

MONOTONICITY OF RATIO BETWEEN THE GENERALIZED LOGARITHMIC MEANS

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Abstract. Let $c > b > a > 0$ be real numbers. Then the function $f(r) = \frac{L_r(a,b)}{L_r(a,c)}$ is strictly decreasing on $(-\infty, \infty)$, where $L_r(a, b)$ denotes the generalized (extended) logarithmic mean of two positive numbers a and b .

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