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Monopoly, Pareto and Ramsey Mark-ups

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Abstract Monopoly prices are too high. It is a price level problem, in the sense that the relative mark-ups have Ramsey optimal proportions, at least for independent constant elasticity demands. I show that this feature of monopoly prices breaks down the moment one demand is replaced by the textbook linear demand or, even within the constant elasticity framework, dependence is introduced. The analysis provides a single Generalized Inverse Elasticity Rule for the problems of monopoly, Pareto and Ramsey.

Keywords monopoly prices · Ramsey prices · inverse elasticity rules

JEL Classification D40 · D60 · L50

1 Introduction

Monopoly prices are too high. It is a price level problem, in the sense that the relative mark-ups have Ramsey optimal proportions, at least for independent constant elasticity demands. By the same token, Ramsey pricing is considered business oriented (Laffont and Tirole 2000, p. 63). This attractive feature of monopoly prices breaks down for variable elasticities. The reason is that both monopoly prices and Ramsey prices are governed by local inverse elasticity rules, but if the elasticities differ at the (low) monopoly output and the (high) Ramsey output, the mark-ups will differ as well. Hoeffler (2006) illustrates this phenomenon using a kinked demand curve. The break-down of the optimality of relative monopoly

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mark-ups already occurs once a constant-elasticity demand is replaced by the textbook linear demand. And worse–remaining within the framework of constant elasticities–the break-down also occurs the moment dependence is introduced. Our counterexamples are strong: even the *orders* of monopoly and Ramsey price components differ! These negative results follow a novel, unifying framework, featuring a Generalized Inverse Elasticity Rule for alternative pricing rules. Hitherto Ramsey rules have been less transparent for interdependent demands (Morhring 1970).

2 Monopoly, Pareto and Ramsey pricing: One rule

I will analyze the alternative pricing rules using a single framework. It is general yet simple and parametrizes the problems of monopoly, Pareto and Ramsey (through variable μ defined below).

Consider an industry with n products. List the prices in the n -dimensional row vector p. Demand is given by the n-dimensional column vector $D(p)$ and revenue is $pD(p)$. Subtraction of costs $C(D(p))$ defines profit. Welfare is the sum of profit and consumers' surplus, the area under the demand curve (or surface): $\int_p^\infty D(\widetilde{p}) \bullet d\widetilde{p}$, where the dot denotes the inner product. Because demand is assumed to be independent of income, this line integral is path-independent. It has the property that its derivative with respect to p_i equals $-D_i(p)$. Ramsey prices maximize welfare subject to the constraint that profit is nonnegative. The Lagrangian function is:

$$\int_p^\infty D(\widetilde{p}) \bullet d\widetilde{p} + (1+\lambda)[pD(p) - C(D(p))],\ \lambda \geqslant 0 \tag{1}$$

Profit enters both the objective function (the term with coefficient 1) and the constraint part (the term with coefficient λ) of the Lagrangian function. Equation 1 is not a Lagrangian function in the narrow sense, because I want to be able to consider cases in which λ is set exogenously. Ignoring the profit constraint defines Pareto optimality. This is encompassed by $\lambda = 0$. Conversely, if all weight in the Lagrangian function is on the constraint function, profit, we have the problem of the monopolist. This is encompassed by $\lambda \to \infty$. The profit constraint is binding in the Ramsey problem. This is the intermediate case, $0 < \lambda < \infty$.

It is convenient to change variable $0 \leqslant \lambda \leqslant \infty$ into $\mu = \lambda/(1+\lambda)$, $0 \leqslant \mu \leqslant 1$. The cases of Pareto, monopoly and Ramsey pricing are then encompassed by $\mu = 0$, 1, and $0 < \mu < 1$, respectively. Now in each case, the first-order conditions are obtained by setting the price derivatives of expression (1) zero. For this purpose it is convenient to use the row vector of relative mark-ups or *Lerner indices*, $L(p) = [p - C'(D(p))]\widehat{p}^{-1}$, where C' is the row vector of marginal costs and the hat transforms a vector to a diagonal matrix. And instead of the matrix of demand derivatives, $D' = (\partial D_i/\partial p_j)_{i,j=1,\ldots,n}$, it is customary to use that of elasticities: $\varepsilon = (\varepsilon_{ij})_{i,j=1,\ldots,n} = [(p_j/D_i)\partial D_i/\partial p_j]_{i,j=1,\ldots,n} = \widehat{D}^{-1}D'\widehat{p}$. The following formula generalizes the Inverse Elasticity Rule (Baumol and Bradford 1970) with respect to demand, costs, and problem setting. It follows Cuthbertson and Dobbs (1996) and solves for the Lerner indices:

Proposition 1 (Generalized Inverse Elasticity Rule) *The first order conditions read* $L_i(p^\mu) = -\mu \sum_{j=1}^{n} p_j D_j(p)(\varepsilon^{-1})_{ji}/[p_i D_i(p)]$. *Here problem identifier* μ *is* 0 (*Pareto prices*), 1 (*monopoly prices*) *or intermediate* (*Ramsey prices*).

Proof The partial derivatives of consumers' surplus with respect to price, organized in a row vector, are given by $-D^\intercal(p)$, where $^\intercal$ is the transposition sign. Setting the price dervative of expression (1) equal to zero: $-D^\intercal(p) + (1+\lambda)[D^\intercal(p) + pD'(p) - C'(D(p))D'(p)] = 0$ or $L(p)\widehat{p}D'(p) = -\mu D^\intercal(p)$. Solving for the Lerner indices, $L(p) = -\mu D^\intercal(p)D'^{-1}(p)\widehat{p}^{-1} = -\mu D^\intercal(p)\widehat{p}\varepsilon^{-1}\widehat{D}^{-1}\widehat{p}^{-1}$. Taking the i-th component completes the proof. □

The i-th (Pareto, monopoly or Ramsey) optimal Lerner index is a weighted average of elements of the inverse elasticity matrix ε^{-1}; the weights are the budget ratios $p_jD_j(p)/[p_iD_i(p)]$. Proposition 1 has two corollaries, both well-known results. Before I present them, I define two concepts. First, demand is *independent* if $\partial D_i/\partial p_j = 0$ for all $i \neq j$. Second, two price vectors have the same *structure*, if the vectors of Lerner indices are proportionate (collinear).

Corollary 1 *Pareto prices equal marginal costs.*

Proof Pareto prices are encompassed by $\mu = 0$. By Proposition 1, the Lerner indices are zero. By definition of the latter, $p = C'$. □

Corollary 2 *Monopoly prices and Ramsey prices have the same structure, if elasticities are constant and demand is independent.*

Proof Independent demand means that D' is a diagonal matrix. Hence ε is a diagonal matrix. Hence ε^{-1} is the diagonal matrix with elements ε_{ii}^{-1}. Recall that elasticities are functions of prices. By independence, they depended only on own prices. By Proposition 1, $L_i(p_i^\mu) = -\mu\varepsilon_{ii}^{-1}$. Because the elasticities are assumed constant, we conclude $L_i(p_i^\mu) = -\mu L_i(p_i^1)$. □

The upshot of Corollary 2 is that monopoly prices may be too high, but their structure (the proportions of the mark-ups) is right. This has the policy implication that regulation can be limited to the price level, leaving the fine-tuning of the mark-ups to the monopolist. Vogelsang and Finsinger (1979) detail the regulatory process which, perhaps surprisingly, holds for interdependent demands. Anyway, a monopolist can and will charge high prices for products with inelastic demand–without eroding the market too much–while a social planner charges high prices for products with inelastic demand, because the allocation is little distorted.

Corollary 2 provides sufficient conditions for the similarity of monopoly and Ramsey prices, but they are not necessary. Non-constant elasticities and demand dependencies are sources which drive a wedge between the structures of monopoly and Ramsey prices, but, at least in principle, these sources may neutralize each other. In other words, monopoly prices and Ramsey prices *may* be similar in industries with complicated demands.

3 Two counterexamples

I will now demonstrate that both the constant elasticities and the independence of demand are critical to the result that monopoly prices have a Ramsey structure. Counterexample 1 presents a violation of constant elasticities (while demand is independent) and Counterexample 2 presents a violation of independence (while all elasticities are constant).

Counterexample 1. Consider two, independent demands. The first has constant elasticity: $D_1(p_1) = p_1^{-2}$. The second demand is from the textbooks, the simple linear $D_2(p_2) = 1 - p_2$. There is a small fixed cost, *f*, and the variable production costs are constant, $\frac{1}{2}$ per unit of (either) output. In market 2, the elasticity is $-\frac{p_2}{1-p_2}$. By Proposition 1, the Generalized Inverse Elasticity Rule reads $\frac{p_1-\frac{1}{2}}{p_1} = \frac{\mu}{2}$, $\frac{p_2-\frac{1}{2}}{p_2} = \mu\frac{1-p_2}{p_2}$. The solution is $p_1^\mu = \frac{1}{2-\mu}$, $p_2^\mu = \frac{\frac{1}{2}+\mu}{1+\mu}$.

First consider the monopoly case, $\mu = 1$. Then $p_1 = 1$, $p_2 = \frac{3}{4}$ and the Lerner indices are $\frac{1}{2}$ and 1/3, respectively. Thus, a monopolist charges a high price (and Lerner index) in the *first* market.

Next consider the Ramsey case, where μ is determined by the profit constraint, $(p_1 - \frac{1}{2})p_1^{-2} + (p_2 - \frac{1}{2})(1 - p_2) = f$. Substitution of the prices yields $\frac{\mu}{2}(2-\mu)+(\frac{\frac{1}{2}+\mu}{1+\mu} - \frac{1}{2})(1 - \frac{\frac{1}{2}+\mu}{1+\mu}) = f$. This equation determines μ as an increasing function of *f*, starting in $\mu = 0$ for $f = 0$ (the Pareto case). For example, $\mu = 1/3 \approx 0.33$ corresponds with $f = 187/576 \approx 0.32$. In this case, $p_1 = \frac{1}{2-\mu} = 0.6$, $p_2 = \frac{\frac{1}{2}+\mu}{1+\mu} = 0.625$ and the Lerner indices are $1/6 \approx 0.17$ and $1/5 = 0.20$, respectively. The prices and Lerner indices are collected in Table 1.

Contrary to a monopolist, a regulator would charge the higher price (and Lerner index) in the *second* market. Not only the proportions of monopoly prices are off, but even the order of price components is reversed.

Counterexample 2. Consider $D_1(p_1) = p_1^{-1}$ and $D_2(p_1, p_2) = p_1^{-1}p_2^{-2}$. (The products are complements.) There is a small fixed cost, *f*, and the variable production costs are constant, $2/3 \approx 0.67$ per unit of output 1 and $\frac{1}{2} = 0.50$ per unit of output 2. $\varepsilon = \begin{pmatrix} -1 & 0 \\ -1 & -2 \end{pmatrix}$ and, therefore, $\varepsilon^{-1} = \begin{pmatrix} -1 & 0 \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$. By Proposition 1, the Generalized Inverse Elasticity Rule reads $L_1 = \frac{p_1-2/3}{p_1} = \mu - \mu p_1^{-1}p_2^{-1}\frac{1}{2}$, $L_2 = \frac{p_2-\frac{1}{2}}{p_2} = \mu/2$. The solution is $p_1^\mu = \frac{1}{6(1-\mu)} + \frac{1-\mu}{2}$, $p_2^\mu = \frac{1}{2-\mu}$. The first price is minimal, in fact $p_1^\mu = 1/\sqrt{3} \approx 0.58$, at $\mu = 1 - 1/\sqrt{3} \approx 0.42$, whilst there $p_2^\mu = \frac{1}{1+1/\sqrt{3}} = 0.63$. The Lerner indices are $L_1 = 1 - \frac{2/3}{p_1^\mu} = 1 - \frac{2}{3}\left[\frac{1}{6(1-\mu)} + \frac{1-\mu}{2}\right]^{-1} = 1 - \frac{4(1-\mu)}{1+3(1-\mu)^2}$, $L_2 = \mu/2$. In the Ramsey case, μ is determined by the profit constraint. One can show that the profit constraint reads $f = \mu$. (The demonstration involves the tedious but straightforward calculation by which variable profit, $(p_1 - 2/3)D_1(p_1) + (p_2 - \frac{1}{2})D_2(p_1, p_2)$, reduces to a simple μ.) For the case of $\mu = f = 1 - 1/\sqrt{3} \approx 0.42$ the Lerner indices reduce to $L_1 = 1 - 2/\sqrt{3} \approx -0.15$ and $L_2 = \frac{1}{2} - \frac{1}{2}/\sqrt{3} \approx 0.21$. For this case, prices and Lerner indices are collected in Table 2.

As is well known, a monopolist ($\mu = 1$) increases the price of the good with unitary demand elasticity (product 1) without limit; the Lerner index for good 1 is a full 1. The monopolist charges a limited price for the good with elastic demand; that Lerner index is

Table 1 Pareto, Ramsey and monopoly prices and Lerner indices in Counterexample 1

Problem	μ	p_1	p_2	L_1	L_2
Pareto	0	0.50	0.50	0	0
Ramsey	0.33	0.60	0.625	0.17	0.20
Monopoly	1	1	0.75	0.50	0.33

Table 2 Pareto, Ramsey and monopoly prices and Lerner indices in Counterexample 2

Problem	μ	p_1	p_2	L_1	L_2
Pareto	0	0.67	0.50	0	0
Ramsey	0.42	0.58	0.63	−0.15	0.21
Monopoly	1	∞	1	1	0.50

only 0.50. A regulator, however, charges a *low* price for the good with relatively inelastic demand (product 1), namely 0.58, which is even below cost. The price cut boosts the demand for the complement, which generates the funds required to defray the fixed cost. As in Counterexample 1, not only the proportions of monopoly prices are off, but even the order of price components is reversed.

Ever since Baumol et al. (1979) it is indeed known that Ramsey prices may involve a cross-subsidy. In Counterexample 2 the second market cross-subsidizes the first. This flow of funds renders the second market vulnerable to entry. In other words, the Ramsey optimum is not sustainable. Indeed, a condition of the Weak Invisible Hand Theorem (by which the Ramsey optimum is sustainable, see Baumol et al. 1977) is not fulfilled (namely weak gross substitutability).

4 Discussion

The examples show that monopoly prices and Ramsey prices can have very different structures, even to the extent that the order of price components is reversed. It shows that the problem of monopoly pricing by a multi-product monopolist is not solved by just reducing all prices by some proportion. An RPI-X price cap for each product need not yield the desired result of approaching Ramsey efficient prices. In the RPI-X price cap system the price cap is allowed to increase at the rate of inflation, measured by the retail price index, less some "X factor" to account for productivity gains or to reduce the regulated firm's rents, but otherwise the regulated firm is allowed to adjust its own prices, unlike rate of return based regulation. When should the regulator be alerted to the fact that only the level but also the structure of the prices better be adjusted?

The first situation that springs to mind is the case of non-constant elasticities. In this case it is quite intuitive that monopoly and Ramsey prices have different structures, because the pricing rules are local and, therefore, may produce different results. From an applied point of view, however, such variations are hard to estimate and the other cause of trouble, the violation of the independence assumption is more serious. To develop some intuition, recall the Generalized Inverse Elasticity Rule (Proposition 1): $L_i(p^\mu) = -\mu \sum_{j=1}^{n} p_j D_j(p)(\varepsilon^{-1})_{ji}/[p_i D_i(p)]$. By definition, the monopoly price vector p^1 has the same structure as the Ramsey price vector p^μ if $L_i(p^1)$ and $L_i(p^\mu)$ are collinear, i.e. if $\sum_{j=1}^{n} p_j D_j(p)(\varepsilon^{-1})_{ji}/[p_i D_i(p)]$ is constant. Clearly, it is not enough if the demand elasticities (ε) are constant, but also the budget shares must be constant. In other words, *income effects between the products* break down the similarity between monopoly and Ramsey prices.

The upshot for regulators is that doubt is shed on standard tools such as RPI-X price caps when a monopolist produces final consumption goods with mutual income effects. The most prominent example is a firm that offers a quality ladder of products. Think of a window producer. Its monopoly prices are not only high–the standard tool to restrain the market and exercise market power–but also take output in a region where the market share of double glazed windows is smaller (because of the negative income effect).

It is possible to analyze this one step further by a first order approximation of the inverse elasticity rule about the benchmark of independent demands, which is not plagued by the dissimilarity problem (Corollary 2). Thus, rewriting the elasticities matrix ε as the sum of diagonal matrix $\widehat{\varepsilon}$–representing the own elasticities–and off-diagonal matrix $\widetilde{\varepsilon}$ –representing the cross elasticities, let the latter be small compared to the former. Then $\varepsilon^{-1} = (\widehat{\varepsilon} + \widetilde{\varepsilon})^{-1} = [\widehat{\varepsilon}(I + \widehat{\varepsilon}^{-1}\widetilde{\varepsilon})]^{-1} = (I + \widehat{\varepsilon}^{-1}\widetilde{\varepsilon})^{-1}\widehat{\varepsilon}^{-1} \approx (I - \widehat{\varepsilon}^{-1}\widetilde{\varepsilon})\widehat{\varepsilon}^{-1}$ and, therefore, the Generalized Inverse Elasticity Rule becomes $L_i(p^{\mu}) \approx \mu \sum_{j=1}^{n} p_j D_j(p)(I - \widehat{\varepsilon}^{-1}\widetilde{\varepsilon})_{ji}/[p_i D_i(p)(-\varepsilon_{ii})] = \mu/(-\varepsilon_{ii}) + \mu \sum_{j \neq i} p_j D_j(p)(-\varepsilon_{jj}^{-1}\varepsilon_{ji})/[p_i D_i(p)(-\varepsilon_{ii})]$. For quality ladders, the commodities are substitutes, hence ε_{ji} ($j \neq i$) are positive. If commodity i is a top-of-the-line product, such as a double glazed windows, the exercise of monopoly power (with its negative income effect) reduces $p_i D_i(p)/p_j D_j(p)$ ($j \neq i$), hence increases the Lerner index disproportionally much. Quality is overprized and a regulator might consider to impose a stiffer cap on high-quality products.

In this case, however, the critical condition of the Weak Invisible Hand Theorem, namely weak gross substitutability, is fulfilled, and a more practical regulation would be to reduce barriers to entry, to introduce the disciplinary sway of potential competition.

5 Conclusion

The Generalized Inverse Elasticity Rule presented in this note encompasses the problems of monopoly, Pareto and Ramsey. The result that monopoly prices are merely too high, while their structure is right in the sense that relative mark-ups have the same proportions as of Ramsey prices, is confined to industries with demands that feature constant elasticities and are independent. Otherwise even the rankings of monopoly and Ramsey price components can be different. If there are income effects, a regulator may consider to cap some prices more strictly than others.

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