Monte Carlo Simulation of Sudden Death Bearing Testing

Brian L. Vlcek
Georgia Southern University, Statesboro, Georgia

Robert C. Hendricks and Erwin V. Zaretsky
Glenn Research Center, Cleveland, Ohio

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Brian L. Vlcek
Georgia Southern University, Statesboro, Georgia

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National Aeronautics and
Space Administration

Glenn Research Center

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Brian L. Vlcek
Georgia Southern University
Statesboro, Georgia 30460−8045

Robert C. Hendricks and Erwin V. Zaretsky
National Aeronautics and Space Administration
Glenn Research Center
Cleveland, Ohio 44135

ABSTRACT

Monte Carlo simulations combined with sudden death testing were used to compare resultant bearing lives to the calculated bearing life and the cumulative test time and calendar time relative to sequential and censored sequential testing. A total of 30 960 virtual 50-mm bore deep-groove ball bearings were evaluated in 33 different sudden death test configurations comprising 36, 72, and 144 bearings each. Variations in both life and Weibull slope were a function of the number of bearings failed independent of the test method used and not the total number of bearings tested. Variation in L_{10} life as a function of number of bearings failed were similar to variations in life obtained from sequentially failed real bearings and from Monte Carlo (virtual) testing of entire populations. Reductions up to 40 percent in bearing test time and calendar time can be achieved by testing to failure or the L_{50} life and terminating all testing when the last of the predetermined bearing failures has occurred. Sudden death testing is not a more efficient method to reduce bearing test time or calendar time when compared to censored sequential testing.

NOMENCLATURE

CD	dynamic load capacity, N (lbf)
C	censored or suspended test
c	stress-life exponent
e	Weibull slope
h	exponent
j	mean order number
L	life, number of stress cycles or hr
L_{10}	10-percent life or life at which 90 percent of a population survives, number of stress cycles or hr
L_{β}	characteristic life or life at which 63.2 percent of population fails, number of stress cycles or hr
m	total number of bearing testers used simultaneously
N	life, number of stress cycles
n	number of bearings or number ($m \times r$) of subgroups with a single failed bearing
Peq	equivalent radial load, N (lbf)
p	load-life exponent
r	number of failures or number of subgroups of m bearings
S	probability of survival, fraction or percent
V	stressed volume, m^3, (in.^{3})
Z_o	depth to the orthogonal shearing stress
ι_o	orthogonal shearing stress, GPa (ksi)

Subscripts

i	i[th] component or bearing
ir	inner race
L	lower limit
n	number of components
or	outer race
re	rolling elements
sys	system
up	upper limit

Abbreviations

calc	calculated
max	maximum
min	minimum

Definitions

calculated life	=	The life obtained using the Lundberg-Palmgren life equations
resultant life	=	The life obtained from the Weibull analysis of bearing systems generated by a Monte Carlo technique

INTRODUCTION

Predicting and experimentally validating rolling-element bearing life is a complex, time- consuming, and costly task. Many investigators believe that there is no alternative to testing an entire fixed population of components to failure when accurately determining the life of their product. Sudden death testing, however, is considered a technique by which product development test time is shortened by running concurrent tests and not necessarily running all of the tests to failure. The question becomes whether the reliability (accuracy and precision) of the resultant life is influenced by the chosen testing method.

One needs to go back to the classic work of L.G. Johnson (1) to find early explanations of methods for reducing testing time. Johnson (1) identifies three methods for reducing test time. In the first method, more specimens are run simultaneously than are intended to fail. For example, Johnson points out that for a Weibull slope of 1.0, the median time to fail 10 out of 20 samples is significantly less than the median time required to fail 10 out of 10 samples. In fact, the first scenario will take 24 percent of the time it takes for the second. This is assuming that there are no replacements of failed specimens, all testers are the same, and all specimens are run simultaneously. Since the width of the confidence band is determined by the number of items failed and not the total number of specimens in a test (1), (2) the number failed is 10 in both cases. As a result, the only difference is that the 10 items having the lowest lives in a lot of 20 is plotted on a Weibull plot instead of all 10 specimens in a lot of 10 (1). Similar observations were made by Vlcek, Hendricks, and Zaretsky in (2).

Johnson (1) describes sequential analysis as a second method of deciding if an improvement in life has been realized at any time during the test series based upon the performance of the latest element in a sequential group of failures. The total number of specimens needed to fail is not known in advance. The investigator decides after each failure whether or not additional testing is required. In this manner, the bare minimum number of runs needed to demonstrate an improvement or worsening of life is conducted. A method similar to this has been used extensively by Zaretsky et al. (3). An estimate of the 50-percent life (L_{50}) is made and test bearings are then run on identical testers until this life is reached. As samples fail or are suspended at the L_{50} life, they are removed, and new samples are mounted and evaluated to the estimated L_{50}. In this manner, at least 50 percent of the samples are typically failed out of the entire available population. In some cases, the L_{50} target may have to be adjusted as dictated by the number of failures encountered.

Sudden death testing is the third method of reducing testing time described by Johnson (1). The total accumulated test time is reduced by not running all specimens to failure. The total number of specimens to be evaluated *n* is divided into equal-sized subgroups according to the number of available experimental testers. Thus, there are *m* specimens in each equal-sized subgroup, and there are a total of *r* subgroups (Table 1(a)). The total number of specimens evaluated *n* equals *m* times *r*.

The specimens in each subgroup are fatigue tested identically and simultaneously on different testers. The first subgroup of specimens is run until the first failure occurs. At this point, the surviving specimens are suspended and

removed from testing. An equal set of new specimens numbering m samples is next tested until the first failure in that subpopulation. This process is repeated until one failure is generated for each of the subgroups. In the end, r failures are generated while $(m-1) \times r$ samples are suspended (Table 1(b)). Thus, the total accumulative test time is the time to fail r specimen times the number of samples concurrently tested m, not the time for n failures. With correct analysis, reliability of the life predicted from r failures is assumed to be comparable to that obtained when failing the entire population. Some sudden death testing results are reported in (4)-(8). The issue remains as to whether life predictions from sudden death testing are equivalent to those obtained with sequentially failed data.

It is time-consuming and cost prohibitive to rigorously examine the advantages of sudden death testing in the physical world. However, sudden death testing can be investigated by the generation of virtual bearing sets for evaluation to an extent that might not be possible with physical testing. Vlcek, Hendricks, and Zaretsky (2) have shown that computer modeling of bearing life based upon Monte Carlo assignment of bearing component median ranks, and probabilistic characteristics based upon Lundberg and Palmgren (9) results in reasonable engineering predictions of bearing life that are relatively easy to determine. These predictions compared favorably to the rolling-element bearing data contained in (10), (11).

The generation of virtual bearing lives demonstrated by Vlcek, Hendricks, and Zaretsky (2) can be used to generate bearing sets, which will then be distributed into combinations of sudden death testing matrices. In view of the aforementioned, it is the objectives of the work reported herein to compare, 1. the resultant bearing lives obtained using sudden death testing to the calculated bearing life, and 2. the cumulative test time and calendar time for sudden death testing relative to that for sequential testing methods.

Table 1.—An n specimen matrix of r subgroups and m samples evaluated (testers in use).
(a) Test matrix. (b) Failure matrix. (c) Test matrix example. (d) Failure matrix example.

PROCEDURE

Bearing Life Analysis

Bearing life testing is expensive and time-consuming. As a result, it is desirable to predict by calculation the life of a bearing with reasonable engineering certainty. In probabilistic life models, the bearing physical characteristics, applied load, operating profile, and environment determine the probability of failure, assuming that the life is represented by a known probability function. W. Weibull (12)–(14) was the first to suggest a reasonable way to estimate material fracture strength with such a probability function. Based upon the work of Weibull (12)–(14) and G. Lundberg and A. Palmgren (9) in 1947 showed that the probability of survival S could be expressed as a power function of the orthogonal shear stress τ_o, life N, depth to the maximum orthogonal shear stress Z_o, and stressed volume V. That is,

$$\ln \frac{1}{S} \sim \frac{\tau_o^c N^e}{Z_o^h} V \tag{1}$$

From Eq. [1], Lundberg and Palmgren (9) derived the following relation:

$$L_{10} = \left[C_D / P_{eq} \right]^p \tag{2}$$

where C_D, the basic dynamic load capacity, is defined as the load that a bearing can carry for one million inner-race revolutions with a 90-percent probability of survival, P_{eq} is the equivalent bearing load, and p is the load life exponent.

Lundberg and Palmgren (9) first derived the relationship between individual component life and system life. A bearing is a system of multiple components each with a different life. As a result, the life of the system is different from the life of an individual component in the system. The system life can be expressed, to a first order, as

$$\frac{1}{L_{10sys}{}^e} = \frac{1}{L_{10ir}{}^e} + \frac{1}{L_{10or}{}^e} \tag{3}$$

where the life of the rolling element by inference is incorporated into the life of each raceway. In properly designed and operated rolling-elements bearings, fatigue of the cage or separator should not occur and, therefore, is not considered in determining bearing life and reliability.

The work of E.V. Zaretsky (15) builds upon the work of Lundberg and Palmgren (9) where using Zaretsky's rule (15)

For radially loaded ball and roller bearings, the life of the rolling element set is equal to or greater than the life of the outer race. Let the life of the rolling element set be equal to that of the outer race.

then with this assumption, Eq. [3] becomes

$$[1/L_{10}]e = [1/L_{ir}]e + 2[1/L_{or}]e \tag{4}$$

where $L_{re} = L_{or}$

The life of a virtual bearing was calculated according to Lundberg and Palmgren (9) with a lubricant life factor from (15). The lives of the inner and outer races were recalculated together with the lives of the balls using Eq. [4].

Bearing Type, Operating Conditions, and Calculated Lives

The virtual bearings in this study were generated using physical bearing parameters. The bearings were a 6010-size (50-mm bore) deep-groove ball bearing. The bearing dimensions and operating conditions are summarized in Table 2. Operating conditions were assumed to be 10 000 rpm using an MIL–L–23699 (tetraester-based) lubricant at 135 °C (275 °F). The load applied to bearing was calculated to result in a maximum Hertz stress of 1.55 GPa (225 ksi) on the inner race of the bearing.

Table 2.—Bearing specifications, operating conditions, and calculated lives used in assembly and Monte Carlo sudden death testing.

Bearing type		Deep-groove ball bearing		
Bore size, mm		50		
Curvatures, percent	Inner race	52		
	Outer race	52		
Ball diameter, mm (in.)		87.3 (11/32)		
Number of balls		14		
Contact angle, deg		0		
Load, N (lbs)		950 (215) radial		
Maximum Hertz stress, GPa (ksi)		1.55 (255)		
Lubricant type		MIL–L–23699		
Surface finish, rms μm (μ in.)	Inner race	7.62×10^{-2} (3)		
	Outer race	7.62×10^{-2} (3)		
	Balls	2.54×10^{-2} (1)		
Operating temperature, °C, (°F)		135 (275)		
Lubricant life factors	Inner race	0.75		
	Outer race	1.05		
Life, hrs	Component	L_{10}	L_{50}	L_β
	Inner race[a]	9547	52123	72500
	Outer race[a]	38188	208448	290000
	Ball[a]	38118	208448	290000
	Bearing[b]	6912	37729	47227
Weibull slope, e		1.11		

[a]Life based on Zaretsky's rule and lubricant life factor *(15)*.
[b]Life based on Lundberg-Palmgren equation *(9)* and lubricant life factor *(15)*.

The lives of the bearing components and the bearing were calculated using a commercial bearing code, COBRA-EHL, incorporating the Lundberg-Palmgren (9) equations (Eqs. [1] and [2] and Zaretsky's rule, Eq. [4]). For purposes of the life analysis, all life factors with the exception of the lubricant life factor were assumed to be equal to one since interest was primarily in the qualitative results. A lubricant life factor, however, was used as a function of lubricant film parameter from (15) for these operating conditions since its effect on the resulting lives of the inner and outer races can be different. Following Lundberg and Palmgren (9), the Weibull slope e was assumed to equal 1.11 for each of the bearing components in this study. This results in a near-exponential distribution of the bearing failure data. The resultant bearing L_{10} life was 6912 hr.

Lundberg and Palmgren (9) incorporated into their analysis a method and distribution function for statistically describing the fatigue life of materials developed by Weibull (12) referred to as the two-parameter Weibull distribution function.

$$\ln \ln \frac{1}{S} = e \ln\left(\frac{L}{L_\beta}\right); 0 < L < \infty;\ 0 < S < 1 \qquad [5]$$

The characteristic lives (L_β) calculated using Eq. [5] for the inner race, the balls, and the outer race were 72 490; 289 000; and 289 000 hr, respectively.

Virtual bearing testing

Whether the data is generated experimentally or analytically, the ultimate goal is to determine the life and characteristics of a larger population from a limited amount of data. Since exhaustive physical testing is economically prohibitive, and in some cases not physically possible, it is extremely desirable to mathematically model the life of a bearing. Probabilistic approaches are based upon the fact that material strength varies from sample to sample as a result of material inhomogeneities and manufacturing variability. The applied loads to a system are also variable as a result of operation loads, mission cycles, and environmental factors. As a result, component life is modeled as a statistical variable. If the applied load is known in terms of duty cycle together with the relevant bearing geometry and operating conditions, the probability of failure is determined assuming that the bearing life is represented by a known probability function.

The life of virtual bearings consisting of multiple components was determined using the method demonstrated in Ref. (2). It was assumed that each bearing was assembled from three separate bins of components, with one bin containing 1000 inner rings, one 1000 rolling element sets, and one 1000 outer rings. The median ranks of the individual components were assigned and then virtual bearing assemblies were created using a Monte Carlo technique. The corresponding life of the bearing components were determined using Eq. [5]. The weakest link theory was applied, that is it was assumed that the life of the shortest-lived component of the system was the life of the system. A linear curve fit of these system lives results in a Weibull plot. Weibull parameters from the plot and Eq. [5] can be used to determine lives at any percentage of survivability.

Vlcek, Hendricks, and Zaretsky (2) determined the L_{10} maximum limit and L_{10} minimum limit for the number of bearings failed using a Weibull-based Monte Carlo method. By fitting the resultant lives for different sized populations of failed bearings, equations were determined for both of these limits:

$$Max\ variation\ \boldsymbol{L_{10}}\ life = calc\ \boldsymbol{L_{10}}\ life\ (1 + 6n^{-0.6}) \qquad [6a]$$

$$Min\ variation\ \boldsymbol{L_{10}}\ life = calc\ \boldsymbol{L_{10}}\ life\ (1 - 1.5n^{-0.33})\quad where\ \boldsymbol{r} > 3 \qquad [6b]$$

$$Min\ \boldsymbol{L_{10}}\ life = 0\quad where\ \boldsymbol{r} \leq 3 \qquad [6c]$$

These curves compared favorably with the 90-percent confidence limits of Johnson (1) at a Weibull slope of 1.5 (2). Additionally, field bearing life data of Harris (10), (11) was compared to these limit curves, and most of the bearing lives fell within these bounds (2). The upper and lower limits are thus assumed to be reasonable bounds and are a function of only the number of bearings failed and the known characteristic life of the bearing. Equations [6a] to [6c] were determined from Monte Carlo simulation of testing all bearings to failure.

Data Analysis

There are multiple statistical methods for determining bearing life estimates from endurance data that for practical engineering purposes, give similar results. These methods differ significantly; however, in their level of complexity to apply and limitations to their application. For example, maximum likelihood estimates (MLE) of Weibull slope, characteristic life L_β (life at which 63.2 percent of the specimens have failed), and L_{10} (life at which 10 percent of the specimens have failed) can be found from an iterative process (16). Confidence limits can be placed upon these values using published tables that are appropriate for a limited number of testing configurations (6), (7). In general, extensive Monte Carlo simulations are required to generate additional entries for the tables found in the open literature. The relative likelihood method can also be employed to estimate statistical intervals (16). Linear regression least-square fits of mean order numbers assigned to experimental lives also result in estimations of Weibull slope, e, characteristic life, L_β, and L_{10} life when plotted on Weibull paper. Confidence limits can be placed upon these values using the method of Johnson (1). For its relative ease of use and engineering application with comparable reliability, we have selected the linear regression least-square-fit method over that of maximum likelihood estimations for use in this study.

Sudden Death Testing Techniques

In sudden death testing, the total number of specimens to be evaluated is divided into equal subgroups that can be evaluated simultaneously. The first subgroup of specimens is run simultaneously until the first failure occurs. The
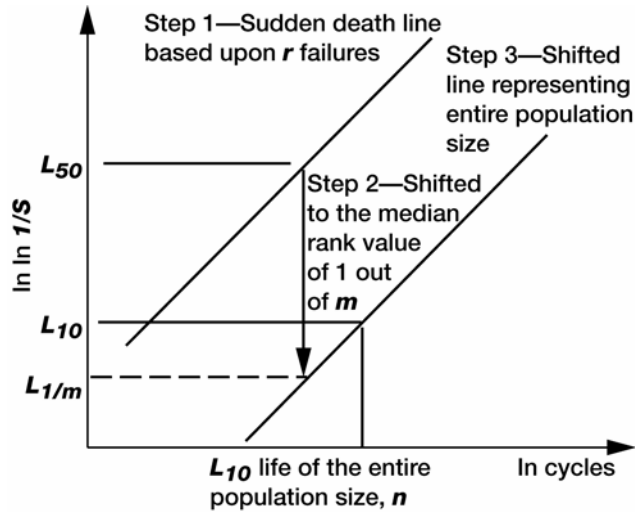
Figure 1.—Schematic of generic Weibull plot of the sudden death line shifted to total population line (Ref. (*1*)).

surviving tests for that subgroup are terminated (i.e., suspended), and new test specimens are mounted in the testers. This process is repeated until all of the subgroups of test specimens in the population have been screened.

In this study, the technique developed by Vlcek, Hendricks, and Zaretsky (2) was used to generate virtual bearing sets that were then analyzed as if they were sudden death tested. Total populations with 36, 72, and 144 deep-groove bearings were generated. The populations were then sequentially broken into subgroups representing all possible combinations of sudden death test series of ($m \times r$), where m was the number of bearing testers used simultaneously and r was the number of sets of m bearings necessary to achieve the total number of bearings n in the total population. The total number of bearings n equals m times r. For example, if 36 bearings were to be evaluated, and there were 4 bearing fatigue testers available, the value of m would be 4 while the value of r would be 9; whereby n equals 36[($m = 4$) $\times$ ($r = 9$)] (Table 1(c)). For this example, the first four bearings were run simultaneously until the first failure occurred. The three surviving tests were suspended, and four new bearings were mounted. This process was repeated until 9 failures occurred, 1 from each of the sets of 4, with ($m - 1$) $\times r$ or 27 suspensions (Table 1(d)).

These nine failures and their corresponding median ranks, one for each subgroup of size m, were then plotted on a Weibull plot. This sudden death line (SDL) represents the distribution of first failures in each subgroup. The SDL of each series was next shifted on its respective Weibull plot so that the curve represented the failures of the total population, not just that of one out of r bearings.

Various methods exist for shifting the SDL line and finding the life of a total population n based upon sudden death testing data where only r failures are considered. The slope and characteristic life of the subpopulation generated during sudden death testing can be found from maximum likelihood estimators of Cohen (17) that are obtained from an iterative process. This life for r samples must be corrected to represent the life of the original population containing n samples. One way to achieve this, as reported by McCool (6), (7) is to multiply the subpopulation life estimator by the number of samples (m) in each equally sized subpopulation raised to the inverse of the slope estimator. Confidence limits can be placed upon these values for a limited number of cases provided in tables in the open literature or by extensive Monte Carlo simulations (6), (7). Houpert (4) also proposes a technique for determining the life of a larger population based upon a subpopulation determined from sudden death testing, but a comparison to our work is beyond the scope of this paper.

For its simplicity of application and relative engineering reliability, we prefer a technique presented by Johnson (1) for shifting the SDL so that the life and characteristics of the larger bearing population (n) can be projected. Figure 1 is a generic Weibull plot that accompanies the following steps and includes many of the elements mentioned:

Step 1—*Plotting the subgroup of r failures on Weibull paper.* To plot the failures, median ranks are assigned to the sequentially ordered lives. Median ranks were defined using

$$\text{Median rank} = (j - 0.3)/(r + 0.4) \qquad \text{[7a]}$$

where j = 1, 2, 3, … r. For sudden death testing, the number of subsets r equals the number tested to failure. A discussion and comparison of median rank definitions are available in (4) and (18). The median ranks along with their corresponding lives are next plotted on Weibull paper. The locus of points is fitted with a linear curve. From the Weibull plot, the Weibull slope, L_{10} life, L_{50} life, and characteristic life, L_β, are determined. This SDL represents the distribution of first failures at the median rank for one failure out of the number of samples (m) in each of the subgroups.

Step 2—*Determining by how much the* SDL *must be shifted to accurately estimate the* L_{10} *life for the entire population*. The median rank (Eq. [7a]) must be determined for the first failure out of the number of bearings simultaneously evaluated (*m*). In the above example, for a [(*m* = 4) × (*r* = 9)] test, the four simultaneous testers are stopped after the first test failure occurs, thus the mean order number *j* equals 1 and the subgroup size *m* equals 4 where it is assumed that in Eq. [7a], *m* equals *r*. The median rank for one out of four, found using Eq. [7b], is 0.1591. This is the value to which the SDL must be shifted (Figure 1). In general

$$\text{First failure median rank (FFMR)} = (1 - 0.3)/(m + 0.4) \qquad [7b]$$

Step 3—*Constructing the total population line (TPL) by shifting the SDL*. At the L_{50} intersection of the SDL, a vertical line is drawn down to the median rank value determined in step 2. Through this point, a line is drawn parallel to the SDL created in step 1. The slopes of both lines are assumed to be equal. Figure 1 is a generic Weibull plot of the SDL and the shifted TPL representative of this technique.

Step 4—*Determining lives from the shifted TPL*. The Weibull slope and lives are read directly from the TPL generated in step 3. Table 3 contains the Weibull slope, L_{10} life, L_{50} life, and characteristic life, L_{β}, for a typical [(*m* = 4) × (*r* = 9)] sudden death test of a virtual deep-groove bearing. The values obtained from the SDL and the shifted TPL are provided.

Table 3.—Comparison between lives obtained from Monte Carlo sudden death testing and that adjusted for total population. Number of failures, 9; number of bearings tested, 36; number of test rigs, 4; assumed Weibull slope, 1.11; resultant Weibull slope, 1.033; bearing size and type, 50-mm bore deep-groove ball bearing.

Bearing population size	Bearing life, hrs			Weibull slope, *e*
	L_{10}	L_{50}	L_{β}	
Sudden death for 9 failures	2354	14569	20772	1.033
Adjusted for total population from sudden death	9003	55729	79453	1.033
Calculated[a] (actual) for total population	6912	37729	47729	1.11

[a]Life based on Zaretsky's rule and lubricant life factor *(15)*.

This process was repeated for each of the 33 combinations reported in Table 4. For each sudden death test combination, 10 different sets of lives were generated and their respective SDL determined. The SDLs were shifted appropriately and the L_{10} lives determined from the TPL. The variation in the maximum and minimum L_{10} life with respect to the calculated L_{10} life and the maximum and minimum Weibull slopes for each of the 10 trial sets are reported in Table 4.

RESULTS AND DISCUSSIONS

Experimental testing will never be eliminated. Where appropriate and economical, physical testing is the method of choice. Sudden death testing techniques have been purported to have the potential to reduce test time and associated costs. An outstanding issue regarding sudden death testing is whether testing a large population but failing only a subset of specimens tested has advantages as compared to testing the entire bearing population to failure. It is not possible to examine this issue by physical testing. Because physical testing is limited and/or cost prohibitive, a Monte Carlo simulation based upon a known Weibull distribution has been demonstrated as a means of obtaining reasonable design limits (2). Merging these techniques has the potential to generate insightful data for test runs that might be impractical or impossible to otherwise run.

Some of the test combinations and total number of virtual bearings evaluated in this study are impractical to evaluate from a physical sense. For example, one would probably not run a total of 144 bearings using only 2 testers. The results of such extremes, however, are included for completeness. The table entries (Table 4) corresponding to those testing combinations that are of "engineering application interest" are presented in bold script. Comparison of the Monte Carlo generated results from this study to upper and lower limits previously correlated to experimental data in (2) provides confidence in the results of the sudden death testing technique.

Table 4.—Summary of minimum and maximum variation from calculated L_{10} life and Weibull slope from Monte Carlo simulation of assembly of 50-mm bore deep-groove ball bearings evaluated in 33 different sudden death test configurations comprising 36, 72, and 144 bearings.

No. of simultaneously operated testers	No. of failures	Variation from calculated L_{10} life, percent[a]		Variation in Weibull slope, e[b]	
		Minimum[c]	Maximum[c]	Minimum[c]	Maximum[c]
(a) 36 bearings, 8 configurations					
18	2	−89	228	0.48	11.93
12	3	−92	303	0.32	3.02
9	4	−39	83	0.92	2.43
6	6	−37	53	0.57	2.43
4	9	−41	120	0.69	1.49
3	12	−55	86	0.73	1.54
2	18	−47	83	0.75	1.50
1	36	−58	67	0.83	1.21
(b) 72 bearings, 11 configurations					
36	2	−88	886	0.39	6.02
24	3	−65	192	0.73	5.58
18	4	−51	93	0.54	1.49
12	6	−57	84	0.70	1.42
9	8	−10	61	0.70	1.78
8	9	−32	58	0.86	1.30
6	12	−26	33	0.78	1.64
4	18	−36	43	0.72	1.28
3	24	−68	116	0.75	1.52
2	36	−33	68	0.99	1.32
1	72	−32	51	0.98	1.26
(c) 144 bearings, 14 configurations					
72	2	−89	55	1.26	9.03
48	3	−81	346	0.58	3.70
36	4	−48	330	0.61	1.21
25	6	−57	94	0.84	2.63
18	8	−42	67	0.82	2.61
16	9	−47	170	0.84	1.73
12	12	−26	66	0.88	1.67
9	16	−25	23	0.91	1.35
8	18	−33	50	0.78	1.61
6	24	−30	55	0.80	1.55
4	36	−24	39	0.90	1.25
3	48	−18	43	0.91	1.30
2	72	−34	25	0.93	1.31
1	144	−16	23	0.99	1.25

[a]Based upon calculated L_{10} life of 6912 hours (from Table 1).
[b]Compare to assumed Weibull slope of 1.11.
[c]Maximum and minimum values of life do not necessarily correlate with maximum and minimum values of Weibull slope.

Endurance Life Variation

For each set of 10 trials for the 33 sudden death test combinations of 6010-size (50-mm bore) deep-groove bearings evaluated, the maximum and minimum L_{10} life was determined and compared to the calculated L_{10} life of 6912 hr. The maximum and minimum values of the L_{10} lives as a percent of the calculated L_{10} life for each combination ($m \times r$) of sudden death bearing tests with r bearing failures was determined as follows:

$$Max\ variation\ from\ calc\ L_{10}\ life = \frac{max\ L_{10} - calc\ L_{10}}{calc\ L_{10}}$$

$$\times 100\ percent \qquad [8a]$$

$$Min\ variation\ from\ calc\ L_{10}\ life = \frac{min\ L_{10} - calc\ L_{10}}{calc\ L_{10}} \times 100\ percent \qquad [8b]$$

These variations are reported in Table 4 for each of the 33 combination of sudden death tests evaluated.

The maximum and minimum L_{10} life from 10 trials of each of the 33 sudden death test configurations studied are plotted in Figure 2. Additionally, the L_{10} life bounding limits from (2) are shown. There were 9 possible combinations of sudden death test configurations for 36 bearings (Figure 2(a)), 11 possible combinations of test configurations for 72 (Figure 2(b)), and 14 possible combinations of test configurations for 144 (Figure 2(c)) deep-groove ball bearings. In 29 out of the 33 possible test combinations, the L_{10} values fell within these upper and lower bounds established in (2). In three of the cases, the maximum L_{10} life value exceeded the upper bound. In all 32 cases the minimum L_{10} life value fell on or above the minimum L_{10} life bound. In only one case did the minimum L_{10} life fall below the lower bound.

The percent variation of the maximum predicted L_{10} life from that of the calculated life is re-plotted in the bar chart of Figure 3(a). The minimum values are shown in Figure 3(b). The variations in life are grouped according to the total number of bearings tested [$n = \{36, 72, 144\}$] and as a function of the number of bearings failed [$r = \{2, 3, 4, 6, 9, 12, 18, 36\}$] or the number of bearing testers simultaneously used. Some variations or scatter in the range between the percent variations obtained for different total numbers of bearings tested at the same number of failures are observed. For sudden death testing the percent variation in the resultant L_{10} life from that calculated is independent of the number of bearings tested and dependent only on the number of bearings failed.

A curve fit of the variation in L_{10} life as a function of the number of bearings failed for the sudden death failed Monte Carlo data results in the following relation:

$$Max\ variation\ L_{10}\ life\ sudden\ death = calc\ L_{10}\ life\ (1+6r^{-0.6}) \qquad [9a]$$

$$Min\ variation\ L_{10}\ life\ sudden\ death = calc\ L_{10}\ life\ (1-1.2r^{-0.33}) \qquad [9b]$$

where r equals the total number of bearings failed in sudden death testing. Equations [6a] and [9a] for sequential testing and sudden death testing, respectively, are equal. The minimum values obtained for sequential testing (Eq. [6b]) is 20 percentage points less than that obtained for sudden death testing (Eq. [9b]). For most engineering applications, this difference is negligible. Accordingly, from an engineering point of view, these two curve fits are equivalent. Since Eq. [9b] is a curve-fit of minimum L_{10} lives from populations of 36, 72, and 144 bearings failed, and this in turn is in agreement with the curve-fit for sequentially generated failures (Eq. [6b]), it can be concluded that the percent variation in life is a function of the number of failed bearings and not the total number of bearings evaluated. This observation is consistent with those of Vlcek, Hendricks, and Zaretsky (2) and Johnson (1). It can also be concluded that the determination of the life is independent of the method of obtaining the data, whether it is sequential or sudden death testing.

Weibull Slope Variation

Lundberg and Palmgren (9) assumed the value of the Weibull slope e in Eq. [5] to be 1.11. Experience has shown that most rolling-element bearing life data exhibit Weibull slopes between 1 and 2. For this analysis, a Weibull slope of 1.11 was assumed for all of the components for each bearing. This should theoretically result in a Weibull slope of 1.11 for the bearing system.
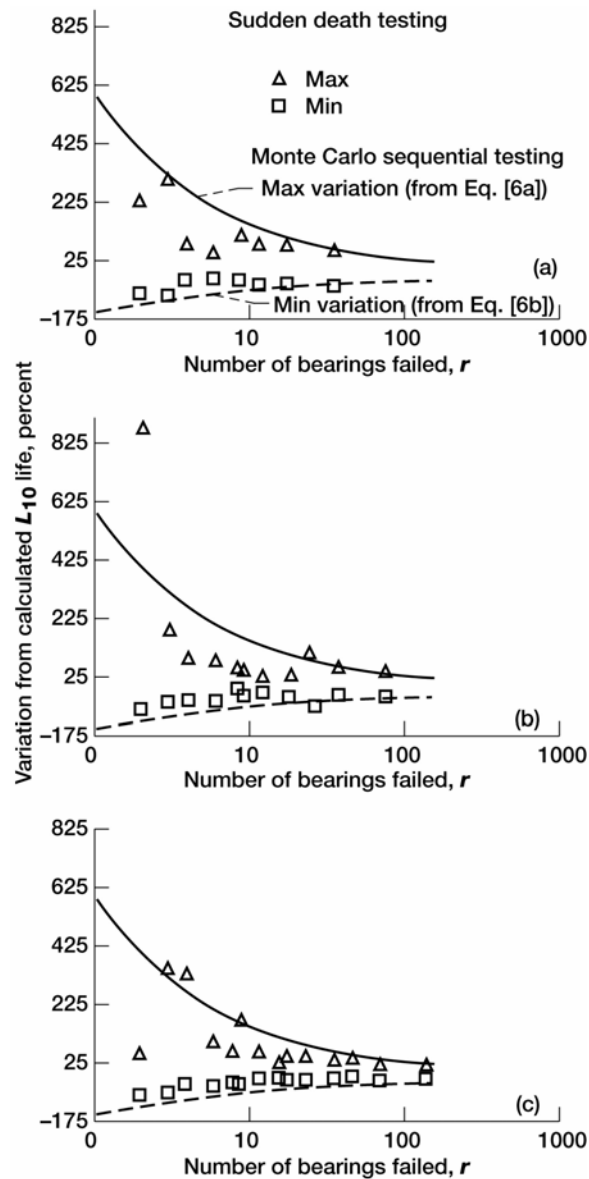
Figure 2.—Maximum and minimum variation of L_{10} lives as percent of calculated L_{10} life for groups of sudden death tested bearings compared to maximum and minimum variation of L_{10} life from sequential testing for 50-mm bore deep-groove ball bearings. (a) Number of bearings tested, n, 36. (b) Number of bearings tested, n, 72. (c) Number of bearings tested, n, 144.
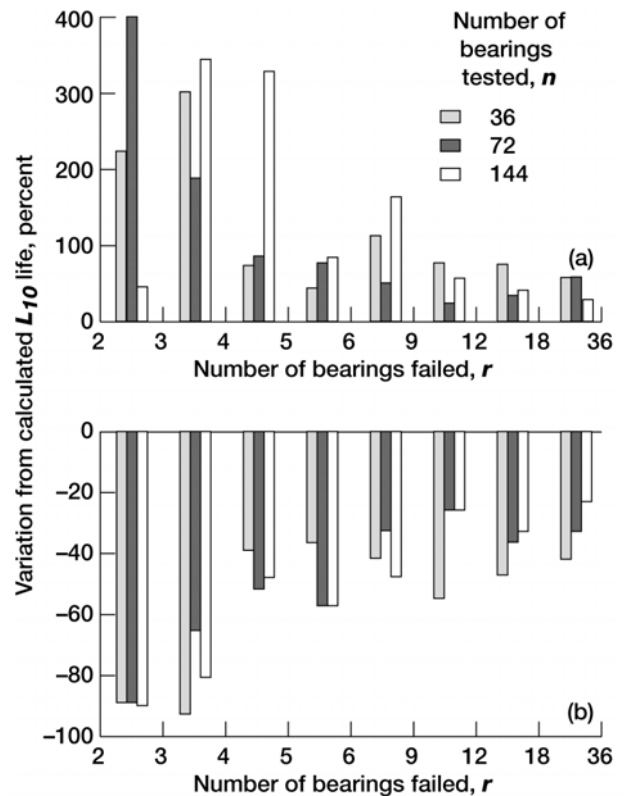

Figure 3.—Maximum and minimum variation of L_{10} lives as percent of calculated L_{10} life as function of number of bearings tested, n, and number of bearings failed, r, for sudden death testing of 50-mm bore deep-groove ball bearings. (a) Maximum variation. (b) Minimum variation.
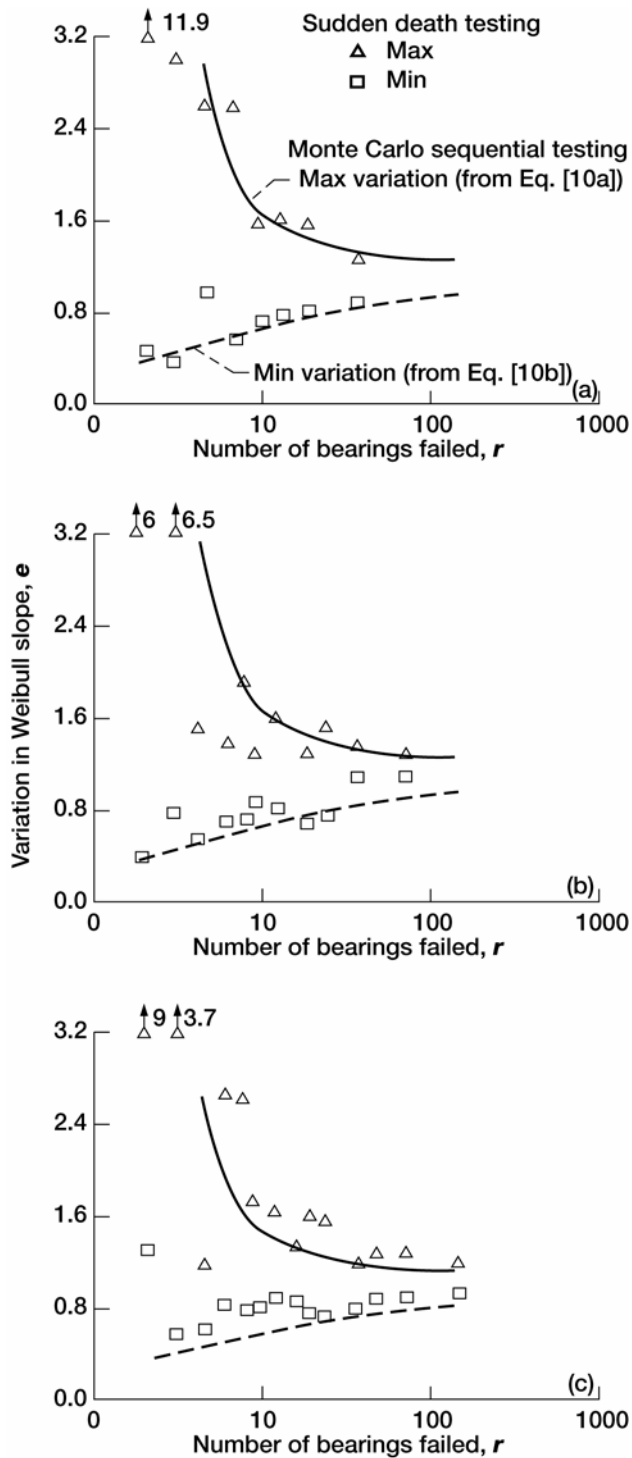
Figure 4.—Variation of Weibull slope *e* for Monte Carlo sudden death testing for each group of 10 bearing trials comprising *n* 50-mm bore deep-groove ball bearings each compared to extremes of Weibull slope from Monte Carlo sequential testing. (a) Number of bearings tested, *n*, 36. (b) Number of bearings tested, *n*, 72. (c) Number of bearings tested, *n*, 144.

The results of the extremes in the Weibull slopes for each group of the 10 bearing trials of *r* bearing failures are summarized in Table 4. The maximum and minimum bearing lives are not necessarily associated with the same trials as the maximum and minimum values of the Weibull slopes.

Vlcek, Hendricks, and Zaretsky (2) show that the relation between the number of bearings that failed and the limits of the Weibull slope is as follows:

$$\text{Max Weibull slope} = 1.2 + 5(\ln r)^{-3} \qquad [10a]$$

$$\text{Min Weibull slope} = 1.11 - 0.95r^{-0.33} \qquad [10b]$$

In this study, the Weibull slope extremes from 10 trials of 33 combinations of sudden death testing are compared to the upper and lower limit bounds from (2). Figure 4 contains the Weibull slope *e* limits as well as the maximum and minimum Weibull slope *e* from 10 trials of each of the 33 sudden death test configurations studied. There were 9 possible combinations of test configurations for 36 bearings (Figure 4(a)), 11 possible combinations of test configurations for 72 (Figure 4(b)), and 14 possible combinations of test configurations for 144 (Figure 4(c)) deep-groove bearings. In 21 out of the 33 possible test combinations, the extreme values of the Weibull slope fell within in these upper and lower bounds. In 11 of the cases, the maximum Weibull slope exceeded the upper bound. Of the 33 minimum values generated, in only one case did the minimum Weibull slope fall below the lower bound. Most of the significant upper bound violations occurred for 30 or less failed bearings.

Comparison of Bearing Test Time

An examination of the data summarized in Table 5 reveals that both in actual bearing test time and in calendar time, sudden death testing requires less time than sequential testing to failure an entire population of *n* bearings using the same number of testers simultaneously. However, where the number of bearings failed *r* is less than *n*, the reliability of the results, as measured by the extent of data variation, is poor as compared to testing the entire population *n* to failure. It was concluded that whether employing sudden death testing or sequential testing, the reliability of the data is a function of the number of bearings failed *r* and not the number of bearings tested *n*. If it is required to fail *r* number of specimens in order to obtain L_{10} estimates of desired reliability, the question remains whether it is advantageous in terms of testing or calendar time to fail (a) all specimens using sequential testing, (b) *r* of *n* specimens using *m* testers, or (c) some other variation of these schemes.

The Monte Carlo sudden death testing results summarized in Tables 4 and 5 incorporated up to 144 testers operating simultaneously. It is recognized by us that most test facilities generally will have less than nine testers of a particular kind and most likely not more than two to eight testers operating simultaneously. Since the reliability of the data is dependent on the number of failures generated r, the issue becomes what is the most time- and cost-efficient way to utilize these assets.

Table 5.—Effect of sudden death testing and sequential testing on bearing test time and calendar time for test completion of 50-mm bore deep-groove ball bearings evaluated in 33 different sudden death configurations comprising 36, 72, and 144 bearings.

No. of simultaneously operated testers	Sudden death testing			Sequential testing		
	Failure index[a]	Bearing test time, hrs.[b]	Calendar time, hrs.[c]	Failure index[a]	Bearing test time, hrs.[b]	Calendar time, hrs.[c]
(a) 36 bearings, 8 configurations						
18	2/36	125.1 x 10³	7.0 x 10³	36/36	1813.3 x 10³	100.7 x 10³
12	3/36	243.8	20.3			151.1
9	4/36	244.6	27.2			201.5
6	6/36	363.6	60.6			302.2
4	9/36	516.7	129.2			453.3
3	12/36	580.4	226.8			604.4
2	18/36	882.0 x 10³	441.0 x 10³			906.6
1	--	--	--	36/36	1813.3 x 10³	1813.3 x 10³
(b) 72 bearings, 11 configurations						
32	2/72	93.4 x 10³	2.6 x 10³	72/72	3640.2 x 10³	101.1 x 10³
24	3/72	246.2	10.3			151.7
18	4/72	257.3	14.3			202.2
12	6/72	393.5	32.8			303.3
9	8/72	484.9	53.9			404.5
8	9/72	535.6	67.0			455.0
6	12/72	712.1	118.7			606.7
4	18/72	1170.0	292.5			910.0
3	24/72	1273.0	424.3			1213.4
2	36/72	1954.9 x 10³	977.5 x 10³			1820.1
1	--	--	--	72/72	3640.2 x 10³	3640.2 x 10³
(c) 144 bearings, 14 configurations						
72	2/144	157.5 x 10³	2.2 x 10³	144/144	7129.9 x 10³	99.0 x 10³
48	3/144	271.2	5.7			148.5
36	4/144	348.1	9.7			198.0
25	6/144	458.7	18.3			285.1
18	8/144	590.6	32.8			396.0
16	9/144	633.5	39.6			445.5
12	12/144	828.9	69.1			594.0
9	16/144	985.9	109.5			792.0
8	18/144	1047.8	131.0			891.0
6	24/144	1463.4	243.9			1188.0
4	36/144	2189.7	547.4			1782.0
3	48/144	2652.9	884.3			2376.0
2	72/144	4008.0 x 10³	2004.0 x 10³			3584.0
1	--	--	--	144/144	7129.9 x 10³	7127.9 x 10³

[a]Number of bearings failed out of number of bearings tested.
[b]Total bearing test time for all bearings tested.
[c]Total calendar time to test all the bearing based upon 100% tester utilization.
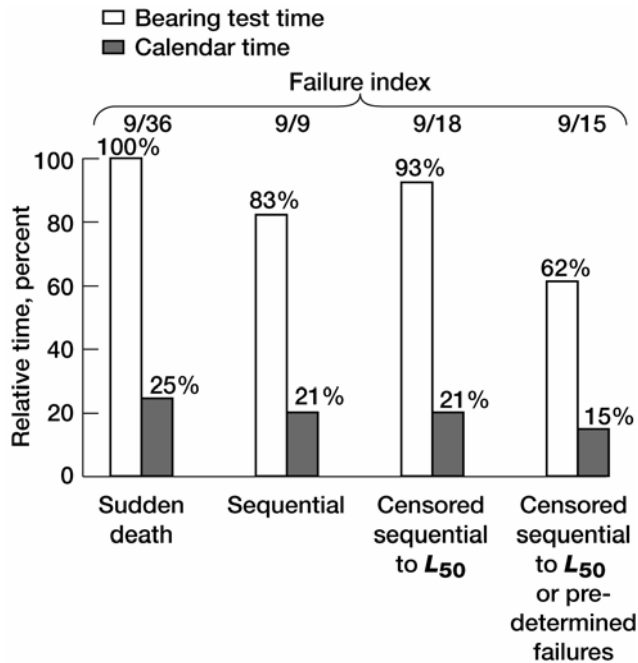
Figure 5.—Comparison of bearing test and calendar time for various testing methods using four testers simultaneously.

In the introduction to this paper, we discussed different testing techniques. Besides sudden death testing and sequential testing the entire population to failure using multiple testers, we also discussed (a) testing bearings to failure or the L_{50} life, whichever comes first and (b) testing to a predetermined number of failures r and then terminate all remaining tests. A combination of these test methods would be to sequentially test to failure or the L_{50} life, whichever comes first, and then terminate all testing when the predetermined number of failures r is achieved.

For a predetermined number of failures r, the test method chosen dictates the number of bearings n to be tested. Where sequential testing to a predetermined number of failures r, the number of bearings required n equal r. For bearings tested to failure or the L_{50} life, whichever comes first, the number of bearings to be tested n equals $2r$. Where testing is conducted to failure or the L_{50} life, whichever comes first, and then terminated when a predetermined number of failures r is achieved, the number of bearings to be tested n is equal to or less than $2r$.

For purposes of example, assume that the required number of failures to satisfy a required reliability is 9. From Table 5, the bearing test time and calendar time are obtained for sudden death testing and sequential testing as well as the failure index (the number of failures r out of those tested n) for 4 and 8 testers, respectively, that are operated simultaneously. These data are summarized in Table 6. Additionally, 10 Monte Carlo simulations were conducted using censored sequential testing where 4 and 8 testers run simultaneously to obtain 9 failures. For each group of testers under the column titled "Censored Sequential Testing," the first line represents the maximum time required to obtain 9 failures out of 18 bearings tested where all bearings are run to failure or the L_{50} life, whichever comes first. The second line represents the minimum time from the 10 Monte Carlo trials where testing was conducted to failure or the L_{50} life, whichever comes first, and then terminated when the 9 failures were obtained.

The data from Table 6 are normalized to the bearing test time for sudden test testing for four testers in Figure 5. From these data it becomes apparent that sudden death testing requires more bearing test time and calendar time then just testing nine bearings sequentially to failure. However, it may be reasonably argued that the differences in time are not significant. It can further be argued that using sudden death testing assures the required number of failures while testing only nine bearings will not assure that nine bearings will fail, within a reasonable calendar time.

Where censored sequential testing of all 18 bearings to failure or the L_{50} life to obtain nine failures is employed, there is a reduction of bearing test time and calendar time from those values obtained from sudden death testing. However, these reductions in time are not significant.

Table 6.—Summary comparison of bearing and calendar time for various methods.

No. of simultaneous operated testers	Sudden death testing			Sequential testing			Censored sequential testing		
	Failure index	Bearing test time, hrs.	Calendar time, hrs.	Failure index	Bearing test time, hrs.	Calendar time, hrs.	Failure index	Bearing test time, hrs.	Calendar time, hrs.
4	9/36	516.7×10^3	129.2×10^3	9/9	429.4×10^3	516.7×10^3	9/18	479.6×10^3	119.9×10^3
	--	--	--		--		9/15	319.6×10^3	79.9×10^3
8	9/72	535.6×10^3	67.0×10^3	9/9	429.4×10^3	516.7×10^3	9/18	516.7×10^3	60.0×10^3
	--	--	--		--	--	9/13	347.8×10^3	43.5×10^3

From Table 6 and Figure 5 it is apparent that reductions as much as 40 percent in both bearing test time and calendar time can be achieved when compared to sudden death testing by testing to failure or the L_{50} life, whichever comes first, and then terminating all testing when the last of the nine failures has occurred. Hence, for required test data reliability, sudden death testing is not a more efficient method to reduce bearing test time or calendar time as compared to sequential testing for a required reliability (accuracy and precision).

These studies were conducted to provide guidance for endurance testing, planning, and execution. Optional test plans will depend on the constraints of a particular project such as relative costs of specimens and testing, number of specimens available, and project objectives.

SUMMARY OF RESULTS

Monte Carlo simulation combined with sudden death testing were used to compare resultant bearing lives to the calculated bearing life and the cumulative test and calendar test time relative to sequential and censored sequential testing. A total of 30 960 virtual bearings of 50-mm bore deep-groove ball bearings were evaluated in 33 different sudden death test configurations compromising 36, 72, and 144 bearings each. The resultant variations in life and Weibull slope were also compared to results obtained from Monte Carlo (virtual) testing of entire populations of up to 1000 bearings and to 51 sets of actual ball and roller bearing data. The following results were obtained:

1. For sudden death testing and sequential testing, variations in both life and Weibull slope were a function of the number of bearings failed independent of the test method used and not the total number of bearings tested. Variations in L_{10} life as a function of number of bearings failed were similar to variations in life obtained from actual bearing data and from Monte Carlo (virtual) testing of entire populations.
2. To achieve a predetermined number of failures for a required reliability, reductions as much as 40 percent in both bearing test time and calendar time can be achieved when compared to sudden death testing by testing to failure or the L_{50} life, whichever comes first, and then terminating all testing when the last of the predetermined bearing failures has occurred.
3. For required test data reliability, sudden death testing is not a more efficient method to reduce bearing test time or calendar time when compared to censored sequential testing. However, sudden death testing will assure that with reasonable certainty a predetermined number of failures will occur.

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Brian L. Vlcek, Robert C. Hendricks, and Erwin V. Zaretsky

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13. ABSTRACT *(Maximum 200 words)*

Monte Carlo simulations combined with sudden death testing were used to compare resultant bearing lives to the calculated hearing life and the cumulative test time and calendar time relative to sequential and censored sequential testing. A total of 30 960 virtual 50-mm bore deep-groove ball bearings were evaluated in 33 different sudden death test configurations comprising 36, 72, and 144 bearings each. Variations in both life and Weibull slope were a function of the number of bearings failed independent of the test method used and not the total number of bearings tested. Variation in L_{10} life as a function of number of bearings failed were similar to variations in lift obtained from sequentially failed real bearings and from Monte Carlo (virtual) testing of entire populations. Reductions up to 40 percent in bearing test time and calendar time can be achieved by testing to failure or the L_{50} life and terminating all testing when the last of the predetermined bearing failures has occurred. Sudden death testing is not a more efficient method to reduce bearing test time or calendar time when compared to censored sequential testing.

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