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"Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium"

by

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Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium*

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Abstract

We compare three pricing mechanisms for monetary economies: bargaining (search equilibrium); price taking (competitive equilibrium); and price posting (competitive search equilibrium). We do this in a framework that, in addition to considering different mechanisms, extends existing work on the microfoundations of money by allowing a general matching technology and endogenous entry. We study how the nature of equilibrium and effects of policy depend on the mechanism. Under bargaining, trades and entry are both inefficient, and inflation implies a first-order welfare loss. Under price taking, the Friedman rule solves the first inefficiency but not the second, and inflation can actually improve welfare. Under posting, the Friedman rule implies first best, and inflation reduces welfare but the effect is second order.

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1 Introduction

We compare three alternative pricing mechanisms – or, three different equilibrium concepts – for monetary economies: bargaining (search equilibrium); Walrasian price taking (competitive equilibrium); and price posting with directed search (competitive search equilibrium). For this comparison, we develop a new model of monetary exchange. The basic physical environment is related to recent search-theoretic models following Lagos and Wright [2002], in that it borrows the assumption that economic activity sometimes takes place in highly centralized markets and sometimes takes place in more or less decentralized markets. The existence of the latter markets generates an essential role for money.[1] The existence of the former markets greatly reduces the complexity or the analysis.[2] Although we borrow the idea of combining decentralized trade with periodic access to centralized markets, we also extend along several dimensions existing models in the literature on the microfoundations of monetary theory.

First, we add heterogeneity in the sense that some agents will always be buyers and others will always be sellers in the decentralized markets. Second, this heterogeneity allows us to adopt a generalized matching technology and to introduce an entry decision by one side of the market; these extensions, which one might think of as being borrowed from labor market theory along the lines of Pissarides [2000], make the analyses of equilibrium and policy

[1] In this context *essential* means that money allows one to achieve outcomes that could not be achieved without it (Kocherlakota [1998]; Wallace [2001]). Essentiality arises from the combination of a double coincidence problem and some form of anonymity (or, in Kocherlakota's language, limited memory). The role of anonymity in monetary theory was emphasized earlier by Levine [1991].

[2] Green and Zhou [1998], Molico [1999], Camera and Corbae [1999], and Zhou [1999] provide examples of models that are quite complicated, mainly because all trade is decentralized and this makes it hard to keep track of the endogenous distribution of money. In the Lagos-Wright framework, periodic access to centralized markets renders this distribution simple. See Shi [1997] for a different but related model.

much more interesting. In particular, free entry allows us to easily analyze effects on the extensive margin (the number of trades) as well as the intensive margin (the amount exchanged in each trade), and the general matching technology allows us to discuss "search externalities" (in the sense that the probability of trade can depend on the numbers and types of agents in the market). In addition to having a more general physical environment, perhaps the key innovation in the paper is to consider the implications of alternative pricing mechanisms. It turns out that the nature of equilibrium and the effects of policy are very different under the different mechanisms.

Under bargaining, the quantity traded in each match and entry by sellers are both inefficient, and although the Friedman Rule is the optimal policy it does not fully correct either inefficiency. In this model inflation implies a first-order welfare loss. Under price taking, the Friedman Rule solves the inefficiency on the intensive margin but not the extensive margin. In this model the effects of policy are ambiguous, and inflation in excess of the Friedman Rule may be optimal – something quite rare in monetary theory. Under posting, the Friedman Rule achieves the first best, and inflation reduces welfare but the effect is second order. The results are interesting for the following reason. Economists have recently come to understand exactly what frictions are necessary for money to be essential, and models based explicitly on these frictions make novel predictions about things like the effects of inflation. But the extent to which the results are due to features of the environment (preferences, information etc.) or to the equilibrium concept (bargaining e.g.) has not been previously analyzed.

Our three mechanisms have of course all been used in different contexts. Dating back to Shi [1995] and Trejos and Wright [1995], most search-based monetary models use bargaining (with exceptions to be noted below). Walrasian pricing is used in monetary theory in, say, overlapping generations models by Wallace [1980] and turnpike models by Townsend [1980]. Competitive search equilibrium, introduced by Moen [1997] and Shimer [1996] in labor search theory, has not been used previously in monetary economics, but the key ingredients of price posting rather than bargaining or price taking, and partially directed rather than random search, fit nicely into the model. Our results in terms of efficiency and the impact of policy under alternative mechanisms have not been noted before mainly because the different mechanisms have not been studied in one environment – bargaining is used in most search models, Walrasian pricing is used in overlapping generations models, etc. The framework here allows one to compare mechanisms holding the environment constant.

This also explains the paper's title. Diamond [1984] introduced a cash-in-advance constraint in the Diamond [1982] model because he wanted to discuss "Money in Search Equilibrium." Although his approach to bargaining was primitive at best, perhaps a bigger problem was that money is imposed exogenously via the cash-in-advance constraint. However, Kiyotaki and Wright [1991,1993] showed that in a very similar environment a role for money can be derived endogenously. Kocherlakota [1998] later clarified exactly what makes money essential in those environments: a double coincidence problem, imperfect enforcement and anonymity. It seems natural to look for a physical environment that incorporates these features, but also allows one to consider alternative mechanisms. Here, in addition to being able to discuss what Diamond wanted, we can also consider "Money in Competitive Equilibrium" and "Money in Competitive Search Equilibrium."

The paper is organized as follows. Section 2 presents the basic assumptions. Section 3 analyzes the model with bargaining. Here we are able to borrow some technical results from Lagos and Wright [2002], although for the case with free entry we still need to prove several things, like existence.

We also show for this case that equilibria are generically not unique, where versions without entry including Lagos-Wright imply uniqueness. Section 4 analyzes competitive equilibrium, where prices are set by a Walrasian auctioneer even though there are search-type frictions, in the spirit of Lucas and Prescott [1974]. An advantage of this model is that it is much more tractable than the bargaining version, even though the main existence and multiplicity results are qualitatively similar. Section 5 analyzes competitive search equilibrium. Our framework is fairly different from existing analyses of competitive search, like Moen [1997] or Shimer [1996], mainly because we have fiat money. Hence for this model we go into more detail concerning the assumptions as well as existence and uniqueness. For each mechanism we derive the welfare and policy conclusions described above. Section 6 concludes by summarizing the results and offering suggestions for future research.

2 The Environment

Time is discrete and continues forever. Each period is divided into two subperiods, day and night, where economic activity will differ. During the day there will be a centralized and frictionless market, while at night trade will be more or less decentralized, depending on which mechanism we adopt, and this will make money essential. There is a continuum of agents divided into two types that differ in terms of when they produce and consume. We find it convenient to call them *buyers* and *sellers*. The difference is the following: while all agents produce and consume during the day, at night buyers want to consume but cannot produce and sellers are able to produce but do not want to consume – a classic double coincidence problem. These assumptions on preferences and technology, combined with the assumption that agents are anonymous, which precludes credit in the decentralized night markets, generate a role for money.[3]

Many other devices could work to generate double coincidence problems, but our set up has one big advantage over the environment in the typical search model. In that model any agent in the decentralized market may end up either buying or selling depending on who they meet, while here sellers can only sell and buyers can only buy. Differentiating two types ex ante allows us to introduce an entry decision by one side and thereby capture the extensive margin in a very simple way. Thus, the measure of buyers is set to 1 and the measure of sellers is $n \geq 0$, and we consider both the case where n is exogenous and the case where sellers can enter at cost k. In any case, there is an intrinsically useless, perfectly divisible, asset called fiat money. The quantity of money per buyer grows at a constant rate γ, so that $M_{+1} = \gamma M$ (we drop the t subscript, writing M for M_t, M_{+1} for M_{t+1}, etc.). New money is injected, or withdrawn if $\gamma < 1$, by lump-sum transfers, or taxes in the centralized market. To reduce notation transfers apply only to buyers.

The von Neuman-Morgenstern instantaneous utility function of a buyer is given by

$$U^b(x, y, q) = v(x) - y + \beta_d u(q), \tag{1}$$

where x is the quantity consumed and y the quantity produced during the day, q is his consumption at night, and $\beta_d \in (0, 1)$ is a discount factor between day and the night. There is also a discount factor between night and the next day, $\beta_n \in (0, 1)$, and we let $\beta = \beta_d \beta_n$.[4] We assume $u(0) = 0$, $u'(0) = \infty$,

[3]Note that those we call buyers will buy at night and those we call sellers will sell at night, but all agents buy and sell during the day; we hope these labels are nevertheless clear. Note also that the double coincidence problem is temporal in nature, as opposed to the double coincidence problem at any point in time in most of the search literature following Kiyotaki and Wright [1989]. In this sense is the model is more in the spirit of recent work by Kiyotaki and Moore [2001], or perhaps the turnpike and overlapping generations models mentioned in the Introduction.

[4]One special case is when agents do not discount between day and night, $\beta_d = 1$, as assumed in some earlier papers. Another is when $\beta_d = \beta_n$, so that the two subperiods could be thought of as even and odd dates.

$u'(q) > 0$, and $u''(q) < 0$. Also, $v'(x) > 0$, $v''(x) < 0$ for all x, and there exists $x^* > 0$ such that $v'(x^*) = 1$. Lifetime utility for a buyer is given by $\sum_{t=0}^{\infty} \beta^t U^b(x_t, y_t, q_t)$.

The instantaneous utility function of a seller is

$$U^s(x, y, q) = v(x) - y - \beta_d c(q). \tag{2}$$

We assume $c(0) = c'(0) = 0$, $c'(q) > 0$ and $c''(q) > 0$. Lifetime utility for a seller is given by $\sum_{t=0}^{\infty} \beta^t U^s(x_t, y_t, q_t)$. Also, we assume $c(q) = u(q)$ for some $q > 0$, and let q^* denote the *efficient* (first best) quantity: $u'(q^*) = c'(q^*)$. From (1) and (2) notice that sellers and buyers have the same preferences over day goods and the same discount factor, although we could relax this with no difficulty. The key difference between buyers and sellers is that the former enjoy consumption at night while the latter produce at night.

In the centralized market the price of goods is normalized to 1 and the price of money is ϕ. As in Lagos-Wright, we will see below that the quasi-linearity in (1) and (2) implies all agents of a given type choose the same money holdings in the centralized market. In the decentralized night market, although the details differ across the models studied below, there will always be some "stochastic rationing" in the following sense: each period buyers get an opportunity to trade with probability $\alpha(n)$ and sellers get an opportunity to trade with probability $\alpha(n)/n$, where $\alpha'(n) > 0$, $\alpha''(n) < 0$, $\alpha(n) \leq \min\{1, n\}$, $\alpha(0) = 0$, $\alpha'(0) = 1$ and $\alpha(\infty) = 1$. This allows for "search externalities" in the sense that trading probabilities will depend on the ratio of sellers to buyers. The function $\alpha(n)$ can be given several interpretations. For now, think of it as the standard specification coming from a constant returns to scale matching technology.[5]

[5] If $\mu(n_b, n_s)$ is the number of meetings when there are n_b buyers and n_s sellers, constant returns implies the arrival rate for a representative buyer is $\mu(1, n_s)/n_b = \alpha(n_s/n_b)$. See Petrongolo and Pissarides [2001] for a survey on matching functions.

2.1 Buyers

A buyer trades at night with probability $\alpha(n)$, in which case he pays $d = d(m)$ dollars for $q = q(m)$ units of goods, where (q, d) in general depends on his money holdings. Let $V^b(m)$ and $W^b(m)$ be the value functions for a buyer with m dollars in the night and day market, respectively (because we focus on steady-state equilibria where the aggregate real money supply is constant no other variables need to be included as arguments of these functions). Bellman's equation for an buyer in the decentralized night market is

$$\begin{aligned} V^b(m) &= \alpha(n)\left\{u\left[q(m)\right] + \beta_n W^b_{+1}\left[m - d\left(m\right)\right]\right\} \\ &\quad + \left[1 - \alpha(n)\right]\beta_n W^b_{+1}(m). \end{aligned} \tag{3}$$

In words, with probability $\alpha(n)$ he gets to trade, buys $q(m)$ and starts the next day with $m - d(m)$ dollars, and with probability $1 - \alpha(n)$ he does not trade and starts the next day with m.

In the centralized day market a buyer's problem is

$$W^b(m) = \max_{\hat{m},x,y}\left\{v(x) - y + \beta_d V^b(\hat{m})\right\} \tag{4}$$

$$\text{s.t. } \phi\hat{m} + x = \phi(m + T) + y, \tag{5}$$

where T is his transfer and $\hat{m}$ is the money he takes into the night market. Substituting for y we have

$$W^b(m) = \max_{\hat{m},x}\left\{v(x) - x + \phi m + \phi(T - \hat{m}) + \beta_d V^b(\hat{m})\right\}. \tag{6}$$

From (6) we see: the maximizing choice of x is x^* where $v'(x^*) = 1$; the maximizing choice of $\hat{m}$ is independent of m; and $W^b(m) = \phi m + W^b(0)$ is linear in m. Assuming V^b is differentiable (it will be) the first order condition for $\hat{m}$ is

$$-\phi + \beta_d V^b_m(\hat{m}) \le 0, \quad = 0 \text{ if } \hat{m} > 0, \tag{7}$$

where V_m^b denotes the derivative. If V^b is strictly concave (it will be under relatively weak conditions) there is a unique solution to (7) and all buyers choose the same $\hat{m}$.[6]

2.2 Sellers

Let $V^s(m)$ and $W^s(m)$ be the value functions for sellers. We will prove below that sellers' terms of trade (q, d) do not depend on the m they carry; for now we take this as given. Since all buyers in the decentralized market hold the same amount of money, say m_b, Bellman's equation for a seller is

$$\begin{aligned} V^s(m) &= \frac{\alpha(n)}{n}\left\{-c\left[q\left(m_b\right)\right]+\beta_n W_{+1}^s\left[m+d\left(m_b\right)\right]\right\} \\ &\qquad +\left[1-\frac{\alpha(n)}{n}\right]\beta_n W_{+1}^s(m)-k. \end{aligned} \tag{8}$$

There are several differences between (8) and (3). First, sellers have a different arrival rate, $\alpha(n)/n$ rather than $\alpha(n)$. Second, they produce and suffer disutility $-c(q)$ in exchange for cash, while buyers spend cash and get to enjoy utility. Also, sellers must pay cost k per period to participate in the night market.

The problem of a seller in the centralized market is

$$W^s(m) = \max_{\hat{m},x,y}\left\{v(x) - y + \beta_d V^s(\hat{m})\right\} \tag{9}$$

$$\text{s.t. } \phi\hat{m} + x = \phi m + y. \tag{10}$$

Substituting for y we have

$$W^s(m) = \max_{\hat{m},x}\left\{v(x) - x + \phi m - \phi\hat{m} + \beta_d V^s(\hat{m})\right\}. \tag{11}$$

[6] Lagos and Wright [2002] provide details on the existence, differentiability, and strict concavity of the value function, as well as conditions guaranteeing the nonnegativity of y, which is implicitly being assumed here, for their version of the model. Those arguments also apply to our model under bargaining. We discuss below how things change under price taking and posting.

It is obvious that, like buyers, sellers choose $x = x^*$; $\hat{m}$ is again independent of m; and $W^s(m) = \phi m + W^s(0)$ is again linear.

To say more, take the first order condition for $\hat{m}$:

$$-\phi + \beta_d V^s_m(\hat{m}) \leq 0, \quad = 0 \text{ if } \hat{m} > 0. \tag{12}$$

From (8), $V^s_m = \beta_n W^s_{m,+1} = \beta_n \phi_{+1}$. Focusing on equilibria where real balances are constant, we have $\phi_{+1} = \phi/\gamma$. The last two observations reduce the first order condition to $-\phi + \beta\phi/\gamma \leq 0$, $= 0$ if $\hat{m} > 0$, which cannot hold in a monetary equilibrium unless $\gamma \geq \beta$. For all $\gamma > \beta$ the solution is $\hat{m} = 0$; for $\gamma = \beta$, any $\hat{m}$ is any solution, but we only consider the limit as $\gamma \to \beta$. Hence, in any equilibrium $\hat{m} = 0$, and since sellers carry no money buyers carry it all, at least as long as (q, d) is independent of sellers' money holdings as we have so far been assuming.

Lemma 1: Given (q, d) is independent of sellers' money holdings, all sellers hold $m = 0$, and thus all buyers hold $m = M$.

While the measure of buyers is exogenous and normalized to 1, regarding the measure of sellers we consider two alternative assumptions. Either it is exogenous at $n = N$, or it is endogenous and determined by a free-entry condition. If sellers do not enter, they produce and consume x^* each day for a payoff of $v(x^*) - x^*$, which we normalize to 0 with no loss in generality. Hence free-entry means $V^s(0) = 0$, which implies after simplification that[7]

$$\frac{\alpha(n)}{n} \left[-c(q) + \beta_n \phi_{+1} d \right] = k. \tag{13}$$

Intuitively, (13) equates the participation cost to the probability of trading multiplied by a seller's surplus from a trade.

[7] A seller with m dollars spends it all in the centralized market whether or not she wishes to participate in the decentralized market. Since $v(x^*) - x^* = 0$ we can rewrite (11) as $W^s(m) = \phi m + \beta_d \max[V^s(0), 0] = \phi m$ since free entry implies $V^s(0) = 0$. From (8), $V^s(0) = \frac{\alpha(n)}{n} \left\{ -c[q(m_b)] + \beta_n \phi_{+1} d(m_b) \right\} - k$, and then $V^s(0) = 0$ yields (13).

2.3 Welfare

We measure welfare by $\mathcal{W} = n(1-\beta)V^s(0) + (1-\beta)V^b(M)$. After simplification this becomes

$$\mathcal{W} = \alpha(n)\left[u(q) - c(q)\right] - kn + \beta_n\left[nv(x_s) + v(x_b) - (nx_s + x_b)\right], \tag{14}$$

where q is the quantity consumed at night, while x_b and x_s are the quantities consumed by buyers and sellers during the day.[8] If a planner could choose (q, n, x_s, x_b), the first-order conditions would be

$$u'(q) - c'(q) = 0, \tag{15}$$

$$\alpha'(n)\left[u(q) - c(q)\right] = k \tag{16}$$

$$v'(x_s) = 1 \tag{17}$$

$$v'(x_b) = 1. \tag{18}$$

From (15), the efficient quantity traded at night is the q^* that equates marginal utility and marginal cost. From (16), the efficient n implies a seller's marginal contribution to the matching process, $\alpha'(n)$, times the total surplus $u(q) - c(q)$ should equal the participation cost k. From (17) and (18), $x_s = x_b = x^*$. Given that $x_s = x_b = x^*$ in any equilibrium considered below, the normalization $v(x^*) - x^* = 0$ implies that welfare can be expressed succinctly for our purposes as

$$\mathcal{W} = \alpha(n)\left[u(q) - c(q)\right] - kn. \tag{19}$$

3 Search Equilibrium (Bargaining)

In this section we study the mechanism used in much of the recent literature on the microfoundations of monetary theory, where (q, d) is determined by

[8] Notice the q consumed by buyers at night necessarily equals the amount produced by sellers, while we could let buyers produce y_b and sellers y_s during the day, subject to $x_b + nx_s = y_b + ny_s$; substituting this into the objective function yields (14).

bilateral bargaining. Figure 1 illustrates the functioning of the decentralized market, where to keep track of things we represent buyers by men and sellers by women. At random some agents are matched, as indicated by a dotted circle, and some are unmatched. In each match the agents bargain bilaterally. The figure shows meetings only between men and women, but that is without loss of generality: given preferences and technology, any meeting between two women or between two men is irrelevant.

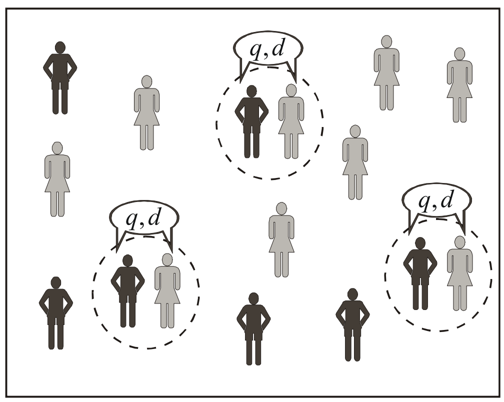

Figure 1: Search equilibrium

3.1 Equilibrium

Consider a meeting in the decentralized market between a buyer with m_b and a seller with m_s. Lemma 1 says that $m_b = M$ and $m_s = 0$, if the terms of trade do not depend on m_s, but we now need to establish that (q, d) indeed does not depend on m_s, and so we write things more generally. We adopt the generalized Nash solution, where $\theta \in (0, 1]$ is the bargaining power of a buyer and threat points are given by continuation values. Because of the linearity of $W^b(m_b)$ and $W^s(m_s)$, this simplifies nicely to

$$\max \left[u(q) - \beta_n \phi_{+1} d\right]^{\theta} \left[-c(q) + \beta_n \phi_{+1} d\right]^{1-\theta} \tag{20}$$

subject to $d \leq m_b$.[9] It is immediate that the solution (q, d) is independent of m_s, as assumed in Lemma 1; hence, in equilibrium we can now be assured that $m_b = M$ and $m_s = 0$.

Moreover, notice that (q, d) depends on m_b iff the constraint $d \leq m_b$ binds. If it does not bind, the first order conditions for (20) are

$$
\begin{aligned}
u'(q) &= c'(q) && (21)\\
\theta\left[-c(q) + \beta_n \phi_{+1} d\right] &= (1-\theta)\left[u(q) - \beta_n \phi_{+1} d\right], && (22)
\end{aligned}
$$

which implies $q = q^*$ and $d = m^*$ where $\beta_n \phi_{+1} m^* = \theta c(q^*) + (1-\theta) u(q^*)$. If the constraint does bind, then q solves the first order condition from (20) with $d = m_b$. Letting $z = \beta_n \phi_{+1} m_b$ denote the buyer's real balances, it will be convenient below to write this first order condition as

$$
z = g(q) = \frac{\theta u'(q) c(q) + (1-\theta) c'(q) u(q)}{\theta u'(q) + (1-\theta) c'(q)}. \qquad (23)
$$

This fully describes decentralized trade under bargaining.

Now consider the centralized market. From (3), given what we have just seen concerning bargaining, if $m_b > m^*$ then $V_m^b(m) = \beta_n \phi_{+1}$ and if $m_b < m^*$ then

$$
V_m^b(m_b) = \alpha(n)\left[u'(q)\frac{\partial q}{\partial m_b} - \beta_n \phi_{+1}\right] + \beta_n \phi_{+1}. \qquad (24)
$$

We claim that $\hat{m} < m^*$. To summarize the argument, which is discussed in more detail in Lagos and Wright [2002], first note that in equilibrium we must have $\phi \geq \beta \phi_{+1}$ since otherwise the problem $\max_{\hat{m}} \left\{-\phi \hat{m} + \beta_d V^b(\hat{m})\right\}$ has no solution.[10] Given this, $-\phi \hat{m} + \beta_d V^b(\hat{m})$ is weakly decreasing for $\hat{m} > m^*$.

[9] The payoffs of the buyer and seller are $u(q) + \beta_n W_{+1}^b(m_b - d)$ and $-c(q) + \beta_n W_{+1}^s(m_s + d)$, and the threat points are $\beta_n W_{+1}^b(m_b)$ and $\beta_n W_{+1}^s(m_s)$. Linearity of $W^b(m_b)$ and $W^s(m_s)$ implies $W_{+1}^b(m_b - d) - W_{+1}^b(m_b) = -\phi_{+1} d$ and $W_{+1}^s(m_s + d) - W_{+1}^s(m_s) = \phi_{+1} d$, leading to (20).

[10] We earlier argued that in a stationary equilibrium where ϕM is constant we have $\gamma \phi_{+1} = \phi$, and also that we must have $\gamma \geq \beta$, which gives the desired result. The argument here is more general: it applies to any equilibrium, and not only to stationary equilibria.

One can also show that $-\phi\hat{m}+\beta_d V^b(\hat{m})$ is strictly decreasing just to the left of m^*; simply compute $\partial q/\partial m_b$, insert it into (24), and let $m_b \to m^*$. This establishes the optimizing choice is $\hat{m} < m^*$.

Inserting (24) into the first order condition $\phi = \beta_d V^b_m(\hat{m})$ and rearranging, we get

$$\frac{\gamma-\beta}{\beta\alpha(n)} + 1 = \frac{u'(q)}{\beta_n \phi_{+1}} \frac{\partial q}{\partial m_b}. \tag{25}$$

Since $m_b < m^*$ we know from the bargaining solution that $\beta_n \phi_{+1} m_b = g(q)$, so $\partial q/\partial m_b = \beta_n \phi_{+1}/g'(q)$, and

$$\frac{\gamma-\beta}{\beta\alpha(n)} + 1 = \frac{u'(q)}{g'(q)}. \tag{26}$$

Given n, this condition determines the steady state q. For future reference let $\tilde{q}^*$ be the solution to (26) when we follow the Friedman Rule and deflate at the rate of time preference: $\gamma = \beta$. Notice that $\tilde{q}^* = q^*$ if $\theta = 1$ and $\tilde{q}^* < q^*$ otherwise.

We make the following assumptions:

Assumption 1: (i) $\lim_{q\to 0} u'(q)/g'(q) = \infty$; (ii) for all $q < q^*$, $u'(q)/g'(q)$ is strictly decreasing.

Part (i) is a standard Inada condition to guarantee existence; part (ii) implies uniqueness when n is exogenous, and is made so that we will know any multiplicity that occurs when n is endogenous must be due to free entry.[11] Also, simplifying the free entry condition (13) using (23), we get

$$\frac{\alpha(n)}{n} \frac{(1-\theta)c'(q)}{\theta u'(q) + (1-\theta)c'(q)} [u(q) - c(q)] = k, \tag{27}$$

and from this it is clear that a necessary condition for $n > 0$ is

[11] Lagos and Wright [2002] establish that a sufficient condition for (ii) is that either θ is not too small or u' is log-concave. This is condition can also be used to prove the value function is strictly concave. Some such condition is required because, under bargaining, unless $\theta = 1$, q is a nonlinear function of m_b that depends on u'''. As we will see later, this is not a problem in models where agents take prices as given.

Assumption 2: $k < \frac{(1-\theta)c'(\tilde{q}^*)}{\theta u'(\tilde{q}^*) + (1-\theta)c'(\tilde{q}^*)} [u(\tilde{q}^*) - c(\tilde{q}^*)]$.

Given $k > 0$, naturally this Assumption requires $\theta < 1$.

We now define equilibrium formally for the model with bargaining. In this definition, and those that follow, when we say an equilibrium we mean a steady-state monetary equilibrium, with $q, n > 0$.

Definition 1 *(i) With $n = N$, a search equilibrium is a pair $(q, z) \in \mathbb{R}^2_+$ satisfying (23) and (26). (ii) With free entry, a search equilibrium is a triple $(q, z, n) \in \mathbb{R}^3_+$ satisfying (23), (26) and (27).*

Note that equilibrium has a recursive structure: with n fixed q is determined by (26), and with free entry (q, n) is determined by (26)-(27), but in either case we can solve for $z = g(q)$ using (23) after we find q. Hence, we concentrate on q and n in what follows.

In the case with $n = N$ equilibrium exists and is unique by Assumption 1. It is easy to see that in this case $q < q^*$ and $q \to q^*$ as $\gamma \to \beta$ iff $\theta = 1$. In the case with n endogenous, equilibrium obtains at the intersection of two upward-sloping curves in (n, q) space defined by (26)-(27), shown as EE and FE in Figure 2. As γ increases EE rotates downward. As the figure suggests, one can show the following: First, there is a $\bar{\gamma} > \beta$ such that equilibrium exists iff $\gamma \leq \bar{\gamma}$. Second, for $\gamma \in (\beta, \bar{\gamma}]$ equilibria are generically not unique, since at both $n = 0$ and $n = n(\tilde{q}^*)$ the FE curve is above the EE curve, where $n(\tilde{q}^*)$ is the value of n that solves (27) at $\tilde{q}^*$. Third, in the limit as $\gamma \to \beta$ the EE curve becomes horizontal at $\tilde{q}^*$ for all $n > 0$, and hence we get a unique (monetary) equilibrium; in terms of Figure 2, the equilibrium with low (q, n) coalesces with the origin at $\gamma = \beta$.[12]

[12] Although these results should be easy to understand from Figure 2, here we provide the formal arguments. Let $\bar{\bar{q}}$ be the value of q that solves (27) when $n = 0$. From Assumption 2, $\bar{\bar{q}} < \tilde{q}^*$. For all $q \in [\bar{\bar{q}}, q^*]$, (27) can be written $n = n(q)$ with $n' > 0$ and $n(\bar{\bar{q}}) = 0$. Let

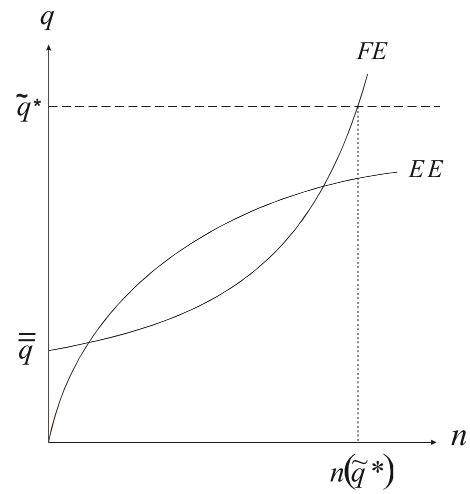

Figure 2: Equilibrium

We will collect these results in a Proposition after we discuss efficiency. To close this subsection we want to comment on fact that, for $\gamma > \beta$, if any (monetary) equilibria exist there must be more than one. It is clear that this multiplicity requires an entry decision, since when $n = N$ is exogenous Assumption 1 guarantees uniqueness. What is interesting is that multiplicity here does not require increasing returns, as is the case in most search models going back to Diamond [1982]. A nonmonetary model with constant returns would not display this multiplicity, even with an entry decision. The difference is that in this model there is a strategic interaction between entry by

$\Gamma(q;\gamma)$ be defined for $q \in \left[\bar{\bar{q}}, q^*\right]$ by

$$\Gamma(q;\gamma) = \beta\alpha\left[n(q)\right]\left[\frac{u'(q)}{g'(q)} - 1\right] - (\gamma - \beta).$$

An equilibrium exists iff there is a $q \in \left(\bar{\bar{q}}, q^*\right]$ such that $\Gamma(q;\gamma) = 0$. Assume first $\gamma = \beta$. Then, $\Gamma(q;\beta) = 0$ iff $q = \bar{\bar{q}}$ or $q = \tilde{q}^*$. As $n\left(\bar{\bar{q}}\right) = 0$, $q = \tilde{q}^*$ is the unique equilibrium with $n > 0$. Assume next $\gamma > \beta$. Then at $q = \bar{\bar{q}}$ and for all $q \geq \tilde{q}^*$, $\Gamma(q;\gamma) < 0$. Consequently, if equilibrium exists it is generically not unique. Define $\bar{\gamma} = \sup\left\{\gamma \geq \beta; \max_{q\in\left[\bar{\bar{q}},q^*\right]} \Gamma(q;\gamma) \geq 0\right\}$. Since $\Gamma(q;\gamma)$ is decreasing in γ, equilibrium exists iff $\gamma \leq \bar{\gamma}$ where $\bar{\gamma} > \beta$.

sellers and money demand by buyers, which can lead to multiple equilibria even with constant returns.[13]

3.2 Welfare

We now analyze efficiency and the effects of changes in inflation. First note that when $n = N$ is exogenous, the unique equilibrium implies $\partial q/\partial\gamma < 0$, and when n is endogenous, the equilibrium with the highest q implies $\partial q/\partial\gamma < 0$ and $\partial n/\partial\gamma < 0$. Notice from (26) that q is efficient iff $\gamma = \beta$ and $\theta = 1$ (i.e. iff the Friedman Rule holds and buyers have all the bargaining power), irrespective of n. If $\theta < 1$ then $q \leq \tilde{q}^* < q^*$ even at $\gamma = \beta$. This is due to a holdup problem that reduces the demand for money: when a buyer brings cash to the decentralized market he is making an investment, but when $\theta < 1$ he is not getting the full return on his investment. This reduces the equilibrium value of money q below the efficient level.

An alternative intuition is displayed in Figure 3, which plots the total surplus from decentralized trade, $S(q) = u(q) - c(q)$, as well as the buyer's share,[14]

$$S^b(q) = \frac{\theta u'(q)}{\theta u'(q) + (1-\theta)c'(q)}\left[u(q) - c(q)\right], \tag{28}$$

as functions of q. The curve $S^b(q)$ reaches a maximum at $q = \tilde{q}^* \leq q^*$, with the inequality strict if $\theta < 1$. Now q increases with m_b, but a buyer will never bring more money than needed to buy the quantity that maximizes $S^b(q)$. If there is an opportunity cost of holding money, which there is when $\gamma > \beta$, he will in fact prefer to buy less than $\tilde{q}^*$. Hence, we have $q \leq \tilde{q}^* < q^*$ whenever $\gamma > \beta$.

[13] We emphasize the interaction of monetary considerations and the entry decision: both are needed for multiplicity here. A related point has been made by Johri [1999], who shows that a version of Diamond [1982] with constant returns can have multiple equilibria once fiat money is introduced in a sensible way.

[14] To derive(28), insert $z = g(q)$ from (23) into $S^b(q) = u(q) - z$ and simplify. The seller's share is defined similarly.

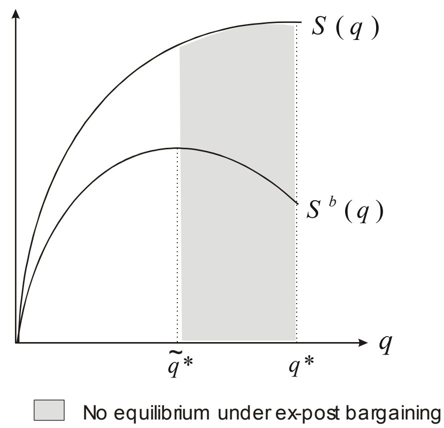

Figure 3: Surpluses

In terms of the extensive margin, comparing (27) and (16) we see that when n is endogenous it is efficient iff

$$\frac{(1-\theta)c'(q)}{\theta u'(q)+(1-\theta)c'(q)}=\eta\left(n\right), \tag{29}$$

where $\eta\left(n\right) = n\alpha'(n)/\alpha(n)$ measures sellers' contribution to the matching process by the elasticity of $\alpha(n)$. This is the familiar Hosios [1990] condition: entry by a group is efficient iff their share of the surplus from matching equals their contribution to matching. It is possible for n to be either too high or too low in equilibrium. This has interesting welfare implications, since entry can either exacerbate or mitigate the cost of inflation, depending on whether n is too low or too high. Although the Hosios condition is well known, notice that things are more complicated here than in typical applications in, say, labor economics, because sellers' share is the left side of (29), not simply the exogenous bargaining weight $1-\theta$. In fact, sellers' share equals $1-\theta$ iff $q=q^*$. However, $q=q^*$ in equilibrium iff $\theta=1$ (and $\gamma=\beta$), but if $\theta=1$ sellers do not enter, so there is no way to achieve $q=q^*$ and $n>0$.

When there are multiple equilibria they can be ranked. First note that under free entry $\mathcal{W}=\alpha(n)S^b(q)$, which is nice because it is separable between

$\alpha(n)$, which captures the extensive margin, and $S^b(q)$, which captures the intensive margin. In equilibrium $S^b(q)$ is increasing in q because $q < \tilde{q}^*$ (see Figure 3), and $\alpha(n)$ is always increasing in n. Hence equilibria with higher (q, n) unambiguously yield higher $\mathcal{W}$, and indeed, we can say that they are better in terms of both the intensive and extensive margin. Finally, consider an increase in γ in the best equilibrium. This rotates EE downward, so n and q both decrease and this unambiguously reduces $\mathcal{W}$. The best policy is therefore the minimum inflation rate, the Friedman Rule $\gamma = \beta$ (we proved earlier that there is no equilibrium with $\gamma < \beta$). Although this is optimal, we reiterate that it cannot support the first best outcome when n is endogenous, because $q = q^*$ requires both $\gamma = \beta$ and $\theta = 1$, while $\theta = 1$ is inconsistent with free entry. In any case, we summarize things as follows.

Proposition 1 *(i) Assume $n = N$. Search equilibrium exists and is unique. The optimal policy is $\gamma = \beta$ and it yields the efficient outcome iff $\theta = 1$. (ii) Assume free-entry. There is a $\bar{\gamma} > \beta$ such that equilibrium exists iff $\gamma \leq \bar{\gamma}$. For all $\gamma \in (\beta, \bar{\gamma})$ equilibrium is not unique. When $\gamma = \beta$ there exists a unique equilibrium. The optimal monetary policy is $\gamma = \beta$ but it can never achieve the efficient outcome.*

4 Competitive Equilibrium (Price Taking)

A few of the results in the previous section – i.e., those for the case $n = N$, although not the more interesting case with n endogenous – have been described in previous papers that use bargaining to determine the terms of trade. However, in either case it is not clear to what extent things are driven by features of the environment such as the double coincidence problem and anonymity assumption, and to what extent things are driven by the use of bargaining as a solution concept. One could follow Wallace's [2001] advice and use a mechanism design approach: in fact, as we vary θ between 0 and 1 in the previous section we trace out the set of (symmetric, stationary) incentive feasible and bilaterally-efficient allocations for a given policy. For our purposes, however, the more interesting issues involve thinking about different sets of institutions that can be used to determine the terms of trade. We especially want to study mechanisms that have been used by others in different contexts, as discussed in the Introduction. Here we consider Walrasian price-taking in the decentralized night market.

The first thing to emphasize is that introducing a Walrasian auctioneer may make the decentralized market less decentralized, but it does not make money inessential as long as we maintain the double coincidence problem and anonymity.[15] The second thing is that one can still capture search-type frictions with Walrasian pricing, and we do so here by assuming there are competitive markets open at night, but agents must queue to randomly get in to these markets and not necessarily all of them succeed.[16] For now, to help compare different models, we assume the same number enter on each side, so that the probabilities of getting in for buyers and sellers are $\alpha(n)$ and $\alpha(n)/n$. The only role this plays is to isolate the price-setting function of the auctioneer from the function of physically moving goods between sellers and buyers, since with equal numbers one can think of every physical exchange as bilateral if so desired; in any case we relax this below.

The situation is depicted in Figure 4. The night market is represented by a dashed circle. Inside the market all agents see the price p announced by the auctioneer. Buyers, again represented by men, observe the price and

[15] See Levine [1991] for an early expression of related ideas, and Temzelides and Yu [2003] for a more recent discussion.

[16] This is similar to the model in Lucas and Prescott [1974], in that markets are competitive but there may be frictions involved in getting into a given market. Things here are more general, however, since we allow two-sided search plus an entry decision on one side. One can also interpret the model in terms of shopping time, as in McCallum and Goodfriend [1987], although again our framework allows two-sided search and entry.

indicate how much they want to buy, q^b, while sellers, again represented by women, observe the price and indicate how much they want to sell, q^s. Goods trade against money here for exactly the same reason goods traded against money in previous section: there is a double coincidence problem and agents are anonymous. Agents outside the market do not trade at night (e.g. there are no bilateral meetings between agents in the two queues). Finally, we emphasize that in this section entry by sellers means entry into the queue; only a fraction of those who enter the queue actually get into the market.

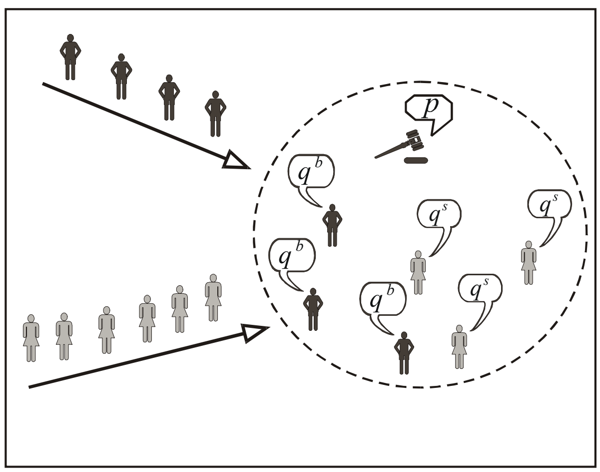

Figure 4: Competitive equilibrium

4.1 Equilibrium

First note that Lemma 1 applies here, so that in equilibrium $m_b = M$ and $m_s = 0$. Now, if a buyer gets in to the night market, he solves $\max_{q^b} \left\{u(q^b) + \beta_n W^b_{+1}\left(m_b - pq^b\right)\right\}$ subject to $pq^b \leq m_b$. The solution satisfies

$$\begin{array}{ll} u'(q^b) = \beta_n p \phi_{+1} & \text{if } m_b \geq m^* \\ q^b = m_b/p & \text{if } m_b < m^* \end{array} \tag{30}$$

(using the linearity of W), where m^* is the level at which the constraint binds, $u'\left(m^*/p\right) = \beta_n p \phi_{+1}$. One can show $m_b < m^*$, for the same reason as in the previous section, and so in equilibrium

$$q^b = M/p. \tag{31}$$

If a seller gets in she solves $\max_{q^s} \left\{-c(q^s) + \beta_n W^s_{+1}(m_s + pq^s)\right\}$. The solution satisfies

$$c'(q^s) = \beta_n p \phi_{+1}. \tag{32}$$

The price clears the market, which with equal numbers requires $q^s = q^b = q$, and so (31) and (32) imply

$$z = qc'(q), \tag{33}$$

where again $z = \beta_n \phi_{+1} M$.

We now determine money demand by buyers in the day market. Given $m_b < m^*$, we have $\partial q^b / \partial m_b = 1/p$ and[17]

$$V^b_m(m_b) = \alpha(n) u'\left(\frac{m_b}{p}\right)\frac{1}{p} + [1 - \alpha(n)]\beta_n \phi_{+1}. \tag{34}$$

Inserting (34) into the first-order condition $\phi = \beta_d V^b_m(m_b)$, using (32), and rearranging we get

$$\frac{u'(q)}{c'(q)} = 1 + \frac{\gamma - \beta}{\beta \alpha(n)}. \tag{35}$$

For a given n (35) determines the equilibrium q, and we note that this coincides with (26) from the previous section iff $\theta = 1$. If n is endogenous, in this model the free entry condition reduces to

$$\frac{\alpha(n)}{n}\left[qc'(q) - c(q)\right] = k. \tag{36}$$

We now have:

Definition 2 *(i) With $n = N$, a competitive equilibrium is a pair $(q, z) \in \mathbb{R}^2_+$ satisfying (33) and (35). (ii) With free entry, a competitive equilibrium is a triple $(q, z, n) \in \mathbb{R}^3_+$ satisfying (33), (35) and (36).*

[17] Notice $V^b_{mm} = \alpha u''/p^2 < 0$ for all $m_b < m^*$ here, and so we do not need any conditions like those discussed in the model with bargaining for the strict concavity of V^b. This is an example of how the price-taking model is much easier than the bargaining model.

Thins are again recursive: we can first determine q and then $z = qc'(q)$. Notice, however, that the condition for z here is different from the previous section, where $z = g(q)$. In particular, even if $\theta = 1$ in the bargaining model, we have $g(q) = c(q)$, and the conditions are the same in the two models iff $c(q)$ is linear. In any case, we focus on q and n.

Assume first $n = N$. Then there exists a unique equilibrium by (35), with $\partial q/\partial \gamma < 0$ and $q \to q^*$ as $\gamma \to \beta$. Assume next n is determined by (36). Then clearly the following restriction is necessary for $n > 0$.

Assumption 2': $k < q^* c'(q^*) - c(q^*)$.

The equilibrium (q, n) is now determined by the intersection of two upward-sloping curves given by (35)-(36). An argument just like the one in the previous section can be used to establish that there exists a threshold $\bar{\gamma} > \beta$ such that equilibrium exists iff $\gamma \leq \bar{\gamma}$, and that if an equilibrium exists it is generically not unique, unless $\gamma = \beta$ in which case there is a unique equilibrium. As in that section, we will collect these results after we discuss policy and welfare.

4.2 Welfare

If $n = N$ the optimal policy is $\gamma = \beta$ and it yields full efficiency. This is in accordance with many models in monetary economics, although not the one in the previous section, where the Friedman Rule was the optimal policy but could not achieve full efficiency. The reason $\gamma = \beta$ implies efficiency here, at least with $n = N$, is that the holdup problem in money demand disappears under competitive pricing.[18] If n is endogenous, we still have $q = q^*$ iff $\gamma = \beta$,

[18] To say it another way, in the model of this section the surplus for a buyer taking p as given is $S^b(q) = u(q) - q\beta_n p \phi_{+1}$. In equilibrium this is equal to $S^b(q) = u(q) - qc'(q)$, which is still maximized at $q < q^*$, but when choosing m_b agents ignore the effect of a change in q on sellers' marginal cost and thus on p. Under the Friedman rule, the function $u(q) - q\beta_n p \phi_{+1}$ reaches a maximum at $q = q^*$.

and comparing (35)-(36) with (15)-(16) we see that n is also efficient iff

$$\eta(n^*) = 1 - \frac{u(q^*) - q^* c'(q^*)}{u(q^*) - c(q^*)}. \tag{37}$$

This is again the Hosios condition. Hence, with free entry, full efficiency is achieved iff the Friedman Rule and the Hosios condition both hold.

There is no reason to expect the Hosios condition to hold, in general, since (37) relates the elasticity of the matching function to properties of preferences. Hence, in equilibrium n is typically inefficient, and it can be either too high or too low. This has interesting implications for policy. Consider the effect of inflation in the neighborhood of $\gamma = \beta$. Differentiating (19) and substituting for k from (36), we have:

$$\left.\frac{d\mathcal{W}}{d\gamma}\right|_{\gamma=\beta} = \frac{\alpha(n)}{n}\left[u(q^*) - c(q^*)\right]\left\{\eta(n) - \left[\tfrac{-c(q^*)+q^*c'(q^*)}{u(q^*)-c(q^*)}\right]\right\}\left.\frac{dn}{d\gamma}\right|_{\gamma=\beta} \tag{38}$$

At the best equilibrium $\partial n/\partial\gamma < 0$; hence, as long as $\eta(n^*) < \frac{-c(q^*)+q^*c'(q^*)}{u(q^*)-c(q^*)}$, welfare is increasing in γ at $\gamma = \beta$. It is easy to construct explicit examples; e.g. if $\alpha(n) = n^a$, so that $\eta = a$ for all n, then $\mathcal{W}$ must be increasing in γ at $\gamma = \beta$ as long as a is small enough.

It is rare in the literature for a deviation from the Friedman Rule to be optimal. The intuition for the result here is as follows. In general, when sellers decide to enter the market they do not take into account "search externalities" in the sense that they impose a "congestion" effect on other sellers and an opposite "thick market" effect on buyers.[19] In equilibrium n may be either too low or too high, but if it is too high then inflation helps welfare because it reduces the sellers' surplus and hence their incentive to enter. Now inflation also reduces the quantity traded in each match, and this hurts welfare along the intensive margin; however, the key observation

[19] The terms "thick market" and "congestion" are standard in the matching literature; all they mean is that when a seller enters he increases $\alpha(n)$ and decreases $\alpha(n)/n$.

is that, because $q = q^*$ at the Friedman rule, this has only a second-order effect.[20]

Proposition 2 *(i) Assume $n = N$. Competitive equilibrium exists and is unique. The optimal policy is $\gamma = \beta$ and it yields the efficient outcome. (ii) Assume free-entry. There exists $\bar{\gamma} > \beta$ such that equilibrium exists iff $\gamma \leq \bar{\gamma}$. For all $\gamma \in (\beta, \bar{\gamma})$ equilibrium is not unique. When $\gamma = \beta$ there exists a unique equilibrium. Equilibrium is efficient iff $\gamma = \beta$ and the Hosios condition (37) holds. If (37) does not hold, optimal policy involves $\gamma > \beta$ iff $\eta(n^*) < \frac{q^* c'(q^*) - c(q^*)}{u(q^*) - c(q^*)}$.*

4.3 Extensions

To close this section we briefly consider some extensions of the model with competitive pricing in order to illustrate the flexibility of the framework, to show it is robust to having unequal numbers of buyers and sellers in the market, and to further develop further our intuition for the inefficiency. Suppose first that all sellers get into the night market with probability 1, while buyers get in with probability $\alpha(n)$, $\alpha' \geq 0$. This may be a natural assumption if, for instance, one wants to interpret buyers as "shopping" among sellers. Also, allowing unequal numbers to potentially get in, market clearing now requires $nq^s = \alpha(n)q^b$.

[20] We know of no previous results in the monetary policy literature based on "search externalities" except Li [1995, 1997] and Berentsen, Rocheteau and Shi [2001], and those results are not especially robust. That is, even with "search externalities" the Friedman Rule is optimal unless special assumptions are made to get around the holdup problem in money demand. Recall from the previous section that $\gamma = \beta$ was always the optimal policy under bargaining even though "search externalities" were present. In that model, inflation is always bad for welfare, even though n may be too big, and we attribute this to the holdup problem. Li gets his results by assuming indivisible goods and money, which avoids holdup problems at the margin, while Berentsen et al. get theirs by invoking a special bargaining solution. In the model of this section we avoid the holdup problem in money demand by the assumption of competitive price taking, which means $q = q^*$ at the Friedman Rule, which means that a little inflation is not very bad on the intensive margin, and hence it may be a net improvement when n is too big.

Under these assumptions, the methods leading to (35) now lead to

$$\frac{u'(q^b)}{c'(q^s)} = 1 + \frac{\gamma - \beta}{\beta\alpha(n)}, \tag{39}$$

while the free-entry condition becomes

$$q^s c'(q^s) - c(q^s) = k. \tag{40}$$

Welfare is $\alpha(n)u(q^b) - nc(q^s) - kn$, and for any given q the optimal n satisfies

$$\alpha'(n)\left[u(q^b) - q^b c'(q^s)\right] + q^s c'(q^s) - c(q^s) = k. \tag{41}$$

If $\alpha' = 0$ then (40) and (41) coincide: equilibrium entry is efficient and the Friedman Rule achieves the first best. But if $\alpha' > 0$, n is inefficiently low because sellers ignore the "thick market" effect of their entry on buyers. Since an increase in γ reduces n, inflation is always bad for welfare.

Assume next that all buyers get in with probability 1 while sellers get in with probability $\xi(n)$, $\xi' \leq 0$, which is quite similar to the Lucas-Prescott [1974] model if we interpret sellers here as workers (selling their labor). Market clearing requires $n\xi(n)q^s = q^b$. Equation (35) becomes

$$\frac{u'(q^b)}{c'(q^s)} = 1 + \frac{\gamma - \beta}{\beta}, \tag{42}$$

and the free-entry condition becomes

$$\xi(n)\left[q^s c'(q^s) - c(q^s)\right] = k. \tag{43}$$

Welfare is $u(q^b) - n\xi(n)c(q^s) - kn$, and the optimal n satisfies

$$\left[\xi(n) + n\xi'(n)\right]\left[q^s c'(q^s) - c(q^s)\right] = k. \tag{44}$$

If $\xi' = 0$ then (43) and (44) coincide: equilibrium entry is efficient and the Friedman Rule achieves the first best. If $\xi' < 0$ however, n is too high and inflation above $\gamma = \beta$ unambiguously improves welfare. This shows that the origin of the inefficiency is the fact that entry generates "congestion" effects that are not internalized by the Walrasian market-clearing price. As pointed out by Moen [1997] and Shimer [1996] in the context of labor markets, one interpretation of this is that there is a missing market that would price the probabilities of trade. In the next section we consider a mechanism that takes care of this.

5 Competitive Search Equilibrium (Posting)

The concept of competitive search equilibrium is based on the idea that some agents can post a price, or more generally, a contract, that specifies the terms at which agents commit to trade. Other agents observe posted prices and choose where to go. Again, there may be "stochastic rationing" – in some versions this is because more buyers may show up at a given seller's location that he has capacity to serve (Burdett et al. [2001]), or in other versions it is because buyers get to choose a location where everyone posts a given price but they still have to search for a seller at that location (Moen [1997]). In any case, there is at least partially directed search, and this generates competition among price setters. As argued in Corbae et al. [2003], directed search does not make money inessential as long as we still have a double coincidence problem and anonymity, but we will see that it will change the way pricing works.[21]

Here we adopt the interpretation of competitive search equilibrium discussed in Mortensen and Wright [2002]. In this version there are agents called *market makers* who can open submarkets where they post the terms of trade

[21] Corbae et al. [2003] allow directed search but do not consider price posting, and the notion of competitive search equilibrium requires a combination of the two. Posting with undirected search has been used in monetary theory by Green and Zhou [1998], Zhou [1999], Curtis and Wright [2000], Head and Kumar [2001], Camera and Winkler [2002], and Jafarey and Masters [forthcoming].

(q, d) and charge participants an entry fee, which will be 0 in equilibrium as the cost of opening a submarket is negligible. Agents direct their search in the sense that they can go to any submarket they like, but within any submarket there is random bilateral matching. Given a menu of (q, d), and expectations about where other agents go which determines the arrival rates, across submarkets, each buyer or seller decides where he or she goes, and in equilibrium expectations must be rational. When designing submarkets market makers take into account the relationship between the posted (q, d) and the numbers of buyers and sellers who choose each submarket, summarized by the ratio n. In equilibrium the set of submarkets is *complete* in the sense that there is no submarket that could be opened and make some buyers and sellers better off, since then a market maker could earn a profit.

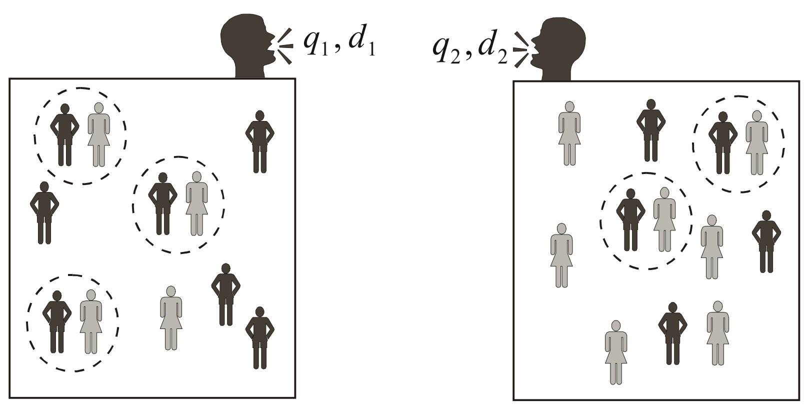

Figure 5: Competitive search equilibrium

The situation is represented in Figure 5, where two submarkets are shown, and in each there is a market maker announcing (q, d). In a submarket the matching process is random, and a meeting is again represented by a circle. The timing of events in a period is as follows. At the beginning of each day, market makers announce the submarkets to be open that night, as described by (q, d), and this implies an expected n in each submarket. Agents then trade in the centralized market during the day, exactly as before, and go to submarkets of their choosing at night. In the submarkets at night agents trade goods and money bilaterally, like in search equilibrium, except they do not bargain: they are bound by (q, d).[22]

5.1 Equilibrium

A market maker can make a profit if he can design a submarket that beats existing submarkets, in the sense of making buyers better off without making sellers worse off. Given (q, d), the market maker can get any number of sellers as long as he matches the market payoff, given by[23]

$$W^s(m_s) = \phi m_s + \beta_d \max_{\omega \in \Omega} \left\{ \frac{\alpha(n)}{n} \left[-c(q) + \beta_n \phi_{+1} d \right] \right\} + \beta W^s_{+1}(0) \tag{45}$$

where $\omega = (q, d, n)$ and Ω is the set of such triples implied by the open submarkets. Thus, a seller with m_s spends it all in the day market and then goes to a submarket ω to maximize her expected surplus. As the choice ω is independent of m_s, at night all sellers obtain the same payoff, and all open submarkets yield sellers the same payoff.

If we let $J_s = \max_{\omega \in \Omega} \left\{ \frac{\alpha(n)}{n} \left[-c(q) + \beta_n \phi_{+1} d \right] \right\}$, any active submarket ω satisfies

$$\frac{\alpha(n)}{n} \left[-c(q) + \beta_n \phi_{+1} d \right] = J_s. \tag{46}$$

[22] Obviously this assumes a certain amount of commitment; this is the essence of posting and competitive search equilibrium. While we could argue about whether this type of commitment is reasonable, we emphasize that logically it does not make money inessential: committing to the terms of decentralized trade is not the same as committing to repayment of credit. We also emphasize that, instead of invoking market makers, it is equivalent for sellers to post (commit to) the terms of trade and then have buyers search across sellers, or for buyers to post (q, d) and then have sellers search. These different stories all generate the same equilibrium conditions.

[23] To derive (45), begin with

$$W^s(m_s) = \phi m_s + \max_{\omega \in \Omega} \beta_d \left\{ \frac{\alpha(n)}{n} \left[-c(q) + \beta_n W^s_{+1}(d) \right] + \left[1 - \frac{\alpha(n)}{n} \right] \beta_n W^s_{+1}(0) \right\}$$

and use the linearity of $W^s_{+1}(m)$. The same method works below for $W^b(m_s)$.

Given (q, d), (46) determines n. Therefore, a market maker designing a submarket at the start of the day maximizes the buyers' payoff,

$$W^b(m_b) = \phi(m_b+T) + \max_{\omega \in \mathbb{R}^3_+} \left\{-\phi d + \beta_d \alpha(n)[u(q) - \beta\phi_{+1} d]\right\} + \beta W^b_{+1}(d) \quad (47)$$

subject to (46). Notice the choice ω is independent of m_b, so all buyers obtain the same payoff, and market makers do not have to cater their submarkets to particular buyers.[24]

Using $\phi = \gamma\phi_{+1}$ and $z = \beta_n \phi_{+1} d$ this problem can be rewritten

$$\max_{\omega \in \mathbb{R}^3_+} \left\{ \alpha(n) \left[u(q) - z\right] - \left(\frac{\gamma - \beta}{\beta}\right) z \right\} \quad (48)$$

$$\text{s.t.} \quad \frac{\alpha(n)}{n} \left[-c(q) + z\right] = J_s. \quad (49)$$

Effectively, market makers maximize buyers' expected surplus minus the opportunity cost of carrying cash, subject to the constraint that ω has to attract sellers.[25] Substituting z from (49) into (48), the first-order conditions with respect to q and n are

$$\frac{\gamma - \beta}{\beta\alpha(n)} + 1 = \frac{u'(q)}{c'(q)} \quad (50)$$

$$\eta(n) \left[u(q) - c(q)\right] = \frac{n}{\alpha(n)} J_s \left\{ 1 + \left[1 - \eta(n)\right] \left(\frac{\gamma - \beta}{\alpha(n)\beta}\right) \right\} \quad (51)$$

where $\eta(n) = \frac{\alpha'(n)n}{\alpha(n)}$. One can show that generically the solution is unique – i.e. there are a countable number of values for J_s such that the solution is not unique – and so any active submarket will have the same ω. Details are in the Appendix (but the idea should be clear from Figure 6 below).

Eliminating J_s using (49), we can write (51) as

$$\eta(n) c'(q) \left[u(q) - z\right] = \left[1 - \eta(n)\right] u'(q) \left[-c(q) + z\right]. \quad (52)$$

[24] It is because of quasi-linear preferences in our model that market makers do not have to cater to particular buyers and sellers with different money holdings at the start of the day. In general, with heterogenous agents, there may have to be many different types of submarkets open in equlibrium (see e.g. Mortensen and Wright [2002]).

[25] The way we write the problem assumes $n > 0$, but of course if $-c(q) + z < J_s$ then $n = 0$; we take care of this more carefully in the Appendix.

Notice that (52) is actually the first order condition from the generalized Nash problem where the seller's bargaining power is $\eta(n)$. Hence, in competitive search equilibrium, the terms of trade *endogenously* satisfy the Hosios condition. Real balances z satisfy a condition analogous to (23) where θ is replaced by $1-\eta$,

$$z = h(q) = \frac{(1-\eta)\, u'(q)c(q) + \eta c'(q)u(q)}{(1-\eta)u'(q) + \eta c'(q)}. \tag{53}$$

Finally, n can either be set to N or endogenized. The free-entry condition is analogous to (27) where θ is replaced by $1-\eta$:

$$\frac{\alpha(n)}{n} \frac{\eta c'(q)}{(1-\eta)u'(q) + \eta c'(q)} \left[u(q) - c(q)\right] = k, \tag{54}$$

Before we define equilibrium, a detail needs mention. It may seem natural to define equilibrium (when n is endogenous) as a triple (q, z, n) satisfying (50), (52) and (53); in general, however, we cannot be sure that all solutions to the first order conditions give the solution to (48) because the second order conditions may not hold here. Hence, we define equilibrium here in terms of the underlying maximization problem.

Definition 3 *(i) With $n = N$, a competitive search equilibrium is a $(q, z) \in \mathbb{R}^2_+$ satisfying (50) and (53). (ii) With free entry, a competitive search equilibrium is a triple $(q, z, n) \in \mathbb{R}^3_+$ that maximizes (48) subject to (49) with $J_s = k$.*

Figure 6 illustrates the determination of competitive search equilibrium. The curve $\tilde{N}(J_s)$, which one can interpret as aggregate demand for sellers by market makers, is the convex hull of the correspondence that gives the value(s) of n emerging from the market maker's problem taking J_s as given.[26]

[26] The reason it is the convex hull of the correspondence is as follows. Supppose, for example, there are eactly two solutions n_1 and n_2 emerging from the market maker's problem, with $n_1 < N < n_2$. Then equilibrium involves some submarkets with $n = n_1$ and others with $n = n_2$, such that the aggregate n equals N.

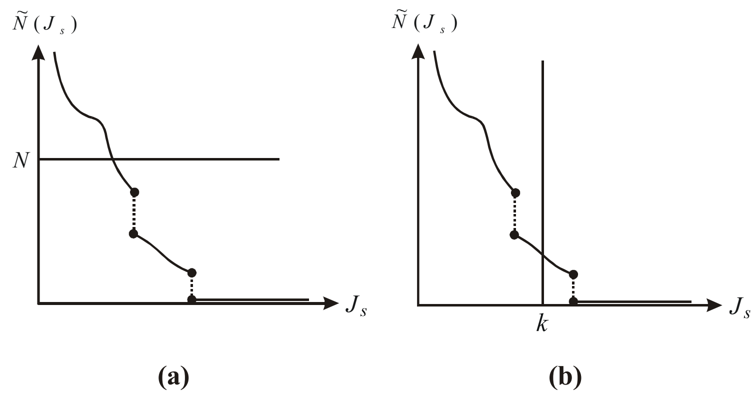

Figure 6: Competitive search equilibrium. (a) No entry. (b) Free entry.

Properties of this correspondence are derived in the Appendix, including the fact that it is strictly decreasing. Without entry, J_s adjusts so that $\tilde{N}(J_s) = N$; with entry, we must have $J_s = k$ and the number of sellers adjusts. Consider first the case $n = N$. In the Appendix we show equilibrium always exists. Further, it is clear from Figure 6 that J_s is uniquely determined, and this implies any multiplicity is payoff irrelevant.

If n is endogenous, the following is necessary for $n > 0$.

Assumption 2": $k < u(q^*) - c(q^*)$.

In the Appendix we show that with free-entry there is a $\bar{\gamma} > \beta$ such that equilibrium exists iff $\gamma \leq \bar{\gamma}$, and if equilibrium exists it is generically unique. The existence result is similar to what we found in the other models, but uniqueness contrasts with the multiplicity found under both bargaining and price taking. This reflects the fact that market makers internalize the strategic complementarity between money demand and entry.

5.2 Welfare

If $n = N$ then equilibrium is efficient iff $\gamma = \beta$, as in competitive equilibrium but not search equilibrium. This is perhaps not so surprising, since the holdup problem associated with bargaining is absent here, just like it is absent in competitive equilibrium. A closer examination of (48) reveals that competitive search equilibrium is equivalent to having buyers and sellers contract (commit to the terms of trade) before matching, and this gets around the holdup problem in the demand for money. As a consequence, if $\gamma = \beta$, so that there is no opportunity cost of holding money, agents carry the efficient amount. Hence competitive search equilibrium, like competitive equilibrium, yields the first best when n is exogenous.

Now suppose n is endogenous. Comparing (50)-(54) with (15)-(16), we see that equilibrium is efficient iff $\gamma = \beta$. Hence, the Friedman Rule implies efficiency along both the intensive margin and the extensive margin. As we noted above, competitive search generates the Hosios condition endogenously. Another way to say it is that entry is efficient because the market maker internalizes the effects of n on arrival rates. As shown in Figure 6, J_s acts as a price that clears the market for sellers. This extends results from the non-monetary literature on competitive search equilibrium (in addition to the papers cited earlier we mention Acemoglu and Shimer [1999] as an important example). However, we emphasize that in a monetary economy competitive search equilibrium is not efficient, in general, but only under the Friedman Rule.

Proposition 3 *(i) Assume $n = N$. Competitive search equilibrium exists. The optimal policy is $\gamma = \beta$ and it implies equilibrium is unique and efficient. (ii) Assume free-entry. There is a $\bar{\gamma} > \beta$ such that equilibrium exists iff $\gamma \leq \bar{\gamma}$. For all $\gamma \in (\beta, \bar{\gamma})$ equilibrium is generically unique. The optimal policy is $\gamma = \beta$ and it implies equilibrium is unique and efficient.*

6 Conclusion

We have considered three different pricing mechanisms for models of monetary exchange: search equilibrium (bargaining), competitive equilibrium (price taking), and competitive search equilibrium (price posting with directed search). We did this in a model that shares features with the recent literature on the microfoundations of monetary economics, but also adds some new features that make our comparisons across mechanisms more interesting, including a particular kind of heterogeneity, a generalized matching technology, and a free entry decision. These features allow for a natural discussion of "search externalities" as well as both intensive and extensive margin effects. We found that efficiency and the effects of policy can depend crucially on the mechanism. We now recapitulate the main results.

The first table below shows the efficiency properties of the mechanisms at the Friedman rule $\gamma = \beta$. Regarding the intensive margin, we have $q = q^*$ in competitive equilibrium and competitive search equilibrium, but $q < q^*$ in search equilibrium given $\theta < 1$. When n is endogenous, we must have $\theta < 1$ or no sellers will enter the market; hence search equilibrium with entry is necessarily inefficient. On the extensive margin, competitive equilibrium as well as search equilibrium imply n is generically inefficient because these mechanisms do not internalize the effects of entry. Efficient n requires the Hosios condition, and this is an unlikely to hold for given exogenous parameters. By contrast, in competitive search equilibrium the relevant condition holds endogenously, so n as well as q are both efficient at the Friedman rule.

	SE	**CE**	**CSE**
Intensive margin: q	$q < q^*$ if $\theta < 1$ $q = q^*$ if $\theta = 1$	$q = q^*$	$q = q^*$
Extensive margin: n	$n \gtrless n^*$	$n \gtrless n^*$	$n = n^*$

The next table investigates the welfare effect of inflation near the Friedman Rule. With n exogenous, in all cases except search equilibrium inflation has only a second-order effect. This is due the envelope theorem: with n fixed, in competitive equilibrium or competitive search equilibrium $\mathcal{W}$ is maximized and $\partial\mathcal{W}/\partial\gamma = 0$ at $\gamma = \beta$. In the case of search equilibrium with $\theta < 1$, the envelope theorem does not apply: $\mathcal{W}$ is maximized but $\partial\mathcal{W}/\partial\gamma < 0$ at $\gamma = \beta$ because we are at a corner solution – $\gamma = \beta$ is the minimum possible inflation rate. In this case γ has a first order effect on $\mathcal{W}$. With n endogenous, $\mathcal{W}$ is decreasing in γ in search equilibrium for any θ. By contrast, γ has an ambiguous effect on $\mathcal{W}$ in competitive equilibrium; it is possible to have $\partial\mathcal{W}/\partial\gamma > 0$. Finally, in competitive search equilibrium, the envelope theorem applies to both q and n, and inflation has only a second order effect even when n is endogenous.

	SE	**CE**	**CSE**
n **exogenous**	$\frac{\partial\mathcal{W}}{\partial\gamma} < 0$ if $\theta < 1$ $\frac{\partial\mathcal{W}}{\partial\gamma} \approx 0$ if $\theta = 1$	$\frac{\partial\mathcal{W}}{\partial\gamma} \approx 0$	$\frac{\partial\mathcal{W}}{\partial\gamma} \approx 0$
n **endogenous**	$\frac{\partial\mathcal{W}}{\partial\gamma} < 0$	$\frac{\partial\mathcal{W}}{\partial\gamma} \gtrless 0$	$\frac{\partial\mathcal{W}}{\partial\gamma} \approx 0$

We do not want to argue that any mechanism is "correct" or "most relevant" here. Competitive search equilibrium may seem appealing since it is efficient given we follow the optimal policy, although it is based on a notion of commitment to the terms of trade that one may find objectionable. Search equilibrium makes explicit holdup problems that may well be important in the real world. Competitive search equilibrium has a big theoretical advantage as a solution concept: it is much easier to work with. In any case, we think the analysis has helped to clarify how equilibrium and the effects of policy depend on different features of the environment and the different mechanisms. Future work could involve quantifying the welfare costs of inflation under each of the mechanisms.

Appendix

Here we verify some claims made in the text about competitive search equilibrium. To begin, write the problem of a market maker as

$$\mathcal{V}(J_s,\gamma)=\max_{\omega\in\Gamma(J_s)}\left\{\alpha(n)\left[u(q)-z\right]-\left(\frac{\gamma-\beta}{\beta}\right)z\right\}, \tag{55}$$

where the constrain set is

$$\Gamma(J_s)=\{\omega:\frac{\alpha(n)}{n}\left[-c(q)+z\right]=J_s \text{ if } -c(q)+z>J_s;\text{ else } n=0\}. \tag{56}$$

To preserve continuity, any ω such that $-c(q)+z=0$ and $n\in[0,\infty]$ is element of $\Gamma(0)$. Denote the set of optimal choices for n by $N(J_s)$. Equilibrium with entry requires $J_s=k$. Equilibrium without entry requires $\{N\}\in\tilde{N}(J_s)$, where $\tilde{N}(J_s)$ is the convex hull of $N(J_s)$.

Part 1. $N(J_s)$ is upper hemi-continuous.

If we reformulate (55) as a choice of q, z and $\alpha=\alpha(n)$, then if a solution exists it needs to be in the compact set

$$\left\{(q,z,\alpha):0\leq\alpha\leq 1,0\leq q\leq q^*,c(q)\leq z\leq u(q)\right\}. \tag{57}$$

From Berge's theorem, and the fact that α^{-1} exists and is continuous, $N(J_s)$ is non-empty and upper hemi-continuous for all $J_s\geq 0$. This implies that $\tilde{N}(J_s)$ is non-empty and upper hemi-continuous. Furthermore, $\mathcal{V}(J_s,\gamma)$ is continuous in (J_s,γ).

Part 2. Any selection from $N(J_s)$ is strictly decreasing in J_s.

Assume the solution to (55) is interior. Substitute $z=J_sn/\alpha(n)+c(q)$ from the constraint into (55) to rewrite the problem as $\max_{(n,q)}\Psi\left(n,q;J_s,\gamma\right)$ where

$$\Psi\left(n,q;J_s,\gamma\right)=\alpha(n)\left[u(q)-c(q)\right]-nJ_s-\left(\frac{\gamma-\beta}{\beta}\right)\left[\frac{n}{\alpha(n)}J_s+c(q)\right] \tag{58}$$

Let $J_s^1 > J_s^0$ and $(n_i, q_i) \in \arg\max_{(n,q)} \Psi(n, q; J_s^i, \gamma)$ for $i = 1, 2$. Then $\Psi(n_0, q_0; J_s^0, \gamma) \geq \Psi(n_1, q_1; J_s^0, \gamma)$ and $\Psi(n_1, q_1; J_s^1, \gamma) \geq \Psi(n_0, q_0; J_s^1, \gamma)$, which implies

$$\left\{\left[\left(\frac{\gamma-\beta}{\beta}\right)\frac{n_1}{\alpha(n_1)} + n_1\right] - \left[\left(\frac{\gamma-\beta}{\beta}\right)\frac{n_0}{\alpha(n_0)} + n_0\right]\right\}\left(J_s^1 - J_s^0\right) \leq 0. \quad (59)$$

Since $n/\alpha(n)$ is strictly increasing in n, this implies $n_1 \leq n_0$. To show the inequality is strict, note from (50) that if $n_1 = n_0$ then $q_1 = q_0$ which is inconsistent with (51).

Part 3. Equilibrium with $n = N$.

From (58), if $J_s = 0$ then $n = \infty$, i.e., $\{\infty\} = \tilde{N}(0)$. Note that the solution for q that maximizes $-\left(\frac{\gamma-\beta}{\beta}\right)c(q) + [u(q) - c(q)]$ is interior from the Inada conditions, which allows us to rule-out any solution (n, q) with $q = 0$. From (51), if $J_s > u(q^*) - c(q^*)$ then there is no interior solution, i.e., $\{0\} = \tilde{N}(J_s)$. Furthermore, any selection from $\tilde{N}(J_s)$ is strictly decreasing in J_s assuming the solution is interior. Therefore, there exists a unique $J_s \leq [u(q^*) - c(q^*)]$ such that $\{N\} \in \tilde{N}(J_s)$.

Part 4. Equilibrium with free entry.

For all γ such that $\mathcal{V}(k, \gamma) > 0$ the solution to the market maker's problem is interior and equilibrium exists. The value function $\mathcal{V}(k, \gamma)$ is continuous and decreasing in γ and, from (58) and Assumption 2", $\mathcal{V}(k, \beta) > 0$. Consequently, there exists a threshold $\bar{\gamma} > \beta$ such that for all $\gamma \in (\beta, \bar{\gamma})$ equilibrium exists. When equilibrium exists it is generically unique. Indeed, given that $\alpha(n)$ is in the compact set $[0, 1]$ and that it is strictly decreasing with J_s, there must be a countable number of values for J_s such that $\tilde{N}(J_s)$ is not a singleton.

References

Acemoglu, Daron and Robert Shimer (1999). "*Holdups and efficiency with search frictions*", International Economic Review 40, 827–849.

Berentsen, Aleksander, Rocheteau, Guillaume and Shi, Shouyong (2001), "*Friedman meets Hosios: Efficiency in search models of money*", mimeo.

Burdett, Kenneth, Shi, Shouyong and Wright, Randall (2001). "*Pricing and matching with frictions*", Journal of Political Economy 109, 1060-1085.

Camera, Gabriele and Corbae, Dean (1999). "*Money and price dispersion*", International Economic Review 40, 985-1008.

Camera, Gabriele and Johannes Winkler (2002). "*International monetary trade and the law of one price*", mimeo.

Corbae, Dean, Temzelides, Ted, and Wright, Randall (2003). "*Directed matching and monetary exchange*", Econometrica 71, 731-756.

Curtis, Elisabeth and Wright, Randall (2000). "*Price setting and price dispersion in a monetary economy; Or, the law of two prices*", mimeo.

Diamond, Peter A. (1982). "*Aggregate demand management in search equilibrium*", Journal of Political Economy 90, 881-94.

Diamond, Peter A. (1984). "*Money in search equilibrium*", Econometrica 52, 1-20.

Green, E. J. and Zhou, Ruilin (1998). "*A rudimentary random-matching model with divisible money and prices*", Journal of Economic Theory 81, 252-271.

Head, Allen and Kumar, Alok (2001). "*Price-posting, price dispersion, and inflation in a random matching model*", mimeo.

Hosios, Arthur J. (1990). "*On the efficiency of matching and related models of search and unemployment*", Review of Economic Studies 57, 279-298.

Howitt, Peter (2000). "*Beyond search: Fiat money in organized exchange*", mimeo.

Jafarey, Saqib and Masters, Adrian (2003). "*Output, prices and the velocity of money in search equilibrium*", Journal of Money, Credit, and Banking, forthcoming.

Johri, Alok (1999). "*Search, money and prices*", International Economic Review 40, 439-54.

Kiyotaki, Nobuhiro, and Moore, John H. (2001). "Evil is the root of all money", The Claredon Lectures, Lecture 1.

Kiyotaki, Nobuhiro, and Randall Wright (1989). "*On money as a medium of exchange*", Journal of Political Economy 97, 927-954.

Kiyotaki, Nobuhiro, and Randall Wright (1991). "*A contribution to the pure theory of money*", Journal of Economic Theory 53, 215-235.

Kiyotaki, Nobuhiro, and Randall Wright (1993). "*A search-theoretic approach to monetary economics*", American Economic Review 83, 63-77.

Kocherlakota, Narayana (1998). "*Money is memory*", Journal of Economic Theory 81, 232-251.

Lagos, Ricardo and Rocheteau, Guillaume (2003). "*Superneutrality and the welfare effects of inflation*", mimeo.

Lagos, Ricardo and Wright, Randall (2002). "*A unified framework for monetary theory and policy analysis*", mimeo.

Lagos, Ricardo and Wright, Randall (2003). "*Dynamics, cycles, and sunspot equilibria in 'genuinely dynamic, fundamentally disaggregative' models of money*", Journal of Economic Theory, forthcoming.

Levine, David (1991). "*Asset trading mechanisms and expansionary policy*", Journal of Economic Theory 54, 148-164.

Li, Victor (1995). "*The optimal taxation of fiat money in search equilibrium",* International Economic Review 36, 927-942.

Li, Victor (1997). "*The efficiency of monetary exchange in search equilibrium",* Journal of Money, Credit and Banking 29, 61-72.

Lucas, Robert E. and Prescott, Edward (1974), "*Equilibrium search and unemployment*", Journal of Economic Theory 7, 188-209.

McCallum, Bennett T. and Marvin S. Goodfriend (1987). "*Demand for Money: Theoretical Studies*", The New Palgrave: A Dictionary of Economics, ed. by J. Eatwell, P. Newman and M. Milgate. London: The Macmillan Press and New York: Stockton Press.

Moen, Espen R. (1997). "*Competitive search equilibrium*", Journal of Political Economy 105, 385-411.

Molico, Miguel, (1999) "*The distribution of money and prices in search equilibrium*", mimeo.

Mortensen, Dale and Wright, Randall (2002). "*Competitive pricing and efficiency in search equilibrium*", International Economic Review 43, 1-20.

Petrongolo, Barbara and Pissarides, Christopher (2001). "*Looking into the black box: A survey of the matching function*", Journal of Economic Literature 39, 390-431.

Pissarides, Christopher (2000). "*Equilibrium unemployment theory*", 2nd edition, MIT Press, Cambridge.

Shi, Shouyong (1995). "*Money and prices: A model of search and bargaining*", Journal of Economic Theory 67, 467-496.

Shi, Shouyong (1997). "*A divisible search model of fiat money*", Econometrica 65, 75-102.

Shimer, Robert (1996). "*Contracts in frictional labor market*", mimeo.

Temzelides, Ted and Yu, Jialin (2003). "*Sufficient conditions for the essentiality of money*", mimeo.

Townsend, Robert (1980). "*Models of money with spatially separated agents*", In J. H. Kareken and N. Wallace (eds.), Models of Monetary Economies. Minneapolis: Federal Reserve Bank of Minneapolis 265-303.

Trejos, A. and R. Wright (1995). "*Search, bargaining, money, and prices*", Journal of Political Economy 103, 118-141.

Wallace, Neil (1980). "*The overlapping generations model of fiat money*", In J. H. Kareken and N. Wallace (eds.), Models of Monetary Economies. Minneapolis: Federal Reserve Bank of Minneapolis 49-82.

Wallace, Neil (2001). "*Whither monetary economics?*", International Economic Review 42, 847-869.

Zhou, Ruilin (1999). "*Individual and aggregate real balances in a random-matching model*", International Economic Review 40, 1009-1038.