collections and the size and financing of the government budget. These changes, in turn, change P and i , thereby shifting the $d + g$ curve and inducing additional changes in the budget and its financing.

Panel 2 of figure 1 shows a (partial) equilibrium position of the asset market. The CM and MM lines are the loci of P, i values that equilibrate the credit market and the money market, respectively.⁵ The slopes of the two lines are given by the elasticities $\epsilon(P, i|CM)$ and $\epsilon(P, i|MM)$. These elasticities are obtained by solving the credit-market equations (7a)–(7c) and the money-market equations (8a)–(8c) separately for P in terms of all the determinants of the demands and supplies of credit and money and expressing the result as a function of i :

$$\epsilon(P, i|CM) = -\frac{\epsilon(CM, i)}{\epsilon(CM, P)} < 0;$$

$$\epsilon(P, i|MM) = -\frac{\epsilon(MM, i)}{\epsilon(MM, P)} > 0.$$

⁵ Note that the equilibrium is a partial equilibrium only. The asset markets are in equilibrium relative to the values of P and i and the prevailing p and y . If the values of p and y are not full stock-flow equilibrium values, generally there is an excess demand or supply of money and credit relative to the equilibrium p, y combination that clears the output market. When expected prices are included in the analysis, the partial equilibrium position of the asset markets must be defined relative to the expected prices. The expected prices may diverge from the actual prices and from the prices implied by the rate of inflation, π , anticipated on the asset markets.

The numerators and denominators of the two elasticities consist of interest-rate and asset-price elasticities of the demand and supply equations for money and credit. Each total elasticity includes the induced change in wealth:

$$\begin{aligned}\epsilon(CM, i) &\sim \bar{\epsilon}(a, i) - \bar{\epsilon}(\sigma, i - \pi) \frac{i}{i - \pi} > 0; \\ \epsilon(MM, i) &= \bar{\epsilon}(m, i) - \bar{\epsilon}(L, i) > 0; \\ \epsilon(CM, P) &= \bar{\epsilon}(a, P) - \bar{\epsilon}(\sigma, P) > 0; \\ \epsilon(MM, P) &= \bar{\epsilon}(m, P) - \bar{\epsilon}(L, P) < 0.\end{aligned}$$

The four elasticities have an immediate economic interpretation. The $\epsilon(CM, i)$ and $\epsilon(CM, P)$ show the response of the credit market's excess supply with respect to interest rates and asset prices. Increases in i and P increase excess supply on the credit market. The $\epsilon(MM, i)$ and $\epsilon(MM, P)$ describe similar elasticities for the money market's excess supply. The money market excess supply increases with interest rates and declines with asset prices. The signs of the four elasticities of excess supply are uniquely determined by the constraints imposed on equations (7) and (8) and the comparative effects of changes in wealth on money and credit. These constraints assure that the MM curve is positively sloped and that the CM curve is negatively sloped. Moreover, the Hotelling conditions imply that $\epsilon(P, i|CM) > \epsilon(P, i|MM)$.

The slopes of the CM and MM curves and the relative size and the direction of changes in the two curves determine whether the feedback from the asset markets to the output market accelerates or decelerates the adjustment of prices and output or reverses the direction of change. The larger the change in P , the larger is the change in $d + g$ induced by the response of the asset market to the disequilibrium on the output market. Any change in the position of the CM or MM curve also changes market interest rates. A rise in i reduces the size of the change in $d + g$ induced by an increase in P ; a sufficiently large increase in i reverses the direction of change, reducing expenditure and the size of the subsequent change in y , p , and dy/y . A decline in interest rates reinforces the effect of a rise in P .

Let $(dP/P)(CM)$ and $(dP/P)(MM)$ denote the relative changes in the positions of the CM and MM curves, respectively, measured by the changes in the vertical intercepts. The combined effect of the two shifts determines the change in P and i . Equations (10a) and (10b) show that the size of the shift in CM and MM depends on the budget deficit, $G - t$, the stocks of financial assets, B and S , and via μ , on the changes in the two stocks required to finance the deficit. In addition, any disequilibrium on the output market shifts the CM and MM curves by changing dy/y and, thereby, changing the demand and supplies for money and credit:

$$\frac{dP}{P}(CM) = \epsilon(P, y|CM) \frac{dy}{y} + \frac{G-t}{B} \left[\mu \epsilon(P, B|CM) + (1-\mu) \epsilon(P, S|CM) \frac{B}{S} \right]; \quad (10a)$$

$$\frac{dP}{P}(MM) = \epsilon(P, y|MM) \frac{dy}{y} + \frac{G-t}{B} \left[\mu \epsilon(P, B|MM) + (1-\mu) \epsilon(P, S|MM) \frac{B}{S} \right]. \quad (10b)$$

The broken lines in panel 2 of figure 1 show an upward shift of CM and MM that moves the intersection to the northeast; P and i increase, and P increases relative to i . The effect of these changes on the output market is shown by the broken lines in panel 1. Since P increases relative to i and $\epsilon(p, P|d+g) > -\epsilon(p, i|d+g)$, as noted earlier, real expenditure increases. The increase is shown by the position of the $(d+g)_2$ curve. At price level p_2 , real expenditure is y_3 , output is y_2 , and excess demand is $y_3 - y_2$. The changes in y and p induce additional changes in P and i . Adjustment continues.

The diagram cannot establish that the relative changes in P and i implied by the shifts of CM and MM are the changes implied by our hypothesis. Since the relative size of changes in P and i is a principal determinant of the direction of change in $d+g$, additional analysis is desirable. To show that, for $dy/y > 0$, the shifts of CM and MM are those shown in the diagram, we analyze the components of $(dP/P)(CM)$ and $(dP/P)(MM)$ in more detail. There are three principal components: (1) the response to a change in output, the output effect; (2) the monetary effect of deficit finance; and (3) the debt effect of deficit finance.

The Output Effect

The output effect is the response of P and i to dy/y . The first terms of $(dP/P)(CM)$ and $(dP/P)(MM)$, $\epsilon(P, y|CM)$ and $\epsilon(P, y|MM)$, determine the size and direction of this response. Each term can be written as the ratio of two elasticities obtained from the solutions of equations (7) and (8):

$$\epsilon(P, y|CM) = \frac{\epsilon(CM, y)}{\epsilon(CM, P)} > 0; \quad \epsilon(P, y|MM) = \frac{\epsilon(MM, y)}{\epsilon(MM, P)}.$$

The denominators were shown above to be responses on the money and credit markets to changes in P . The numerators are, similarly, the responses of excess supplies on the money and credit markets to changes in y . Each numerator combines the responses of the demand and supply for

money or credit to the changes in W_h , e , p , and y^* resulting from the adjustment of the output market:

$$\begin{aligned}\epsilon(CM, y) &= \epsilon(y^*, y) \{ [\epsilon(\sigma, W_h) - \epsilon(a, W_h)] \epsilon(W_h, y^*) \\ &\quad + [\epsilon(\sigma, e) - \epsilon(a, e)] \epsilon(e, y^*) \} \\ &\quad + \epsilon(\sigma, p) \epsilon(p, y) > 0; \\ \epsilon(MM, y) &= \epsilon(y^*, y) \{ [\epsilon(L, W_h) - \epsilon(m, W_h)] \epsilon(W_h, y^*) \\ &\quad + [\epsilon(L, e) - \epsilon(m, e)] \epsilon(e, y^*) \} \\ &\quad + \epsilon(L, p) \epsilon(p, y).\end{aligned}$$

Both the numerator and denominator of $\epsilon(P, y|CM)$ are positive, so the effect of changes in output on the CM curve is unambiguous. The CM curve rises and falls with dy/y . The shift of the CM curve raises i and P in periods of expansion or recovery, as shown in figure 1, and lowers i and P during recessions.

The denominator of $\epsilon(P, y|MM)$ was shown to be negative. The sign of the numerator depends mainly on $\epsilon(L, e)$ and $\epsilon(p, y)$. If the demand for money is very responsive to changes in the expected yield on real capital—if real capital and money are close substitutes— $\epsilon(L, e)$ is large and negative. Whenever the (negative) sign of $\epsilon(L, e)$ dominates the sign of $\epsilon(MM, y)$, $\epsilon(P, y|MM)$ is positive. Increases in output shift the MM curve in the direction shown in figure 1. Rising output raises P and lowers i ; falling output lowers P and raises i . The size of the shift in the MM curve depends on the extent to which anticipations of the future are affected. If most of the change in output is expected to persist, $\epsilon(y^*, y)$ increases, and the size of the shift in the MM curve increases also.

A sustained rise in output raises the rate of capacity utilization. At high rates of utilization, $\epsilon(p, y)$ becomes large, as shown by the slope of the s curve in panel 1. Beyond some point, the rise in $\epsilon(p, y)$ changes the sign of $\epsilon(MM, y)$ from negative to positive; $\epsilon(P, y|MM)$ becomes negative. Further increases in prices and output shift the MM curve to the right, increasing i and reducing P .

Interest rates and asset prices generally rise in periods of expansion and fall during contractions. Two conditions are needed to assure this result. One is $|\epsilon(MM, P)| > \epsilon(CM, P)$. The Hotelling conditions and the comparative effect of wealth on money and credit imply that this condition is met.⁶ The second is $\epsilon(CM, y) > \epsilon(MM, y)$. We know from the discussion just above that within the range of capacity utilization observed during mild cycles, the sign of $\epsilon(MM, y)$ depends on both $\epsilon(L, e)$ and

⁶ The $\epsilon(L, W_n)$ exceeds $\epsilon(\sigma, W_n)$. The former is approximately unity; the latter contains two offsetting components in our construction of the σ -function. One is the effect of wealth on the public's holdings of securities; the other is the effect of wealth on borrowing.

$\epsilon(p,y)$, so that $\epsilon(MM,y)$ is likely to be negative and smaller than $\epsilon(CM,y)$. This is the case shown in figure 1. However, it is not the only possible outcome under our hypothesis. In the recovery from a major recession or in the late stages of an expansion, the relative size of the shifts in the *CM* and *MM* curves or the direction of change may differ from those shown.

Following a major recession, output is at very low levels relative to capacity, and $\epsilon(p,y)$ is very small. Under these conditions, the shift of the *MM* curve, $(dP/P)(MM)$ may be larger than the shift of the *CM* curve, $(dP/P)(CM)$, so that interest rates fall and asset prices rise during the early stages of a recovery. As output rises relative to the fixed capital stock, $\epsilon(p,y)$ rises and $\epsilon(MM,y)$ falls; the size of the shift of the *MM* curve declines and the size of the shift of the *CM* curve increases; P and i once again rise and fall with dy/y , as in figure 1.

In the late stages of an expansion $\epsilon(p,y)$ eventually dominates $\epsilon(MM,y)$, changing the latter elasticity from negative to positive and turning $(dP/P)(MM)$ negative. Interest rates rise relative to the asset price, P ; in the limit, interest rates rise and P remains constant.

The conditions required for rising interest rates and constant asset prices cannot persist. Before the limiting point is reached, the effects of P and i on expenditure cancel. Expenditure remains stationary, and the output effect is exhausted. For any given K , B , and S , the output effect and the output adjustment produce a convergent movement of aggregate expenditure, and of the *CM* and the *MM* curves, toward a consistent, maintainable stock-flow equilibrium. Any increase in y^* contributes to the convergence by shifting the supply curve upward and to the left.

The output effect implies that, generally, asset prices and interest rates rise in periods of expansion and fall in contractions. Any effect of anticipations of inflation or of deflation on market rates and asset prices adds to the output effect and increases the size of the changes. However, even if anticipations of price change form and decay as slowly as some empirical evidence suggests, our hypothesis implies that changes in output can produce the observed pattern of changes in interest rates and asset prices by changing $\epsilon(CM,y)$ relative to $\epsilon(MM,y)$.

The Monetary Effect of Deficit Finance

The second group of terms in equations (10a) and (10b) makes the size and direction of the changes in *CM* and *MM* depend on the budget deficit and the portions of the deficit financed by issuing base money and bonds. In this and the following subsection, we hold output and the deficit constant and consider two polar cases; $\mu = 1$, the deficit is financed by issuing base money; and $\mu = 0$, the deficit is financed by issuing bonds. Later, we relax these constraints, allowing the deficit to change as output and

prices change and combining the effects of financing the deficit with the output effect.

Issuing base money to finance the deficit increases the stocks of money and credit. The *CM* curve in figure 1 shifts to the left, and the *MM* curve shifts to the left. Interest rates fall. The direction of change in asset prices depends on the relative size of the two shifts. From equations (10a) and (10b) above, we see that the direction of the change in *P* depends on two elasticities, $\epsilon(P, B|CM)$ and $\epsilon(P, B|MM)$:

$$\epsilon(P, B|CM) \sim -\frac{1}{\epsilon(CM, P)} < 0;$$

$$\epsilon(P, B|MM) = -\frac{1}{\epsilon(MM, P)} > 0.$$

Since $|\epsilon(MM, P)| > \epsilon(CM, P)$, the change in *P* induced by the shift of the *MM* curve is the smaller of the two; $(dP/P)(MM) < |(dP/P)(CM)|$. Financing a deficit by issuing money raises asset prices and lowers interest rates. The monetary effect of deficit finance reinforces the output effect whenever rising output and a budget deficit occur together. At these times deficit finance increases the shift of $d + g$ and the size of subsequent adjustments on the output market.

We can confirm the effect of the base on interest rates and asset prices by solving the two asset-market equations simultaneously. The solutions are denoted $\epsilon(P, B|AM)$ and $\epsilon(i, B|AM)$ to show that they combine

$$\epsilon(P, B|AM) \sim \frac{\epsilon(MM, i) - \epsilon(CM, i)}{\epsilon(CM, i)\epsilon(MM, P) - \epsilon(MM, i)\epsilon(CM, P)} > 0 \quad \text{and}$$

$$\epsilon(i, B|AM) \sim \frac{\epsilon(CM, P) - \epsilon(MM, P)}{\epsilon(CM, i)\epsilon(MM, P) - \epsilon(MM, i)\epsilon(CM, P)} < 0,$$

all of the changes in excess demand and supply on the credit and money markets induced by the movement from one asset-market equilibrium to another. The denominators are negative. The numerator of $\epsilon(P, B|AM)$ is independent of the magnitude of the elasticities and depends only on their relative size. A very large interest elasticity of the demand function for money contains no implication for the effectiveness or ineffectiveness of monetary policy.⁷

⁷ The reason is exactly the same as the reason that led us to reject as impossible a liquidity trap in the demand function for money (Brunner and Meltzer 1968, p. 18). The approximation in the formulas results from the neglect of wealth terms. Let equations (7) and (8) be linear in the logarithms. Differentiating by *B* shows that the omitted terms are very small. For example $\epsilon(CM, i)$ in the numerator of $\epsilon(P, B|AM)$ is multiplied by $1 + [\epsilon(m, W_n) - \epsilon(L, W_n)](1 + \omega)B/W_n$. The expression is reduced toward unity by the low value of B/W_n and does not exceed 1.05.

The Debt Effect of Deficit Finance

Financing the deficit by issuing debt to the public shifts the *CM* curve to the right and the *MM* curve to the right. Both shifts have the same effect on market interest rates. Interest rates rise. The effect of the two shifts on asset prices cannot be determined unambiguously from equations (10a) and (10b). The simultaneous solution of the asset-market equations shows that *P* rises in response to an increase in *S* under rather general conditions.⁸ If the effect of debt finance on interest rates is large relative to the effect on asset prices, the debt effect decelerates the adjustment of the output market. For the (partial) effect of issuing debt to *reduce* $d + g$, a more stringent condition must be met.⁹

Preliminary Conclusion

In the introduction we listed a number of propositions that distinguish our framework from the standard *IS-LM* paradigm. The basis for several of these propositions is now clearer. One reason that the interest elasticities of the expenditure and demand for money functions are neither necessary nor sufficient for determining the relative responses to fiscal and monetary policies (proposition 1) is that changes in the base and in the stock of debt, whether the result of open-market operations or deficit finance, change asset prices and shift the demand for money, the expenditure function, and the asset-market curves. These shifts, a result of the interaction between the markets for assets and output, also explain why the real balance effect is not necessary or sufficient for a positive response of output to a change in the base (proposition 2). The dominant change in wealth results from the change in asset prices relative to the price of new output (proposition 3).

We have also shown that a constant deficit financed by issuing debt raises market interest rates under our hypothesis (proposition 4). And we have worked throughout with a system in which the stock of money—currency and demand deposits—is an endogenous variable dependent on the monetary base, interest rates, asset prices, and other variables. Yet, there is no point at which any main conclusion of our analysis would be altered if the stock of money was a constant multiple of the base and independent of any feedback from the output or asset markets (proposition 7).¹⁰

⁸ The effect of *S* on *i* and *P* is given by $\epsilon(i, S|AM) \sim [\epsilon(CM, P)\epsilon(L, W_n)(vS/W_n - \epsilon(MM, P)]/den > 0$, $\epsilon(P, S|AM) = [\epsilon(CM, i)\epsilon(L, W_n)(vS/W_n) - \epsilon(MM, i)\epsilon(\sigma, S)]/den$, where *den* is the determinant of the asset market matrix and is shown in the denominator of $\epsilon(P, B|AM)$ in the text. The *den* is negative: $\epsilon(P, S|AM) > 0$ if $\epsilon(CM, i)/\epsilon(MM, i) < (W_n/vS)$.

⁹ The condition is $\bar{\epsilon}(d, P)[dP(S)/P] < -\bar{\epsilon}(d, i)[di(S)/i]$, where $dP(S)/P$ and $di(S)/i$ are the effects of debt finance on *P* and *i*.

¹⁰ One reason that the issue about the endogeneity of money persists is that the

The interaction of the asset and output market discussed in this section produces a movement toward an output-market equilibrium. We have shown that this movement converges and that the speed of convergence depends on the method chosen to finance the deficit, the choice of μ . The larger the value of μ , the larger the increase in money and the greater the size of the feedback from the asset markets to expenditure and output. The smaller the value of μ , the slower the speed of convergence on the output market.

The choice of μ also affects the equilibrium position of the asset market, both directly and by changing the speed of convergence of the output market. It is clear that, with a given budget deficit and all other conditions unchanged, a low value of μ implies a slow process of adjustment to the full stock-flow equilibrium—a long lag in the adjustment of output to the deficit (proposition 6).

However, we have not followed the adjustment of the asset market to a new equilibrium or shown that *both* the asset and output markets converge to a consistent and sustainable equilibrium. In the following section we continue our discussion of the stock-flow interaction and develop the role of the budget more fully.

Short- and Long-Run Equilibrium

A disequilibrium in the output market disturbs the equilibrium of the asset market and sets off a process that moves the output market to a new equilibrium position. In the previous section we developed the response elasticities of the output- and asset-market equations that determine the direction and the speed of adjustment. Throughout that discussion we held the budget deficit constant and ignored the effect on the size of the deficit of changes in output, the price level, and other variables induced by the adjustment of the asset and output markets. Generally, we treated

discussion rarely separates three distinct meanings of "endogeneity." One is the meaning used in the text. This interpretation implies that the feedbacks to the stock of money, through the effect of interest rates, asset prices, and other variables on the money multiplier, do not change the qualitative implications obtained from the analysis. Both time series analysis (Brunner and Meltzer 1968) and spectral analysis (Turner 1972) suggest that the feedbacks via the multiplier are small. A second meaning of "endogeneity of money" is that the base depends on *current* income, interest rates, or asset prices. At times, the argument is made that the central bank cannot control the base. The most common form of this argument applies to an open economy and states that short-term capital movements prevent a country from controlling the base. This argument implies that in the short run, $dB_2 = -dB_1$. Studies by Fratianni (1971) and Neumann (1971) for Italy and Germany provide evidence that in these open economies, the feedback via dB_2 does not entirely offset dB_1 within a year. A third meaning of endogeneity is that a central bank can only control the base if it relinquishes short-term "control" of market interest rates. This meaning improperly mixes "control" and "endogeneity" and focuses attention on the motives of central bankers. In our analysis, whatever central bankers do and for whatever reason they do it, the choices they make are entirely described by μ , ν and dB_2 .

each of the asset markets separately and did not make use of the properties of a simultaneous solution.

We now extend the analysis to include adjustments on the asset and output markets resulting from induced changes in the deficit. The response elasticities and the constraints required by our hypothesis move the system toward short- and long-run equilibrium positions. In this section we develop the dynamic implications more fully and analyze the properties of the equilibrium position. To bring out some main differences in implications, we reduce the asset- and output-market equations to two relations in the familiar i, y plane.

Panel 1 of figure 2 shows the asset market (AM) and output market (OM) relations as positively and negatively sloped curves, respectively. The AM relation is obtained by solving the credit and money market equations simultaneously for i and P , at the prevailing level of output. Corresponding to each position of the output market—whether an equilibrium or disequilibrium position—there are values of i and P that clear the asset markets. The slope of the AM curve, expressed as an elasticity, is given by $\epsilon(i, y|AM)$, the elasticity of i with the respect to y . The position of the curve depends on K , B and S :

$$\epsilon(i, y|AM) = \frac{\epsilon(CM, y)\epsilon(MM, P) - \epsilon(MM, y)\epsilon(CM, P)}{\epsilon(CM, i)\epsilon(MM, P) - \epsilon(MM, i)\epsilon(CM, P)} > 0.$$

The slope of the AM curve, $\epsilon(i, y|AM)$, is the “output effect” on interest rates discussed in the previous section. We concluded there that output, interest rates, and asset prices generally rise and fall together, so a positive slope of the AM curve is expected. The denominator is negative. Its components are the four elasticities of excess-supply on the credit and money markets. A negative sign for the numerator is assured by two inequalities discussed previously: (1), $|\epsilon(MM, P)| > \epsilon(CM, P)$; and (2), $\epsilon(CM, y) > |\epsilon(MM, y)|$. The Hotelling condition implies the first inequality. The second inequality is generally expected to hold. Even if it does not hold, the numerator remains negative unless

$$\frac{|\epsilon(MM, y)|}{\epsilon(CM, y)} > \frac{|\epsilon(MM, P)|}{\epsilon(CM, P)}.$$

The main implication of this condition is that the AM curve is negatively sloped only if the demand function for money is considerably more responsive to the expected yield on real capital than to the price of existing real capital.¹¹ This seems unlikely, and a positive slope seems assured at all values of y . The AM curve, therefore, shows a “flatter,” but never a “flat,” segment at low levels of output.

¹¹ The reason is that $\epsilon(L, e)$ is a component of $\epsilon(MM, y)$ and $\epsilon(L, P)$ is a main component of $\epsilon(MM, P)$. A large negative value for $\epsilon(MM, y)$ occurs only if $\epsilon(L, e)$ is very large and $\epsilon(p, y)$ is very small.

The AM curve is the closest analogy we can find in our system to the familiar LM curve. A principal difference between the two is that the slope of AM does *not* determine the size of the response to fiscal policy. Any change in government expenditure, or in any other variable that disturbs the equilibrium (or disequilibrium) position of the output market, also shifts the position of the AM line. The adjustment to the disturbance depends on the slope, or elasticity, $\epsilon(i, y|AM)$, but is not determined solely by this slope. The financing of fiscal policy changes B and S , and therefore P , and shifts the position of the AM curve.

The OM curve in figure 2 is a locus of equilibrium positions for the output market and is obtained by restricting our previous analysis in two ways. (1) Every point on the OM line is a position of flow equilibrium, $y = d + g$. (2) We replace P in the expenditure function with the solution for P obtained (jointly with i) from the simultaneous solution of the two asset-market equations. The substitution makes the slope as well as the position of the OM curve depend on the solution for P :

$$\begin{aligned} \epsilon(i, y|OM) = & [1 - (1 - \gamma) \{ \epsilon(d, p) \epsilon(p, y) + \epsilon(d, P) \epsilon(P, y|AM) \\ & + \epsilon(y^*, y) [\epsilon(d, W_h) \epsilon(W_h, y^*) + \epsilon(d, e) \epsilon(e, y^*)] \}] / \\ & (1 - \gamma) \epsilon(d, i) \frac{i}{i - \pi} < 0. \end{aligned}$$

The denominator of $\epsilon(i, y|OM)$ is negative. The sign of the numerator depends on three terms inside the braces. The first is negative. The second depends on $\epsilon(P, y|AM)$, the "output effect" on asset prices that we discussed in the previous section. We noted there that the output effect is generally positive. However, we also noted that $\epsilon(P, y|AM)$ declines as output expands because rising capacity utilization raises one component, $\epsilon(p, y)$. The increase in $\epsilon(p, y)$ also increases the absolute value of the first term inside the braces. As a result, $\epsilon(i, y|OM)$ becomes increasingly negative as output expands. The third group of terms is positive but relatively small in the short run. As $\epsilon(y^*, y)$ rises, the third term partly offsets the increase in the negative value of the first two terms.

The first term dominates the numerator if the private expenditure (d) function is homogeneous of zero degree in all nominal values. The reason is that homogeneity of zero degree implies that $\epsilon(d, p) = - [\epsilon(d, P) + \epsilon(d, W_n) + \epsilon(d, W_h)] > 0$. We conclude that the numerator is positive. The OM curve is more negatively sloped at high than at low levels of output. If rising output is accompanied by rising anticipations of inflation, π increases. The increase in π increases the change in interest rates obtained with a given change in output.

The slope of the OM curve does not fully describe the response to a change in the base resulting from monetary policy operations or the financing of a budget deficit. Every monetary policy operation and financ-

ing of fiscal policy alters the equilibrium value of P that clears the asset market. Changes in B or S also shift OM by changing nominal W_n . These real balance and real indebtedness effects are weighted by $(1 + \omega)(B/W_n)$ and vS/W_n , respectively, and are, therefore, much smaller than the relative price and wealth effects induced by changes in P .

Each OM line divides the i, y plane into three distinct sections. The points on any OM line are positions of equilibrium in the output market, given the prevailing budget position and asset price level that determine the position of the line. At points above and to the right of the line, $y > d + g$, the adjustment of the output market reduces output relative to equilibrium output defined by the line. At points below any line, $y < d + g$, adjustment of the output market raises output. Every change in the position of the OM line changes the equilibrium position and the division of the i, y space.

The third relation shown in figure 2 is obtained from the budget equation. Panel 2 shows the nominal deficit, $G - t$, as a function of real output. Since we treat real government expenditure for goods and services, g , as a policy variable, g is given. The slope of the curve in panel 2 depends on three factors: (1) the effect of price changes on nominal expenditure, (2) the effect of price and output changes on tax collections, and (3) the size of interest payments, GI , required to service the outstanding debt:

$$\begin{aligned} d(G - t) = & \{ \epsilon(p, y) p g - [\epsilon(t, p) \epsilon(p, y) + \epsilon(t, y)] t \\ & + \epsilon(GI, i) \epsilon(i, y | AM) GI \} \frac{dy}{y} \\ & + [\epsilon(GI, i) \epsilon(i, S | AM) + \epsilon(GI, S)] GI \frac{(1 - \mu)(G - t)}{S} \\ & + \epsilon(GI, i) \epsilon(i, B | AM) GI \frac{\mu(G - t)}{B}. \end{aligned}$$

The slope of the curve in panel 2 reflects the point so often made in discussions of built-in stabilizers and the full-employment budget concept: The deficit depends on the level of output, increasing as output falls and decreasing as output rises. However, in our analysis, the relation between output and the deficit also depends on the method chosen to finance a deficit (or surplus). A reduction of B , $dB = \mu(G - t)$, or increase in S , $dS = (1 - \mu)(G - t)$, shifts the curve to the right, increasing the deficit or surplus at a given level of output; an increase in B or reduction in S moves the curve to the left. The method of financing the deficit also changes the slope of the curve by changing i and interest payments, GI . An increase in GI makes the curve steeper, accelerating the deficit, and a decrease makes the curve flatter.

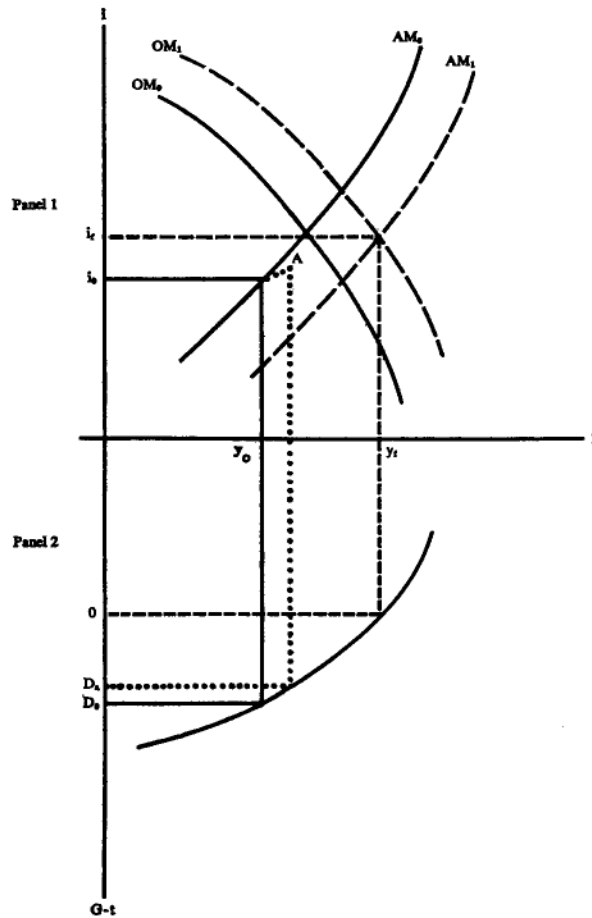


FIG. 2

Our discussion of the AM , OM , and $G - t$ curves brings out the interdependence of the system. Every change in the output market changes y and p , shifts the position of the AM curve, and changes the size of the budget deficit or surplus. Every change in the (partial) equilibrium of the asset markets changes output and the financing of the deficit. Every change in the deficit or surplus changes real expenditure, tax collections and the amounts of base money, and debt issued or withdrawn. These changes affect both the output and asset markets.

To simplify analysis of the interactions, we assume that the economy is closed, $dB_2 = 0$; μ and g are constant, and $v = 0$. We relax some of these restrictions presently.

Suppose some random event reduces output. Let the point i_0, y_0 in figure 2 be the position reached after the event. The output market is in dis-

equilibrium. Output, y_0 , is less than y_f , the level of output that maintains stock-flow equilibrium. At i_0, y_0 , output is also below the short-run equilibrium position of the output market given by the solid line, OM_0 . Since y_0 is below OM_0 , $d + g$ exceeds y_0 at prevailing prices and interest rates, and output rises. If B , S , and K remain unchanged, adjustment proceeds along the AM_0 line toward the intersection with OM_0 .

The asset market is in short-run equilibrium at prices i_0, P_0 , and output y_0 . The decline in output from y_f to y_0 reduces asset prices and market interest rates below the long-run equilibrium position at the intersection of AM_1 and OM_1 . At output y_0 , the budget deficit is D_0 and is found by following the solid lines to $G - t = D_0$ on panel 2.

Adjustment of the output market raises y and p . The dotted line from i_0, y_0 to the point A in panel 1 shows the direction of change in y and i . Note that the adjustment does not proceed along the AM line to the nearest OM line. The reason is that the financing of the budget deficit, D_0 , increases B and S , changing P and i and shifting the AM line to the right (direction of higher y). The size of the changes in P and i depend on the elasticities discussed in the previous section and on the choice of μ . The OM line also shifts to the right (direction of higher y).

The point A is an equilibrium position for the asset market and lies on an AM line (not shown) but not on an OM line. At A , real expenditure exceeds output; prices and output continue rising. The budget deficit is now D_a and requires smaller addition to the base and the stock of securities. The adjustment continues.¹²

Where does the adjustment come to an end? The AM curve continues to shift as long as new issues of debt and base money must be absorbed. Any change in the position of AM changes P and i . The OM curve cannot remain fixed if P changes, and P cannot remain fixed if output, the price level, or the budget change. Equations (10a) and (10b) above show that $dP/P = 0$ when $dy/y = 0$ and $G = t$. In every position of stock-flow equilibrium, the budget must be balanced.

The broken lines of figure 2 show a stock-flow equilibrium. The budget is balanced, so there are no issues of debt and money to shift the position of the AM curve. With the asset market in equilibrium, P and i are constant, and the position of the OM curve is no longer disturbed by changes in P ; with the output market in equilibrium, the asset market is no longer disturbed by changes in y and p . Once there is a full stock-flow equilibrium, the interest rate and level of output remain constant. In figure 2, this occurs only at the intersection of the broken AM and OM lines and with a balanced budget.

Both the final position reached by the economy and the speed of adjust-

¹² In fig. 2 and our discussion, we neglect the effect of i and S on GI and of GI on the slope of the budget line.

ment depend on the way in which the deficit is financed (proposition 5). If the deficit is financed by issuing debt, $\mu = 0$, the budget line in panel 2 moves to the right. If $\mu = 1$, the deficit is financed by issuing base money, and the budget line moves to the left. The *total* change of y and p from $y_0 p_0$ to the long-run stock-flow equilibrium, $y_f p_f$, increases as μ declines from one to zero. Since real resources are fixed, the effect of a larger issue of debt is a larger increase in p .¹³

The speed of adjustment also depends on the choice of μ . The elasticity of output with respect to the base is a multiple of the elasticity of output with respect to the stock of securities. Hence, the response of y and p per unit time—and the speed of adjustment to equilibrium—declines as μ falls.

A more formal demonstration makes the conditions for equilibrium somewhat clearer. There are five equilibrium conditions in our system:

$$\begin{aligned} y &= d(i, P, y \dots) + g; \\ p &= p(y \dots); \\ a(i, y, P \dots)B &= \sigma(i, P, y, p, S \dots); \\ m(i, y, P \dots)B &= L(i, P, y, p, \dots); \\ G(y, i, S, g \dots) &= t(y, p \dots). \end{aligned}$$

For given anticipations and policy variables and given K , the long-run stock-flow equilibrium has five equations in six endogenous variables, y, p, i, P, B , and S .¹⁴ Let D^* be the cumulative deficit. Starting from some initial period with stocks of base money and securities, B_0 and S_0 , the equilibrium stocks of base money and debt are $B = B_0 + \mu D^* + v$; $S = S_0 + (1 - \mu)D^* - v$, where μ and v are policy variables and v is now defined relative to the initial period. Using these two equations to replace B and S reduces the number of endogenous variables to five, y, p, i, P , and D^* . The necessary condition for long-run stock-flow equilibrium is satisfied.

We may now relax the constraints on B_2, μ, v , and g . In an open economy

¹³ The analysis implies that inflation or deflation can occur without any change in B . The size of the price change induced by a change in S depends on the elasticity $\epsilon(p, y)$ and would be modified if we allowed for the effect on prices and output of capital accumulation and anticipations. But the choice of μ affects the price level even if these effects are admitted. Price-level changes of this kind have not been important. Our analysis of inflation, presented at the Universities-National Bureau Conference on Secular Inflation, analyzes the issue in more detail and explains why most inflations or deflations have resulted from changes in money.

¹⁴ Our system requires adjustment to deal with a maintained inflation in which prices rise at a steady rate. Anticipations of inflation have to be included in the equations for the asset and output markets. Tax rates have to be included in the tax function so that tax rates can be reduced. If tax rates are progressive and taxes are not reduced, the government budget equations generate a surplus that disturbs the stock-flow equilibrium. The reason is that, with g fixed, $p g$ rises at the rate of inflation and tax collections rise at a greater rate.

y and d must be redefined to include exports and imports. With fixed exchange rates, financial flows induced by the balance-of-payments position change B_2 . Changes in B_2 affect the system in much the same way as changes in B_1 , by shifting the AM and OM lines, changing P, i, p, y , and D^* . If policy makers change μ , the composition of nominal wealth and the method of financing the deficit change. The AM and OM lines in figure 2 shift with changes in μ , just as they do with constant μ and the changes in B and S we have considered. In addition, the choice of μ affects the position of the $G - t$ curve by changing i and S . The equilibrium prices of assets and output and the equilibrium interest rate also depend on the choice of μ . An increase in the (average) value of μ raises the price level and reduces the market rate of interest in the short run. Open-market purchases or sales that are independent of deficit finance change v , thereby changing B and S in opposite directions and, again, shifting the AM and OM curves. Increases or decreases in g change expenditure and therefore the prices of assets and output and the size of the budget deficit. Further consideration of the responses to B_2, μ, v , and g , however, requires analysis of monetary and fiscal policies, deficit finance, and operating decisions of the central bank, issues that take us beyond the scope of the present paper.

Conclusion

The framework developed in this paper differs from the standard *IS-LM* framework in several principal ways. There are two asset markets and three prices—the prices of real assets, financial assets, and current output. Wealth owners are permitted to choose between money, bonds, real capital, and current expenditure. The real value of the outstanding stock of government debt does not equal the (discounted) present value of future tax liabilities. Costs of adjustment and costs of acquiring information prevent the output market from adjusting immediately. Excess demand or supply on the output market drives prices and output up or down but does not instantaneously restore equilibrium.

Differences in the framework produce differences in implications. Some of the principal implications are stated in the introduction and discussed in our preliminary conclusion above. Others concern the transmission of fiscal and monetary policies, the role of the credit market in the determination of interest rates, and the determination of equilibrium and disequilibrium values for prices and output.

We discard the notion that fiscal policies work “directly” while monetary policies work “indirectly.” Both types of policy change the relative prices of assets and output. Relative price changes set off a process of adjustment that continues until a new stock-flow equilibrium is reached.

Interest rates typically rise in periods of economic expansion and fall in

contraction. The *IS-LM* analysis explains cyclical changes in market rates by introducing anticipations of inflation or deflation. Our analysis recognizes that changes in the anticipated rate of inflation can explain the observed movements of interest rates, but anticipations are not required for the explanation. Changes in the banks' demand for earning assets (credit) and the public's supply of earning assets to banks offer an alternative—and to us a more credible—explanation of the changes observed during periods of mild inflation or constant prices that characterize much of the peacetime history of the United States.

A main failure of the standard analysis is the failure to determine both prices and output when the economy is not at "full employment" output. Keynes resolved the problem by making prices and money wages "rigid downward" so that his system determines real output when expenditure is below full employment output and determines prices when expenditure is above full employment. A central point of his analysis is that the distribution of the effect of monetary and fiscal policy between prices and output depends on the rate of capacity utilization, but the point is made only by denying any effect of price changes on output. Recent attempts to resolve the problem introduce costs of search, adjustment, and acquisition of information in the labor market and the supply function for output. Our analysis builds on these important developments, adds an explicit analysis of the asset markets and the interaction of stocks and flows, and offers a consistent explanation of stocks and flows, relative and absolute prices.

Appendix

Definition of Symbols

Variables Determined by the Model

Flows or changes in stocks:

y = aggregate real (private sector) output

d = aggregate real private expenditure

t = nominal value of tax accruals

dB_1 = change in security portfolio of central bank

dS = change in stock of government debt outstanding

G = government expenditure at nominal value

$\gamma = g/d + g$

GI = interest payments on government debt

Stocks:

W_h = nominal human wealth

W_n = market value of nonhuman wealth

L = stock of money

σ = stock of bank credit

B = monetary base

S = nominal stock of debt outside the government sector at par value

Prices and yields:

- i = market rate of interest
- P = price of existing capital
- p = price level of new production
- e = expected real net yield on real capital per unit of real capital
- n = expected real net yield on real capital

Variables Taken as Given in the Analysis

Policy variables:

- μ = proportion of a deficit (surplus) financed by issuing (withdrawing) base money
- v = changes in the central bank portfolio that are independent of activities financing the budget deficit or surplus
- g = real value of government purchases

Predetermined values and anticipations:

- π = anticipated rate of price change
- y^* = anticipated real income
- ω = net worth multiplier for the banking system
- B_2 = other sources of the monetary base (mainly foreign exchange and gold)
- K = existing stock of real capital
- i_d = interest rate paid on bank deposits

References

- Alchian, A., and Allen, W. R. *University Economics*. Belmont, Calif.: Wadsworth, 1967.
- Brunner, K. "A Survey of Selected Issues in Monetary Theory." *Schweizerische Zeitschrift für Volkswirtschaft und Statis.* 107 (Winter 1971): 1-146.
- Brunner, K., and Meltzer, A. H. "The Role of Financial Intermediaries in the Transmission of Monetary Policy." *A.E.R.* 53 (May 1963): 372-82.
- . "Liquidity Traps for Money, Bank Credit and Interest Rates." *J.P.E.* 76 (January/February 1968): 1-37.
- . "A Monetarist Framework for Aggregative Analysis." Proceedings of the First Konstanz Conference on Monetary Theory and Policy. *Kredit und Kapital* (1972): in press.
- Christ, C. "A Simple Macroeconomic Model with a Government Budget Restraint." *J.P.E.* 76 (January/February 1968): 53-67.
- Fratianni, M. "The Monetary and Fiscal Approach to Inflation: The Italian Case." Paper presented at the Universities-National Bureau Conference on Secular Inflation, November 1971.
- Hicks, J. R. "Mr. Keynes and the Classics: A Suggested Interpretation." *Econometrica* 5 (April 1937): 147-59. Reprinted in *Readings in the Theory of Income Distribution*, edited by W. Fellner and B. F. Haley. Homewood, Ill.: Irwin, 1951.
- Keynes, J. M. *The General Theory of Employment, Interest and Money*. London: Macmillan, 1936.
- Lucas, R., and Rapping, L. "Real Wages, Employment and Inflation." *J.P.E.* 77 (September/October 1969): 721-54.
- Metzler, L. "Wealth, Saving and the Rate of Interest." *J.P.E.* 59 (April 1951): 93-116.

- Neumann, M. J. M. "The Monetary and Fiscal Approach to Inflation: The German Case." Paper presented at the Universities-National Bureau Conference on Secular Inflation, November 1971.
- Patinkin, D. *Money, Interest and Prices*. 2d ed. New York: Harper & Row, 1965.
- Pesek, B., and Saving, T. R. *Money, Wealth and Economic Theory*. New York: Macmillan, 1967.
- Phelps, E. "Money Wage Dynamics and Labor Market Equilibrium." *J.P.E.* 76, pt. 2 (August 1968): 687-711.
- Silber, W. L. "Fiscal Policy in IS-LM Analysis: A Correction." *J. Money, Credit and Banking* 2 (November 1970): 461-72.
- Tobin, J. "A General Equilibrium Approach to Monetary Theory." *J. Money, Credit and Banking* 1 (February 1969) 15-29.
- Turner, T. "A Spectral Analysis of the Money and Credit." *J. Money, Credit and Banking* (1972): in press.
- Wicksell, Knut. *Lectures on Political Economy*. London: Routledge & Kegan Paul, 1935.
- Zwick, B. "The Adjustment of the Economy to Monetary Changes." *J.P.E.* 79 (January/February 1971): 77-96.
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