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MONODROMIES OF HYPERELLIPTIC FAMILIES OF GENUS THREE CURVES

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Abstract. A complete list of the monodromies of degenerations of genus three which are not realized as the monodromies of any hyperelliptic families of genus three is given. We also prove that all the other monodromies of genus three are realized as the monodromies of certain hyperelliptic families.

Introduction. Let $\phi\colon S \to \Delta$ be a proper surjective holomorphic map from a nonsingular complex surface S to a small disk $\Delta := \{t \in \boldsymbol{C} \mid |t| < \varepsilon\}$ such that $\phi^{-1}(t)$ is a nonsingular curve of genus $g \geq 2$ for each $t \in \Delta^* := \Delta \setminus \{0\}$. We call (ϕ, S, Δ) a *degeneration of curves* of genus g. If all $\phi^{-1}(t)$ for $t \in \Delta^*$ are hyperelliptic curves, we call (ϕ, S, Δ) a *hyperelliptic family*. We set $X := \phi^{-1}(0)$ and call it the *special fiber* of S. If the reduced scheme of X has normal crossings as singularities and any (-1)-curve in the special fiber intersects the other components at at least three points, (ϕ, S, Δ) is said to be *normally minimal*. Two degenerations (ϕ, S, Δ) and (ϕ', S', Δ') are said to be *topologically equivalent* if there exist orientation-preserving homeomorphisms $\psi : S \to S'$ and $\overline{\psi} : \Delta \to \Delta'$ satisfying $\phi' \circ \psi = \overline{\psi} \circ \phi$.

Let $\mathcal{T}_g :=$ {normally minimal degenerations of genus g}/$\sim$, where $\sim$ is the topological equivalence. For an element of $\mathcal{T}_g$, we can uniquely determine the topological monodromy (sometimes called the monodromy, for short) as a conjugacy class in the mapping class group of genus g. The monodromy of a degeneration is a conjugacy class of a pseudo-periodic map of negative type (cf. [MM1], [Ni1], [Ni2], [Im], [ES], [ST], [AMO] etc.). Conversely, any conjugacy class of a pseudo-periodic map of negative type is realized as the monodromy of a certain degeneration (cf. [MM2]). In [AI], using the theory of Harvey and Wiman (cf. [Ha], [Wi]) and the list of the stable curves of genus three in [F], we classified the monodromies of degenerations of curves of genus three together with their topological types of moduli points.

In this paper, we completely classify the monodromies of degenerations of genus three that cannot be realized as the monodromies of any hyperelliptic families of genus three (Theorem 1.8). Moreover, we prove that all the other monodromies of genus three are realized as the monodromies of certain hyperelliptic families. For the classification, we define an operation called the "inverse of Horikawa's canonical resolution". Using this operation, we easily see that the closure of the hyperelliptic locus $\overline{\boldsymbol{H}_3}$ in the Deligne-Mumford compactification $\overline{\boldsymbol{M}_3}$ of the moduli space of genus three curves does not intersect the strata of the stable curves

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of types (D), (H), (I), (M), (O) (cf. Corollary 1.6). In order to prove the existence of hyperelliptic families with monodromies not listed in Theorem 1.8, we give, for each monodromy, the defining equation of a hyperelliptic family with the monodromy.

When we deal with the monodromies of hyperelliptic families, we need to deal carefully with the data of monodromies called the screw number.

The conjugacy class of a pseudo-periodic map $f\colon \Sigma_g \to \Sigma_g$ of negative type of genus g (i.e., the monodromy of a degeneration) can be determined by the following data (cf. [MM2]): (i) an admissible sysytem of cut curves $\mathcal{C} = \coprod C_i$ on Σ_g, (ii) an action of f on the oriented graph induced by the admissible system, (iii) the valency data of the stabilizer of each component of $\Sigma \setminus \mathcal{C}$, (iv) the screw number of f around each neighborhood of C_i.

For example, there exist conjugacy classes $[f_1]$ and $[f_2]$ of pseudo-periodic maps of negative type such that $[f_1]$ can be realized as the monodromy of a hyperelliptic family but not $[f_2]$, although their data (i), (ii) and (iii) coincide.

In Section 1, we first classify the monodromies among those listed in [AI] that cannot be realized as monodromies of any hyperelliptic family of genus three (cf. the list in Theorem 1.8). More precisely, for each monodromy $[f]$ listed in Theorem 1.8, we prove that any family whose monodromy is $[f]$ cannot be obtained by Horikawa's canonical resolution of any double covering of $\boldsymbol{P}^1 \times \Delta$. We also show in Theorem 1.8 that all monodromies in [AI] not listed in Theorem 1.8 can be realized as the monodromies of certain hyperelliptic families. In Section 2, we prove this by constructing families whose monodromies are not listed in Theorem 1.8.

In this paper, We adopt the same terminology for topological monodromies of genus three as in [AI].

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1. Possibility for the existence.

1.1. Inverse of canonical resolution. We first review Horikawa's canonical resolution of singularities appearing in double coverings of a surface (cf. [Ho1, §2]). Let $\phi\colon S \to \Delta$ be a normally minimal hyperelliptic family of genus g. By the same argument as in [Ho2, §1], we see that S is bimeromorphic to a double covering $\psi_0\colon S_0 \to W_0 := \boldsymbol{P}^1 \times \Delta$ branched along a divisor B_0 of W_0. More precisely, there exists a line bundle F_0 on $\boldsymbol{P}^1 \times \Delta$ such that the line bundle $[B_0]$ associated to B_0 is isomorphic to $F_0^{\otimes 2}$ and that S_0 is realized in the total space of F_0 as a double covering of $\boldsymbol{P}^1 \times \Delta$. Let π_0 be the second projection of W_0. We set $\Gamma_t := \pi_0^{-1}(t)$, $\tilde{B}_0 := B_0 - \Gamma_0$ when Γ_0 is a component of B_0, and $\tilde{B}_0 := B_0$ otherwise. The pair (S_0, B_0) satisfies the following conditions:

(i) The intersection number $B_0 \cdot \Gamma_t$ is equal to $2g + 2$.

(ii) If the local intersection number $I_P(\tilde{B}_0, \pi_0^{-1}(t))$ of $\tilde{B}_0$ and Γ_t at P is greater than one, then P is on Γ_0.

We define τ_i, $\tilde{\tau}_i$, π_i, B_i, $\tilde{B}_i$, F_i, $\tilde{\Gamma}_i$, E_i and ψ_i inductively as follows: Let τ_0 be the identity map of W_0. We choose a *bad point* P_{i-1} on B_{i-1}, that is, P_{i-1} is a singular point or satisfies $I_{P_{i-1}}(\tilde{B}_{i-1}, ((\tau_0 \circ \cdots \circ \tau_{i-1})^* \Gamma_0)_{\mathrm{red}}) \geq 2$, where $\tilde{B}_{i-1}$ is the strict transform of $\tilde{B}_0$ by $\tau_0 \circ \cdots \circ \tau_{i-1}$. Let $\tau_i \colon W_i \to W_{i-1}$ be the blowing-up at P_{i-1}. We denote the multiplicity of B_{i-1} at P_{i-1} by $m_{P_{i-1}}$. Let E_i be the exceptional set of τ_i. We define $\tilde{\Gamma}_{i-1}$ as the reduced scheme of $(\tau_0 \circ \cdots \circ \tau_{i-1})^* \Gamma_0$. We set $B_i := \tau_i^* B_{i-1} - 2[m_{P_{i-1}}/2]E_i$ and $F_i := \tau_i^* F_{i-1} - [m_{P_{i-1}}/2]E_i$, where $[m_{P_{i-1}}/2]$ is the greatest integer not exceeding $m_{P_{i-1}}/2$. Since $[B_i] \simeq F_i^{\otimes 2}$, we can take a double covering $\psi_i \colon S_i \to W_i$ branched along B_i in the total space of F_i, and naturally define a bimeromorphic map $\tilde{\tau}_i \colon S_i \to S_{i-1}$ (cf. [Ho1, §2]). We set $\pi_i := \pi_{i-1} \circ \tau_i$. Repeating this process at all bad points, we obtain a sequence of blowing-ups $W_r \xrightarrow{\tau_r} \cdots \to W_1 \xrightarrow{\tau_1} W_0$ satisfying the following properties:

(a) B_r is nonsingular.

(b) $\Theta := (\tau_1 \circ \cdots \circ \tau_r)^*(\Gamma_0)$ and the strict transform of $\tilde{B}_0$ intersect each other transversally.

S_r is nonsingular by (a). The reduced scheme of the special fiber of S_r is a normal crossing divisor by (b). We obtain the original normally minimal model $\phi \colon S \to \Delta$ by the composite of the blowing-downs of suitable (-1)-curves successively on S_r. We call the above process *Horikawa's canonical resolution* (the canonical resolution, for short). In this paper, we always use r as the length of the sequence of the blowing-ups that satisfies the conditions (a) and (b).

Conversely, choosing a component E_r' of $(\tau_1 \circ \cdots \circ \tau_r)^*(\Gamma_0)$ whose self-intersection number is -1, we consider the blowing-down $\tau_r' \colon W_r \to W_{r-1}'$ which contracts E_r' to a point P'. We set $B_r' := B_r - E_r'$ when E_r' is a component of B_r, and $B_r' := B_r$ otherwise. Let $m_{P'}$ be the intersection number $E_r' \cdot B_r'$. Since $(\tau_r')_*(B_r + 2[m_{P'}/2]E_r')$ is isomorphic to $(\tau_r')_*(F_r + [m_{P'}/2]E_r')^{\otimes 2}$, we can take the double covering $\psi_{r-1}' \colon S_{r-1}' \to W_{r-1}'$ branched along $(\tau_r')_* B_r$ and naturally define a morphism $\tilde{\tau}_r' \colon S_r \to S_{r-1}'$. Repeating this process, we finally obtain a sequence of blowing-downs $W_r \xrightarrow{\tau_r} \cdots \to W_1' \xrightarrow{\tau_1} W_0'$ and a double covering $\psi_r' \colon S_0' \to W_0' = \boldsymbol{P}^1 \times \Delta$ such that S_0' is bimeromorphic to S_r. We call this process an *inverse of Horikawa's canonical resolution*. Note that if the multiplicity of a component E of $(\tau_1 \circ \cdots \circ \tau_r)^* \Gamma_0$ is one, we can find an inverse of Horikawa's canonical resolution such that $(\tau_1' \circ \cdots \circ \tau_r')_* E$ is $\boldsymbol{P}^1$, i.e., we can consider $(\tau_1' \circ \cdots \circ \tau_r')_* E$ to be Γ_0. We call this an *inverse of Horikawa's canonical resolution associated to* E. Let C be a prime divisor of S that is a component of $\phi^{-1}(0)$. Let Z be the set of points that are the images of the exceptional curves of $\tilde{\tau} \colon S_r \to S$. Let $\Pi(C) := \overline{\psi_r \circ \tilde{\tau}^{-1}(C - (C \cap Z))}$ denote the closure of $\psi_r \circ \tilde{\tau}^{-1}(C - (C \cap Z))$ in W_r. $\Pi(C)$ is also a prime divisor on W_r. Assume that C' is another component of $\phi^{-1}(0)$ satisfying $\Pi(C) \cap \Pi(C') = \emptyset$. Since the dual graph of Θ is connected, there exists a subdivisor $D_{CC'} = \sum a_i E_i$ of Θ that satisfies the following conditions (we use the same symbol E_i for the strict transform of E_i on W_r):

(i) $\Theta \geq D_{CC'}$ and $\Theta \not\geq D_{CC'} + E_i$ for all E_i $(a_i \neq 0)$.

(ii) $\mathrm{Supp}(\Pi(C)) \cap D_{CC'} \neq \emptyset$ and $\mathrm{Supp}(\Pi(C')) \cap D_{CC'} \neq \emptyset$.

(iii) $D_{CC'} \not\geq \Pi(C)$ and $D_{CC'} \not\geq \Pi(C')$.

(iv) Supp($D_{CC'}$) is connected.

Since the dual graph of Θ has no loop, $D_{CC'}$ is uniquely determined. We set $D_{CC'} = 0$ when Π(C) intersects Π(C'). We call the divisor $D_{CC'}$ the *bridge* between $\Pi(C)$ and $\Pi(C')$.

1.2. Periodic case. Let $\Delta' \to \Delta$ be a totally ramified cover of degree d branched at the origin. Let S_d be the nonsiglular model of $S \times_\Delta \Delta'$ and $\phi_d\colon S_d \to \Delta'$ the natural morphism. Let f be a representative of the monodromy of (ϕ, S, Δ) (namely, f is a pseudo-periodic map and its conjugacy class $[f]$ in the mapping class group is the monodromy of (ϕ, S, Δ)). Then the monodromy of (ϕ_d, S_d, Δ') is $[f^d]$. In Lemma 1.4 of [AI], we classified the conjugacy classes of periodic maps of genus g $(1 \le g \le 3)$. The data for the conjugacy class of a periodic map $[f]$ consists of two invariants: the period and the total valency. The period n is the smallest positive integer such that f^n is isotopic to the identity.

We introduce the notion of the valency originally defined by Nielsen ([Ni1]). By Kerchhoff's theorem (cf. [Ke]), for each periodic homeomorphism f, there exist a Riemann surface Σ_g of genus g and an analytic automorphism $\bar{f}\colon \Sigma_g \to \Sigma_g$ isotopic to f. For each point P on Σ_g, we denote by r_P the cardinality of the orbit of P under $\bar{f}$, and let $l_P := n/r_P$. Let δ_P be the smallest nonnegative integer such that $\bar{f}^{r_P}$ is the rotation of angle $2\pi\delta_P/l_P$ near each point in the orbit. Denote by s_P the smallest positive integer satisfying $\delta_P s_P \equiv 1 \pmod{l_P}$ if $\delta_P \ne 0$, and set $s_P := 0$ when $\delta_P = 0$. The symbol s_P/l_P is called the *valency* of the orbit of P.

Note that the valencies of all but a finite number of orbits are zero. The set of the positive valencies is called the *total valency* of $\bar{f}$ and expressed as the formal sum $\sum s_P/l_P$ of symbols.

We define the *total valency of a periodic homeomorphism* f as the total valency of $\bar{f}$. It is well-known that the conjugacy class of a periodic map is determined by its period and total valency.

For instance, (1) in [AI, p. 202], $n = 14$; $11/14+5/7+1/2$ means that there exist three orbits $\mathcal{O}_1 = \{P_1\}$, $\mathcal{O}_2 = \{P_2, \bar{f}^2(P_2)\}$, and $\mathcal{O}_3 = \{P_3, \bar{f}^2(P_3), \cdots, \bar{f}^6(P_3)\}$ such that $\bar{f}$ is the rotaion of angle $2\pi \times 9/14$ near P_1, $\bar{f}^2$ is the rotaion of angle $2\pi \times 3/7$ near each point in $\mathcal{O}_2$ and $\bar{f}^7$ is the rotaion of angle π near each point in $\mathcal{O}_3$.

We use the same symbols as in [AI, p. 202 and p. 203]. To avoid confusion with another number in another paragraph in [AI, p. 203], we denote for example by (i1) the monodromy (1) in [AI, p. 202], $n = 14$; $11/14+5/7+1/2$.

Let S be a family of genus three with periodic monodromy (i1) and $\bar{f}$ as above. Taking a base change of degree seven, we obtain a family $S_7 \to \Delta$ whose monodromy is a periodic map with $n = 2$; $1/2+1/2+1/2+1/2+1/2+1/2+1/2+1/2$, because the points in $\mathcal{O}_1$ and $\mathcal{O}_3$ are the fixed points of the involution $\bar{f}^7$. Repeating this calculation for all periodic maps of genus three, we obtain the following:

LEMMA 1.1. *By taking a base change of suitable degree, all periodic maps of genus three are obtained from those with*

(i1) $n = 14$, $11/14 + 5/7 + 1/2$,

(i7) $n = 12,\ 11/12 + 7/12 + 1/2$,
(i9) $n = 12,\ 11/12 + 3/4 + 1/3$,
(i13) $n = 9,\ 8/9 + 4/9 + 2/3$,
(i20) $n = 8,\ 1/8 + 5/8 + 1/4$,
(i22) $n = 8,\ 3/8 + 3/8 + 1/4$,
(i28) $n = 7,\ 1/7 + 2/7 + 4/7$,
(i44) $n = 4,\ g' = 1,\ 1/2 + 1/2$,
(i47) $n = 2,\ g' = 2$ *and* $\Pi\colon \Sigma_g \to \Sigma_{g'}$ *is an unramified covering.*

PROOF. We set $m_1 =$ (i1), $m_7 :=$ (i7), $m_9 :=$ (i9), $m_{13} :=$ (i13), $m_{20} :=$ (i20), $m_{22} :=$ (i22), $m_{28} :=$ (i28). Then, by elementary calculations, we obtain the following equations:

$(m_1)^2 =$ (i31), $(m_1)^3 =$ (i3), $(m_1)^4 =$ (i25), $(m_1)^5 =$ (i6),
$(m_1)^6 =$ (i29), $(m_1)^7 =$ (i43), $(m_1)^8 =$ (i30), $(m_1)^9 =$ (i5),
$(m_1)^{10} =$ (i26), $(m_1)^{11} =$ (i4), $(m_1)^{12} =$ (i32), $(m_1)^{13} =$ (i2),
$(m_7)^2 =$ (i33), $(m_7)^3 =$ (i39), $(m_7)^4 =$ (i45), $(m_7)^7 =$ (i8),
$(m_7)^9 =$ (i40), $(m_9)^2 =$ (i34), $(m_9)^3 =$ (i36), $(m_9)^4 =$ (i42),
$(m_9)^5 =$ (i11), $(m_9)^6 =$ (i46), $(m_9)^7 =$ (i12), $(m_9)^8 =$ (i41),
$(m_9)^{10} =$ (i35), $(m_9)^{11} =$ (i10), $(m_{13})^2 =$ (i16), $(m_{13})^4 =$ (i18),
$(m_{13})^5 =$ (i17), $(m_{13})^6 =$ (i42), $(m_{13})^7 =$ (i15), $(m_{13})^8 =$ (i14),
$(m_{20})^2 =$ (i37), $(m_{20})^3 =$ (i19), $(m_{22})^3 =$ (i24), $(m_{22})^6 =$ (i38),
$(m_{22})^5 =$ (i23), $(m_{22})^7 =$ (i21), $(m_{28})^3 =$ (i27).

□

LEMMA 1.2. *Let E be a component of $(\tau_1 \circ \cdots \circ \tau_i)^*(\Gamma_0)$ whose multiplicity α is greater than or equal to two. Assume that E intersects at least three distinct components E_{j_1}, E_{j_2}, E_{j_3} of $(\tau_1 \circ \cdots \circ \tau_i)^*(\Gamma_0)$. Let $\hat{E}_{j_i}$ $(i = 1, 2, 3)$ be mutually distinct maximal connected subdivisors of $(\tau_1 \circ \cdots \circ \tau_i)^*(\Gamma_0)$ such that their supports do not contain E and that $\hat{E}_{j_i} \geq E_{j_i}$. In any inverse of the canonical resolution, at least one of the $\hat{E}_{j_i}$ $(i = 1, 2, 3)$ is contracted before E.*

PROOF. Let $\tau_{r'}$ be the blowing-up such that the strict transform of $E_{r'}$ by $\tau_{r'+1} \circ \cdots \circ \tau_i$ is E. If none of the $\hat{E}_{j_i}$ $(i = 1, 2, 3)$ are contracted before E in any inverse of the canonical resolution, three distinct nonzero subdivisors $(\tau_{r'} \circ \cdots \circ \tau_i)(\hat{E}_{j_i})$ $(i = 1, 2, 3)$ intersect at a point on $W_{r'-1}$. However, considering the process of the canonical resolutions, we see that the singularities of the reduced scheme of $(\tau_1 \circ \cdots \circ \tau_s)^*(\Gamma_0)$ are ordinary double points for all s, a contradiction. □

COROLLARY 1.3. *In the notation as above, there exists a subdivisor D of $(\tau_1 \circ \cdots \circ \tau_i)^*(\Gamma_0)$ such that one of $\hat{E}_{j_1}$, $\hat{E}_{j_2}$, $\hat{E}_{j_3}$ coincides with αD.*

PROPOSITION 1.4. *There exist no hyperelliptic families whose topological monodromies are* (i9), (i10), (i11), (i12), (i13), (i14), (i15), (i16), (i17), (i18), (i19), (i20), (i28), (i34), (i35), (i36), (i37), (i41), (i42).

PROOF. By the argument in the proof of Lemma 1.1, we see that by taking a base change of suitable degree, the periodic monodromies listed above become one of the following: (i28) $n = 7$; $1/7+2/7+4/7$, (i37) $n = 4$; $1/4+1/4+1/4+1/4$, (i42) $n = 3$; $1/3+1/3+1/3+1/3+2/3$. Thus, it suffices to prove that there exist no hyperelliptic families whose monodromies are one of them.

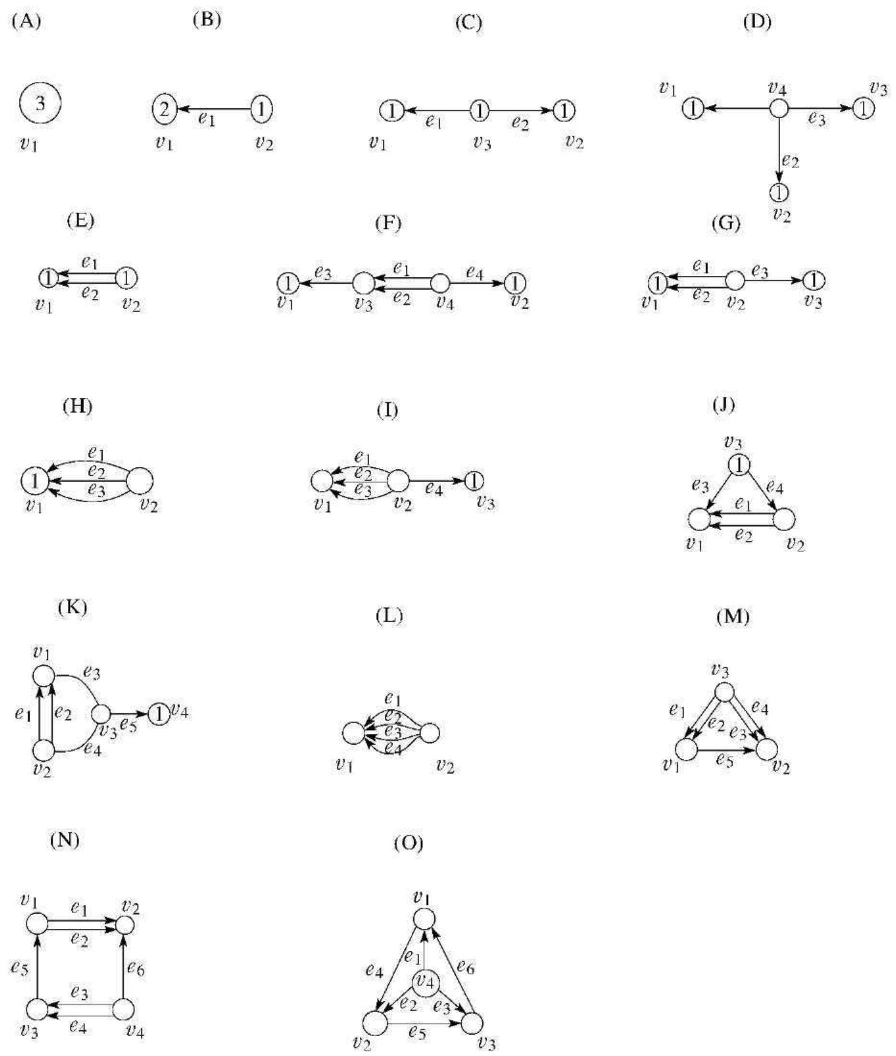
Assume that there exists a hyperelliptic family S whose monodromy is (i28). Let C_0, C_1, C_2, C_3 be the components of the special fiber of S whose multiplicities are 7, 1, 2 and 4, respectively, and C_0 intersects C_1, C_2 and C_3. Since their multiplicities are distinct, $\Pi(C_i)$ are all distinct. Since the multiplicity of each $\Pi(C_i)$ is not a multiple of seven and the bridge $D_{C_0C_i}$ intersects $\Pi(C_0)$ at a point, Lemma 1.2 implies the nonexistence. Case (i42) is similar to Case (i28).

Assume that there exists a hyperelliptic family S whose monodromy is (i37). Let C_0, $C_1, \dots, C_4$ be the components of the special fiber of S whose multiplicities are 4, 1, ..., 1, respectively. If $\Pi(C_0)$ is a component of B_r, $\Pi(C_0)$ intersects at least four components. It contradicts Lemma 1.2. Thus we may assume that $\Pi(C_0)$ is not a component of B_r with multiplicity four, $\Pi(C_1) = \Pi(C_2)$ and $\Pi(C_3) = \Pi(C_4)$ with multiplicity one, respectively. If the bridge $D_{C_0C_1}$ intersects B_r or contains a component of B_r, $\psi_r^*(D_{C_0C_1})$ is connected. In this case, $\tilde{\tau}(\psi_r^*(D_{C_0C_1}))$ is a point, and C_0, C_1 and C_2 intersect at a point, a contradiction. Thus, if $D_{C_0C_1} \neq 0$, all components of $D_{C_0C_1}$ are not components of B_r and do not intersect B_r. Since the multiplicities of the components of $\psi_r^*(D_{C_0C_1})$ are greater than or equal to five, the multiplicities of the components of $D_{C_0C_1}$ are greater than or equal to five. Then, by any inverse of the canonical resolution, $D_{C_0C_1}$ is contracted before $\Pi(C_0)$ and $\Pi(C_1)$ are contracted. It means $D_{C_0C_1} = 0$ because, by our definition of the canonical resolution, we do not blow up at non-bad points. By the same argument, $\Pi(C_0)$ intersects $\Pi(C_3)$ at a point. Thus, there exists no component of $(\tau_1 \circ \cdots \circ \tau_r)^*(\Gamma_0)$ whose self-intersection number is -1, a contradiction. □

1.3. Non-periodic case. By the semistable reduction theorem (cf. [DM]), there exists a branched cover $\Delta' \to \Delta$ totally ramified over the origin with degree d such that $S_d \to \Delta'$ is a semistable family. We call it a *semistable model* of $\phi\colon S \to \Delta$. Let $S_d \to S'$ be a composite of the blowing-downs of (-2)-curves so that S' is free from (-2)-curves. We call $S' \to \Delta'$ a *stable model* of $\phi\colon S \to \Delta$. We sometimes call the special fiber of a (semi)stable model of $\phi\colon S \to \Delta$ the *(semi)stable model* of the special fiber of $\phi\colon S \to \Delta$. We introduce the weighted graphs (A) through (O) as the dual graphs of the stable curves in Table 1 (cf. Table 2 in [AI]). A vertex v corresponds to a component of a stable curve and an edge corresponds to an intersection of two components. Let $g(v)$ and $\rho(v)$ be the genus and the number of singular points of the component v, respectively. The number inside a small circle in Table 1 means $g(v) + \rho(v)$. We omit the number when it is zero. For instance, the graph (B) represents six stable curves, that is, v_1 has genus i_1 and $2 - i_1$ singular points while v_2 has genus i_2 and $1 - i_2$ singular points ($0 \leq i_1 \leq 2$, $0 \leq i_2 \leq 1$). We write the stable curves $B_{i_1i_2}$ ($0 \leq i_1 \leq 2$, $0 \leq i_2 \leq 1$) for short. Figures (A) to (O) in Table 1 can be regarded as the dual graphs of stable curves of genus three. Furthermore, if we replace each edge of these graphs by a chain of (-2)-curves, these graphs can be regarded as the weighted dual graphs of semistable curves of genus three. We call a chain of (-2)-curves a $\boldsymbol{P}^1$*-chain* at the edge. The number of components of a $\boldsymbol{P}^1$-chain is called the *length* of the $\boldsymbol{P}^1$-chain.

PROPOSITION 1.5. *There exists no hyperelliptic family of genus three whose special fiber of the stable model has a topological type of either* (D), (H), (I), (M) *or* (O) *in Table* 1.

TABLE 1. Stable curves of genus 3.



PROOF. Since the degeneration obtained by a base change of a hyperelliptic family is a hyperelliptic family, we may assume the hyperelliptic family $\phi\colon S \to \Delta$ is (semi)stable. The vertices of the graphs are regarded as the corresponding irreducible components for the simplicity of notation.

Assume that there exists a hyperelliptic family whose special fiber is (D). Since the multiplicity of each v_i is one, $\Pi(v_i)$ is not a component of B_r. Since v_i $(1 \leq i \leq 3)$ intersects v_4 at a point, the bridge $D_{v_i v_4}$ between $\Pi(v_i)$ and $\Pi(v_4)$ intersects v_4 at one point P_i. Moreover, each P_i is contained in B_r. It means that v_4 is the double covering of $\boldsymbol{P}^1$ branched at least at three points. This contradicts the fact that v_4 is $\boldsymbol{P}^1$.

Assume that there exists a hyperelliptic family whose special fiber is (H). $\Pi(v_1)$ does not coincide with $\Pi(v_2)$, because v_1 is not homeomorphic to v_2. Since v_1 is connected with v_2 by three $\boldsymbol{P}^1$-chains, there exist at least two bridges between $\Pi(v_1)$ and $\Pi(v_2)$, a contradiction.

Assume that there exists a hyperelliptic family whose special fiber is (I). Since v_3 is not homeomorphic to v_2, we have $\Pi(v_2) \neq \Pi(v_3)$. $\Pi(v_2)$ intersects $D_{v_2 v_3}$ at a point on B_r, because v_2 is connected with v_3 by a $\boldsymbol{P}^1$-chain. Since $\Pi(v_2)$ intersects B_r, we see that $\Pi(v_1) \neq \Pi(v_2)$. Since v_1 is connected with v_2 by three $\boldsymbol{P}^1$-chains, there exist at least two bridges between $\Pi(v_1)$ and $\Pi(v_2)$, a contradiction.

Assume that there exists a hyperelliptic family whose special fiber is (M). Since the dual graph of Θ has no loop, we have $\Pi(v_1) = \Pi(v_2)$. Since $\Pi(v_1) \neq \Pi(v_3)$, there exist at least two bridges between $\Pi(v_3)$ and $\Pi(v_1)$, a contradiction.

Assume that there exist a hyperelliptic family whose special fiber is (O). We may assume $\Pi(v_1) = \Pi(v_2)$ and $\Pi(v_3) = \Pi(v_4)$. In view of the configuration of (O), there exist at least two bridges between $\Pi(v_1)$ and $\Pi(v_3)$, a contradiction. □

COROLLARY 1.6. *Let $\overline{\boldsymbol{M}_3}$ be the Deligne-Mumford compactification of the moduli space of curves of genus three. The closure of the hyperelliptic locus $\overline{\boldsymbol{H}_3}$ in $\overline{\boldsymbol{M}_3}$ does not intersect the loci of the stable curves whose topological types are* (D), (H), (I), (M) *and* (O).

PROOF. Let C be a stable curve whose moduli point is on $\overline{\boldsymbol{H}_3}$. According to [HM, Theorem 3.160], there exists an admissible double cover $\pi : C \to B$ of a stable 8-marked curve B of genus 0. In the proof of Proposition 1.5, we showed that we cannot construct stable curves of type (D), (H), (I), (M) and (O) as a double cover of genus 0 curves. □

PROPOSITION 1.7. *There exists no (semi)stable hyperelliptic family whose special fiber of the stable model has a topological type of either* (E), (F), (G), (J), (K) or (N), *if the following conditions are satisfied*:

(i) *The length of the $\boldsymbol{P}^1$-chains at e_1 and e_2 are mutually distinct in the cases* (E), (F) *and* (G).

(ii) *The length of the $\boldsymbol{P}^1$-chains at e_3 and e_4 are mutually distinct in the cases* (J) *and* (K).

(iii) *The length of the $\boldsymbol{P}^1$-chains at e_5 and e_6 are mutually distinct in the case* (N).

PROOF. If there exists a family whose special fiber is (E) or (G) satisfying the above condition (i), then we see that $\Pi(v_1) \neq \Pi(v_2)$ and there exist at least two bridges between $\Pi(v_1)$ and $\Pi(v_2)$, a contradiction. In the case (F), considering the brideges between $\Pi(v_3)$ and $\Pi(v_4)$, we have the same contradiction. By the same argument, we can prove the non-existence of families with monodromies (J) or (K) satisfying the above condition (ii).

We assume that there exists a family whose special fiber is (N) satisfying the condition (iii). Assume that $\Pi(v_1) = \Pi(v_2)$. Since Θ has no loop, we may assume that $\Pi(v_3) = \Pi(v_4)$. From the condition of (iii), we know that $D_{v_1v_3} \neq D_{v_2v_4}$, although $D_{v_1v_3}$ and $D_{v_2v_4}$ are the bridges between $\Pi(v_1)$ and $\Pi(v_3)$, a contradiction to the unicity of the bridge. Thus, we may assume that $\Pi(v_1) \neq \Pi(v_2)$. Since Θ has no loop, we may assume that $\Pi(v_1) = \Pi(v_3)$ and $\Pi(v_2) = \Pi(v_4)$. In this case the double covering $\psi_r^*(D_{v_1v_2})$ must be two distinct $\boldsymbol{P}^1$-chains between v_1 and v_2. On the other hand, $\psi_r^*(D_{v_3v_4})$ must be two distinct $\boldsymbol{P}^1$-chains between v_3 and v_4. Since $D_{v_1v_2} = D_{v_3v_4}$, $\psi_r^*(D_{v_1v_2})$ must be four distinct $\boldsymbol{P}^1$-chains, a contradiction to the fact that ψ_r is a map of degree two. □

Choosing a topological monodromy, we define an integer $-K_{e_i}$ for each edge as the sum of the valencies and the screw number at the edge ([MM1], [AI, p. 201]). These integers play a very important role when we deal with the monodromies of hyperelliptic families. In the following theorem, c denotes a nonnegative integer.

THEOREM 1.8. *There exist no hyperelliptic families satisfying the following conditions*:

(i) *The topological type of the stable model is one of the types* (D), (H), (I), (M) *and* (O).

(ii) *The topological type of the semistable model is one of the types* (E), (F), (G), (J), (K), (N) *satisfying the conditions* (i), (ii) *and* (iii) *in Proposition* 1.7.

(iii) *The monodromy satisfies one of the following*:

A_3: (i9), (i10), (i11), (i12), (i13), (i14), (i15), (i16), (i17), (i18), (i19), (i20), (i28), (i34), (i35), (i36), (i37), (i41), (i42).

A_2: (iii2), (iii3), (iii7), (iii9), (iii11), (iii13), (iii14), (iii16), (iii18), (iii28).

A_1: (viii5), (viii12).

A_0: (xv2), (xv7).

B_{1i} $(i = 0, 1)$: $V_1 = \{$(vii4), (vii7), (vii8)$\}$.

B_{0i} $(i = 0, 1)$: $V_1 =$ (xiv3).

$C_{111}, C_{101}, C_{001}$: Id, $V_3 = \{$(va4), (va5)$\}$.

C_{111}, C_{001}: II(1,1) $V_3 = \{$(vb1), (vb2)$\}$.

E_{11}: II(0,1), $V_1 = \{$(vb1)$\}$, $V_2 = \{$(vb1, 3, 5)$\}$, $K_{e_1} = 2c - 1$.

E_{11}: II(0,1), $V_1 = \{$(vb1)$\}$, $V_2 = \{$(vb2, 4, 6)$\}$, $K_{e_1} = 2c$.

E_{10}: II(0,1), $V_1 = \{$(vb1, 3, 5)$\}$, $V_2 = \{$(xiib1)$\}$, $K_{e_1} = 2c - 1$.

E_{10}: II(0,1), $V_1 = \{$(vb2, 4, 6)$\}$, $V_2 = \{$(xiib2)$\}$, $K_{e_1} = 2c + 1$.

E_{10}: II(0,1), $V_1 = \{$(vb2, 4, 6)$\}$, $V_2 = \{$(xiib1)$\}$, $K_{e_1} = 2c$.

E_{10}: II(0,1), $V_1 = \{$(vb1, 3, 5)$\}$, $V_2 = \{$(xiib2)$\}$, $K_{e_1} = 2c$.

E_{00}: $\Pi(0,1)$, $V_1 = V_2 = \{(\text{xiibs})\}$, $K_{e_1} = 2c - 1$.
E_{00}: $\Pi(0,1)$, $V_1 = \{(\text{xiibs}_1)\}$, $V_2 = \{(\text{xiibs}_2)\}$, $s_1 \neq s_2$, $K_{e_1} = 2c$.
$F_{ij}(i, j = 0, 1)$: $\Pi(0,1)$, $K_{e_1} = 2c - 1$.
$G_{1i}(i = 0, 1)$: $\Pi(0,1)$, $V_1 = \{(\text{vb}1, 3, 5)\}$, $K_{e_1} = 2c - 1$.
$G_{1i}(i = 0, 1)$: $\Pi(0,1)$, $V_1 = \{(\text{vb}2, 4, 6)\}$, $K_{e_1} = 2c$.
$G_{0i}(i = 0, 1)$: $\Pi(0,1)$, $V_1 = \{(\text{xiib}1)\}$, $K_{e_1} = 2c - 1$.
$G_{0i}(i = 0, 1)$: $\Pi(0,1)$, $V_1 = \{(\text{xiib}2)\}$, $K_{e_1} = 2c$.
J_1: $\Pi(1,4)$, $V_1 = \{(\text{vb}1, 3, 5)\}$, $K_{e_1} = 2c$.
J_1: $\Pi(1,4)$, $V_1 = \{(\text{vb}2, 4, 6)\}$, $K_{e_1} = 2c - 1$.
J_1: $\Pi(1,6)$, $V_1 = \{(\text{vb}1, 3, 5)\}$, $K_{e_1} = 2c - 1$.
J_1: $\Pi(1,6)$, $V_1 = \{(\text{vb}2, 4, 6)\}$, $K_{e_1} = 2c$.
J_0: $\Pi(1,4)$, $V_1 = \{(\text{xiib}1)\}$, $K_{e_1} = 2c$.
J_0: $\Pi(1,4)$, $V_1 = \{(\text{xiib}2)\}$, $K_{e_1} = 2c - 1$.
J_0: $\Pi(1,6)$, $V_1 = \{(\text{xiib}1)\}$, $K_{e_1} = 2c - 1$.
J_0: $\Pi(1,6)$, $V_1 = \{(\text{xiib}2)\}$, $K_{e_1} = 2c$.
K_i $(i = 0, 1)$ $\Pi(1,4)$, $K_{e_1} = 2c$. K_i $(i = 0, 1)$ $\Pi(1,6)$, $K_{e_1} = 2c - 1$.
Moreover, all monodromies listed in [AI] *except those listed above can be realized as monodromies of certain hyperelliptic families.*

PROOF. We prove the existence of the families by giving examples of the equations in Section 2. Since we have too many cases, we write down the proof of nonexistence only for several typical cases. We call a subdivisor Z of the special fiber a $\boldsymbol{P}^1$*-chain* if all the components of Z are nonsingular rational curves and its dual graph is linear.

Assume that there exists a normally minimal hyperelliptic family whose topological monodromy is A_2: (iii2). Let $X = 4C_0 + 8C_1 + 7C_2 + 6C_3 + 5C_4 + 4C_5 + 3C_6 + 2C_7 + 5L_1 + 2L_2 + Z_1 + \cdots + Z_k$ be the special fiber of the family. C_i, L_i, Z_i are all rational curves. C_i intersects C_{i+1}, and $C_i \cdot C_j = 0$ if $|i - j| \geq 2$. C_1 intersects L_1, and L_1 intersects L_2. $Z_1 + \cdots + Z_k$ is a $\boldsymbol{P}^1$-chain connecting C_7 with L_2. Since the multiplicities of C_1, C_2 and L_1 are distinct, $\Pi(C_1)$, $\Pi(C_2)$ and $\Pi(L_1)$ are all distinct. Moreover, by Lemma 1.2, we see that $\Pi(C_1)$ is a component of B_r with multiplicity four. Note that $D_{C_1C_2} + 4\Pi(C_1) + D_{C_1L_1}$ is the bridge $D_{C_2L_1}$ and $\tilde{\tau}(\psi_r^*(D_{C_2L_1})) = 8C_1$. On the other hand, since $X - 4C_0 - 8C_1 - 7C_2 - 5L_1$ intersects C_2 and L_1 at a point, respectively, the bridge $D_{C_2L_1}$ at least contains $\Pi(L_2)$. Thus, L_2 is a component of $\tilde{\tau}(\psi_r^*(D_{C_2L_1}))$, a contradiction.

In the cases A_2: (iii3), (iii7), (iii9), (iii11), (iii13), (iii14), (iii16), the special fibers have loops and we obtain the same contradiction if we assume the existence of hyperelliptic families with the monodromies.

We take this opportunity to point out that the picture of (iii7) in [AI, p. 217] is incorrect. The sequence (3, 2, 1) should read (4, 3, 2, 1).

Assume that there exists a hyperelliptic family with monodromy C_{111} $(i, j = 0, 1)$: Id, $V_1 = V_2 = (\text{iv}1)$, $V_3 = (\text{va}4)$. Let $X = 3C_0 + 2C_1 + 2C_2 + 2C_3 + C_4 + Z_1 + Z_2 + L_1 + L_2$ be the special fiber of the family, where C_i are nonsingular rational curves, L_i are elliptic curves and each Z_i is the $\boldsymbol{P}^1$-chain that connects C_i with L_i. Since the multiplicity of C_0 is odd, $\Pi(C_0)$ is not a component of B_r. Since $\Pi(L_1) \neq \Pi(L_2)$, we see that $\Pi(C_i)$ $(i = 0, 1, 2, 3)$ are mutually distinct. Thus, $\Pi(C_0)$ intersects three distinct bridges $D_{C_0C_1}$, $D_{C_0C_2}$ and $D_{C_0C_2}$ at a point on B_r, respectively. It contradicts the fact that C_0 is a nonsingular rational curve. The other cases of C_{ij1} $(i, j = 0, 1)$: Id, V_3 = (va4, 5) can be proved by the same argument.

Assume that there exists a hyperelliptic family with monodromy A_2: (iii18). Let C_0 be the component with multiplicity two. The dual graph of the special fiber has a loop, and we see that $\Pi(C_0)$ is not a component of B_r and has multiplicity two. Since C_0 intersects other components at six distinct points, $\Pi(C_0)$ intersects at least three components D_1, D_2 and D_3 at three distinct points. For each D_i, let $\overline{D_i}$ be the connected maximal subdivisor of Θ containing D_i but not $\Pi(C_0)$. Each $\overline{D_i}$ has a component with multiplicity one, because the multiplicities of the components other than C_0 are one. It contradicts the assertion of Lemma 1.2.

The cases A_2: (iii28), A_1: (viii5), A_0: (xv2), (xv7), B_{1i} $(i = 0, 1)$: V_1 = {(vii4), (vii7), (vii8)} can be proved by the same argument.

Assume that there exists a hyperelliptic family with monodromy B_{0i} $(i = 0, 1)$, V_1 = (xiv3). Let $D = 2C_0 + 2(\sum Z_j) + C_1 + C_2$ be the subdivisor of the special fiber X as shown in [AI, p. 220, (xiv3)]. C_i and Z_j are nonsingular rational curves and the dual graph of $C_0 + \sum Z_j$ is a loop. By the configuration of X, we see that $\Pi(C_i)$ are mutually distinct and not components of B_n. Assume that $D_{C_0C_1} \neq 0$. Since $\psi_r^*(D_{C_0C_1})$ is contracted to a point at which C_0 intersects C_1, the multiplicity of the component $\overline{D'}$ of $\psi_r^*(D_{C_0C_1})$ intersecting $\tilde{\tau}^*(C_0)$ is odd. On the other hand, the component D' of $D_{C_0C_1}$ intersecting $\Pi(C_0)$ is a component of B_r, because C_0 intersects C_1 at a point. It contradicts the fact that the multiplicity of $\overline{D'} = \psi_r^*(D')$ is odd. Thus, we see that $D_{C_0C_1}$ =0, a contradiction to the configuration of the special fiber.

If there exists a hyperelliptic family S whose monodromy is A_1: (viii12), the configuration of the special fiber X is as shown in Figure 1. We consider $\psi_0\colon S_0 \to \Delta \times \boldsymbol{P}^1$ as in §1.1. Let Γ_0' be the strict transform of Γ_0 in W_r. Since the multiplicity of each component of X is greater than one, Γ_0 is a component of B_0. If not, the multiplicities of $\psi_r^*(\Gamma_0')$ is one, and we cannot contract $\psi_r^*(\Gamma_0')$. Assume that $\Pi(v_1)$ is not a component of B_r. Then, $\Pi(v_1)$ intersects the bridge $D_{v_1v_4}$ between $\Pi(v_1)$ and $\Pi(v_4)$ at a branch point of ψ_r. If $\Pi(v_2) \neq \Pi(v_3)$, then $\Pi(v_1)$ intersects the bridge between $\Pi(v_1)$ and $\Pi(v_i)$ $(i = 2, 3)$ at the branch points

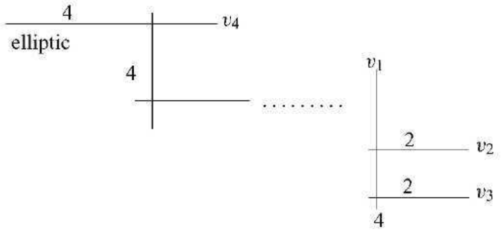


FIGURE 1.

of ψ_r. It contradicts the fact that v_1 is a nonsingular rational curve. We may also assume that $\Pi(v_2) = \Pi(v_3)$. Since the multiplicity of $\psi_r^*(\Gamma_0')$ is equal to two, there exists a bridge $D_{v_2\psi_r^*(\Gamma_0')_{\text{red}}}$ of Θ that does not contain $\Pi(v_1)$ as a component. However, the dual graph of $\psi_r^*(\Gamma_0' + D_{v_2\psi_r^*(\Gamma_0')_{\text{red}}} + \Pi(v_2) + \Pi(v_1))$ has a loop because Γ_0' is a component of B_r, a contradiction to the configuration of X. Moreover, we see that each component of Θ with multiplicity one is a component of B_r by the same argument.

Assume that $\Pi(v_1)$ is a component of B_r. Then the multiplicity of $\Pi(v_1)$ is equal to two and $\Pi(v_2) \neq \Pi(v_3)$. We also asssume that we can find an inverse of the canonical resolution $W_r \xrightarrow{\tau_r} \cdots \to W_{r'} \xrightarrow{\tau_{r'}} W_{r'-1} \to \cdots \xrightarrow{\tau_1} W_0$ satisfying (i) the strict transform of the exceptional set $E_{r'}$ of $\tau_{r'}$ by $\tau_{r'+1} \circ \cdots \circ \tau_r$ is $\Pi(v_1)$, (ii) $\tau_{r'}$ is the blowing-up at a point at which two components E and E' of $(\tau_1 \circ \cdots \circ \tau_{r'-1})^* \Gamma_0$ intersect. In this paragraph and the next, we use the same symbols E, E' and $E_{r'}$ for the divisors on W_r that are the strict transforms of E, E' and $E_{r'}$, respectively. Since E and $\Pi(v_1)$ are components of B_r, there exists a maximal nonzero connected subdivisor D of Θ connecting E with $\Pi(v_1)$. The component D' of D intersecting $\Pi(v_1)$ is not a component of B_r. Moreover, since the multiplicities of $\tau_{r'}^*(E)$ and $E_{r'}$ are one and two, respectively, the multiplicity of D' is odd. Thus, the multiplicity of $\psi_r^*(D')$ is odd. On the other hand, the multiplicities of the components of the special fiber are all even and we cannot make a component having odd multiplicity by any sequence of the blowing-ups, a contradiction.

Thus, for each inverse of the canonical resolution, we can find the blowing-up $\tau_{r'}$ satisfying (ii), and (iii) $(\tau_{r'+1} \circ \cdots \circ \tau_r)(\Pi(v_1))$ is a point on $E_{r'}$ that is not on any other components of $(\tau_1 \circ \cdots \circ \tau_{r'})^*(\Gamma_0)$. If $E_{r'}$ is a component of B_r, we have the same contradiction as in the previous paragraph. Thus, we may assume that $E_{r'}$ is not a component of B_r. Since the dual graph of the special fiber has no loop, we see that the divisor connecting $\Pi(v_1)$ with E, E' and $\Pi(v_1)$ intersects $E_{r'}$ at three distinct branch points of ψ_r. It contradicts the fact that the nonrational component of the special fiber has multiplicity four.

LEMMA 1.9. *The monodromy of a hyperelliptic family whose special fiber is as in Figure* 2 *is* E_{11}: II(0,1), $V_1 = $ (vb1), $V_2 = $ (vb6) .

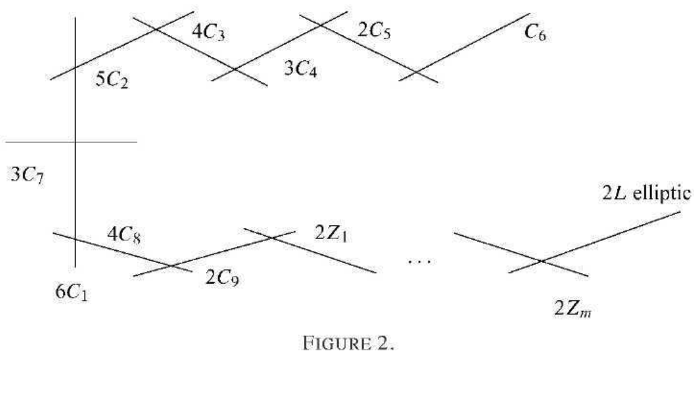


FIGURE 2.

Since the special fiber of a hyperelliptic family with monodromy C_{111}: II(1,1), $V_3 =$ (vb1), $V_1 = V_2 =$ (iv1) has the same configuration of the special fiber as that of a family with monodromy E_{11}: II(0,1), $V_1 =$ (vb1), $V_2 =$ (vb6), we obtain the following:

COROLLARY 1.10. *There exists no hyperelliptic family with monodromy* C_{111}: II(1,1), $V_3 =$ (vb1), $V_1 = V_2 =$ (iv1) .

PROOF OF LEMMA 1.9. Assume that there exists a hyperelliptic family with the special fiber $X = 6C_1 + 5C_2 + 4C_3 + 3C_4 + 2C_5 + C_6 + 3C_7 + 4C_8 + 2C_9 + 2\sum Z_j + 2L$ as in Figure 2. Note that C_i and Z_j are nonsingular rational components and L is an elliptic curve. By Lemma 1.2, we see that $\Pi(C_1)$ is a component of B_r. Assume that the bridge $D_{C_1C_2} \neq 0$. Since $\psi_r^*(D_{C_1C_2})$ is contracted to a point at which C_1 intersects C_2, each multiplicity of a component of $\psi_r^*(D_{C_1C_2})$ is written as $6a + 5b$ with positive integers a and b. We see that the multiplicities of the components of $D_{C_1C_2}$ are at least eight, because the multiplicities of the components of $\psi_r^*(D_{C_1C_2})$ are greater than or equal to eleven if odd, and sixteen, otherwise. Then, by any inverse of the canonical resolution, the bridge $D_{C_1C_2}$ is contracted to a point $P \in W_{r'}$ before $\Pi(C_1)$ and $\Pi(C_2)$ are contracted. If P is a bad point, $\psi_r^*(D_{C_1C_2})$ is not contracted, because $\psi_r^*(D_{C_1C_2})$ is the resolution graph of the singular point $\psi_{r'}^{-1}(P)$. Thus, we see that P is not a bad point, a contradiction to the process of the canonical resolution. (According to our definition of the canonical resolution, we do not blow up a non-bad point.) By the same argument, we see that $D_{C_1C_7} = D_{C_1C_8} = D_{C_iC_{i+1}} = 0$ $(1 \leq i \leq 5)$ and $\Pi(C_8)$ is not a component of B_r.

Assume that $\Pi(C_9)$ is not a component of B_r. Since $\Pi(C_8)$ also is not a component of B_r, we have $D_{C_8C_9} \neq 0$. Since the multiplicities of the components of $\psi_r^*(D_{C_8C_9})$ are greater than or equal to six, those of the components of $D_{C_8C_9}$ are greater than or equal to three. Thus, by any inverse of the canonical resolution, $\Pi(C_8) + D_{C_8C_9}$ is contracted before $\Pi(C_1)$ and $\Pi(C_9)$ are contracted. Especially, $\Pi(C_8)$ is contracted before $\Pi(C_1)$ and $\Pi(C_9)$. Since the multiplicities of $\Pi(C_1)$ and $\Pi(C_8)$ are three and four, there should exist a component of $D_{C_8C_9}$ having multiplicity one, a contradiction. Thus, we see that $\Pi(C_9)$ is a component of B_r with multiplicity one and $D_{C_8C_9} = 0$.

Assume that we can find an inverse of the canonical resolution $W_r \overset{\tau_r}{\to} \cdots \to W_{r'} \overset{\tau_{r'}}{\to} W_{r'-1} \to \cdots \overset{\tau_1}{\to} W_0$ such that $\tau_{r'}$ is a blowing-up at the point at which two components E and E' of $(\tau_1 \circ \cdots \circ \tau_{r'-1})^*(\Gamma_0)$ intersect and $(\tau_{r'+1} \circ \cdots \circ \tau_{r'})\Pi(L)$ is a point Q on $E_{r'}$. We use the same symbols E, E' and $E_{r'}$ for the strict transform of them on W_r. We may assume that E is a component of D_{C_9L}. By the configuration of X, we see that E and E' are components of B_r. If we assume that $E_{r'}$ is a component of B_r, $\psi_r^*(E_{r'})$ is not contracted by $\tilde{\tau}$, because the multiplicities of $\psi_r^*(E_{r'})$ and $\psi_r^*(E')$ are four and two, respectively, a contradiction to the configuration of X. If we assume that $E_{r'}$ is not a component of B_r, we see that $\psi_r^*(E_{r'})$ is not a nonsingular rational curve because the dual graph of X has no loop and the divisor connecting $E_{r'}$ with E, E' and $\Pi(L)$ intersects $E_{r'}$ at three distinct branch points of ψ_r. It contradicts the assumption $\Pi(L) \neq E_{r'}$.

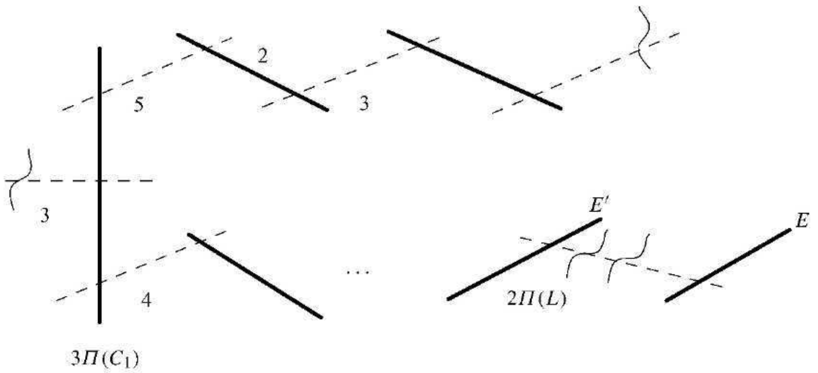


FIGURE 3.

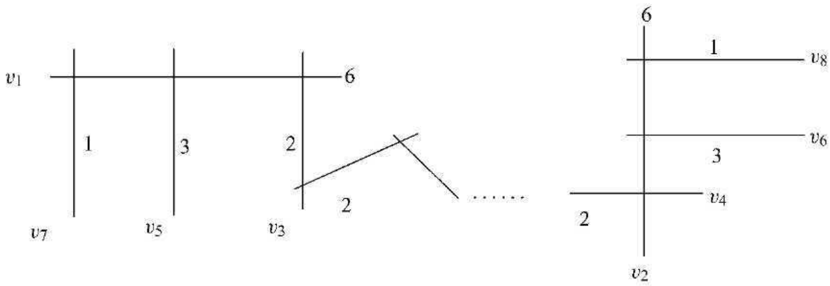


FIGURE 4.

Thus, we cannot find an inverse of the canonical resolution satisfying the above condition and we obtain the figure of Θ as shown in Figure 3. In Figures 3 and 5, the dotted lines mean the components of Θ that are not the components of B_r. The solid lines mean the components of B_r, and the waves mean $\tilde{B}_r$. By an inverse of the canonical resolution associated to E', we obtain the equation

$$y^2 = t(x-1)\{(x-1)^3 - t^{3k+1}\}(x^2 - t)(x^2 + t)$$

for a double covering of $\boldsymbol{P}^1 \times \Delta$. By a base change of degree 6, we see that the stable model of this family is E_{11}. □

By the same argument, we can prove the nonexistence of hyperelliptic families with monodromies C_{111}: II(1,1), $V_3 = (\text{vb}1, 2)$, $V_1 = V_2 = (\text{iv}s)$ $(2 \le s \le 8)$.

We continue the proof of Theorem 1.8. Assume that there exists a hyperelliptic family whose monodromy is E_{11}: II(0,1), $V_1 = V_2 = (\text{vb}2)$, K_{e_1} odd. The special fiber X is as in Figure 4. In this case, the number of components of nonsingular rational curves between v_1 and v_2 is even. By Lemma 1.2, we know that $\Pi(v_1)$ and $\Pi(v_2)$ are components of B_r. Then, we may assume that $\Pi(v_1) \neq \Pi(v_2)$. Assume that $\Pi(v_3)$ is a component of B_r with multiplicity one. Since $\psi_r^*(D_{v_1 v_3})$ is contracted to a point at which v_1 intersects v_3,

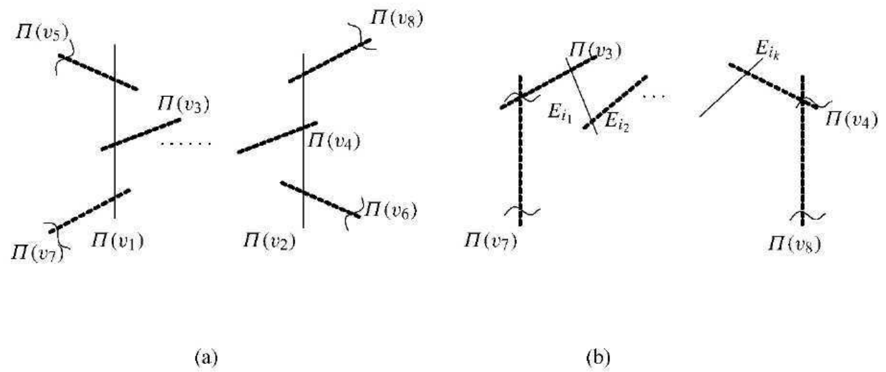


FIGURE 5.

the multiplicities of the components $D_{v_1v_3}$ are greater than three. Thus, by an inverse of the canonical resolution, $\Pi(v_1)$ and $\Pi(v_3)$ cannot be contracted before $D_{v_1v_3}$ is contracted. Let $W_r \xrightarrow{\tau_r} \cdots \to W_{r'} \xrightarrow{\tau_{r'}} W_{r'-1} \to \cdots \xrightarrow{\tau_1} W_0$ be an inverse of the canonical resolution such that the composite of the blowing-downs $\tau_{r'} \circ \cdots \circ \tau_r$ contracts $D_{v_1v_3}$ but not $\Pi(v_1)$ and $\Pi(v_3)$. Let P be the point at which $(\tau_{r'} \circ \cdots \circ \tau_r)(\Pi(v_1))$ and $(\tau_{r'} \circ \cdots \circ \tau_r)(\Pi(v_3))$ intersect. Since $\Pi(v_1)$ and $\Pi(v_2)$ are components of B_r, $\psi_{r'-1}(P)$ is a singular point of $S_{r'-1}$ and $\psi_r^*(D_{v_1v_3})$ is the exceptional set of the resolution of $\psi_{r'-1}(P)$, a contradiction to the fact that $\psi_r^*(D_{v_1v_3})$ is contracted by $\tilde{\tau}$. Therefore, $\Pi(v_3)$ is not a component of B_r. By the same argument, $\Pi(v_4)$ also is not a component of B_r. Since we do not blow up a non-bad point as in our previous argument, we see that $\Pi(v_1)$ intersects $\Pi(v_3)$ at a point. By the same argument, $\Pi(v_1)$ intersects $\Pi(v_l)$ $(l = 5, 7)$ at a point, respectively and $\Pi(v_2)$ intersects $\Pi(v_{l'})$ $(l' = 4, 6, 8)$ at a point, respectively. Then the configuration of Θ is as in Figure 5, (a).

Let $\sum_{j=1}^{k} a_{i_j} E_{i_j}$ be the bridge between v_3 and v_4 such that $\Pi(v_3)E_{i_1} = 1$, $\Pi(v_4)E_{i_k} = 1$, $E_{i_j}E_{i_{j+1}} = 1$ $(1 \leq j \leq k-1)$ and $E_{i_j}E_{i_{j'}} = 0$ $(|j - j'| \geq 2)$. Consider an inverse of the canonical resolution associated to $\Pi(v_8)$. First, we contract $\Pi(v_5)$ and $\Pi(v_6)$, then contract $\Pi(v_1)$ and $\Pi(v_2)$. Then the configuration of the image of Θ by the above contractions is as in Figure 5, (b). We use the same name for the components of Θ after contractions. Assume $a_{i_1} \neq 1$. Since we cannot contract $\Pi(v_3)$ in the next step, there exists j' such that $a_{i_{j'}} = 1$ and after some steps of blowing-downs, $E_{i_{j'}}$ intersects $\Pi(v_3)$ at a point. If the point at which $E_{i_{j'}}$ intersects $\Pi(v_3)$ is a singular point of the branch locus, $\psi_r^*(\sum_{j=1}^{j'})$ cannot be contracted by $\tilde{\tau}$, a contradiction to the process of the canonical resolution. Thus, $a_{i_1} = 1$. By the same argument we see that $a_{i_k} = 1$. If $a_{i_2} > 2$, $\psi_r^*(E_{i_2})$ is contracted by $\tilde{\tau}$. Let j' be the smallest integer greater than two such that $\psi_r^*(E_{i_{j'}})$ is not contracted by $\tilde{\tau}$. Since $a_{i_{j'}}$ is less than or equal to two, all E_{i_j} $(1 < j < j')$ are contracted to a point P on E_{i_1} before E_{i_1} and $E_{i_{j'}}$ by any inverse of the canonical resolution. If P is a singular point of the branch locus, all $\psi_r^*(E_{i_j})$ $(1 < j < j')$ are not contracted, a contradiction to the process of the canonical resolution. Thus we have $a_{i_2} = 2$ and E_{i_2} is not a component of B_r. Repeating this argument, we obtain that E_{i_j} is a component of B_r with multiplicity one when j is odd, and E_{i_j} is not a component of B_r and has multiplicity two, otherwise. Then the number of components of nonsingular rational curves between v_1 and v_2 is odd.

The other cases are proved by similar arguments. □

2. Construction of families. In this section, we complete the proof of Theorem 1.8 by constructing hyperelliptic families whose monodromies are listed in [AI], but not listed in Theorem 1.8. More precisely, for each monodromy $[f]$, we give an equation for a double covering S_0 of $\boldsymbol{P}^1 \times \Delta$ whose monodromy of the nonsingular model is $[f]$. Indices which appear in the table of symbols and equations are positive integers unless we mention their range. Let α, α_i $(1 \le i \le 4)$ be mutually distinct real numbers which are not integers.

Let x be the inhomogeneous coordinate of $\boldsymbol{P}^1$ and t the coordinate of Δ. For example, we give an equation for S_0 whose topological monodromy is (A_3) as follows:

$$(A_3) \quad y^2 = x(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)(x-7)\,.$$

We introduce some symbols for simplicity.

$F_3(x,t,k) := x^3 - t^k$,
$F_{12}(x,t,K_1,K_2) := (x - t^{K_1})(x^2 - t^{K_2})$,
$\tilde{F}_3(x,t,K,L) := (x^2-t)^3 - t^K x^L$,
$\tilde{F}_4(x,t,K,L) := (x^2-t)^4 - t^K x^L$,
$\tilde{F}_{12}(x,t,K_1,K_2,L_1,L_2) := \{(x^2-t) - t^{K_1}x^{L_1}\}\{(x^2-t)^2 - t^{K_2}x^{L_2}\}$,
$\tilde{F}_{13}(x,t,K_1,L_1,K_2,L_2) := \{(x^2-t) - t^{K_1}x^{L_1}\}\{(x^2-t)^3 - t^{K_2}x^{L_2}\}$,
$\tilde{F}_{22}(x,t,K,L) := \{(x^2-t)^2 - t^K x^L\}\{(x^2-t)^2 + t^K x^L\}$.

Let c be a positive integer. We fix a pair of integers (K', L') satisfying $2K' + L' - 6 = c$, $K' > 0$ and $0 \le L' < 5$. We set $\tilde{F}_3^c := \tilde{F}_3(x,t,K',L')$. Similarly, fixing a pair (K', L') satisfying $2K' + L' - 8 = c$, $K' > 0$ and $0 \le L' < 8$, we set $\tilde{F}_4^c := \tilde{F}_4(x,t,K',L')$. Fixing a pair (K', L') satisfying $2K' + L' - 4 = c$, we set $\tilde{F}_{22}^c := \tilde{F}_{22}(x,t,K',L')$. Let c_1 and c_2 be positive integers. We fix two pairs of integers (K_1', L_1') and (K_2', L_2') satisfying $2K_1' + L_1' - 2 = c_1$, $2K_2' + L_2' - 4 = c_2$, $K_1' > 1$, $0 \le L_1' \le 1$, $K_2' > 1$ and $0 \le L_2' \le 3$. We set $\tilde{F}_{12}^{c_1,c_2} := \tilde{F}_{12}(x,t,K_1',L_1',K_2',L_2')$. Fixing two pairs of integers (K_1', L_1') and (K_2', L_2') satisfying $2K_1' + L_1' - 2 = c_1$ and $2K_2' + L_2' - 6 = c_2$, we set $\tilde{F}_{13}^{c_1c_2} := \tilde{F}_{13}(x,t,K_1',L_1',K_2',L_2')$. We also define the following symbols using the above ones:

$f_1(x,t,k,l) := x^3 - \alpha_1 t^{6(k-1)+3l}$. $f_2(x,t,k,l) := F_3(x,t,6k+3l-1)$.
$f_3(x,t,k,l) := F_3(x,t,6k+3l-5)$. $f_4(x,t,k,l) := F_3(x,t,6k+3l-2)$.
$f_5(x,t,k,l) := F_3(x,t,6k+3l-4)$. $f_6(x,t,k,l) = F_{12}(x,t,2k+l,4k+2l-1)$.
$f_7(x,t,k,l) := F_{12}(x,t,2k+l,4k+2l-3)$. $f_8(x,t,k,l) := F_3(x,t,6k+3l-3)$.
$g_1(x,t,k) := x^5 - \alpha_2 t^{10(k-1)}$. $g_2(x,t,k) := x^5 - t^{10k-7}$.
$g_3(x,t,k) := x^5 - t^{10k-3}$. $g_4(x,t,k) := x^5 - t^{10k-1}$.
$g_5(x,t,k) := x^5 - t^{10k-9}$. $g_6(x,t,k) := x(x^4 - t^{8k-1})$.
$g_7(x,t,k) := x(x^4 - t^{8k-3})$. $g_8(x,t,k) := x(x^4 - t^{8k-7})$.

$g_9(x,t,k) := x(x^4 - t^{8k-5})$. $g_{13}(x,t,k) := x^5 - t^{10k-4}$.
$g_{15}(x,t,k) := x^5 - t^{10k-6}$. $g_{17}(x,t,k) := x^5 - t^{10k-2}$.
$g_{19}(x,t,k) := x^5 - t^{10k-8}$. $g_{20}(x,t,k) := x(x^2 - t^{4k-1})(x^2 + t^{4k-1})$.
$g_{21}(x,t,k) := x(x^2 - t^{4k-3})(x^2 + t^{4k-3})$. $g_{24}(x,t,k) := x^5 - t^{10k-5}$.
$h_1(x,t,k,l) := F_{12}(x,t,2k,4k+l+1)$. $h_2(x,t,k,l) := F_{12}(x,t,2k-1,4k+l-1)$.
$\sigma_1(x,t,k,l) := (x^3 - t^{6k})(x^2 - t^{4k+l+1})$. $\sigma_2(x,t,k,l) := (x^3 - t^{6k-2})(x^2 - t^{4k+l})$.
$\sigma_3(x,t,k,l) := (x^3 - t^{6k-4})(x^2 - t^{4k+l-2})$. $\sigma_5(x,t,k,l) := (x^3 - t^{6k-1})(x^2 - t^{4k+l})$.
$\sigma_6(x,t,k,l) := (x^3 - t^{6k-5})(x^2 - t^{4k+l-2})$. $\sigma_9(x,t,k,l) := (x^3 - t^{6k-3})(x^2 - t^{4k+l-2})$.
$\tau_1(x,t,k_1,k_2) := F_{12}(x,t,2,k_1+4)\{(x-2t^2)^2 - t^{k_2+4}\}$.
$\tau_2(x,t,k_1,k_2) := (x-2t)(x^2 - t^{k_1+2})\{(x-t)^2 - t^{k_2+2}\}$.
$\tau_4(x,t,k_1,k_2) := (x-t^2)\{(x^2-t^3)^2 - t^{k_1}x^{k_2}\}$, $(2k_1+3k_2-10 \geq 1)$.
$\tau_5(x,t,k_1,k_2) := x\{(x^2-t)^2 - t^{k_1}x^{k_2}\}$ $(2k_1+k_2-4 \geq 1)$.
$\theta_1 := \tilde{F}_3^{6k-3}$. $\theta_1'(x,t,k) := \tilde{F}_3^{6k}$. $\theta_2(x,t,k) := \tilde{F}_3^{6k-4}$.
$\theta_2'(x,t,k) := \tilde{F}_3^{6k-1}$ $\theta_3(x,t,k) := \tilde{F}_3^{6k-2}$. $\theta_3'(x,t,k) := \tilde{F}_3^{6k+1}$.
$\theta_4(x,t,k) := \tilde{F}_3^{6k-5}$. $\theta_4'(x,t,k) := \tilde{F}_3^{6k-2}$. $\theta_5(x,t,k) := \tilde{F}_3^{6k-1}$.
$\theta_5'(x,t,k) := \tilde{F}_3^{6k+2}$. $\theta_6(x,t,k) := \tilde{F}_{12}^{2k-1,\,4k-3}$. $\theta_6'(x,t,k) := \tilde{F}_{12}^{2k,\,4k-1}$.
$\theta_7(x,t,k) := \tilde{F}_{12}^{2k,\,4k-1}$. $\theta_7'(x,t,k) := \tilde{F}_{12}^{2k+1,\,4k+1}$. $\theta_8(x,t,k) := \tilde{F}_3^{6k}$.
$\theta_8'(x,t,k) := \tilde{F}_3^{6k+3}$. $\omega_1(x,t,k) := x^4 - \alpha_1 t^{4(k-1)}$. $\omega_1'(x,t,k) = \tilde{F}_4^{4k}$.
$\omega_2(x,t,k) := x^4 - t^{4k-1}$. $\omega_2'(x,t,k) := \tilde{F}_4^{4k-1}$. $\omega_3(x,t,k) := x^4 - t^{4k-3}$.
$\omega_3'(x,t,k) := \tilde{F}_4^{4k-3}$. $\omega_4(x,t,k) := (x-t^k)(x^3 - t^{3k-1})$.
$\omega_4'(x,t,k) := \tilde{F}_{13}^{k,3k-1}$. $\omega_5(x,t,k) := (x-t^k)(x^3 - t^{3k-2})$.
$\omega_5'(x,t,k) := \tilde{F}_{13}^{k,3k-2}$. $\omega_6(x,t,k) := (x^2 - t^{2k-1})(x^2 + t^{2k-1})$.
$\omega_6'(x,t,k) := \tilde{F}_{22}^{2k-1}$.
$\Gamma_1(x,t,k) = (x-t^k)(x^3 - t^{3k-2})$. $\Gamma_2(x,t,k) := (x-t^k)(x^3 - t^{3k-1})$.
$\Gamma_3(x,t,k) := x^4 - t^{4k-3}$. $\Gamma_4(x,t,k) := x^4 - t^{4k-1}$.
$\Gamma_5(x,t,k) := x^4 - \alpha t^{4k-4}$. $\Gamma_6(x,t,k) := x^4 - t^{4k-2}$.
$\rho_1(x,t,k,l) := (x^2 - \alpha t^{2l})(x^2 - t^{2l+k})$. $\rho_2(x,t,k,l) := (x^2 - t^{2l+1})(x^2 - t^{k+2l+1})$.
$\eta_1(x,t,k,l) = \tilde{F}_{13}^{l+1,6k+3l+3}$. $\eta_2(x,t,k,l) = \tilde{F}_{13}^{l+1,6k+3l+2}$. $\eta_3(x,t,k,l) = \tilde{F}_{13}^{l+1,6k+3l-2}$.
$\eta_4(x,t,k,l) = \tilde{F}_{13}^{l+1,6k+3l+1}$. $\eta_5(x,t,k,l) = \tilde{F}_{13}^{l+1,6k+3l-1}$.
$\eta_6(x,t,k,l) = \tilde{F}_{12}^{2k+l+1,4k+2l+1}\{(x^2-t) - t^{k_1}x^{l_1}\}$, $(2k_1+l_1 = l+2)$ $(l_1 \leq 2)$.
$\eta_7(x,t,k,l) = \tilde{F}_{12}^{2k+l,4k+2l-1}\{(x^2-t) - t^{k_1}x^{l_1}\}$, $(2k_1+l_1 = l+2)$ $(l_1 \leq 2)$.
$\eta_8(x,t,k,l) = \tilde{F}_{13}^{l+1,6k+3l}$.

We first give examples in the semistable and periodic cases. We then give examples of hyperelliptic families whose topological monodromies are neither periodic nor semistable. In the periodic cases, it suffices to construct hyperelliptic families whose topological monodromies are (i1), (i7), (i22), (i44) and (i47). We give two or three equations for the same symbol of the topological monodromies classified in [AI] according to the difference of their screw numbers. In the following equations, k, k_i are positive integers and l, l_i are nonnegative integers unless otherwise specified.

THE CASES OF SEMISTABLE CURVES.

(A_3) $y^2 = x(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)(x-7)$.

(A_2) $y^2 = (x^2 - t^k)(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)$.

(A_1) $y^2 = (x^2 - t^{k_1})\{(x-1)^2 - t^{k_2}\}(x-2)(x-3)(x-4)(x-5)$.

(A_0) $y^2 = (x^2 - t^{k_1})\{(x-1)^2 - t^{k_2}\}\{(x-2)^2 - t^{k_3}\}(x-3)(x-4)$.

(B_{21}) $y^2 = (x^3 - t^{6k})(x-1)(x-2)(x-3)(x-4)(x-5)$.

(B_{20}) $y^2 = (x - t^{2k})(x^2 - t^{4k+k_1})(x-1)(x-2)(x-3)(x-4)(x-5)$.

(B_{11}) $y^2 = (x^3 - t^{6k_1})\{(x-1)^2 + t^{k_2}\}(x-2)(x-3)(x-4)$.

(B_{10}) $y^2 = (x - t^{2k_1})(x^2 - t^{4k_1+k_2})\{(x-1)^2 + t^{k_3}\}(x-2)(x-3)(x-4)$.

(B_{01}) $y^2 = (x^3 - t^{6k_1})\{(x-1)^2 + t^{k_2}\}\{(x-2)^2 + t^{k_3}\}(x-3)$.

(B_{00}) $y^2 = (x - t^{2k_1})(x^2 - t^{4k_1+k_2})\{(x-1)^2 + t^{k_3}\}\{(x-2)^2 + t^{k_4}\}(x-4)$.

(C_{111}) $y^2 = (x^3 - t^{6k_1})\{(x-1)^3 - t^{6k_2}\}(x-3)(x-4)$.

(C_{110}) $y^2 = (x^3 - t^{6k_1})\{(x-1)^2 - t^{k_2}\}\{(x-2)^3 - t^{6k_3}\}$.

(C_{011}) $y^2 = (x - t^{2k_1})(x^2 - t^{4k_1+k_2})\{(x-1)^3 - t^{6k_3}\}(x-3)(x-4)$.

(C_{001}) $y^2 = (x + t^{2k_1})(x^2 - t^{4k_1+k_2})(x - 2 + t^{2k_3})\{(x-2)^2 - t^{4k_3+k_4}\}(x-3)(x-4)$.

(C_{010}) $y^2 = (x + t^{2k_1})(x^2 - t^{4k_1+k_2})(x - 1 + t^{2k_3})\{(x-2)^3 - t^{6k_4}\}$.

(C_{000}) $y^2 = (x + t^{2k_1})(x^2 - t^{4k_1+k_2})\{(x-2)^2 - t^{k_3}\}(x - 3 + t^{2k_4})\{(x-3)^2 - t^{4k_4+k_5}\}$.

(E_{11}) $y^2 = (x^4 - t^{4k})(x-1)(x-2)(x-3)(x-4)$.

(E_{01}) $y^2 = (x^2 - t^{2k_1})(x^2 - t^{2k_1+k_2})(x-1)(x-2)(x-3)(x-4)$.

(E_{00}) $y^2 = (x^2 - t^{2k_1})(x^2 - t^{2k_1+k_2})\{(x-1)^2 - t^{k_3}\}(x-3)(x-4)$.

(F_{11}) $y^2 = (x - t^{k_1})(x^3 - t^{3k_1+6k_2})\{(x-1)^3 - t^{6k_3}\}(x-2)$.

(F_{01}) $y^2 = (x - t^{k_1})(x^3 - t^{3k_1+6k_2})(x - 1 + t^{2k_3})\{(x-1)^2 - t^{4k_3+k_4}\}(x-2)$.

(F_{00}) $y^2 = (x - 1 + t^{2k_1})\{(x-1)^2 - t^{4k_1+k_2}\}(x - \alpha t^{k_3})(x - t^{k_3+2k_4})(x^2 - t^{2k_3+4k_4+k_5})(x-2)$.

(G_{11}) $y^2 = (x - t^{k_1})(x^3 - t^{3k_1+6k_2})(x-1)(x-2)(x-3)(x-4)$.

(G_{10}) $y^2 = (x - t^{k_1})(x - t^{k_1+2k_2})(x^2 - t^{2k_1+4k_2+k_3})(x-1)(x-2)(x-3)(x-4)$.

(G_{01}) $y^2 = (x - t^{k_1})(x^3 - t^{3k_1+6k_2})\{(x-1)^2 - t^{k_3}\}(x-2)(x-3)$.

(G_{00}) $y^2 = (x - \alpha t^{k_1})(x - t^{k_1+2k_2})(x^2 - t^{2k_1+4k_2+k_3})\{(x-1)^2 - t^{l_2}\}(x-2)(x-3)$.

(J_1) $y^2 = (x^4 - t^{4k_1})\{(x-1)^2 - t^{k_2}\}\{(x-2)^2 - t^{k_3}\}$.

(J_0) $y^2 = (x^2 - t^{2k_1})(x^2 - t^{2k_1+k_2})\{(x-1)^2 - t^{k_3}\}\{(x-2)^2 - t^{k_4}\}$.

(K_1) $y^2 = (x - t^{k_3})(x^3 - t^{3k_3+6k_4})\{(x-1)^2 - t^{k_1}\}\{(x-2)^2 - t^{k_2}\}$.

(K_0) $y^2 = (x - t^{k_3})(x - t^{k_3+2k_4})(x^2 - t^{2k_3+4k_4+k_5})\{(x-1)^2 - t^{k_1}\}\{(x-2)^2 - t^{k_2}\}$.

(L) $y^2 = (x^2 - t^{k_1})\{(x-1)^2 - t^{k_2}\}\{(x-2)^2 - t^{k_3}\}\{(x-3)^2 - t^{k_4}\}$.

(N) $y^2 = \{(x - t^{k_1})^2 - t^{2k_1+k_2}\}\{x^2 - t^{2k_1+k_3}\}\{(x-1)^2 - t^{k_4}\}\{(x-2)^2 - t^{k_5}\}$.

THE PERIODIC CASES.

(i1) $y^2 = (x^7 - t^9)(x-1)$.

(i7) $y^2 = x(x^6 - t^7)(x-1)$.

(i22) $y^2 = x^8 + t^3$.

(i44) $y^2 = t\{(x^4 - t)(x^4 + t)\}$.

(i47) $y^2 = t\{(x^2 - \alpha_1 t)(x^2 - \alpha_2 t)(x^2 - \alpha_3 t)(x^2 - \alpha_4 t)\}$.

Next, we give examples of hyperelliptic families whose monodromies are neither periodic nor semistable.

THE CASES WHERE THE STABLE MODEL IS A_2.

(iii4) $y^2 = t(x^6 - t^5)(x^2 - t)$, $y^2 = (x^6 - t^5)\{(x-1)^2 + t^l\}$.

(iii5) $y^2 = (x^6 - t)\{(x-1)^2 + t^l\}$.
(iii6) $y^2 = x(x^5 - t^4)\{(x-1)^2 + t^l\}$ $y^2 = tx(x^2 - t)(x^5 - t^4)$.
(iii8) $y^2 = x(x^5 - t)\{(x-1)^2 + t^l\}$.
(iii10) $y^2 = x(x^5 - t^2)\{(x-1)^2 + t^l\}$.
(iii12) $y^2 = tx(x^2 - t)(x^5 - t^3)$, $y^2 = x(x^5 - t^3)\{(x-1)^2 + t^l\}$.
(iii15) $y^2 = t(x^3 + t^2)(x^3 - t^2)(x^2 - t)$, $y^2 = (x^3 - t^2)(x^3 + t^2)\{(x-1)^2 + t^l\}$.
(iii17) $y^2 = (x^3 - t)(x^3 + t)\{(x-1)^2 + t^l\}$.
(iii19) $y^2 = (x^2 - t)(x^2 + t)(x^2 - 2t)\{(x-1)^2 + t^l\}$.
(iii20) $y^2 = tx(x^5 - t^2)\{(x-1)^2 + t^l\}$.
(iii21) $y^2 = t(x^5 - t^2)(x^2 - t)(x-1)$, $y^2 = tx(x^5 - t^3)\{(x-1)^2 + t^l\}$.
(iii22) $y^2 = tx(x^5 - t)\{(x-1)^2 + t^l\}$.
(iii23) $y^2 = (x^2 - t)(x^5 - t)(x-1)$, $y^2 = tx(x^5 - t^4)\{(x-1)^2 + t^l\}$.
(iii24) $y^2 = t(x^6 - t)\{(x-1)^2 + t^l\}$.
(iii25) $y^2 = (x^2 - t)(x^6 - t)$, $y^2 = t(x^6 - t^5)\{(x-1)^2 + t^l\}$.
(iii26) $y^2 = t(x^3 - t)(x^3 + t)(x^2 - t)$, $y^2 = t(x^3 - t^2)(x^3 + t^2)\{(x-1)^2 + t^l\}$.
(iii27) $y^2 = t(x^3 - t)(x^3 + t)\{(x-1)^2 + t^l\}$.
(iii29) $y^2 = t(x^2 - t^k)(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)$.
(iii30) $y^2 = t(x^2 - t)(x^2 + t)(x^2 - 2t)\{(x-1)^2 + t^l\}$.

THE CASES WHERE THE STABLE MODEL IS A_1.

(viii2) $y^2 = t(x^4 - t)(x^2 + t^k)\{(x-1)^2 + t^l\}$.
(viii3) $y^2 = t(x^4 - t^3)(x^2 - t^{k+1})\{(x-1)^2 + t^l\}$. $y^2 = (x^4 - t^3)(x^2 - t^{k+1})(x^2 - t)$
(viii4) $y^2 = (x^2 - t^{k_1+1})(x^2 - t)(x^2 + t)\{(x-1)^2 - t^{k_2}\}$.
(viii6) $y^2 = t(x^2 - t^{k_1})\{(x-1)^2 - t^{k_2}\}(x-2)(x-3)(x-4)(x-5)$.
(viii7) $y^2 = t(x^2 - t^{k+1})(x^2 - t)(x^2 + t)\{(x-1)^2 + t^l\}$.
(viii8) $y^2 = (x^2 - t)(x^2 + t)\{(x^2 - 2t)^2 - t^k x^l\}$, $(2k + l - 2 \geq 1)$.
(viii9) $y^2 = (x-1)(x^2 - t)(x - t)\{(x^2 - 2t)^2 - t^k x^l\}$, $(2k + l - 2 \geq 1)$.
(viii10) $y^2 = t(x-1)(x^2 - t)(x - t)\{(x^2 - 2t)^2 - t^k x^l\}$, $(2k + l - 2 \geq 1)$.
(viii11) $y^2 = t(x^2 - t)(x^2 + t)\{(x^2 - 2t)^2 - t^k x^l\}$, $(2k + l - 2 \geq 1)$.

THE CASES WHERE THE STABLE MODEL IS A_0.

(xv3) $y^2 = t(x^2 - t^{k_1})\{(x-1)^2 + t^{k_2}\}\{(x-2)^2 + t^{k_3}\}(x-3)(x-4)$.
(xv4) $y^2 = t(x^2 - t)\{(x^2 - 2t)^2 - t^{k_1} x^{l_1}\}\{(x-1)^2 + t^{l_2}\}$, $(2k_1 + l_1 - 2 \geq 1)$.
(xv5) $y^2 = (x^2 - t)\{(x^2 - 2t)^2 - t^{k_1} x^{l_1}\}\{(x-1)^2 + t^{l_2}\}$, $(2k_1 + l_1 - 2 \geq 1)$.
(xv6) $y^2 = tx(x-1)\{(x^3 - t)^2 - t^k x^l\}$, $(3k + l \geq 7)$.
(xv8) $y^2 = x(x-1)\{(x^3 - t)^2 - t^k x^l\}$, $(3k + l \geq 7)$.

THE CASES WHERE THE STABLE MODEL IS B_{21}.

B_{21}: $V_1 = (\mathrm{ii}s_1)$, $V_2 = (\mathrm{iv}s_2)$. $y^2 = g_{s_1}(x, t, k_1) f_{s_2}(x - 1, t, k_2, 0)$.

When B_{21}: $V_1 = (\mathrm{ii}s_1)$, $V_2 = (\mathrm{iv}s_2)$ and the screw number is special (not appearing in the above equation), examples of their equations are as follows (we write (ii2)-(iv2) instead of writing $V_1 =$ (ii2), $V_2 =$ (iv2) for simplicity):

(ii2)-(iv2) $y^2 = t(x^3 - t^2)(x^5 - t^2)$. (ii2)-(iv6) $y^2 = tx(x^2 - 1)(x^5 - t^2)$.
(ii3)-(iv2) $y^2 = t(x^5 - t^2)\{(x-1)^3 - t^2\}$. (ii3)-(iv4) $y^2 = t(x^5 - t^2)\{(x-1)^3 - t\}$.
(ii3)-(iv5) $y^2 = t(x^3 - t^2)(x^5 - t^3)$. (ii3)-(iv6) $y^2 = t(x^5 - t^2)(x-1)\{(x-1)^2 - t\}$.

(ii3)-(iv8) $y^2 = t(x^5 - t^2)(x-1)(x-2)(x-3)$.
(ii4)-(iv2) $y^2 = t(x^5 - t^4)\{(x-1)^3 - t^2\}$. (ii4)-(iv3) $y^2 = t(x^3 - t)(x^5 - t)$.
(ii4)-(iv4) $y^2 = t(x^5 - t^2)\{(x-1)^3 - t\}$. (ii4)-(iv5) $y^2 = t(x^3 - t)(x^5 - t^3)$.
(ii4)-(iv6) $y^2 = t(x^5 - t^4)(x-1)\{(x-1)^2 - t\}$. (ii4)-(iv7) $y^2 = tx(x^2 - t)(x^5 - t)$.
(ii4)-(iv8) $y^2 = t(x^5 - t^4)(x-1)(x-2)(x-3)$.
(ii6)-(iv2) $y^2 = tx(x^4 - t^3)\{(x-1)^3 - t^2\}$. (ii6)-(iv4) $y^2 = t(x^4 - t^3)(x^3 - t)(x-1)$.
(ii6)-(iv5) $y^2 = t(x^4 - t)(x^3 - t^2)(x-1)$.
(ii6)-(iv6) $y^2 = tx(x^4 - t^3)(x-1)\{(x-1)^2 - t\}$.
(ii6)-(iv7) $y^2 = tx(x^2 - t)(x^4 - t)(x-1)$.
(ii6)-(iv8) $y^2 = tx(x^2 - t^3)(x-1)(x-2)(x-3)$.
(ii7)-(iv2) $y^2 = tx(x^4 - t)\{(x-1)^3 - t^2\}$. (ii7)-(iv4) $y^2 = tx(x^4 - t)\{(x-1)^3 - t\}$.
(ii7)-(iv5) $y^2 = tx(x^4 - t)(x-1)\{(x-1)^2 - t\}$.
(ii7)-(iv8) $y^2 = tx(x^4 - t)(x-1)(x-2)(x-3)$. (ii9)-(iv2) $y^2 = tx(x^4 - t^3)(x^3 - t)$.
(ii9)-(iv4) $y^2 = (x^4 - t)(x^3 - t)(x-1)$.
(ii9)-(iv6) $y^2 = x(x^4 - t)(x^2 - t)(x-1)$. (ii13)-(iv2) $y^2 = t(x^5 - t)\{(x-1)^3 - t^2\}$.
(ii13)-(iv4) $y^2 = t(x^5 - t)\{(x-1)^3 - t\}$.
(ii13)-(iv6) $y^2 = t(x^5 - t)(x-1)\{(x-1)^2 - t\}$.
(ii13)-(iv8) $y^2 = t(x^5 - t)(x-1)(x-2)(x-3)$. (ii15)-(iv2) $y^2 = t(x^5 - t^4)(x^3 - t)$.
(ii15)-(iv4) $y^2 = (x^5 - t)(x^3 - t)$. (ii15)-(iv6) $y^2 = t(x^5 - t^4)(x^2 - t)(x-1)$.
(ii17)-(iv2) $y^2 = t(x^5 - t^2)\{(x-1)^3 - t\}$. (ii17)-(iv4) $y^2 = t(x^5 - t^3)\{(x-1)^3 - t\}$.
(ii17)-(iv5) $y^2 = (x^5 - t^3)(x^3 - t)$. (ii17)-(iv6) $y^2 = t(x^5 - t^3)(x-1)\{(x-1)^2 - t\}$.
(ii17)-(iv7) $y^2 = (x^5 - t^3)(x^2 - t)(x-1)$.
(ii17)-(iv8) $y^2 = t(x^5 - t^3)(x-1)(x-3)(x-3)$. (ii19)-(iv2) $y^2 = t(x^5 - t^2)(x^3 - t)$.
(ii20)-(iv2) $y^2 = tx(x^2 - t)(x^2 + t)\{(x-1)^3 - t^2\}$.
(ii20)-(iv4) $y^2 = tx(x^2 - t)(x^2 + t)\{(x-1)^3 - t\}$.
(ii20)-(iv5) $y^2 = (x^3 - t^2)(x^2 - t)(x^2 + t)(x-1)$.
(ii20)-(iv6) $y^2 = tx(x^2 - t)(x^2 + t)(x-1)\{(x-1)^2 - t\}$.
(ii20)-(iv8) $y^2 = tx(x^2 - t)(x^2 + t)(x-1)(x-2)(x-3)$.
(ii21)-(iv2) $y^2 = tx(x^2 - t)(x^2 + t)(x^3 - t)$. (ii24)-(iv2) $y^2 = tx(x^2 - t)(x^2 + t)(x^3 - t)$.
(ii24)-(iv4) $y^2 = t(x^3 - t)(x-1)(x-2)(x-3)(x-4)(x-5)$.
(ii24)-(iv6) $y^2 = tx(x^2 - t)(x-1)(x-2)(x-3)(x-4)(x-5)$.

THE CASES WHERE THE STABLE MODEL IS B_{20}.

B_{20}: $V_1 = (\text{ii}s_1)$, $V_2 = (\text{xi}s_2)$. $y^2 = g_{s_1}(x, t, k_1)h_{s_2}(x-1, t, k_2, l)$.

When the screw number at e_1 is special, we need the following equations in addition to those above:

(ii6)-(xi2) $y^2 = tx(x^4 - t^3)\{(x-1)^2 - t^l\}(x-2)$.
(ii13)-(xi2) $y^2 = t(x^5 - t)\{(x-1)^2 - t^l\}(x-2)$.
(ii17)-(xi2) $y^2 = t(x^5 - t^3)\{(x-1)^2 - t^l\}(x-2)$.
(ii20)-(xi2) $y^2 = tx(x^2 - t)(x^2 + t)\{(x-1)^2 - t^l\}(x-2)$.

THE CASES WHERE THE STABLE MODEL IS B_{11}.

B_{11}: $V_1 = (\text{vii}s_1)$, $V_2 = (\text{iv}s_2)$. $y^2 = f_{s_2}(x-1, t, k, 0)\sigma_{s_1}(x, t, k_1, l)$.

THE CASES WHERE THE STABLE MODEL IS B_{10}.
B_{10}: $V_1 = (\text{vii}s_1)$, $V_2 = (\text{xi}s_2)$. $y^2 = \sigma_{s_1}(x-1, t, k_1, l_1)h_{s_2}(x, t, k_2, l_2)$.

THE CASES WHERE THE STABLE MODEL IS B_{01}.
B_{01}: $V_1 = (\text{xiv}s_1)$, $V_2 = (\text{iv}s_2)$. $y^2 = \tau_{s_1}(x, t, k_1, k_2)f_{s_2}(x-1, t, k_3, 0)$.

THE CASES WHERE THE STABLE MODEL IS B_{00}.
B_{00}: $V_1 = (\text{xiv}s_1)$, $V_2 = (\text{xi}s_2)$. $y^2 = \tau_{s_1}(x, t, k_1, k_2)h_{s_2}(x-1, t, k_3, l)$.

THE CASES WHERE THE STABLE MODEL IS C_{111}.
C_{111}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{iv}s_2)$, $V_3 = (\text{va1})$.
$y^2 = f_{s_1}(x, t, k_1, 0)f_{s_2}(x-1, t, k_2, 0)(x-2)(x-3)$. ($k_i \geq 2$ when $s_1 = 1$ or $s_2 = 1$.)
C_{111}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{iv}s_2)$, $V_3 = (\text{va2})$.
$y^2 = (x^2 - t^3)f_{s_1}(x, t, k_1, 1)f_{s_2}(x-1, t, k_2, 0)$. ($k_1 \geq 0$ when $s_1 = 2, 4, 6, 8$.)
C_{111}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{iv}s_2)$, $V_3 = (\text{va3})$.
$y^2 = (x^2 - t)f_{s_1}(x, t, k_1, 1)f_{s_2}(x-1, t, k_2, 0)$.
C_{111}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{iv}s_2)$, $V_3 = (\text{va6})$.
$y^2 = (x^2 - t^2)f_{s_1}(x, t, k_1 + 1, 0)f_{s_2}(x-1, t, k_2, 0)$.
C_{111}: II(1,1), $V_1 = V_2 = (\text{iv}s_1)$, $V_3 = (\text{vb3})$. $y^2 = tx(x-1)\theta_{s_1}(x, t, k)$.
C_{111}: II(1,1), $V_1 = V_2 = (\text{iv}s_1)$, $V_3 = (\text{vb4})$. $y^2 = x(x-1)\theta_{s_1}(x, t, k)$.
C_{111}: II(1,1), $V_1 = V_2 = (\text{iv}s_1)$, $V_3 = (\text{vb5})$. $y^2 = (x^2 - 2t)\theta'_{s_1}(x, t, k)$.
C_{111}: II(1,1), $V_1 = V_2 = (\text{iv}s_1)$, $V_3 = (\text{vb6})$. $y^2 = t(x^2 - 2t)\theta'_{s_1}(x, t, k)$.

THE CASES WHERE THE STABLE MODEL IS C_{110}.
C_{110}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{iv}s_2)$, $V_3 = (\text{xiia1})$.
$y^2 = f_{s_1}(x, t, k_1, 0)f_{s_2}(x-2, t, k_2, 0)\{(x-1)^2 - t^{k_3}\}$.
C_{110}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{iv}s_2)$, $V_3 = (\text{xiia2})$.
$y^2 = tf_{s_1}(x, t, k_1, 1)f_{s_2}(x-2, t, k_2, 1)\{(x-1)^2 - t^{k_3}\}$. ($k_1 \geq 0$ when $s_1 = 2, 4, 6, 8$.)
C_{110}: II(1,1), $V_1 = V_2 = (\text{iv}s_1)$, $V_3 = (\text{xiib1})$. $y^2 = (x^2 - t^{k_1+1})\theta'_{s_1}$.
C_{110}: II(1,1), $V_1 = V_2 = (\text{iv}s_1)$, $V_3 = (\text{xiib2})$. $y^2 = t(x^2 - t^{k_1+1})\theta'_{s_1}$.

THE CASES WHERE THE STABLE MODEL IS C_{101}.
C_{101}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{va1})$.
$y^2 = f_{s_1}(x, t, k_1, 0)h_{s_2}(x-1, t, k_2, l)(x-2)(x-3)$.
C_{101}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{va2})$.
$y^2 = (x^2 - t^3)h_{s_2}(x, t, k_2 + 1/2, l)f_{s_1}(x-1, t, k_1, 0)$.
C_{101}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{va3})$.
$y^2 = (x^2 - t)h_{s_2}(x, t, k_2 + 1/2, l)f_{s_1}(x-1, t, k_1, 0)$.
C_{101}: Id, $V_1 = (\text{iv}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{va6})$.
$y^2 = (x^2 - t^2)h_{s_2}(x, t, k_2, l)f_{s_1}(x-1, t, k_1, 0)$.

THE CASES WHERE THE STABLE MODEL IS C_{100}.
C_{100}: Id $V_1 = (\text{iv}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{xiia1})$.
$y^2 = f_{s_1}(x-1, t, k_1, 0)h_{s_2}(x, t, k_2, l)\{(x-2)^2 - t^{k_3}\}$. ($k_1 \geq 2$ when $s_1 = 1$.)

C_{100}: Id $V_1 = (\text{iv}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{xiia2})$.
$y^2 = tf_{s_1}(x-1, t, k_1, 1)h_{s_2}(x, t, k_2 + 1/2, l)\{(x-2)^2 - t^{k_3}\}$.

THE CASES WHERE THE STABLE MODEL IS C_{001}.

C_{001}: Id, $V_1 = (\text{xi}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{va1})$.
$y^2 = h_{s_1}(x-1, t, k_1, l)h_{s_2}(x, t, k_2, l)(x-2)(x-3)$.
C_{001}: Id, $V_1 = (\text{xi}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{va2})$.
$y^2 = t(x^2 - t)h_{s_1}(x, t, k_1, l_1)h_{s_2}(x-1, t, k_2 + 1/2, l_2)$.
C_{001}: Id, $V_1 = (\text{xi}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{va3})$.
$y^2 = (x^2 - t)h_{s_1}(x, t, k_1 + 1/2, l_1)h_{s_2}(x-1, t, k_2, l_2)$.
C_{001}: Id, $V_1 = (\text{xi}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{va6})$.
$y^2 = (x^2 - t^2)h_{s_1}(x, t, k_1, l)h_{s_2}(x-1, t, k_2, l)$.
C_{001}: II(1,1), $V_1 = V_2 = (\text{xi1})$, $V_3 = (\text{vb3})$. $y^2 = tx(x-1)\tilde{F}_{12}^{2k_1-1, 4k_1+k_2-2}$.
C_{001}: II(1,1), $V_1 = V_2 = (\text{xi2})$, $V_3 = (\text{vb3})$. $y^2 = tx(x-1)\tilde{F}_{12}^{2k_1, 4k_1+k_2}$.
C_{001}: II(1,1), $V_1 = V_2 = (\text{xi1})$, $V_3 = (\text{vb4})$. $y^2 = x(x-1)\tilde{F}_{12}^{2k_1-1, 4k_1+k_2-2}$.
C_{001}: II(1,1), $V_1 = V_2 = (\text{xi2})$, $V_3 = (\text{vb4})$. $y^2 = x(x-1)\tilde{F}_{12}^{2k_1, 4k_1+k_2}$.
C_{001}: II(1,1), $V_1 = V_2 = (\text{xi1})$, $V_3 = (\text{vb5})$. $y^2 = \{(x^2 - 2t) - t^2x\}\tilde{F}_{12}^{2k_1, 4k_1+k_2-2}$.
C_{001}: II(1,1), $V_1 = V_2 = (\text{xi2})$, $V_3 = (\text{vb5})$. $y^2 = \{(x^2 - 2t) - t^2x\}\tilde{F}_{12}^{2k_1-1, 4k_1+k_2-2}$.
C_{001}: II(1,1), $V_1 = V_2 = (\text{xi1})$, $V_3 = (\text{vb6})$. $y^2 = t\{(x^2 - 2t) - t^2x\}\tilde{F}_{12}^{2k_1, 4k_1+k_2}$.
C_{001}: II(1,1), $V_1 = V_2 = (\text{xi2})$, $V_3 = (\text{vb6})$. $y^2 = t\{(x^2 - 2t) - t^2x\}\tilde{F}_{12}^{2k_1-1, 4k_1+k_2-2}$.

THE CASES WHERE THE STABLE MODEL IS C_{000}.

C_{000}: Id, $V_1 = (\text{xi}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{xiia1})$.
$y^2 = h_{s_1}(x, t, k_1, l)h_{s_2}(x-1, t, k_2, l)\{(x-2)^2 - t^k\}$.
C_{000}: Id, $V_1 = (\text{xi}s_1)$, $V_2 = (\text{xi}s_2)$, $V_3 = (\text{xiia2})$.
$y^2 = th_{s_1}(x, t, k_1 + 1/2, l_1)h_{s_2}(x-1, t, k_2 + 1/2, l_2)\{(x-3)^2 - t^k\}$.
C_{000}: II(1,1), $V_1 = V_2 = (\text{xi1})$, $V_3 = (\text{xiib1})$. $y^2 = (x^2 - t^{k_1+1})\tilde{F}_{12}^{2k_1, 4k_1+k_2}$.
C_{000}: II(1,1), $V_1 = V_2 = (\text{xi2})$, $V_3 = (\text{xiib1})$. $y^2 = (x^2 - t^{k_1+2})\tilde{F}_{12}^{2k_1-1, 4k_1+k_2-2}$.
C_{000}: II(1,1), $V_1 = V_2 = (\text{xi1})$, $V_3 = (\text{xiib2})$. $y^2 = t(x^2 - t^{k_1+1})\tilde{F}_{12}^{2k_1, 4k_1+k_2}$.
C_{000}: II(1,1), $V_1 = V_2 = (\text{xi2})$, $V_3 = (\text{xiib2})$. $y^2 = t(x^2 - t^{k_1+1})\tilde{F}_{12}^{2k_1-1, 4k_1+k_2-2}$.

THE CASES WHERE THE STABLE MODEL IS E_{11}.

E_{11}: Id, $V_1 = (\text{va}s_1)$, $V_2 = (\text{va}s_2)$. $y^2 = \omega_{s_1}(x, t, k_1)\omega_{s_2}(x-1, t, k_2)$.
E_{11}: II(0,1), $V_1 = (\text{vb}s_1)$, $V_2 = (\text{vb}s_2)$. $y^2 = t\Gamma_{s_1}(x, t, k_1)\Gamma_{s_2}(x-1, t, k_2)$.
We have to give more examples when the screw number at e_1 is special.
E_{11}: II(0,1), $V_1 = (\text{vb1})$, $V_2 = (\text{vb4})$. $y^2 = x(x^4 - t)(x^3 - t)$.
E_{11}: II(0,1), $V_1 = V_2 = (\text{vb2})$. $y^2 = x(x^3 - t^2)(x^3 - t)(x-1)$.
E_{11}: II(0,1), $V_1 = (\text{vb2})$, $V_2 = (\text{vb4})$. $y^2 = (x^4 - t^3)(x^3 - t)(x-1)$.
E_{11}: II(0,1), $V_1 = (\text{vb2})$, $V_2 = (\text{vb6})$. $y^2 = (x^4 - t^2)(x^3 - t)(x-1)$.
E_{11}: II(1,2), $V_1 = V_2 = (\text{va}s)$. $y^2 = t\omega'_s$.

E_{11}: II(1,3), $V_1 = V_2 =$ (va$s$). $y^2 = \omega'_s$.

THE CASES WHERE THE STABLE MODELS IS E_{10}.

E_{10}: Id, $V_1 =$ (vas_1), $V_2 =$ (xiias_2). $y^2 = \omega_{s_1}(x, t, k_1)\rho_{s_2}(x-2, t, k_2, l)$.
E_{10}: II(0,1), $V_1 =$ (vbs_1), $V_2 =$ (xiibs_2). $y^2 = t\Gamma_{s_1}(x, t, k_1)\rho_{s_2}(x-1, t, k_2, l)$.
We have to give more examples when the screw number at e_1 is special.
$V_1 =$ (vb2), $V_2 =$ (xiib2). $y^2 = x(x^3 - t^2)(x^2 - t)\{(x-1)^2 - t^{k-1}\}$.
$V_1 =$ (vb4), $V_2 =$ (xiib2). $y^2 = x(x^3 - t^2)(x^2 - t)\{(x-1)^2 - t^{k-1}\}$.

THE CASES WHERE THE STABLE MODEL IS E_{00}.

E_{00}: Id, $V_1 =$ (xiias_1), $V_2 =$ (xiias_2). $y^2 = \rho_{s_1}(x, t, k_1, l_1)\rho_{s_2}(x-1, t, k_2, l_2)$.
E_{00}: II(0,1), $V_1 =$ (xiibs_1), $V_2 =$ (xiibs_2). $y^2 = t\rho_{s_1}(x, t, k_1, l_1)\rho_{s_2}(x-1, t, k_2, l_2)$.
E_{00}: II(1,2), $V_1 = V_2 =$ (xiia1).
$y^2 = t\{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x^2-t)^2 - t^{k_2}x^{l_2}\}$, $(2k_1 + l_1 - 4 = 2k,\ 2k_2 + l_2 - 4 = 2k + l)$.
E_{00}: II(1,2), $V_1 = V_2 =$ (xiia2).
$y^2 = t\{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x^2-t)^2 - t^{k_2}x^{l_2}\}$, $(2k_1 + l_1 - 4 = 2k_3 - 1,\ 2k_2 + l_2 - 4 = 2k_3 + k_4)$.
E_{00}: II(1,3), $V_1 = V_2 =$ (xiia1).
$y^2 = \{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x^2-t)^2 - t^{k_2}x^{l_2}\}$, $(2k_1 + l_1 - 4 = 2k,\ 2k_2 + l_2 - 4 = 2k + l)$.
E_{00}: II(1,3), $V_1 = V_2 =$ (xiia2).
$y^2 = \{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x^2-t)^2 - t^{k_2}x^{l_2}\}$, $(2k_1 + l_1 - 4 = 2k_3 - 1,\ 2k_2 + l_2 - 4 = 2k_3 + k_4)$.

THE CASES WHERE THE STABLE MODEL IS F_{11}.

F_{11}: Id, $V_1 =$ (ivs_1), $V_2 =$ (ivs_2).
$y^2 = (x - \alpha t^{l+1})(x-2)f_{s_2}(x, t, k_1, l+1)f_{s_1}(x-1, t, k_2, 0)$.
($k_1 \geq 2$ when $s_1 = 1$ or $s_2 = 1$.)
F_{11}: II(0,1), $V_1 =$ (ivs_1), $V_2 =$ (ivs_2).
$y^2 = t(x - \alpha t^{l+1})(x-2)f_{s_2}(x, t, k_1, l+2)f_{s_1}(x-1, t, k_2, 0)$.
F_{11}: II(2,1), $V_1 = V_2 =$ (iv$s$). $y^2 = t\eta_s$.
F_{11}: II(2,2), $V_1 = V_2 =$ (iv$s$). $y^2 = \eta_s$.

THE CASES WHERE THE STABLE MODEL IS F_{10}.

F_{10}: Id, $V_1 =$ (ivs_1), $V_2 =$ (xis_2).
$y^2 = (x - t^{l_1+1})f_{s_1}(x, t, k_1, l_1 + 1)h_{s_2}(x-1, t, k_2 + 1, l_2)(x-2)$.
F_{10}: II(0,1), $V_1 =$ (ivs_1), $V_2 =$ (xis_2).
$y^2 = t(x - t^{l_1+1})f_{s_1}(x, t, k_1, l_1 + 2)h_{s_2}(x-1, t, k_2 + 1/2, l)(x-2)$.

THE CASES WHERE THE STABLE MODEL IS F_{00}.

F_{00}: Id, $V_1 =$ (xis_1), $V_2 =$ (xis_2).
$y^2 = (x - t^{l_1+1})f_{s_1}(x, t, k_1 + (l_1+1)/2, l_2)h_{s_2}(x-1, t, k_2, l_3)(x-2)$.
F_{00}: II(0,1), $V_1 =$ (xis_1), $V_2 =$ (xis_2).
$y^2 = t(x - t^{l_1+1})h_{s_1}(x, t, k_1 + (l_1+1)/2, l_2)h_{s_2}(x-1, t, k_2 + 1/2, l_3)(x-2)$.
F_{00}: II(2,1), $V_1 = V_2 =$ (xi1). $y^2 = t\tilde{F}_{12}^{2k_2+k_1, 4k_2+2k_1+k_3}(x^2 - t - t^{k_4}x^l)$, $(2k_4 + l - 2 = k_1)$.
F_{00}: II(2,1), $V_1 = V_2 =$ (xi2).
$y^2 = t\tilde{F}_{12}^{2k_2+k_1-1, 4k_2+2k_1+k_3-2}(x^2 - t - t^{k_4}x^l)$, $(2k_4 + l - 2 = k_1)$.

F_{00}: II(2,2), $V_1 = V_2 =$ (xi1). $y^2 = \tilde{F}_{12}^{2k_2+k_1,4k_2+2k_1+k_3}(x^2-t-t^{k_4}x^l)$, $(2k_4+l-2=k_1)$.
F_{00}: II(2,2), $V_1 = V_2 =$ (xi2).
$y^2 = \tilde{F}_{12}^{2k_2+k_1-1,4k_2+2k_1+k_3-2}(x^2-t-t^{k_4}x^l)$, $(2k_4+l-2=k_1)$.

THE CASES WHERE THE STABLE MODEL IS G_{11}.
G_{11}: Id, $V_1 =$ (vas_1), $V_3 =$ (ivs_2).
$y^2 = f_{s_2}(x,t,k_1,0)(x-1)\omega_{s_1}(x-2,t,k_2)$. ($k_2 \geq 2$ when $s_2 = 1$.)
G_{11}: II(0,1), $V_1 =$ (vbs_1), $V_3 =$ (ivs_2).
$y^2 = t\Gamma_{s_1}(x,t,k_1)(x-1)f_{s_2}(x,t,k_2,1)$. ($k_2 \geq 0$ when $s_2 = 2,4,6,8$.)

THE CASES WHERE THE STABLE MODEL IS G_{10}.
G_{10}: Id, $V_1 =$ (vas_1), $V_3 =$ (xis_2). $y^2 = \omega_{s_1}(x,t,k_1)(x-1)h_{s_2}(x-2,t,k_2,l)$.
G_{10}: II(0,1), $V_1 =$ (vbs_1), $V_3 =$ (xis_2). $y^2 = t\Gamma_{s_1}(x,t,k_1)(x-1)h_{s_2}(x-2,t,k_2+1/2,l)$.

THE CASES OF THE STABLE MODELS ARE G_{00}.
G_{00}: Id, $V_1 =$ (xiias_1), $V_3 =$ (xis_2). $y^2 = \rho_{s_1}(x,t,k_1,l_1+1)(x-1)h_{s_2}(x-2,t,k_2,l_2)$.
G_{00}: II(0,1), $V_1 =$ (xiibs_1), $V_3 =$ (xis_2).
$y^2 = t\rho_{s_1}(x,t,k_1,l_1+1)(x-1)h_{s_2}(x-2,t,k_2+1/2,l_2)$.

THE CASES OF THE STABLE MODELS ARE G_{01}.
G_{01}: Id, $V_1 =$ (xiias_1), $V_3 =$ (ivs_2). $y^2 = \rho_{s_1}(x,t,k_1,l+1)(x-1)f_{s_2}(x-2,t,k_2,0)$.
G_{01}: II(0,1), $V_1 =$ (xiibs_1), $V_3 =$ (ivs_2). $y^2 = t\rho_{s_1}(x,t,k_1,l+1)(x-1)f_{s_2}(x-2,t,k_2,1)$.

THE CASES WHERE THE STABLE MODEL IS J_1.
J_1: Id, $V_1 =$ (va$s$).
$y^2 = \omega_s(x,t,k_1)\{(x-1)^2-t^{k_2}\}\{(x-2)^2-t^{k_3}\}$. ($k_1 \geq 2$ when $s = 1$.)
J_1: II(0,1), $V_1 =$ (va$s$).
$y^2 = \omega_s(x,t,k_1)\{(x^2-t)^2-t^{k_2}x^l\}$, $(2k_2+l-4 \geq 1)$. ($k_1 \geq 2$ when $s = 1$.)
J_1: II(1,4), $V_1 =$ (vb$s$). $y^2 = t\Gamma_s(x-1,t,k_1)\{(x^2-t)^2-t^{k_2}x^l\}$, $(2k_2+l-4 \geq 1)$.
J_1: II(1,6), $V_1 =$ (vb$s$). $y^2 = t\Gamma_s(x,t,k_1)\{(x-1)^2-t^{k_2}\}\{(x-2)^2-t^{k_3}\}$.

THE CASES WHERE THE STABLE MODELS IS J_0.
J_0: Id, $V_1 =$ (xiia$s$). $y^2 = \rho_s(x,t,k_1,l+1)\{(x-1)^2-t^{k_2}\}\{(x-2)^2-t^{k_3}\}$.
J_0: II(0,1), $V_1 =$ (xiia$s$). $y^2 = \rho_s(x,t,k_1,l_1+1)\{(x^2-t)^2-t^{k_2}x^l\}$, $(2k_2+l-4 \geq 1)$.
J_0: II(1,4), $V_1 =$ (xiib$s$). $y^2 = t\rho_s(x-1,t,k_1,l_1)\{(x^2-t)^2-t^{k_2}x^l\}$, $(2k_2+l-4 \geq 1)$.
J_0: II(1,6), $V_1 =$ (xiib$s$). $y^2 = t\rho_s(x,t,k_1,l_1)\{(x-1)^2-t^{k_2}\}\{(x-2)^2-t^{k_3}\}$.

THE CASES WHERE THE STABLE MODEL IS K_1.
K_1: Id, $V_4 =$ (iv$s$). $y^2 = (x-t^{l_1+1})f_s(x,t,k_1+1,l_1+1)\{(x-1)^2-t^{k_2}\}\{(x-2)^2-t^{k_3}\}$.
K_1: II(0,1), $V_4 =$ (iv$s$).
$y^2 = \{(x^2-t)^2-t^{k_1}x^{l_1}\}\{(x-1)-t^{l_2+1}\}f_s(x-1,t,k_2+1,l_2+1)$, $(2k_1+l_1-4 \geq 1)$.
K_1: II(1,4), $V_4 =$ (iv$s$).
$y^2 = t\{(x^2-t)^2-t^{k_1}x^{l_1}\}\{(x-1)-t^{l_2+1}\}f_s(x-1,t,k_2,l_2+2)$, $(2k_1+l_1-4 \geq 1)$.
K_1: II(1,6), $V_4 =$ (iv$s$).
$y^2 = t(x^2-t^{k_1})\{(x-1)^2-t^{k_2}\}\{(x-2)-t^{l_1+1}\}f_s(x-3,t,k_3,l_1+2)$.

THE CASES WHERE THE STABLE MODEL IS K_0.

K_0: Id, $V_4 = $ (xi$s$). $y^2 = (x - t^{k_1})h_s(x, t, k_1/2 + k_2, l)\{(x-1)^2 - t^{k_3}\}\{(x-2)^2 - t^{k_4}\}$.

K_0: II(0,1), $V_4 = $ (xi$s$).
$$y^2 = \{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x-1) - t^{k_2}\}h_s(x-1, t, k_2/2 + k_3, l)\ (2k_1 + l_1 - 4 \geq 1).$$

K_0: II(1,4), $V_4 = $ (iv$s$).
$$y^2 = t\{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x-1) - t^{k_2}\}h_s(x-1, t, k_3 + (k_2+1)/2, l_2)\ (2k_1 + l_1 - 4 \geq 1).$$

K_0: II(1,6), $V_4 = $ (iv$s$).
$$y^2 = t(x^2 - t^{k_1})\{(x-1)^2 - t^{k_2}\}\{(x-2) - t^{k_3}\}h_s(x-3, t, k_4 + (k_3+1)/2, l).$$

THE CASES WHERE THE STABLE MODEL IS L.

L: II(0,1), $V_1 = V_2 = $ (xb).
$$y^2 = (x^2 - t^{k_1+1})\{(x^2-t)^2 - t^{k_2}x^{l_1}\}\{(x-1)^2 + t^{l_2})\},\ (2k_2 + l - 4 \geq 1).$$

L: II(0,2), $V_1 = V_2 = $ (xc).
$$y^2 = \{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x^2-t)^2 - 2t^{k_2}x^{l_2}\}.\ (2k_1 + l_1 - 4 \geq 1, 2k_2 + l_2 - 4 \geq 1)$$

L: II(1,5). $y^2 = t\{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x^2-2t)^2 - t^{k_2}x^{l_2}\}, (2k_1 + l_1 - 4 \geq 1, 2k_2 + l_2 - 4 \geq 1)$.

L: II(1,7). $y^2 = t\{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{x^2 - t^{l_2}\}\{(x-1)^2 + t^{l_3}\}, (2k_1 + l_1 - 4 \geq 1)$.

L: II(1,8). $y^2 = t(x^2 - t^{k_1})\{(x-1)^2 - t^{k_2}\}\{(x-2)^2 - t^{k_3}\}\{(x-3)^2 - t^{k_4}\}$.

L: III(0,1), $V_1 = $ (vd$s$) $y^2 = \{(x^3-t)^2 - t^{k_1}x^{l_1}\}\{(x-1)^2 - t^l\},\ (3k_1 + l_1 - 6 \geq 1)$.
$$y^2 = \{(x^3-t)^2 - t^{k_1}x^{l_1}\}(x^2 - t^{k_2+1}),\ (3k_1 + l_1 - 6 \geq 1).$$

L: IV(0,1), $V_1 = $ (ve). $y^2 = \{(x^4-t)^2 - t^{k_1}x^{l_1}\}, (4k_1 + l_1 - 8 \geq 1)$.

L: IV(1,1), $V_1 = $ (ve). $y^2 = t\{(x^4-t)^2 - t^{k_1}x^{l_1}\}, (4k_1 + l_1 - 8 \geq 1)$.

L: VI(1,1), $V_1 = $ (vd$s$). $y^2 = t\{(x^3-t)^2 - t^{k_1}x^{l_1}\}\{(x-1)^2 - t^{k_2-1}\}, (3k_1 + 2l_1 - 6 \geq 0)$.

THE CASES WHERE THE STABLE MODEL IS N.

N: II(0,1).
$$y^2 = \{(x^2-t)^2 - t^{k_1}x^{l_1}\}\{(x-1)^2 - t^{k_2+2k_3}\}\{(x-1-t^{k_3})^2 - t^{k_4+2k_3}\}, (2k_1 + l_1 - 4 \geq 1).$$

N: II(0,2). $y^2 = \{(x^2 - t^{2k_1-1})^2 - t^{k_2+4k_1-4}x^{l_1}\}\{((x^2-1)^2 - t)^2 - t^{k_3}x^{l_2}\}$,
$$(2k_2 + l_1 - 4 \geq 1, 2k_3 + l_2 - 4 \geq 1).$$

N: II(2,3). $y^2 = t\{(x^2 - t^{2k_1-1})^2 - t^{k_2+4k_1-4}x^{l_1}\}\{((x^2-1)^2 - t)^2 - t^{k_3}x^{l_2}\}$,
$$(2k_2 + l_1 - 4 \geq 1, 2k_3 + l_2 - 4 \geq 1).$$

N: II(2,4). $y^2 = t\{(x - t^{k_1})^2 - t^{2k_1+k_2}\}\{x^2 - t^{2k_1+k_3}\}\{(x-1)^2 - t^{k_4}\}\{(x-2)^2 - t^{k_5}\}$.

N: II(2,5).
$$y^2 = t\{(x-1)^2 - t^{k_1}\}\{(x-2)^2 - t^{k_2}\}\{(x^2 - t^{2k_3-1})^2 - t^{4k_3+k_4-4}x^{l_2}\}, (2k_4 + l_2 - 4 \geq 1).$$

N: II(2,7). $y^2 = t\{((x^2-t) - t^{k_1}x^{l_1})^2 - t^{k_2}x^{l_2}\}\{(x^2-t)^2 - t^{k_3}x^{l_3}\}$,
$$(2k_3 - 4k_1 + l_3 - 2l_1 \geq 0, 2k_2 + l_2 - 4k_1 - 2l_1 \geq 1).$$

N: II(2,8). $y^2 = \{((x^2-t) - t^{k_1}x^{l_1})^2 - t^{k_2}x^{l_2}\}\{(x^2-t)^2 - t^{k_3}x^{l_3}\}$,
$$(2k_3 - 4k_1 + l_3 - 2l_1 \geq 0, 2k_2 + l_2 - 4k_1 - 2l_1 \geq 1).$$

N: IV(2,1). $y^2 = t\{(x^2-t)^2 - t^{k_1}x\}^2 - t^{k_2}x^l, (2k_1 - 3 \geq 0, 2k_2 + l - 8 \geq 0)$

N: IV(2,2). $y^2 = \{(x^2-t)^2 - t^{k_1}x\}^2 - t^{k_2}x^l, (2k_1 - 3 \geq 0, 2k_2 + l - 8 \geq 0)$.

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