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MONTE CARLO SIMULATION OF INELASTIC NEUTRINO SCATTERING IN DUMAND

Arthur Roberts
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

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ABSTRACT

Detailed Monte Carlo calculations simulating the detection in the DUMAND 1-km^3 optical detector of inelastic neutrino scattering by nucleons at 2 TeV and above show that the measurement of the y distribution is subject to systematic errors due to a) experimental errors and intrinsic fluctuations which produce errors in the energy determinations of hadronic cascade and muon; b) uncertainty in the exact amount of antineutrino fraction in the cosmic-ray neutrino flux. The nature of these errors is explored, and methods for removing them from the data developed. The remaining uncertainties are those in the evaluation of the errors in energy determination, and in the antineutrino contamination. It appears that these errors, not statistical ones, will eventually govern the accuracy of the y distributions obtained. Nonetheless, the effect of the boson propagator on the y distribution is so marked that we can find no plausible scenario in which the residual errors cast doubt on whether or not the propagator effect is present.

I. Introduction

An extensive Monte Carlo simulation is in progress to determine the capabilities of the proposed DUMAND optical array for studying the y-distribution in inelastic muon-neutrino scattering from nucleons, in the energy range of about 3 to 30 TeV. This region is of special interest, since it is there that the propagator due to the W-boson should manifest itself clearly, provided only that the W boson mass lie in the expected range of 40 to 100 GeV/c^2. The analysis is being carried out in three stages:

1. A study of errors in the determination of muon and cascade energies due to intrinsic fluctuations of energy loss.
2. A study of experimental errors in the determination of muon and cascade energies by DUMAND.
3. A study of the effect on the analysis of y distributions of errors in energy determination of muons and nuclear cascade, regardless of their source; and methods for correcting such effects.

Of these three subjects we have made a good start on the first and third. The second awaits the completion of Monte Carlo programs now being written, to yield the signals to be expected from a given sensor array for typical muon and neutrino events; and also a study of how to analyze the data to obtain muon and cascade energies. It is the most complex of the three, involving as it does both a complete array simulation and an entire analysis program, which may well compare in complexity to a bubble-chamber analysis system (and may perhaps borrow from existing ones.)

We have already reported briefly on progress to date in evaluating the possible error in muon energy due to inherent fluctuations, in an earlier paper[1] in this volume. We have not as yet tackled the cascade, but since the energies are high and the Cerenkov light plentiful, we expect the errors due to fluctuations to be smaller than the muon errors.

This paper will report on progress to date on the third stage, the effect of fluctuations and experimental errors on the measurement of the y distribution. The importance of that problem was first suggested to us by T. K. Gaisser.

II. Theory

The effect of the propagator on the y distribution - i. e. the degree of inelasticity in neutrino-nucleon collisions - has been described by Gaisser and Halprin.[2] Further studies by Halprin[3] and by Halprin and Oakes[4] have discussed such corrections to the original results as the effect of scaling violations and the effect of modifying the quark distributions originally assumed. They find these effects to be small, and not to modify in any way the original result[2] that the effect of the propagator is to alter the y distribution as the CM energy of the interaction passes through the threshold of W-production, in a striking and unmistakable fashion. The relevant curves are shown in Fig. 1, which shows the change of shape, indicating increasing elasticity, as the energy increases from accelerator values toward 100 TeV. There is, in addition to the change in y distribution, a corresponding flattening of the cross section, but that is much more difficult to determine experimentally.

Two experimental factors complicate the analysis of the results. One is that the neutrino spectrum is a steeply falling one; the integral spectrum in the TeV region is decreasing about as $E^{-2.5}$. It is well known that in apparatus with finite resolution, steep slopes in the measured quantity tend to wash out, and can be restored only by a careful de-convolution of the observed spectrum, using the known resolution function of the detector. Of course, peaks narrower than the resolution width tend to disappear, but that problem, at least, we are spared; the spectrum is a simple steep monotonic function.

The other factor complicating the analysis is the problem of determining the antineutrino component in the cosmic-ray neutrino spectrum. Unlike accelerator experiments, in which the neutrino sign is determined both by the incident beam and by the sign of the outgoing muon in a magnetic spectrometer, DUMAND has no direct way of identifying the muon sign. Consequently it measures an admixture of antineutrinos which must be determined if the y distribution is to be significant. This is especially important because the y distribution for antineutrinos contains the factor $(1-y)^2$, which emphasizes low y-values (see Fig. 2), precisely the effect of the boson propagator. Thus an unexpectedly large antineutrino component could mask the desired effect. If the magnitude of antineutrino component is as we expect, relatively independent of energy, then the data at low energies, before the effect of the propagator becomes large, can be used to determine the antineutrino admixture.

The most dangerous scenario is that in which the propagator effect is absent (no boson in the expected mass range) and the antineutrino component maliciously increases in abundance with energy in just such a way as to counterfeit the presence of the expected boson. Even in that case an independent check can be applied. The neutrinos arise from pion and kaon decays, primarily from those in which the only other decay product is the muon. The +/- sign ratio of the muons must therefore carry the required information about the neutrino signs. The desired data can be unfolded, at least in principle, from a knowledge of the muon +/- ratio as a function of energy, the pion and kaon x distributions up to the energies of the primaries responsible for the neutrinos being studied (i. e. the 20 - 500 TeV range), and the pion/kaon ratios . All these values are in hand up to ISR energies (i. e. about 1.5 TeV), and (Feynmann) scaling appears to be well observed. Thus we have in principle a basis for ruling out the pathological behavior stipulated. The analysis has not yet been carried out in detail.

III. Procedure For Monte Carlo Calculations

We have used the equations of Gaisser and Halprin, integrated over x, to find the y distribution at several different energies. The y distribution changes only slowly with energy; thus values representing the y distribution for many different energy bins, each a factor of 2 in width, were stored in a table and used to provide weighting for random numbers selected to represent the y-values of events. Thus in any given energy bin a random number distribution reproduces the calculated y-distribution.

Similarly, the neutrino spectrum is represented by an exponential with a slope of $E^{-1.5}$; this differs from the integral neutrino spectrum by one power of E, because the number of events observed at a given energy is weighted by the interaction cross section, which is itself closely proportional to E over the energy range considered. A random number chooses the neutrino energy, according to this spectrum, within energy limits set by the programmer.

The program first selects a neutrino energy, and finds the energy bin in which it is located. Another random number then determines the y for that event, and calculates the muon and cascade energies. We now assume a gaussian distribution of errors in determination of muon and cascade energies, from whatever cause, measurement or fluctuation. Actually the errors are gaussian on a logarithmic energy scale,[1] which approximates the true situation; random numbers determine the errors, and thus give new "measured" values for the cascade and muon. From these the "measured" neutrino energy and the y value are calculated. The cascade and muon standard deviations are independent data supplied to the program.

IV. Results

Initially we took, as a simple measure of the shape of the y distribution, the ratio of the number of events in the upper half of the distribution to the number in the lower half, i.e. $R = N(y > 0.5)/N(y < 0.5)$. Plotted in that way, the results look very favorable; see Fig. 3. The ratio appeared to be not far from correct, and displaced in the direction one would expect because of the steeply falling spectrum. The latter is moved to higher energies by the convolution of the energy resolution. But an equal number of events deposited in the next higher and next lower bins have entirely different effects; in the lower bin a number of events has been added to a much larger population, but in the higher energy bin the same number has been added to a much smaller one. As regards spectrum shape, the overall effect with a power spectrum is to displace it to a higher value. We can numerically evaluate the effect by constraining all the events to lie within one bin and observing the numbers that are transferred to other bins by the error assumed. With that information, the deconvolution can be carried out. For large errors - 50% standard deviations in both hadron and muon energies - population of each bin is increased 28% by this effect; the deficit appears in the lowest energy portion of the spectrum.

More important than the shift in population is the change in the y distribution. Events which are moved to another bin are usually those with a greater than average deviation from the mean. They carry with them a correspondingly distorted y value. Thus, if the "measured" muon energy is much larger than the true value, the y-value will be too small. If the "measured" cascade energy is too large, the y-value will be too large. Thus events shifted to another bin carry correspondingly shifted y-values with them. This effect is shown in Figs. 4 - 6. These figures show data with zero antineutrino content, in which all neutrinos were constrained to be in the energy bin 2-4 TeV. Separate runs with cascade and muon errors (standard deviations, errors gaussian on logarithmic scale) each of 0.3 E or 0.5 E gave the residual spectra shown; the missing events are of course in other bins, principally the closest ones. As expected, the bin is depleted of large y-values by errors in cascade energy (Fig. 4a), and of small y-values by errors in muon energy (Fig. 4b). Fig. 4b). Fig. 4c shows the effect of errors in both cascade and muon energy; the bin has now been more uniformly depleted. Figs. 5 a-c show a plot of events "transferred" to the next higher energy bin, 4-8 TeV, in which there now appear only those events from the lower energy bin which were "measured" as belonging to the next bin. In Fig. 5a, the effect of the cascade error is shown, and in Fig. 5b the effect of the muon error: these distributions are both sharply peaked, but at opposite ends of the y distribution, just as predicted above; and in the last case, Fig. 5c shows the effect of errors in both quantities together, with a transferred spectrum sharply peaked at both ends. In similar fashion Figs. 6 a-c show the 1-2 TeV bin, the adjacent one on the low energy side. The distribution is the obverse of the one on the opposite side of the original energy; but as we mentioned earlier the effect of these events is far smaller, since they are much diluted by the greater indigenous population.

V. Correction Procedures

We have examined only the redistribution of events from a given bin to the nearest neighbors; a few events land even further away, especially with large errors. Similar runs can be made for neutrino spectra confined to each of the other bins in turn, the results varying only as the y distribution changes and/or the antineutrino composition may be expected to change. From these runs a set of correction factors can be evolved that allow the original spectrum to be reconstituted from the data. This procecure has been carried out successfully; it does in fact reconstitute the spectrum and also the corresponding y distributions, with errors compatible with the statistics of the run.

Now, we may ask, what is the effect on this correction procedure and its results, of a misjudgment in the measurement error of either the cascade or the muon? We have seen in Figs. 4-6 the importance of the magnitude of the error. Suppose we correct for a 30% error when in fact the error is 50% or 10%?

The result of such a trial is shown in Fig. 7. Here we have used cascade and muon standard deviations of 0.5 E, the curve shows the corrected values obtained when the errors are correctly assessed. It also shows what happens if the errors are incorrectly assessed at 0.3 each instead of 0.5.

We note that the correct evaluation of the errors yields a curve indistinguishable from the original, except that it lies slightly higher. On the other hand, the correction with the wrong values results in systematic errors most noticeable at the two ends of the spectrum. If the error had been overestimated instead of underestimated, the curvature of the final data would be reversed.

Figure 8 shows the result of a similar erroneous evaluation of the fraction of interactions due to antineutrinos; the correct fraction was taken to be 0.15, and data for 0.15 and 0.22 are both corrected on this assumption. The value 0.15 chosen is not far from the actual one; it assumes a $\nu/\bar{\nu}$ ratio of 2, and a cross-section ratio of 3. No possible error in this case would yield a flat distribution.

The adequacy of the data obtained from a 1-km^3 DUMAND array to carry out the proposed experiment was discussed at some length in the first DUMAND proposal. We give in Table 1 a somewhat modified summary of the data expected from such an array in one year's operation, using only the simple ratio test described above, and not the more accurate fitting procedure which will be appropriate in actual operation. We have also assumed, pessimistically, that the antineutrino component will not be known to better than 50%.

The region above $y \approx 0.8$ will probably not be accessible to measurement, since it will be subject to errors from the presence of neutral current interactions, which all look like $y = 1$ events; and from a cut-off at low muon energies which prevents accurate energy determination, or even observation.

Table 1. Expected Precision of Y Determination. Statistics and errors of measurement of the y distributions expected in one year of operation of DUMAND G (1 km^3). The event numbers quoted are from the integral spectrum (i. e., all events of energy E_ν and above). The difference between measured and total events includes estimated corrections for fiducial volume, unmeasurable events from various sources, etc.

E_ν TeV	No. of Events Per Year Total	Meas.	Expected R = $y>.5/y<.5$	Systematic Error if $\bar{\nu}$ Admixture = $0.15 \pm .075$	Total Error	No. of Std. Deviations from R = 1
4	3600	2000	0.720 ± .03	±0.04	±0.05	6
10	700	400	0.630 ± .06	±0.04	±0.07	6

From this table the remarkable result emerges that except at the very highest energies, the results to be expected from the DUMAND array on the y distribution will not be limited in accuracy by a paucity of events. Rather, the limitation is likely to be systematic error, due to uncertainties in the antineutrino component and in the experimental errors in energy determination. As we learn more about the equipment, these can be expected to decrease with time.

VI. Summary

The Monte Carlo procedures we have outlined give us the means for deconvolving the measured distributions to find the true ones, provided we know what the "measurement" errors are. By "measurement" we mean both the inherent fluctuations in the energy loss as well as the actual measurement errors. At present we do not yet know how accurately we will be able to estimate these "measurement" errors. The errors due to fluctuations are rather well understood, and we should be able to calculate them. The errors in the measurement procedure depend on the results on the second part of the Monte Carlo program, which has not as yet been carried out; we cannot yet say, for a given sensor array, what its precision of measurement will be. We think it unlikely that the errors - the standard deviation of the energy determination of the cascade and the muon, - will be larger than 50 percent, and there is a good chance that the cascade measurement, at least, may be considerably better. We note, in addition, that even with a generously large error in the determination of both these quantities, the basic trend of the y distribution remains. In fact, it has not been possible to find a scenario in which the boson propagator, if present in the expected region, will remain undiscovered by the data.

REFERENCES

[1] A. Roberts, "Monte Carlo Study of the Measurement of the Energy of Muons and Cascades in a DUMAND array," Proc. 1978 DUMAND Summer Workshop, SIO, La Jolla (in press), Vol. 1

[2] T. Gaisser and A. Halprin, Proc. 15th Intl. Cosmic Ray Conf., MN, 265 (1977) Plovidiv, Bulgaria

[3] A. Halprin, "The Nature of y Distributions in Neutrino Reactions at DUMAND Energies," DUMAND 1978, Vol. 2 (in press)

[4] A. Halprin and R. J. Oakes, ibid.

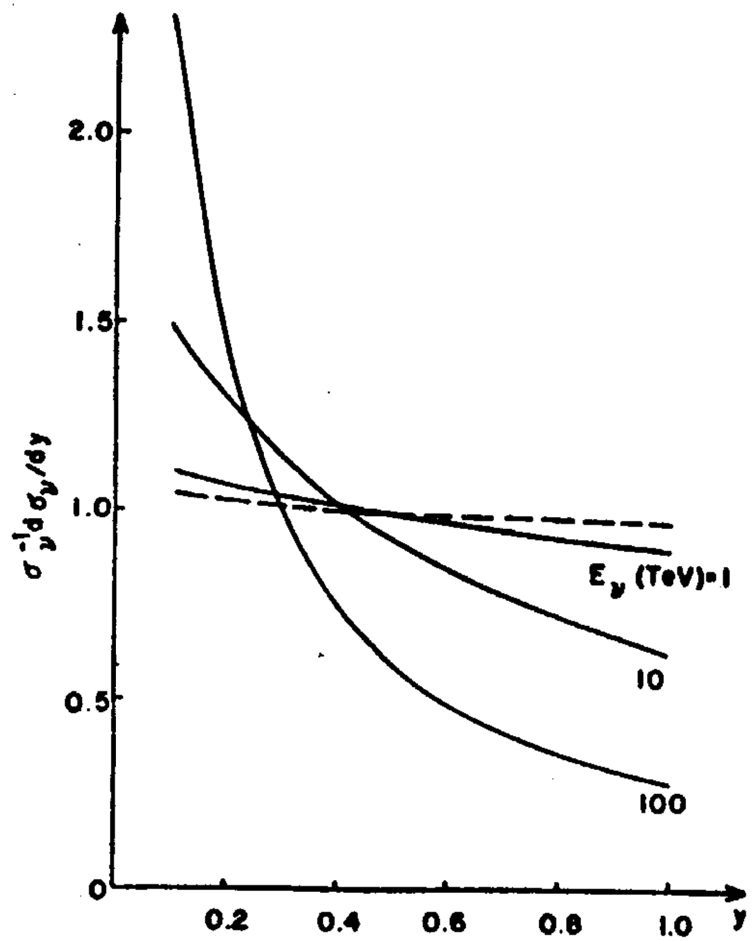

Fig. 1. The y-distribution, as calculated by Gaisser and Halprin (ref.2) for a W mass of 80 Gev/c^2, at neutrino energies of 1, 10, and 100 TeV. The dotted line shows the effect of assuming an infinite boson mass (point interaction.)

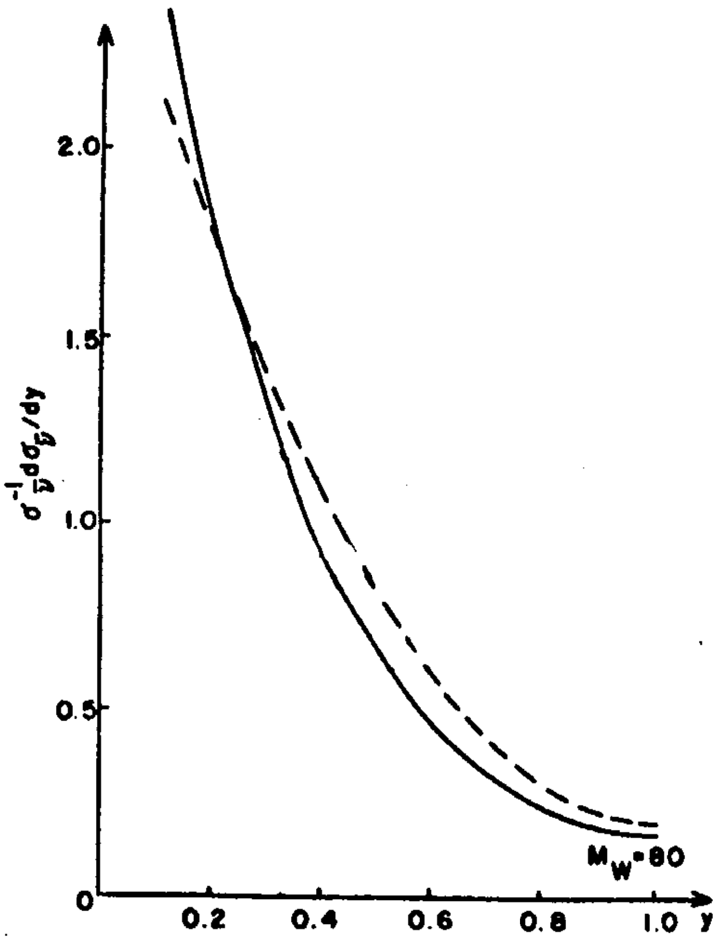

Fig. 2. The y-distribution for antineutrinos, calculated the same way, showing the strong effect of the $(1-y)^2$term. The antineutrino energy is 10 TeV.

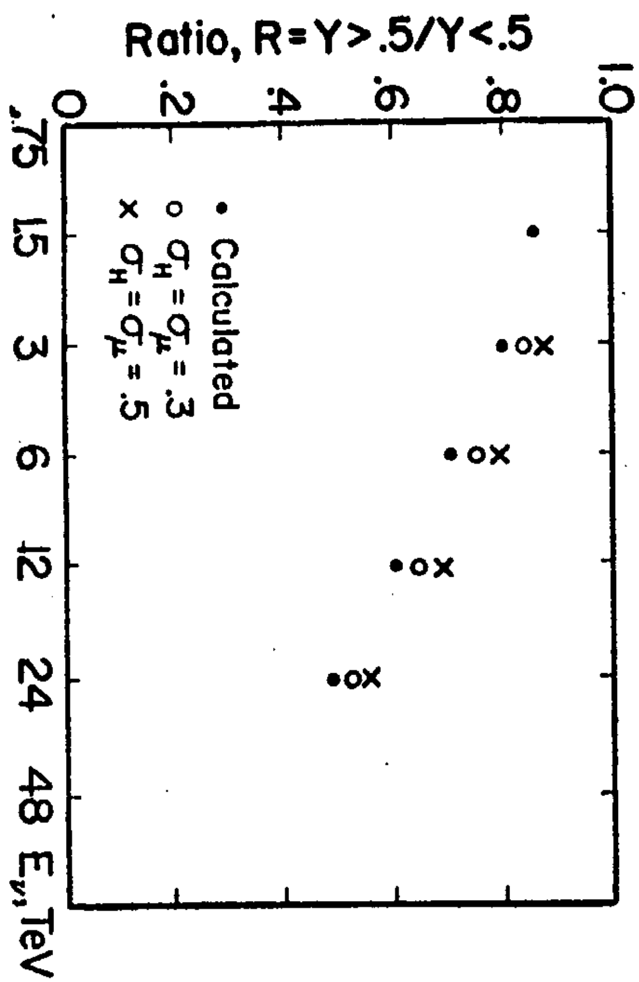

Fig. 3. Comparison of y-distributions: the true (calculated) value for the ratio R of the number of counts with y > 0.5 to the number with y < 0.5, a measure of the shape of the distribution. Also shown: the values obtained with errors of 30% and 50% in both hadron and muon energy.

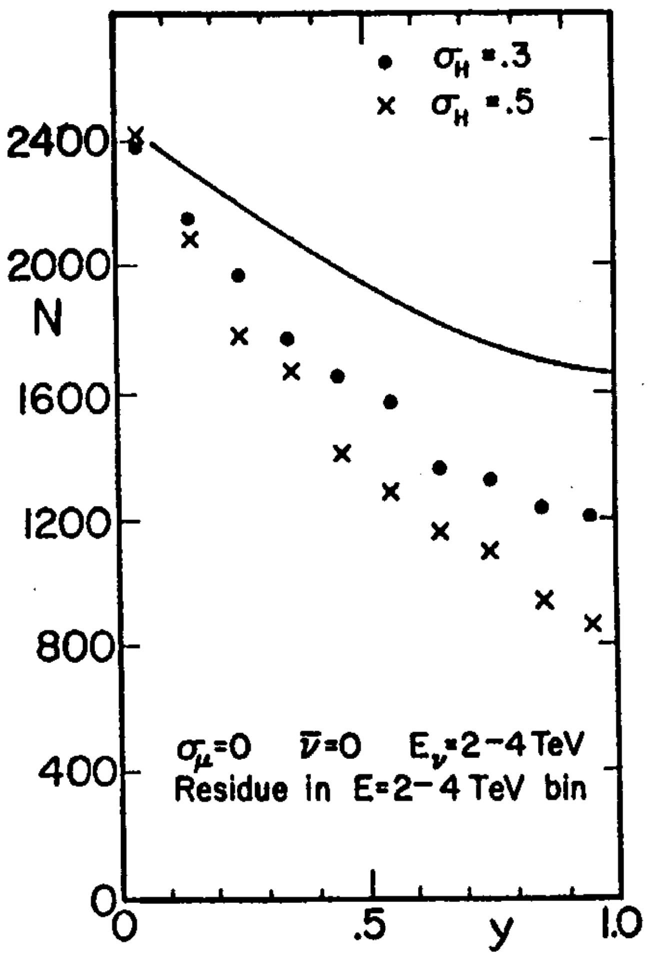

Fig. 4a. The y-distribution in the energy range 2-4 TeV: the full line is the true value. The residue after events are removed from the bin by random errors in hadronic energy with $\sigma = .3E$ and $.5E$ is plotted. Note loss of events with large y.

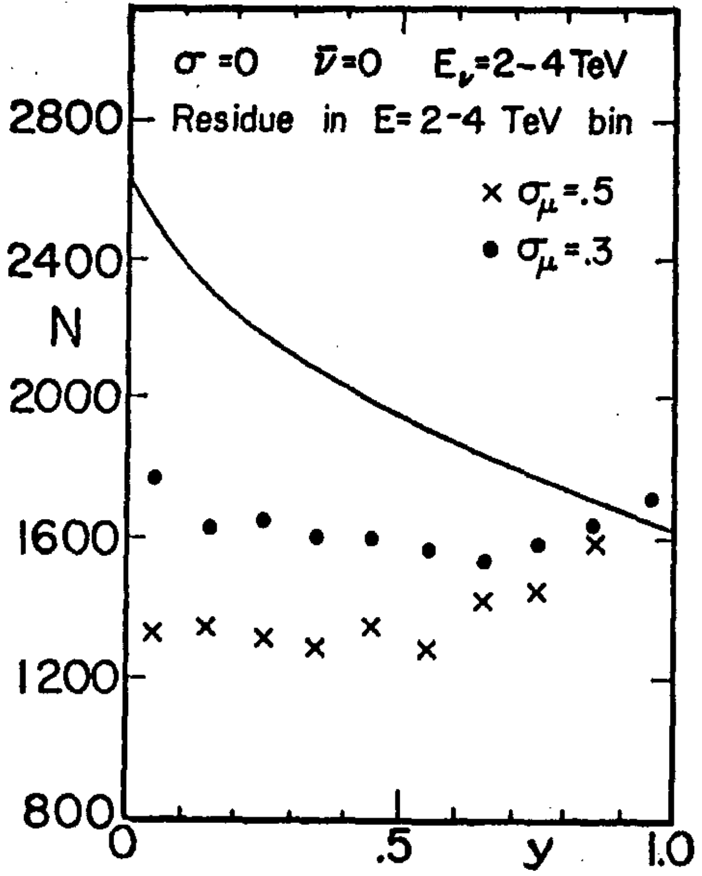

Fig. 4b. Same as Fig 4a, for similar errors in the muon energy. Now events with small y are missing.

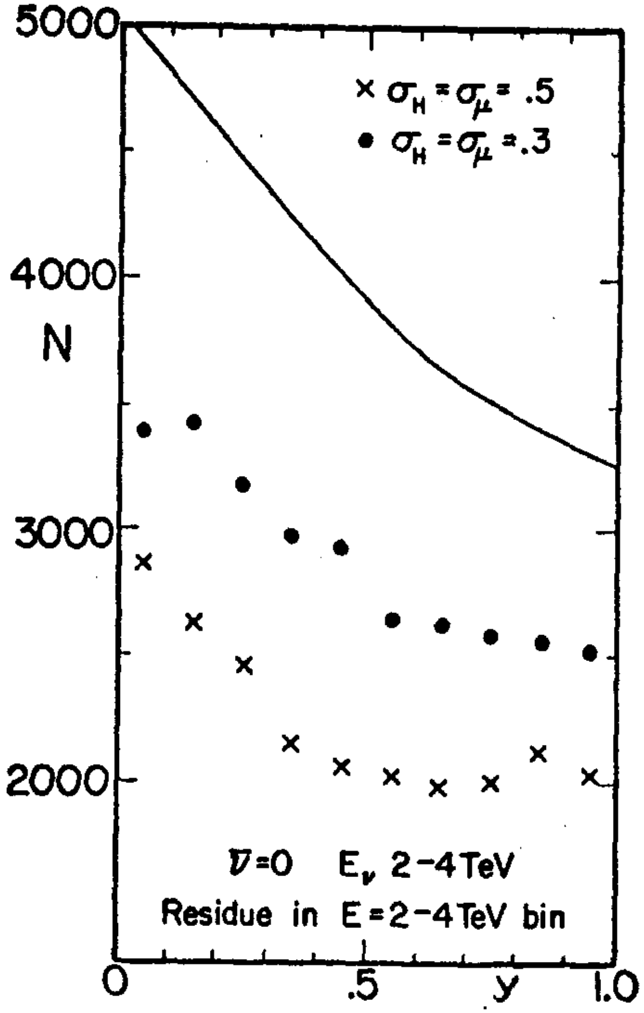

Fig. 4c. Errors in both hadron and muon energy yield a fairly uniform depletion throughout the distribution.

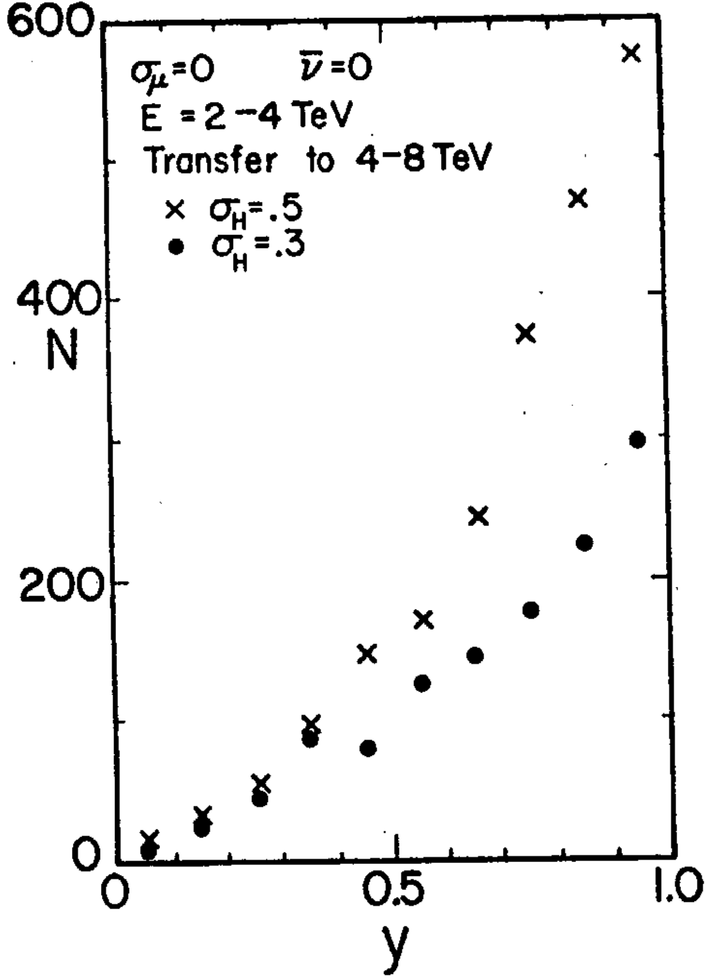

Fig. 5a. The y-distribution of those events missing from 4a that are transferred to the next higher energy bin, 4-8 TeV, by the experimental error. The very strong bias toward large y will greatly distort the observed y-distribution if not corrected.

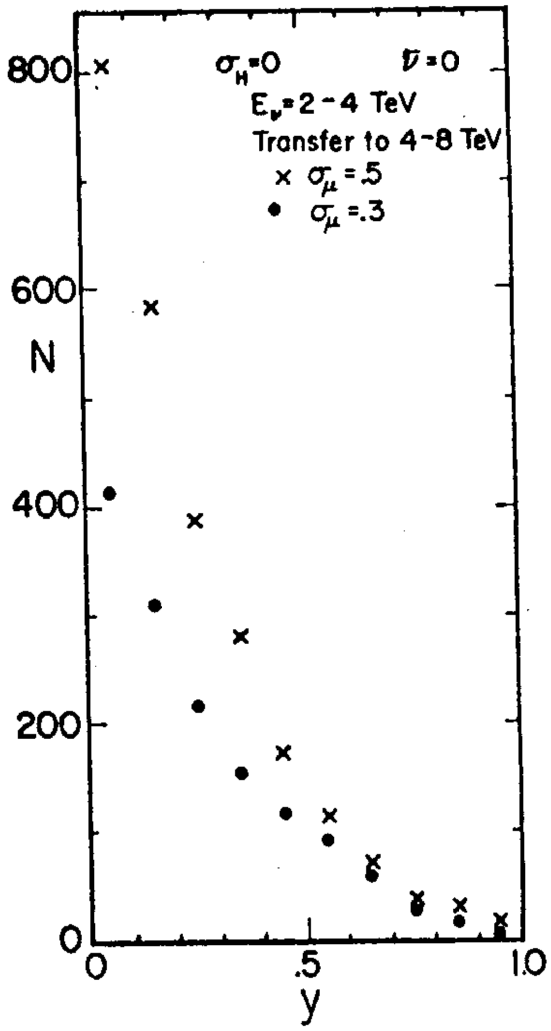

Fig. 5b. The same as 5a, for errors in muon energy; the distortion accents small y values.

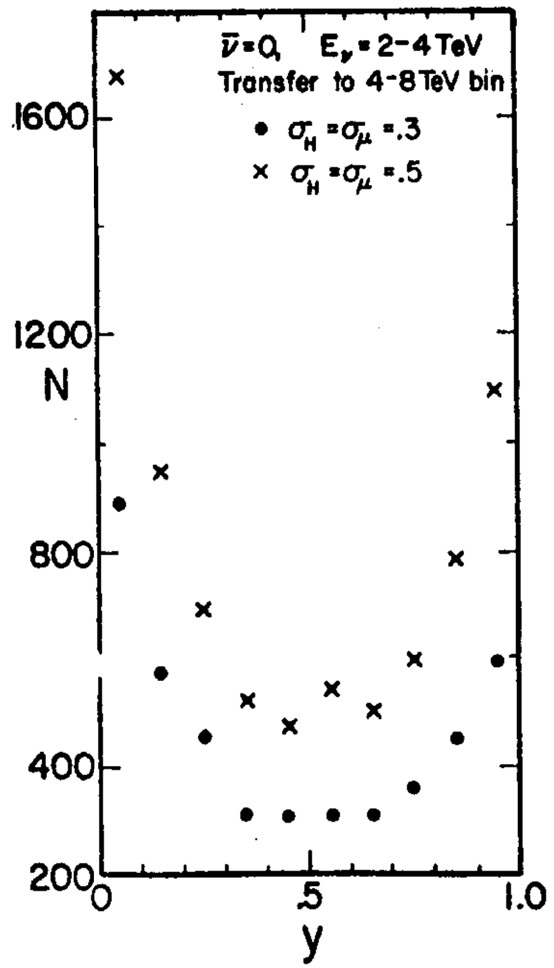

Fig. 5c. The same, with errors in both hadron and muon providing a distribution peaked at both ends.

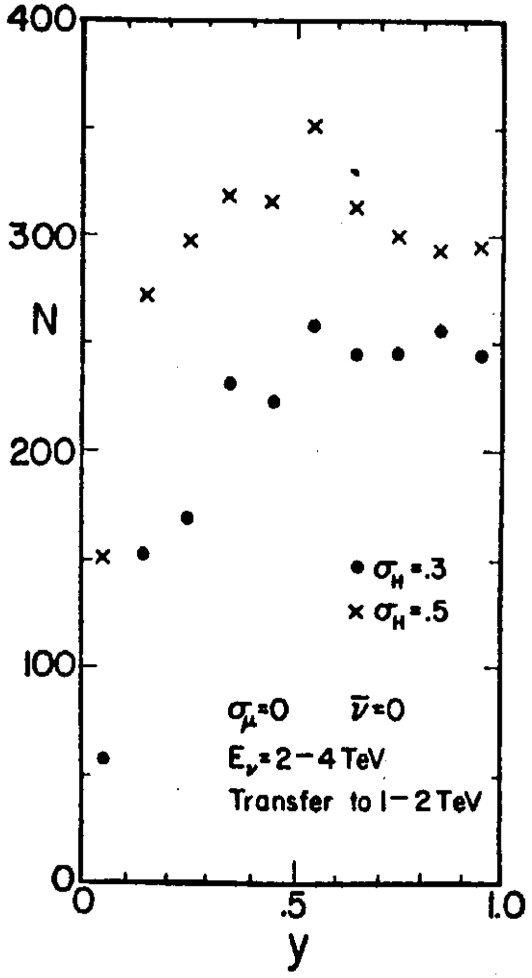

Fig. 6a. The y-distribution of events transferred by experimental error in hadronic energy to the 1-2 TeV bin. Again large y-values are emphasized.

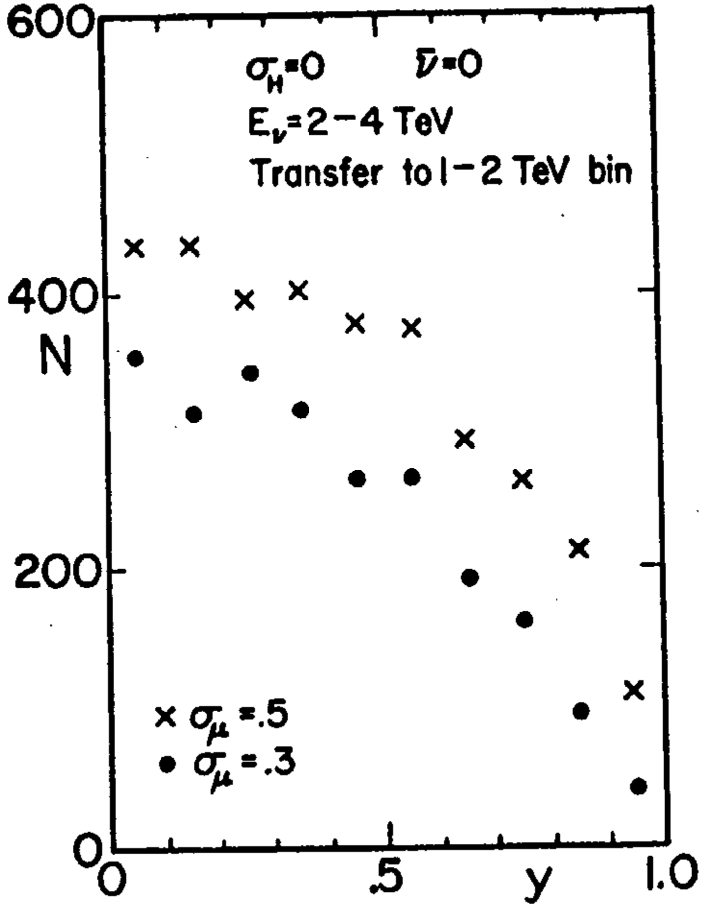

Fig. 6b. The same, for similar muon errors; now small y-values are emphasized.

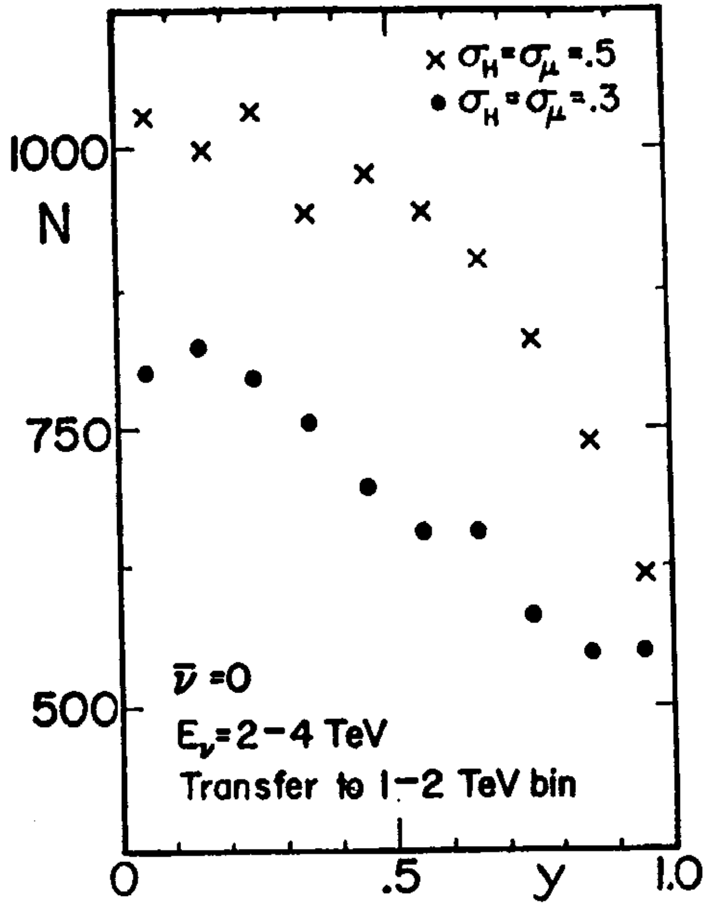

Fig. 6c. Errors in both hadron and muon give this distribution for events transferred to 1-2 TeV.

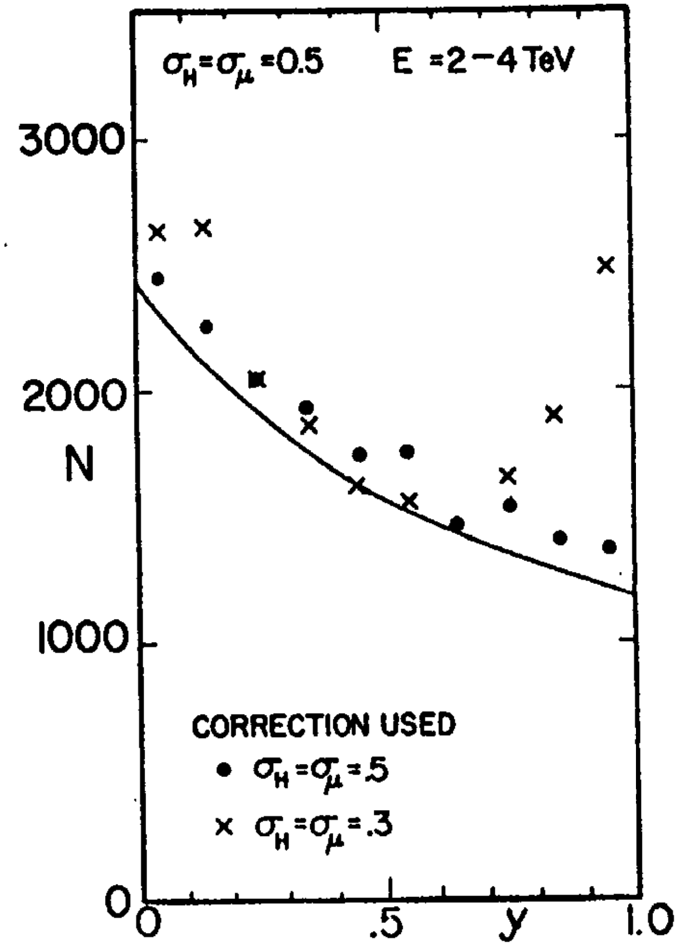

Fig. 7. The effect of correcting the errors described in Figs. 4-6. The full line is the true distribution; circles show data for errors of 50% in both hadron and muon energy, but corrected accordingly. Crosses show the effect of incorrectly estimating measurement error: the error is corrected as if it were .3 instead of .5

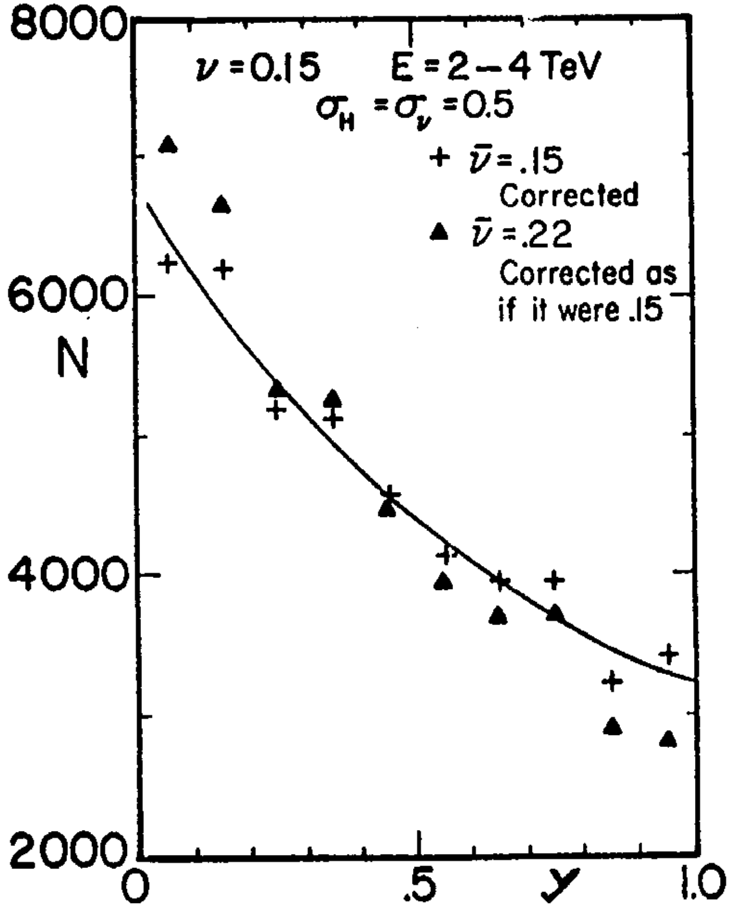

Fig. 8. This figure is similar to Fig. 7, but shows a distribution containing 15% antineutrinos. Crosses show values rected using true antineutrino fraction, triangles the effect of incorrectly estimating fraction as 0.15 when it was in fact 0.22 (a 50% error).