

Moral Hazard and Non-Exclusive Contracts*

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ABSTRACT

This paper studies equilibria for economies with hidden action in which the set of contracts marketed in equilibrium is determined by the interaction of financial intermediaries in environments in which the agents' contractual relationships with competing intermediaries cannot be monitored (or are not contractible upon). We argue that such environments might constitute an appropriate model of markets for unsecured loans.

We fully characterize equilibrium allocations and contracts for such economies. Depending on the parameters of the economy, the optimal action choice is or is not sustained in equilibrium. Whenever it is, agents necessarily enter into multiple contractual relationships and intermediaries make positive profits, even if the intermediation market operates under free entry conditions. Such implications are consistent with several stylized facts of unsecured loans market.

Finally, we study and discuss the welfare properties of equilibria.

Keywords: asymmetric information, exclusivity, efficiency.

JEL: D82, D61, G20.

1 Introduction

Models of contracts with asymmetric information are usually models of exclusive contractual relationships. In other words, it is assumed that a party in a contract can enforceably restrict the other party's participation in contractual relationships with other agents.¹ As a consequence agents cannot undo the incentive effects of one contract by engaging in additional contractual relationships with other agents or institutions. In terms of informational requirements, exclusive contracts effectively require that the institutions which design the contracts are able to perfectly monitor the agents' trades with other institutions. Also, courts can enforce exclusive contracts only if agents' trades are observable and verifiable, which requires a rich institutional setting allowing for some centralized information about trades.

Enforceability of exclusive contracts is a strong assumption, and while it is a very useful benchmark, there are many interesting economic environments in which technologies to monitor trades are either quite costly or tightly regulated, and exclusive contracts are difficult to enforce. Only rarely do debt covenants in financial contracts include in fact explicit exclusivity clauses (Asquith-Wizman (1990), Smith-Warner (1979)).

In particular, markets for unsecured or partially secured loans do not seem to operate even implicitly through exclusive contracts. Consumer credit markets are a clear example: for instance, in the U.S. consumers hold several credit cards and are constantly solicited to open new accounts; also consumers often finance the acquisition of several durable goods, like cars, furniture, electronic appliances, with distinct debt contracts. Information sharing among lenders is frequently absent or imperfect in most countries for small business transactions (see Jappelli-Pagano (2000)).²

¹See e.g., the survey of contract theory by Hart-Holmstrom (1987). Even general equilibrium analysis of economies with asymmetric information rely heavily on exclusivity assumptions: see e.g., Prescott-Townsend (1984), Townsend (1987) for theoretical foundations, and Atkinson-Lucas (1992), Townsend (1994), for applied analysis.

²Other markets which could be appropriately modelled as characterized by non-exclusive contractual relationships include farm credit markets in less developed countries, where private moneylenders and family-related informal financial transactions interact and compete with banks and other formal financial institutions (the success of various microfinance programs like the Grameen Bank in Bangladesh is often explained in terms of their ability to partially relax the informational constraints on exclusivity by means of extensive monitoring of trades; see e.g., Morduch (1997)). Also the market for private lending to governments and other international institutions shares some aspects of non exclusivity: lack of information sharing across banks has been reported after the Latin American debt crisis in the 1970s (see Kletzer (1984)) as well as after the Asian crisis in the 1990s (see Radelet-Sachs (1998)). Finally, non-exclusivity, in the specific form of the inability of large firms to monitor the portfolio positions of their managers, has possibly important effects on the managerial incentive structure induced by stock-based compensation schemes, as documented by Ofek-Yermack (2000).

Unsecured and partially secured loan markets constitute a relevant component of credit markets in general. In the U.S., for instance, revolving consumer loans (mostly credit card loans) are unsecured, and account for more than a third of outstanding consumer credit. Moreover, a consistent share of consumer credit in general, including e.g., automobile, mobile home and education loans is only partially secured (Ausubel (1997)).³

What characterizes unsecured or partially secured credit contracts is of course default risk. In the credit card market, for instance, delinquency rates (the percentage of accounts 30 or more days past due) exhibits peaks of about 3.5 percent; similarly, chargeoff rates (the percentage of outstanding balances written off as uncollectible) peak at about 5.0 percent (Ausubel (1997)).

In this paper we study in particular economies with moral hazard in the form of hidden action, to capture some abstract features of credit markets in which the borrowers need to take actions to prevent insolvency, and private default provisions are either contractually specified, or determined by courts. In such economies, agents choose a costly action to limit the probability of insolvency. The action is agents' private information and, more specifically, affects the probability distribution of the agent's future income. Default is not strategic, as we interpret an unsuccessful realization of the agent's future income shock as a state of insolvency and default. Financial intermediaries issue contracts whose default provisions insure agents on their outcome realization, without being able to condition on their action choice.

We analyze economies in which the action is dichotomous ('high' or 'low'), although we briefly discuss some of the implications of the corresponding model with a richer domain for the action variable. For instance, when the agent is a consumer, the high action might consist of taking an extra job opportunity, or, when the agent is an entrepreneur, of an indivisible investment.

For this class of economies we are able to characterize equilibria with non-exclusivity. We show that for an open set of economies, in particular those with a relatively high cost of action, there exist only inefficient equilibria in which the low action is implemented. For those economies the effects of non-exclusivity are quite severe: in equilibrium the agents choose the low action even if both at the incentive constrained optimal contract and at the autarchic (no-trade) allocation they would choose high action. These equilibria occur because of the intermediaries' incentive to fully insure agents conditionally on their undertaking the low action whenever other intermediaries provide insurance at more favorable terms for the agents. As a consequence high action can only be sustained, if at all, as an equilibrium when

³Also, the fraction of unsecured loans, for firms with less than 500 employees (such firms generate 51 percent of private GNP), amounts to about 31 percent of all loans (from the National Survey of Small Business Finances).

contracts can be designed to prevent entry of other contracts which adversely affect incentives for the incumbents. We show that equilibria which implement high action can be sustained for an open set of economies, and require the presence of ‘latent contracts’, i.e., contracts that are not actively traded in equilibrium, but whose presence reduces the profitability of equilibrium deviations for potential entrants. Latent contracts serve the purpose of restricting the entry of other contracts which would have negative incentive effects on the incumbents, and thereby moderate the effects of non-exclusivity. On the other end, latent contracts guarantee rents to the incumbents in equilibria which support agents’ ‘high’ action.

As a consequence we show that the high action can be implemented in equilibrium, and intermediaries necessarily make positive profits. Furthermore, in equilibrium agents actually engage in multiple contractual relationships with different lenders. Multiple contracts are necessary to prevent the active intermediaries from deviating to contracts guaranteeing even higher profits. Latent contracts take the form of available lines of credit at high interest rates (fair conditionally on the agents’ low action).

Some existing evidence suggests that both supra-normal profits and multiple credit relationships with different lenders may characterize some markets for unsecured loans. The credit card market provides our best example, as data are readily available for this market. In fact, agents in general hold several cards, on average more than 7 per household in the U.S. (more than 9 for those households which hold at least one; Evans-Schmalensee (1999)). Also, as documented in detail by Ausubel (1991, 1997), since the deregulation of the credit card market in 1982, the profits of credit card companies in the U.S. range from 3 to 5 times higher than the standard profits in the banking industry (interest rates are about 3 times the cost of funds adjusted for default risks).⁴

Latent contracts, i.e. available credit lines at high interest rates, also seem to characterize credit card markets in the form of frequent mailing and telemarketing solicitations (in 1995 the number of direct-mail solicitations from credit card issuers totalled more than 2 per month per American household on average; Ausubel (1997)).

Several pieces of evidence also suggest that the main implications of our analysis might be consistent with the observed structure of unsecured credit markets other than credit card markets. Multiple credit sources are for instance documented by Petersen-Rajan (1994) for the U.S. market for unsecured loans to small businesses; they also present some evidence of a ‘pecking order’ in credit sources which can

⁴More conservative estimates of the profitability of the credit card market in the ’80s emerge from the case study of the Discover Card program, in Lapuerta-Myers (1997). No consensus is reached in the literature on the profitability of the credit card industry in the long run; see Evans-Schmalensee (1999).

be interpreted as evidence of the availability of credit lines at high interest rates. In Europe multiple credit sources are more prevalent: Detragiache-Garella-Guiso (1997) documents for instance in detail the Italian case. Also, Jappelli-Pagano (2000) provides a cross-country analysis of credit markets, bankruptcy institutions, private credit bureaus and public credit registers. Multiple credit relationships for small business are common, especially, as our analysis implies, in those countries in which information sharing institutions among lenders (private credit bureaus and public credit registers) are either recent or only partially developed. Furthermore, default rates are about twice as high on average in such countries, which is also consistent with the implications of our analysis of economies with non-exclusive contracts.

Finally, we show that non-exclusivity has important welfare effects. When contractual relationships are non-exclusive, two forms of moral hazard arise. First, agents' choice of action is private information. Second, agents' choice of trades is also private information. Not surprisingly, we show that equilibrium allocations are inefficient from the point of view of a planner who does not control the effort choice, but does control the agents' trade. On the other hand, consider a planner facing the same observability constraints that intermediaries face; that is, the planner controls neither the effort choice, nor the agents' trade. We show that equilibria are efficient from the point of view of such a planner, that is, there is no other feasible allocation (feasible with respect to the observability constraints) which is preferred to an equilibrium allocation by both the agents and the intermediaries. We say that equilibria are *incentive constrained inefficient* but *third best efficient*. This is true even though, as we noted, in equilibrium intermediaries are not perfect competitors and entry is prevented via latent contracts.

1.1 Related Literature

The analysis of hidden action economies in *non-exclusive* environments has been pioneered by R. Arnott and J. Stiglitz in a sequence of unpublished papers in the early '80s (their work is now collected in Arnott-Stiglitz (1993)) and by the enlightening comments on their work by Hellwig (1983).⁵ Our paper is mostly related to this line of work. We study the same class of economies as Arnott-Stiglitz (1993) and Hellwig (1983), but we postulate a larger contract space which allows for 'negative insurance' contracts (i.e., insurance contracts which pay in the high income state). We see no compelling reason to exclude negative insurance from the contract space. Even though negative insurance contracts are not traded in equilibrium, in fact, we

⁵But see also Bizer-de Marzo (1992), Helpman-Laffont (1975), Jaynes (1978), Pauly (1968). The analysis of equilibria with non-exclusivity is also related to the analysis of 'common agency' in the contract theory literature; see e.g., Bernheim-Whinston (1986).

show that their presence in the contract space substantially reduces the set of contracts which are sustained as an equilibrium of the game played by intermediaries. As a consequence, the equilibrium set has a simpler structure, providing for sharper predictions. We will analyze more precisely the relationships between our paper and Arnott-Stiglitz's and Hellwig's after introducing our results, in Section 3.3.

Our paper contributes to the literature started by Arnott-Stiglitz's and Hellwig's papers also by studying the question of constrained efficiency of equilibria. While non-exclusivity clearly imposes on the economic environment additional constraints with respect to incentive compatibility, and hence in general equilibria will not be incentive constrained Pareto efficient, it is not clear if equilibria are third best Pareto efficient, once the constraints due to the non-exclusivity are explicitly considered. We will answer the question affirmatively, in the sense that a planner facing the same observability constraints that intermediaries face at equilibrium cannot find an allocation which is preferred to an equilibrium allocation by both the agents and the intermediaries.

Kahn-Mookherjee (1998) also study non-exclusive contracts in moral hazard economies with hidden action. The structure of the game intermediaries and agents play in their model, though, is quite different from ours (from Arnott-Stiglitz's and Hellwig's as well); in particular, in Kahn-Mookherjee's economy, agents design their own contracts (intermediaries can only either accept or reject the agents' offers), they make contractual decisions sequentially, and their contractual portfolios are observable, even if not contractible upon. Such differences in the strategic interactions of agents and intermediaries have a crucial effect on equilibrium allocations and contracts. In our economy, because intermediaries in equilibrium exploit the rents due to the presence of 'latent contracts', intermediaries necessarily make positive profits and agents face distorted insurance prices at equilibrium. On the contrary, intermediaries make zero profits and agents face fair insurance prices in equilibrium in Kahn-Mookherjee's environments. In Section 3.3, after introducing our results, we will identify the modelling aspects which are responsible for the different results we obtain with respect to Kahn-Mookherjee.

Parlour-Rajan (1999) independently study a model of strategic default in unsecured credit market economies with non-exclusive contractual relationships.⁶ Our analysis has many elements in common with theirs, including the existence of equilibria with positive profits. By concentrating on strategic default their model, differently from our hidden action model, has the property that in equilibrium default is never observed, thereby contradicting the evidence regarding most unsecured credit markets, and, most importantly, making it impossible to derive any implications linking the institutional characteristics of unsecured credit markets (e.g., regulatory constraints, the existence of information sharing institutions) with default rates

⁶We thank a referee for this reference.

(some implications of our analysis for default rates are discussed in the conclusions).⁷

Helpman-Laffont (1975) (see also Bisin-Gottardi (1999)) study competitive equilibria in economies with hidden action. In their set-up linearity of prices captures a strong form of non-exclusivity: each intermediary has no control over agents' trades, not even over trades of its own contracts. In the set-up of the present paper instead intermediaries control agents' trades in the contracts they themselves issue.

Finally, our paper is also in part related to the literature on Cournot convergence to competitive equilibria; in Bisin-Gottardi-Guaitoli (1999) we develop the analysis of the present paper to study the issue in detail (see also Hellwig (2000) and Segal-Whinston (2001)).

2 The Economy

The general economy we study lasts two periods, $t \in \{0, 1\}$. It is populated by a continuum of ex-ante identical agents, indexed by $i \in I$ with total measure 1, and by a finite number of financial intermediaries, indexed by $h \in H$; we will in general think of H as large as we want to model intermediation markets in which entry is free (requiring H to be finite is only for the purpose to avoid technical difficulties of no substantial relevance in the analysis). Agents are risk averse. They value consumption in period 0 and 1, c_0 and c respectively, and action e :

$$u(c_0) + u(c) - e$$

(units are chosen so that e is measured in utils, without loss of generality). We assume that $u : \mathfrak{R}_+ \rightarrow \mathfrak{R}$ is twice differentiable, strictly increasing, strictly concave and $\lim_{c \rightarrow 0} u(c) = -\infty$. The action is chosen in $t = 0$, is private information, and can take two values, $e \in \{a, b\}$ (but we will be careful in discussing which results are robust to the introduction of a richer support of the action variable). Without loss of generality, $a > b$. The choice of action affects the probability distribution of the uncertain income of the agents at time 1, a random variable w^i which is i.i.d. across agents $i \in I$, whose realization is publicly observable, and which takes values w_H, w_L , with $w_H > w_L$ (from now on we drop the apex i whenever confusion should not arise). Let π_a (resp. π_b) denote the probability of income w_H given action a (resp. b). Assume $\pi_a > \pi_b$. The reader will have noticed that H (resp. a) takes

⁷The characterization of equilibria is also substantially different, e.g., no latent contracts arise in Parlour-Rajan (1999)'s analysis, as in their analysis the possible out of equilibrium deviations by intermediaries are limited by the fact that if default is induced both the incumbents' and the entrants' profits are negatively affected. In our set-up, on the contrary, a richer strategy space of intermediaries allows entrants to offer contracts which affect negatively only the incumbents (by inducing low action on the part of the agents), as the entrants can control the implicit rate of return required on the contracts so as to guarantee themselves non-negative profits.

the interpretation of the ‘high income state’ (resp. ‘high action’). We think of the high action as an action which helps preventing insolvency, that is reducing the probability of the low income state.

As standard in moral hazard environments, we use the properties of large economies. In particular, the Law of Large Numbers allows us to identify π_e with the fraction of agents which observe the realization w_H when producing action e (see Al-Najjar (1995), Sun (1998)).

Prior to the beginning of time, intermediaries strategically design contracts. Each intermediary h can design and issue J^h contracts, and J is the set of contracts issued overall. A contract prescribes a set of transfers from the intermediary to the buyer (possibly negative) conditionally on publicly observable variables. Formally, a contract $j \in J$ is a vector d^j representing the payoff at each date, 0 and 1, and state, H and L . Intermediaries can also make a contract divisible allowing agents to buy fractions $\lambda_j \in [0, 1]$. A complete specification of contract j is represented by $D^j = \{d^j, \Lambda_j\}$, where Λ_j (the set of admissible λ_j) is either $\{0, 1\}$ (indivisible) or $[0, 1]$ (divisible).⁸ Let $D^h = (D^j)_{j=1}^{J^h}$ denote the set of contracts issued by intermediary h .

Intermediaries maximize profits. Contractual relationships are non-exclusive, as intermediaries cannot condition payoffs to the agents’ trading positions.

In such an economy contracts have both an insurance and a credit component which interact with non-exclusivity. This is indeed the case in markets for unsecured and partially secured credit, which motivate our analysis. Consider for instance the credit card market (see Ausubel (1997)). In this market, because contracts are non-exclusive, agents accumulate credit by engaging in multiple contractual relationships (multiple credit cards). Agents are in general exposed to private income risks and, as a consequence, they frequently let credit card accounts go delinquent. Credit card companies often simply chargeoff the delinquent accounts, thereby extending a form of private insurance provision to their debtors. Credit limits and implicit insurance provisions are designed by credit card companies for an environment in which contracts are non-exclusive and agents hold multiple credit accounts.

In order to simplify the analysis, while capturing the fundamental properties of unsecured and partially secured credit markets, we consider two special cases, and deal separately with credit limits and private insurance provisions.

In the first economy, which we call the *pure insurance* economy, private default provisions are contractually specified, but we restrict credit limits at time $t = 0$ to be exogenous. In fact, without further loss of generality, we normalize the agents’ borrowing position at 0.⁹ Therefore, such an economy is simply a standard insurance

⁸As it turns out, only one type of contract will be made divisible in equilibrium (the latent contracts in the high action equilibria); we will often refer to a contract only by its transfers d^j , intended to be indivisible unless specified otherwise, to save on notation.

⁹The normalization in fact amounts to letting w denote income net of repayment of the amount

economy with moral hazard. In the second economy we analyze, which we call the *credit* economy, we restrict instead state L to represent the default state, in which agents consume an exogenous amount w_L , that is, their endowment or a consumption provision exogenously determined by bankruptcy laws. On the other hand, in this economy credit limits are determined endogenously, agents borrow to finance consumption at time 0, and repay their debt at time 1 in state H .

In fact, the insurance and the credit economy turn out to be essentially equivalent in terms of our analysis and results. Therefore we proceed first with the analysis of the pure insurance economy because it is this economy that has been studied in the literature to which we mean to compare our results: Arnott-Stiglitz (1993), Hellwig (1983) and Kahn-Mookerjee (1998). In Section 4 we will then show how all of our results extend to the unsecured credit economy. We will also provide some simulations to show that the results extend to the general economic environment introduced in this section in which both the insurance and the credit dimension are jointly analyzed.

3 Insurance Economy

Consider the pure insurance economy, in which agents only consume at time 1. We first introduce the definition of equilibrium we shall use in this paper: *equilibrium with non-exclusivity*. Given the set of contracts issued by intermediaries, agents choose which contracts to buy. This determines their consumption allocation. Agents also choose an action. Anticipating the choices of agents, as a function of the set of contracts they are allowed to trade, intermediaries strategically choose which contracts they issue, to maximize profits.¹⁰

The problem solved by agents can be formally described as follows. Each agent chooses an action $e \in \{a, b\}$, portfolio choices $\lambda = \{\lambda_j \in \Lambda_j\}_{j \in J}$, and consumption $c = (c_L, c_H)$, to maximize:

$$\pi_e u(c_H) + (1 - \pi_e) u(c_L) - e \tag{1}$$

subject to

$$c = w + \sum_{j \in J} \lambda_j d^j, \text{ given } D^j, j \in J. \tag{2}$$

Note that if $\Lambda_j = \{0, 1\}$ agents can either buy or not buy contract j ; they cannot buy just a fraction, although they can buy multiples if the same contract is issued

borrowed at time 0, and similarly to let d^j denote contract's j payoff net of repayment of the amount borrowed at time 0.

¹⁰We restrict the definition of equilibrium to the symmetric case in which all agents behave identically. This is just for the sake of notation, and we do not in fact make such an assumption in the analysis.

by more than one intermediary. In other words, while we model *non-exclusivity* as a form of inability of each intermediary to observe agents' trades with other intermediaries, we assume that each intermediary does observe its own aggregate trades with each agent.

The problem solved by intermediaries can be described as follows. Intermediary $h \in H$ chooses $D^h = (D^j)_{j=1}^{J^h}$ to minimize:¹¹

$$\sum_{j \in J^h} \left(\pi_e d_H^j + (1 - \pi_e) d_L^j \right) \lambda_j \quad (3)$$

subject to:

$$e, \lambda \text{ solve (1)-(2), } (D^{h'})_{h' \neq h} \text{ given.} \quad (4)$$

Note that intermediaries effectively play a simultaneous game by choosing the structure of contracts they trade.

Definition 1 (Equilibrium) *An equilibrium with non-exclusivity is an array*

$$\langle e, \lambda, c, (D^h)_{h \in H} \rangle$$

such that

- i) $\langle e, \lambda, c \rangle$ maximize (1) subject to (2) given $D = (D^h)_{h \in H}$;
- ii) D^h minimizes (3) subject to (4), for any $h \in H$.

This definition of equilibrium (with a strategic component in the intermediaries' choices) has been used in different contexts by Rotschild-Stiglitz (1976), Arnott-Stiglitz (1993) and many others.

3.1 Characterization

We are now ready for the characterization of equilibria with non-exclusivity.¹² We will first derive conditions under which equilibria implement the low action (Proposition 1). We will then show that in general, whenever equilibria with non-exclusivity implement the high action, *i*) agents trade multiple contracts at the equilibrium; *ii*) such trades are necessarily sustained as an equilibrium by other contracts which are issued but not traded (and are therefore called *latent contracts*, following Hellwig's

¹¹We implicitly assume that intermediaries have large enough income to avoid bankruptcy issues. Note also that they do not need to be risk averse: profits in fact are deterministic because of the Law of Large Numbers.

¹²All proofs are collected in the Appendix.

terminology); *iii*) latent contracts operate as a barrier to entry and hence guarantee positive profits to intermediaries (Propositions 2 and 3).

We illustrate our analysis and most of our results graphically. Consider Figure 1. Point w is the no trade allocation at which consumption coincides with income in each state. The straight lines from w represent the fair price lines (equivalently, the zero profit lines): the steeper line has slope $-(1 - \pi_b)/\pi_b$, the (negative of) the fair price of insurance conditionally on action b , while the less steep line has slope $-(1 - \pi_a)/\pi_a$, the (negative of) the fair price of insurance conditionally on action a .

Let C denote the subset of consumption allocations $(c_L, c_H) \geq 0$ which satisfy

$$c_L \geq w_L$$

$$\frac{1 - \pi_a}{\pi_a} \leq \left| \frac{c_H - w_H}{c_L - w_L} \right| \leq \frac{1 - \pi_b}{\pi_b}.$$

It contains all allocations which involve positive insurance on the part of the agents at prices which are not more expensive than the fair prices when the agents choose action b , and not cheaper than the fair prices when action a is chosen. In figure 1, C coincides with the area contained between the two (zero profit) lines from the no trade allocation w . Any equilibrium allocation must be contained in C (otherwise either profits are negative or there is always room for a profitable contract to be issued).

Proposition 1 *Define $\Delta = a - b$. If*

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) - u(\pi_b c_H + (1 - \pi_b)c_L) - \Delta < 0, \text{ for any } c \in C \quad (5)$$

then there exists a unique equilibrium allocation with non-exclusivity characterized by:

$$c_H = c_L = \pi_b w_H + (1 - \pi_b)w_L \quad \text{and} \quad e = b.$$

We call these equilibria *low action equilibria*. With the help of figure 1 we can provide an illustration of the proposition. Consider an allocation like A in the figure, at which agents choose action a . Such allocation can never be sustained as an equilibrium for the preferences represented in the figure. Rather than consuming allocation A and choosing action a , agents would strictly prefer to buy additional insurance at the price $(1 - \pi_b)/\pi_b$, reaching allocation B, and choose action b . But Proposition 1 requires this to hold *for any consumption point in C* (condition (5)). Under this condition then no allocation in C with high action can represent an equilibrium, because at such an allocation any intermediary could make positive profits with a contract selling insurance at a price slightly higher than $(1 - \pi_b)/\pi_b$. On the other hand, insurance contracts at the price $(1 - \pi_b)/\pi_b$ can never make losses.

We conclude then that, under the conditions of Proposition 1, an equilibrium is necessarily represented by action b and the consumption allocation which is preferred by agents facing a fair insurance price conditional on action b , $(1 - \pi_b)/\pi_b$. Such an allocation is obviously c^b in the figure, the full insurance allocation conditional on action b .

< Figure 1 >

Condition (5) is satisfied for a robust set of economies, which therefore have only low action equilibria. To see this, consider an economy with logarithmic preferences, $u(c) = \ln(c)$. For such an economy, condition (5) can be written as

$$\exp\{-\Delta\} \left(\frac{c_H}{c_L}\right)^{\pi_a} < (1 - \pi_b) + \pi_b \left(\frac{c_H}{c_L}\right), \quad \text{for all } c \in C$$

It can be immediately shown that such a condition is in fact satisfied for Δ high enough. Also, locally perturbing preferences around the logarithmic formulation, condition (5) can be satisfied for a robust set of economies.

Proposition 2 *Suppose that, for an open subset of consumption allocations $(c_L, c_H) \in C$,*

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) - u(\pi_b c_H + (1 - \pi_b)c_L) - \Delta \geq 0; \quad (6)$$

then any equilibrium allocation in pure strategies satisfies

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) - u(\pi_b c_H + (1 - \pi_b)c_L) - \Delta = 0 \quad (7)$$

*with $e = a$.*¹³

We call these equilibria *high action equilibria*. Equation (7) defines the locus of allocations such that, when associated with the high action, agents are as well off as they would be buying the optimal level of additional insurance at price $(1 - \pi_b)/\pi_b$ and switching to $e = b$. The properties of high action equilibria can be illustrated with the help of figure 2, in the case of logarithmic preferences (but see the Appendix for formal proofs).

< Figure 2 >

¹³The case in which (6) is satisfied for a zero measure subset of C is non-generic in the parameters of our economy, as it is immediately demonstrated by locally perturbing Δ . We refer to the Appendix for a discussion of this case.

With logarithmic preferences, property (7) becomes

$$\exp\{-\Delta\} \left(\frac{c_H}{c_L}\right)^{\pi_a} = (1 - \pi_b) + \pi_b \left(\frac{c_H}{c_L}\right).$$

For Δ small enough such an equation has two solutions in c_H/c_L , represented in the figure by line $\overline{(7)}$ and line $\overline{(7)}$.¹⁴ We proceed now to show that any high action equilibrium must lie on $\overline{(7)}$. From any allocation in the interior of the cone, in fact, another allocation on $\overline{(7)}$ preferred by agents can be reached with a contract making non-negative profits. The same is obviously true for allocations on $\overline{(7)}$. Finally, any allocation lying outside of the cone satisfies $\pi_a u(c_H) + (1 - \pi_a)u(c_L) - u(\pi_b c_H + (1 - \pi_b)c_L) - \Delta < 0$, and hence cannot be supported as an equilibrium with high action.

An allocation such as c^a on $\overline{(7)}$ may be supported as an equilibrium even if divisible contracts offering positive or negative insurance at price $(1 - \pi_b)/\pi_b$ are also issued. At any allocation on $\overline{(7)}$, consider again c^a for instance, agents are by construction indifferent between choosing action a , and moving to allocation c^b with action b . But since allocation c^b is the most preferred allocation which can be reached from c^a with a contract which allows the agents to buy any amount of insurance (a divisible contract) at price $(1 - \pi_b)/\pi_b$, we can assume that such contracts would remain untraded if issued (latent); moreover, we can assume that such contracts are in fact issued as they guarantee non-negative profits to the issuer.

Latent contracts are a necessary component of equilibrium. Consider an agent trading two contracts to consume c^a in equilibrium: the first takes him/her from w to m , and the second from m to c^a . Without the latent contract, another intermediary could offer a contract which takes the agent from m to some point c^a' , which the agent prefers to c^a , and possibly to the allocation he/she can reach buying all three contracts offered. But with the latent contracts, as the agent reaches a point outside the area defined by (7), he/she can do even better by getting more insurance and switching to action b . If, and only if, the latent contracts are issued, then, the intermediary offering the deviation contract makes negative profits.

Let J_1 denote the subset of the set of contracts J which contains contracts actively traded in equilibrium, i.e., contracts j such that $\lambda_j > 0$. Our main characterization result regarding high action equilibria is the following.

Proposition 3 *At a high action equilibrium, each agent actively trades multiple contracts:*

$$\text{the cardinality of } J_1 \text{ is } > 1,$$

¹⁴A condition on the parameters is needed to guarantee that the cone from the origin generated by these two lines does not include the no trade point, w , as in the figure; such a cone, however, always lies north-west of the 45° line.

all of which guarantee positive profits to the intermediaries offering them:

$$\pi_a d_H^j + (1 - \pi_a) d_L^j < 0, \forall j \in J_1.$$

To illustrate the first result of Proposition 3, note that for an allocations like c^a in Figure 2 to represent an equilibrium, it is necessary that agents actually trade multiple contracts; there are two in this case: the first takes agents from w to m , and the second from m to c^a . Such contracts are constructed, in fact, to prevent the two intermediaries which are active in equilibrium from deviating and charging a higher price for their contract. They have the property that agents are indifferent between buying either one or both of them. Therefore, a deviation which worsens the terms of trade of one of the contracts would have the effect that such a contract is not traded. The crucial property which prevents the deviations of the incumbent intermediaries is that the indifference curve for action a through c^a cuts the implicit insurance price line, connecting the no trade allocation point w with the consumption allocation c^a , twice: at c^a and at m . A single intermediary offering both contracts and allowing agents to consume at the same allocation c^a , with no other active intermediary, would not sustain the equilibrium as he/she could deviate and increase profits by charging a higher price for insurance.¹⁵ As for the latent contracts, there must be at least two intermediaries, each selling a divisible contract for a large enough quantity of insurance: agents can then always buy the optimal amount in response to any entry and the intermediaries selling the latent contracts do not have profitable deviations.

We can finally see on Figure 2 why positive profits are necessary in equilibrium. The only allocation sustained by contracts making zero profits and lying on (7) is c_0^{tb} . To be an equilibrium for some number of active intermediaries $n > 1$, as we just argued, the indifference curve should cut the fair price line, with slope $(1 - \pi_a)/\pi_a$, twice: at $c_0^{tb} = w + d$ and at $w + \frac{n-1}{n}d$.¹⁶ In the limit, as $n \rightarrow \infty$, the indifference curve should be tangent to the fair price line. But with action $e = a$, indifference curves have a slope $(1 - \pi_a)/\pi_a$ at full insurance and a steeper slope in the region of underinsurance, as at c_0^{tb} . In this region, then, they can only cut the zero profit line once from above. Therefore c_0^{tb} cannot be supported as a high action equilibrium with any number n of active intermediaries. The same argument used to show that multiple contractual relationships are necessary at high action equilibria imply that

¹⁵In general there can be many equilibria with different numbers of active intermediaries: in the limit as such number goes to infinity, each intermediary offers a negligible quantity of insurance and the price of insurance equates the (negative of the) tangent to the indifference curve at the equilibrium allocation.

¹⁶We are here implicitly assuming that each of the n intermediaries offers an identical contract to the agents. This is the case since in equilibrium agents must be indifferent between buying the n contracts or any $n - 1$ of them.

c_0^{tb} cannot be supported by only one active intermediary; such an intermediary would deviate to a contract with positive profits, along (7).

High action equilibria do exist. We can easily show this by considering an economy with logarithmic preferences, $u(c) = \ln c$. By Proposition 2 a high action equilibrium necessarily lies on (7). In the logarithmic case, we have repeatedly argued, (7) is the lower edge of the cone defined by

$$\exp\{-\Delta\} \left(\frac{c_H}{c_L}\right)^{\pi_a} \leq (1 - \pi_b) + \pi_b \left(\frac{c_H}{c_L}\right)$$

We will first construct a limit equilibrium, in which the number of active intermediaries, n , tends to infinity.¹⁷ In this case each agent's indifference curve at the equilibrium allocation must be tangent to the price of insurance. To demonstrate the existence of such an equilibrium, therefore, it suffices to show that there exists an allocation on (7) with this property, that is, such that the indifference curve through the allocation is tangent to the price line from the endowment w to the allocation. But, as a consequence of homotheticity of preferences, the marginal rate of substitution is constant along any ray from the origin, in particular along (7). Moreover, the marginal rate of substitution along (7) is greater in absolute value than $(1 - \pi_a)/\pi_a$, since (7) is steeper than the full insurance line. It is also smaller than $(1 - \pi_b)/\pi_b$, whenever the cone defined by (7) has a non-empty interior (the generic case). This is the case since, when the cone has a non-empty interior, (7) is less steep than the line $c_H = \frac{(1-\pi_b)\pi_a}{\pi_b(1-\pi_a)}c_L$, the singular solution of (7) when the cone collapses into a line, along which the marginal rate of substitution is $(1 - \pi_b)/\pi_b$. We conclude that it is always possible to find a price line from the endowment w , with slope p in absolute value, $\frac{1-\pi_a}{\pi_a} < p < \frac{1-\pi_b}{\pi_b}$, which cuts (7) tangentially at the indifference curve; and therefore a high action equilibrium exists in the limit case with $n \rightarrow \infty$.

Equilibria with a finite number of active intermediaries, finite n , can now be constructed for lower insurance prices, with the only caveat that integer constraints must be satisfied.

Note also that our analysis of the logarithmic economy demonstrates that in this case aggregate equilibrium profits for intermediaries increase with the number of intermediaries which are active at equilibrium.

We turn now to discuss how our characterization of equilibria for economies with dichotomous action extends to economies with continuous action variables, in which for instance the action variable takes the interpretation of costly effort in the

¹⁷This example is essentially taken from Hellwig (1983). We report it here for completeness, as Hellwig's paper is not published and might not be easily available to the reader.

success of an investment. Our analysis concerning low effort equilibria is immediately extended to economies with a continuous hidden action variable and non exclusive contractual relationships. If the support of the action variable is continuous and connected, e.g., the interval $[b, a]$, and the dependence of the probability distribution of income on the action is smooth, in fact low effort equilibria, characterized by the full insurance allocation and the minimal action b , arise precisely under condition (5).

The characterization of equilibria in which an action strictly higher than b is supported is difficult. Our analysis of the dichotomous case shows though that high effort equilibria are supported when a discontinuity arises in the agents' choice of action as their allocation is parametrically varied. The construction of high effort equilibria in the dichotomous case in fact requires latent contracts, which operate by inducing 'large' discontinuous shifts in the agents' behavior as a consequence of the entry of a contract which deviates from the equilibrium. Such discontinuities follow in general from the non-convexity of the agents' choice problem. More precisely, our analysis indicates that high effort equilibria could be supported, for economies in which the unobservable action lies in a continuous domain, provided assumptions on the map from the action to the probability distribution of income (concavity of the preference index is naturally maintained) are made which guarantee that the agents' objective function lacks concavity in the appropriate range of consumption allocations (the lack of connectedness of the domain of the action variable would also do, as would the lack of continuity of the probability distribution of income as a function of the action).

3.2 Welfare

We are now ready to study the welfare properties of equilibria with non-exclusivity. In the class of economies we study in this paper two forms of moral hazard arise. First, the choice of action e on the part of the agents is private information. Second, the choice of trades on the part of the agents is also private information. We introduce therefore two different appropriate definitions of optimality.

At an *incentive constrained optimum* the planner does not control the effort choice, but does control the agents' trade. If equilibria with non-exclusivity were incentive constrained optimal, then the unobservability of trades, the non-exclusivity, would not matter in equilibrium. We will show that in general non-exclusivity matters; it introduces an externality in equilibrium, and equilibria are not incentive constrained optimal.

At a *third-best optimum*, instead, the planner controls neither the effort choice, nor the agents' trade. The planner faces in fact the same observability constraints that intermediaries face in equilibrium.

More precisely, at an incentive constrained optimum the planner chooses the agents' action, the consumption allocation and the set of contracts to be traded, to maximize agents' utility subject to: the definition of consumption, equation (9), the condition which guarantees a given aggregate amount k of profits to intermediaries, equation (10), and the incentive compatibility constraint, equation (11). We are only interested in the non-trivial case in which the high action, $e = a$, can be supported at an incentive constrained optimum. We restrict attention to this case:¹⁸ the incentive constraint guarantees that agents have no incentives, at the prescribed consumption allocation, to deviate and choose instead $e = b$.

Definition 2 (Incentive constrained optimum) *An incentive constrained optimum supporting $e = a$ is an array*

$$\langle c, d \rangle$$

which maximizes

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) - a \quad (8)$$

subject to:

$$c = w + d \quad (9)$$

$$\pi_a d_H + (1 - \pi_a)d_L = -k \quad (10)$$

and

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) - \pi_b u(c_H) + (1 - \pi_b)u(c_L) - \Delta \geq 0. \quad (11)$$

Note that the amount of aggregate intermediaries' profits, k , parametrizes the frontier of incentive constrained optima.

An incentive constrained optimum supporting action $e = a$ cannot be decentralized by an equilibrium with non-exclusivity. In other words, non-exclusivity has relevant welfare effects, because it introduces an externality. Figure 3 illustrates graphically this point for the incentive constrained optimum associated with zero profits, but the argument extends to $k > 0$. Let u^e denote the indifference curve conditional on action e being chosen; we have drawn them so that u^a and u^b have the same expected utility, and hence the unconditional indifference curve is represented by the lower envelope of u^a and u^b . With c^{ic} we denote the incentive constrained optimum allocation. Since $\frac{1-\pi_b}{\pi_b} > \frac{1-\pi_a}{\pi_a}$, the marginal rate of substitution at any

¹⁸A sufficient condition for the existence of incentive constrained optima supporting $e = a$, for instance, is the following:

$$\pi_a u(w_H) + (1 - \pi_a)u(w_L) - u(\pi_b w_H + (1 - \pi_b)w_L) \geq \Delta.$$

point is higher on u^b than on u^a . Any point in the shaded area in the figure can then be reached from c^{ic} with a contract making positive profits, and is preferred by agents to c^{ic} . This proves that c^{ic} cannot be an equilibrium with non-exclusivity.¹⁹

< Figure 3 >

Equilibria with non-exclusivity then are not in general incentive constrained optimal, for those economies in which constrained optimal allocations support the high action, $e = a$. Non-exclusivity introduces an externality which is distinct from the externality caused by the non-observability of the action e .

At a third-best optimum, the planner does not control the agents' trades. Therefore the incentive compatibility constraint is more restrictive in this case: it must guarantee that agents do not have an incentive to deviate jointly from the prescribed action and from the prescribed consumption allocation. We continue restricting attention to the case in which the high action, $e = a$, can be supported at an optimum; the condition in footnote (18) is sufficient also for this the case of third-best optimum. When contemplating a deviation to $e = b$, agents anticipate being able to supplement the prescribed consumption allocation with any feasible trade at fair odds, that is they anticipate facing insurance opportunities at price $(1 - \pi_b)/\pi_b$.²⁰

Definition 3 (Third-best optimum) *A third-best optimum supporting $e = a$ is an array*

$$\langle c, d \rangle$$

which maximizes (8) subject to: equation (9), equation (10), and

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) - u(\pi_b c_H + (1 - \pi_b)c_L) - \Delta \geq 0. \quad (12)$$

¹⁹It may be interesting to characterize the payoffs $\{d^{j'}\}$ of the contract that makes positive profits when coupled with the incentive constrained optimal contract. Since agents when buying only $\{d^j\}$ in equilibrium are indifferent between $e = a$ and $e = b$ (the incentive constraint is easily shown to be binding), there exists a contract $d^{j'}$ with the following properties:

- $d_L^{j'} > 0$, $d_H^{j'} < 0$, and $\pi_b d_H^{j'} + (1 - \pi_b)d_L^{j'} = -\epsilon$ (small enough),
- agents prefer $d^j + d^{j'}$ to d^j .

The first property states that contract $d^{j'}$ offers positive insurance and makes profits ϵ (i.e., the price of the insurance is less than fair for the agents). The second property guarantees that agents nonetheless prefer to buy the combination of contracts d^j and $d^{j'}$ rather than the incentive constrained optimal contract d^j by itself.

As a consequence, if both $\{d^j, d^{j'}\}$ are issued, agents choose action $e = b$, the intermediary issuing d^j makes negative profits, while the one issuing $d^{j'}$ makes positive profits.

²⁰The implicit price of insurance in case of deviation is arbitrary. While this is a delicate issue, it has no impact on our analysis.

Note that the incentive constraint, equation (12), coincides with the condition which guarantees existence with high action equilibria, equation (6). Therefore, for those economies for which low action equilibria arise, no third-best optimum supporting the high action, $e = a$, exists. In that case then any third-best optimum can only support action b , and the planner cannot do any better than offering full insurance at fair prices. We conclude that low action equilibria are third-best optimal. Note that even though third-best optimal, the inefficiency associated with *low action equilibria* is quite severe. For a robust set of economies, for instance, the following three properties hold simultaneously: *i*) all equilibria are *low action* (hence they implement $e = b$); *ii*) at the incentive constrained optimum allocation, $e = a$; *iii*) if no contract were offered (i.e., at autarchy), agents would choose $e = a$.²¹

The following result, the main one of this section, concerns third best optimal allocations which support the high action.

Proposition 4 *High action equilibria with non-exclusivity are third-best optimal.*

In order to understand the third-best optimality of high action equilibria it is convenient to characterize the third-best optimum frontier. The frontier is parametrized by the aggregate profits of intermediaries, $k \geq 0$. Consider figure 4 (the figure is drawn for logarithmic preferences, but the argument extends). It is straightforward to see from the definition that third-best allocations supporting the high action must belong to C , and lie on (7) . Furthermore, moving along (7) north-east of A , towards B , one moves along the third-best frontier in the direction of allocations preferred by agents. In other words, allocation B in figure 4 is a third-best optimum associated with smaller profits k than allocation A (c_0^{tb} is the third-best optimal allocation preferred by agents, associated with zero profits for intermediaries, $k = 0$). We conclude that high action equilibria are third-best optimal, as, by Proposition 2, they must necessarily lie on (7) .

This result may require an explanation. Proposition 2 shows that high action equilibria are supported by latent contracts and require positive profits for the intermediaries. Therefore, high effort equilibria never decentralize c_0^{tb} . As a consequence, high effort equilibria induce a distortion of the equilibrium price of insurance; more specifically, agents face in equilibrium less than fair insurance odds. Consider figure 4, for instance: an equilibrium allocation such as B is supported by an implicit insurance price $q > \frac{1-\pi_a}{\pi_a}$. How can third-best optimality be maintained despite this price distortion? The answer is that relative prices do not determine the agents' consumption allocation at equilibrium, the incentive constraint does (the incentive

²¹We leave the details of the proof of this statement to the reader. It follows from noticing that *ii*) and *iii*) hold generically for economies with logarithmic preferences for which (5) (and hence *i*) are satisfied, and that the result is maintained under small perturbations of preferences.

constraint (12) is binding at any high action equilibrium allocation). In other words, prices do not matter as agents' insurance positions are 'rationed' in equilibrium.

We can illustrate this point with the help again of figure 4. The implicit insurance price agents face at allocation B is q , and intermediaries make profits k (in the figure, k is the length of the segment BC multiplied by $(1 - \pi_a)$). Suppose now that agents pay the lump-sum amount k to intermediaries out of their budget set, but face no distortion in the implicit insurance price. The budget set of agents after the lump-sum transfer to intermediaries is represented by the line passing through $w' = w - \frac{1}{1-\pi_a}k$, with slope $(1 - \pi_a)/\pi_a$. The resulting third-best optimum allocation is therefore B , and the price distortion does not introduce any inefficiency.²²

3.3 Discussion

We have shown that, in a hidden action model with non-exclusivity, the optimal action is not implemented in equilibrium for an open set of economies, and, for the economies in which it is implemented, agents have multiple contractual relationships and intermediaries make positive profits.

These results differ from the those reached by Kahn-Mookherjee (1998) (henceforth KM), Arnott-Stiglitz (1993) and Hellwig (1983) (henceforth ASH). In this section we attempt to explain such differences.

The possibility of either low or high action equilibria is a robust result, as it arises in all the different models (ours, as well as KM and ASH). All these papers, as we do, consider economies in which the action choice is dichotomous. But high action equilibria in KM are always associated with zero profits for intermediaries, a result opposed to ours.

It is the different structure of the game that generates different equilibria. There are two main features which distinguish the strategic interaction of intermediaries postulated by KM and ours: while we study economies in which intermediaries design contracts simultaneously, KM *i*) postulate a sequence of bilateral trades between each agent and the intermediaries, and *ii*) allow agents to design contracts, which agents themselves offer to the intermediaries in a pre-specified sequence (offers are take-it-or-leave-it, each intermediary can only accept or reject an offer).

We want to argue that it is the agents' bargaining power, due to their ability to design contracts and make take-it-or-leave-it offers, which drives KM's zero profit result (differently from what they expected; see footnote 5 and section 6 in KM). The sequential nature of the offers is certainly not sufficient to generate zero

²²Even in the general economy in which agents consume at time $t = 0$ high action equilibria are third-best efficient. The distortion on the price of insurance has no effect on the marginal condition which determines borrowing and lending, again because agents are 'rationed' in the insurance market.

profits in equilibrium. Suppose in fact we maintain KM's sequential structure, but modify the contract design mechanism, so as to allow the intermediaries to make take-it-or-leave-it offers to the agents, rather than the opposite. In this case, in a subgame perfect equilibrium, it will never be the case that the first intermediary offers the contract which makes zero profits; he could in fact offer any contract which corresponds to a Nash equilibrium of the sequential game studied in our paper, thereby guaranteeing positive profits. In fact, the sequential structure of offers allows the first intermediary in the sequence to do better in general and extract all surplus, thereby exploiting both the barriers to entry which, as our analysis has demonstrated, endogenously arise in markets for non-exclusive contracts, and the bargaining power due to his position in the sequence of offers.

In other words, our and KM's analysis of markets for non-exclusive contracts can be interpreted to show that: *i*) competition of intermediaries in the design of contracts (and implicitly in their prices) is not sufficient to drive profits to zero; but *ii*) if (and only if) agents are endowed with the whole bargaining power in the contractual relationships, intermediaries in equilibrium cannot exploit any market power by constructing barriers to entry.

As we argued in the Introduction, ASH study the same economy as we do, with intermediaries simultaneously designing contracts. They restrict the strategy space of intermediaries to contracts which offer positive insurance, i.e., to contracts whose payoff in state L is non-negative, $d_L \geq 0$. Even though contracts offering negative insurance are never traded in equilibrium, the restriction of the strategy space in ASH is not without loss of generality. We argue that such restriction expands the set of equilibria which can be supported by latent contracts. Suppose in fact that some allocation, e.g., c^A in figure 5, is supported by a latent contract which, added to the allocation c^A , induces an allocation c^B (by construction the agent is indifferent consuming c^A with action a and consuming c^B with action b).

< Figure 5 >

Suppose also that at c^B the agent is over-insured, i.e., $c_H^B < c_L^B$, as is the case in figure 4, where c^B is below the 45° line. Such a latent contract is never part of an equilibrium if negative insurance contracts are allowed, because such a contract can be profitably introduced that would be added to c^B by the agents, *i*) inducing action b on the part of the agents, and thereby *ii*) negative profits for the intermediary trading the latent contract (in figure 5, e.g., such contract supports the allocation c^C when added to c^B). In ASH's environment, on the contrary, many equilibria are supported by such latent contracts (see Hellwig (1983), in particular, for a characterization).

4 Credit Economy

Consider now the credit economy, in which agents in state L consume w_L . The problem solved by agents can be formally described as follows. Each agent chooses an action $e \in \{a, b\}$, portfolio choices $\lambda = \{\lambda_j \in \Lambda_j\}_{j \in J}$, and consumption $c = (c_0, c_H)$, to maximize:

$$u(c_0) + \pi_e u(c_H) + (1 - \pi_e)u(w_L) - e \quad (13)$$

subject to

$$c_0 = w_0 + \sum_{j \in J} \lambda_j d_0^j \quad (14)$$

$$c_H = w_H + \sum_{j \in J} \lambda_j d_H^j. \quad (15)$$

Intermediaries offer contracts characterized by a loan at time 0 in amount d_0^j , requiring a repayment d_H^j when the agent is not in default, that is, in state H . Formally, intermediary $h \in H$ chooses $D^h = (D^j)_{j=1}^{J^h}$ to minimize:²³

$$\sum_{j \in J^h} (d_0^j + \pi_e d_H^j) \lambda_j \quad (16)$$

subject to

$$e, \lambda \text{ solve (1)-(2), } (D^{h'})_{h' \neq h} \text{ given.}$$

We assume $w_0 < \frac{1}{\pi_b} w_H$ to guarantee that agents will borrow in equilibrium rather than lend.

We argue that the pure insurance economy studied in the previous section and the credit economy introduced here are equivalent. More specifically, for the credit economy as well, equilibria are either low or high action; high action equilibria are sustained by latent contracts, which guarantee positive profits for the intermediaries, and by multiple active contractual relationships; moreover equilibria are incentive constrained inefficient but third best efficient.

All this should become apparent to the reader by studying, for the credit economy, the condition analogous to (6), that is, the incentive constraint associated with third best optima. We will derive here this condition and then limit our analysis to pointing out the formal equivalence between the pure insurance and the credit economy.

²³We maintain the assumption that intermediaries have large enough income to avoid bankruptcy issues (see footnote 11).

For the credit economy, latent contracts are contracts which provide the agent with credit at a price $1/\pi_b$. Such contracts necessarily guarantee non-negative profits for the intermediaries offering them. An equilibrium allocation with high action therefore has the property that it must be (weakly) preferred to the allocation an agent could reach by choosing action $e = b$ and buying additional credit at price $1/\pi_b$. Formally,

$$u(c_0) + \pi_a u(c_H) \geq \max u(c_0 + d_0) + \pi_b u(c_H - \frac{1}{\pi_b} d_0) - \Delta'$$

where $\Delta' = a - b + (\pi_a - \pi_b)u(w_L)$ denotes the relative costs associated with the high action in this economy, an exogenous constant as in the case of the pure insurance economy. It is also straightforward to show that the optimal deviation when choosing the low action $e = b$, called d_0 , is such that $c_0 + d_0 = c_H - \frac{1}{\pi_b} d_0$; that is, following a deviation to the low action, agents use latent contracts to smooth perfectly consumption at time 0 and at time 1 in state H . It follows that the incentive compatibility constraint, i.e, the condition analogous to (6), for the credit economy can be written as:

$$u(c_0) + \pi_a u(c_H) - (1 + \pi_b)u(\frac{1}{1 + \pi_b} c_0 + \frac{\pi_b}{1 + \pi_b} c_H) - \Delta' \geq 0. \quad (17)$$

The formal analysis of equilibrium characterization now proceeds essentially as in the case of the insurance economy, but in the space of consumption allocations (c_0, c_H) rather than in the space (c_L, c_H) . Let C' denote the set of allocations $(c_0, c_H) \geq 0$ which satisfy

$$c_0 \geq w_0$$

$$\frac{1}{\pi_a} \leq \left| \frac{c_H - w_H}{c_0 - w_0} \right| \leq \frac{1}{\pi_b}.$$

Then, for parameter configurations such that there exists no allocation $(c_0, c_H) \in C'$ satisfying (17), the only equilibrium is a low action equilibrium with perfect smoothing: $e = b$, $c_0 = c_H$. Otherwise, as in the pure insurance case, we have high action equilibria. Such equilibria must satisfy conditions similar to those derived for the pure insurance economy. First, the incentive constraint (17) will be binding (otherwise another profitable credit contract can be introduced with agents still choosing the high action). Furthermore, in order to limit the market power of active intermediaries, multiple contracting is needed such that a single intermediary does

not contribute a net surplus to the agent (the local satiation or double intersection between the price line and the indifference curve that we have seen in Proposition 3). At any such equilibrium intermediaries necessarily make positive profits. In fact, the marginal rate of substitution between c_0 and c_H is always greater than $1/\pi_a$ above the 45° line, i.e., at all incentive compatible allocations. Therefore the zero profit point on the third best frontier cannot satisfy the ‘local satiation’ condition.

Finally, it is straightforward to show that high action equilibria are third best but not second best efficient.

5 The General Economy with Credit and Insurance

As we argued in the Introduction, contractual relationships in markets for unsecured credit have both relevant credit and insurance components. We have studied separately an insurance and a credit economy for the sake of simplicity. Analytical results for the economy with both credit and insurance, that is for the general economy introduced in Section 2, are hard to derive. In this section we briefly report on some computations we have performed for specific parameter values, with the aim of showing that our analysis carries through for this more complex class of economies.

Consider logarithmic preferences, $u(c) = \ln c$, and fix arbitrary parameter values: $(w_0, w_H, w_L, \pi_a, \pi_b, a - b) = (5.0, 12.0, 2.0, .5, .2, .1)$.

We then proceed as follows.²⁴ We first solve for the agent’s equilibrium choice of borrowing at time $t = 0$, d_0 , as a function of (c_L, c_H) , and conditionally on the agent also choosing the high action $e = a$. We then substitute the solution into the incentive constraint, the constraint analogous to equation (6), and into preferences. By looking at such indirect utility functions and incentive constraints, effectively we reduce the consumption space to (c_L, c_H) . This procedure has the advantage that results can be directly compared to those of the insurance economy, analyzed in detail in Section 3. The incentive constrained region in the space (c_L, c_H) , for our parametric example, is shown in Figure 6. In the insurance economy, with logarithmic preferences, this set is a cone from the origin; in the general economy here it is still a cone, but translated from the origin. Also, indirect indifference curves on the space (c_L, c_H) are regularly shaped and quasi-concave.

< Figure 6 >

Finally, we have computed an equilibrium, in the limit case when $n \rightarrow \infty$ (the

²⁴We only sketch our procedure here. Details are available in an Appendix posted at <http://www.econ.nyu.edu/~user/bisina>.

number of active intermediaries), following the example by Hellwig reported in our analysis of the insurance economy.

At the equilibrium (see figure 7) the implicit insurance price, the slope of the line from $w - d_0$ to the equilibrium allocation c^* , is less than $(1 - \pi_a/\pi_a)$ in absolute value, and therefore intermediaries make positive profits. Also, as in the pure insurance economy (and the credit economy), there is no equilibrium at which intermediaries make zero profits.

< Figure 7 >

6 Conclusions

Our analysis of the hidden action model in economies in which contractual relationships are non-exclusive has shown that equilibria in such environments have several stylized properties that have been associated with unsecured credit markets. In particular, when the high action is supported, agents enter in equilibrium multiple contractual relationships and intermediaries guarantee themselves positive profits.

We conclude here by discussing some possible extensions with the intent of better understanding the implications of this class of models with respect to default rates and the size of the market.

A model with endogenous borrowing and lending positions can generate in equilibrium a negative correlation between the amount borrowed and the action chosen by the agents, as a consequence of exogenous changes in the rate of return on borrowing and the insurance price. An exogenous reduction in the rate of return and in the price of insurance induces higher borrowing and possibly higher insurance in equilibrium. In such a model, in fact, agents smooth perfectly their consumption over time: non-exclusivity implies that any equilibrium contract must be immune to deviations in the form of pure borrowing and lending contracts, whose rate of return is constant and hence independent of the agents' action choice. This in turn might tighten the agents' incentive constraint and make it more difficult to sustain a high action. In an economy with a continuous action variable and with enough non-convexity of the agents' choice problem to support in equilibrium actions higher than the minimal action, the agents' choice of action at equilibrium might be reduced. Since default rates are in our interpretation measured by the probability of the low state, this would imply higher default rates.

The other fundamental stylized fact in the credit card industry is indeed the contemporaneous increase of the size of the market and of default rates. Delinquency rates on credit cards exhibited cyclical peaks of about 2.75 percent before 1982, and of about 3.5 percent after; similarly, chargeoff rates exhibited peaks of about 3.5

percent before 1982, and of about 5.0 percent after (Ausubel (1997)). Such a behavior of default rates in the credit card market, as a consequence of the burgeoning of the market after deregulation, could then be explained in the context of this model. The direct effect of the deregulation has been in fact to reduce the interest rates of credit card borrowing from usury rates, thereby increasing the amount borrowed by agents in equilibrium, from essentially an autarchic equilibrium in which credit cards operated as means of payment rather than a credit instrument.

Similarly, such a model can explain the observed correlation between default rates and the lack of information sharing institutions (Jappelli-Pagano (2000)). Such institutions in fact better support quasi-exclusive contractual relationships and as a consequence can more easily support high action choices, by restricting the set of contracts available, and hence lower default rates.

Appendix

We first sketch here a proof of existence in mixed strategies for completeness. The proof follows the lines of Bisin (1998).

We let for simplicity $\Lambda_j = \{0, 1\}$, for all j , and leave the general case to the reader. The agents' optimal choice in problem (1)-(2) is described by a mapping from $\{d^h\}_{h \in H}$ into (e, λ, c) . Let this mapping be denoted ψ . Then clearly, under the assumptions on preferences, ψ is upper-hemi-continuous.

We can now restrict the set of feasible contracts, without loss of generality as follows: $d_L^j \in [-w_L, +w_L]$, $d_H^j \in [-w_H, +w_H]$, $\forall j \in J$. The strategy space of the game played by intermediaries is then compact, and their payoff function is continuous in (e, λ, c) . Intermediaries rationally anticipate the map ψ of $\{d^h\}_{h \in H}$ into (e, λ, c) . In fact, because there is a continuum of agents $i \in I$, intermediaries rationally anticipate the convex hull of ψ (see Bisin (1998)). Intermediaries' profits are then effectively an upper-hemi-continuous convex valued correspondence, since they are a continuous function of ψ . The main theorem in Simon-Zame (1990) allows finally to show that, for some selection of the intermediaries' profit correspondence, a Nash equilibrium in mixed strategies exists. \diamond

Proof of Proposition 1. Consider the allocations (c_L, c_H) satisfying

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) < \pi_b u\left(c_H - \frac{1 - \pi_b}{\pi_b} \tilde{d}\right) + (1 - \pi_b)u(c_L + \tilde{d}) - \Delta, \quad (18)$$

where

$$\tilde{d} = \operatorname{argmax} \left\{ \pi_b u\left(c_H - \frac{1 - \pi_b}{\pi_b} d\right) + (1 - \pi_b)u(c_L + d) \right\}.$$

By concavity of preferences, \tilde{d} is the full insurance contract (conditionally on action b); substituting in (18), we obtain condition (5). No allocation (c_L, c_H) with high action satisfying (5) can be sustained as an equilibrium because, for some small positive ϵ , d' s.t. $\pi_b d'_H + (1 - \pi_b)d'_L = -\epsilon$ is a strict best reply to any contract d such that $c = w + d$. Only the full insurance allocation, $c_H = c_L = \pi_b w_H + (1 - \pi_b)w_L$, with action b can be sustained in equilibrium. The equilibrium contracts $D = \{D^j\}_{j \in J}$ sustaining such allocation satisfy: $\pi_b d_H^j + (1 - \pi_b)d_L^j = 0$, and $\Lambda_j = [0, 1]$, $\forall j \in J$. A contract D' such that $i)$ $\pi_b d_H' + (1 - \pi_b)d_L' = 0$, $\Lambda' = [0, 1]$, and $ii)$ $d_L' \geq 0$ ('positive insurance'), is in fact a weak best reply to D . \diamond

Proof of Proposition 2. If an open set of consumption allocations (c_L, c_H) in C satisfy (6), there exists a non-empty set of allocations for which (7) is satisfied (by continuity, since (5) is always satisfied when $c_L = c_H$). No allocation with high action for which (5) is satisfied can be an equilibrium (from Proposition 1). Suppose

(6) is satisfied with strict inequality at some allocation (c_L, c_H) and sustained by contracts

$$c = w + \sum_{j \in J} d^j.$$

In the region of underinsurance the marginal rate of substitution is greater than the zero profit rate, i.e.,

$$\frac{1 - \pi_a}{\pi_a} \frac{u'(c_L)}{u'(c_H)} > \frac{1 - \pi_a}{\pi_a} \quad \text{if } c_L < c_H.$$

There is always then another profitable contract, d' , such that $U(c + d', a) > U(c, a)$ and $U(c + d', a) > U(c + d', b)$.

However an allocation (c_L, c_H) which satisfy (7) could be sustained by a set of contracts $\{d^j\}$ including not only those needed to reach the consumption point, i.e., $\{d^j\}_{j \in J_1}$ s.t.

$$c = w + \sum_{j \in J_1} d^j,$$

but also ‘latent’ contracts that deter entry, i.e., $\{d^j\}_{j \in J_2 = J - J_1}$ s.t.

$$\max_{\lambda_j \in [0,1]} U(c + \sum_{j \in J_2} \lambda_j d^j, b) = U(c, a).$$

If $U(c + d', a) > U(c, a)$, then,

$$\max_{\lambda_j \in [0,1]} U(c + d' + \sum_{j \in J_2} \lambda_j d^j, b) > U(c + d', a).$$

Here the following Lemma applies.

Lemma 1 *The set of contracts sustaining an equilibrium with $e = a$ must include ‘latent’ contracts $\{d^j\}_{j \in J_2}$ s.t. $\lambda_j = 0, \forall j \in J_2$; moreover any latent contract must satisfy $\pi_b d_H^j + (1 - \pi_b) d_L^j = 0, \forall j \in J_2$.*

Proof of Lemma 1. If no latent contracts are issued, there always exists another profitable contract, for any proposed allocation c . Suppose latent contracts provide additional insurance at a price $|d_H^j/d_L^j| < (1 - \pi_b)/\pi_b$, for any $j \in J_2$. Agents will be indifferent between the candidate equilibrium allocation c , with high action a , and the best allocation they can reach with the latent contracts, $\hat{c} = \operatorname{argmax} U(c', b)$ s.t. $c' = c + \sum_{j \in J_2} \lambda_j d^j$, which will be in the region of overinsurance (where the marginal rate of substitution with low action is less than $(1 - \pi_b)/\pi_b$). But then there exists a contract d' selling negative insurance (i.e., $d_L' < 0, d_H' > 0$) with $|d_H'/d_L'| < |d_H^j/d_L^j| < (1 - \pi_b)/\pi_b$, such that $U(\hat{c} + d', b) > U(\hat{c}, b) = U(c, a)$ (agents

strictly prefer the low action allocation with latent contracts and the negative insurance). Also, agents prefer this to adding the negative insurance to c with high action, i.e., $U(\hat{c} + d', b) > U(c + d', a)$. If such a contract is introduced, agents will then buy all the contracts and choose low action. Any contract selling positive insurance with a slope less than $(1 - \pi_b)/\pi_b$ will make losses, and d' , being the negative of such a contract, will make positive profits. Suppose now the aggregate quantity of insurance offered by such latent contracts is rationed so that agents cannot reach the region of overinsurance, but only allocations either of underinsurance or of full insurance. Then, in the first case, additional positive insurance at a price slightly higher than $(1 - \pi_b)/\pi_b$ is a profitable deviation; while, in the second case, any intermediary selling part of the latent can unilaterally deviate to a profitable contract, since the remaining quantity of the latent contracts will be insufficient to trigger the agents' reaction (even in the limit with an infinite number of intermediaries selling each an infinitesimal amount of the latent contracts, given that the full insurance point is not a point of tangency, i.e., of local satiation). A profitable deviation exists, then, for any candidate equilibrium supported by latent contracts at a price less than $(1 - \pi_b)/\pi_b$. The only equilibrium contracts that survive are those supported by latent contracts at a price $|d_H^j/d_L^j| = (1 - \pi_b)/\pi_b$, for any $j \in J_2$. To trigger agents' reaction for any possible entry, these contracts will be available in any quantity (i.e., they have to be divisible) up to a large enough maximum. To be a best reply for intermediaries, the number of intermediaries selling the latent contracts must be, say, $m \geq 2$, with the aggregate quantity offered by any $m - 1$ intermediaries large enough to satiate agents for any possible deviations (e.g., $d_H^j = -w_H$ for any $j \in J_2$). \diamond

Any equilibrium allocation sustained by latent contracts (according to Lemma 1) must then satisfy (7), by definition: more precisely, it must belong to $\underline{(7)}$, the subset of points c such that $c_L > c'_L$ for any c' in (7) with $\pi_b c'_H + (1 - \pi_b) c'_L = \pi_b c_H + (1 - \pi_b) c_L$. \diamond

Proof of Proposition 3. Latent contracts deter entry, but also intermediaries that are active in equilibrium must be prevented from deviating and charging a higher price for their part of the aggregate insurance. If $U(c, a) > U(c - d^j, a)$ for some $j \in J_1$, there is some contract $d^{j'}$ (with a higher price) which is more profitable than d^j . The equilibrium requires $U(c, a) = U(c - d^j, a)$ for all $j \in J_1$. With n active intermediaries, such a condition implies $d^j = \frac{1}{n} |c - w|$: isoprofit lines have a slope $(1 - \pi_a)/\pi_a$, less steep than the indifference curve, so no deviation is profitable.

We will now show that positive profits are necessary in equilibrium: a point of intersection of (7) with the zero profit line, as c_0^{tb} in Figure 2, cannot be an equilibrium with non-exclusivity. To be an equilibrium for some number of active intermediaries

$n > 1$, the indifference curve should cut the zero profit line twice, at $c_0^{tb} = w + d$ and at $w + \frac{n-1}{n}d$ (or, in the limit as $n \rightarrow \infty$, be tangent). But with action $e = a$, indifference curves have a slope $(1 - \pi_a)/\pi_a$ at full insurance and a steeper slope in the region of underinsurance. In this region, then, they can only cut the zero profit line once from above. Therefore c_0^{tb} cannot be supported as a high action equilibrium with n active intermediaries. Nor can it be supported with $n = 1$: a configuration in which only one intermediary sells the active zero profit contract and others offer latent contracts, in fact, is not an equilibrium configuration since the active intermediary could do better, given the equilibrium choice of the others, by charging a higher price to reach a point along (7), associated with positive profits. \diamond

Notice that the arguments in the proofs of Propositions 2 and 3 require that for an open subset of consumption allocations $(c_L, c_H) \in C$,

$$\pi_a u(c_H) + (1 - \pi_a)u(c_L) - u(\pi_b c_H + (1 - \pi_b)c_L) - \Delta \geq 0.$$

In the case in which, on the contrary, this condition holds for a zero measure subset of allocations $c \in C$ (and therefore holds necessarily with equality, by continuity), equilibria which do not satisfy $d^j = \frac{1}{n}|c - w|$, $\forall j \in J_1$ might exist. Moreover, in this case, both equilibria which support low action and equilibria which support high action, with or without positive profits, might exist. This set of economies is non-generic, as it can immediately be proved using perturbations of the parameter Δ .

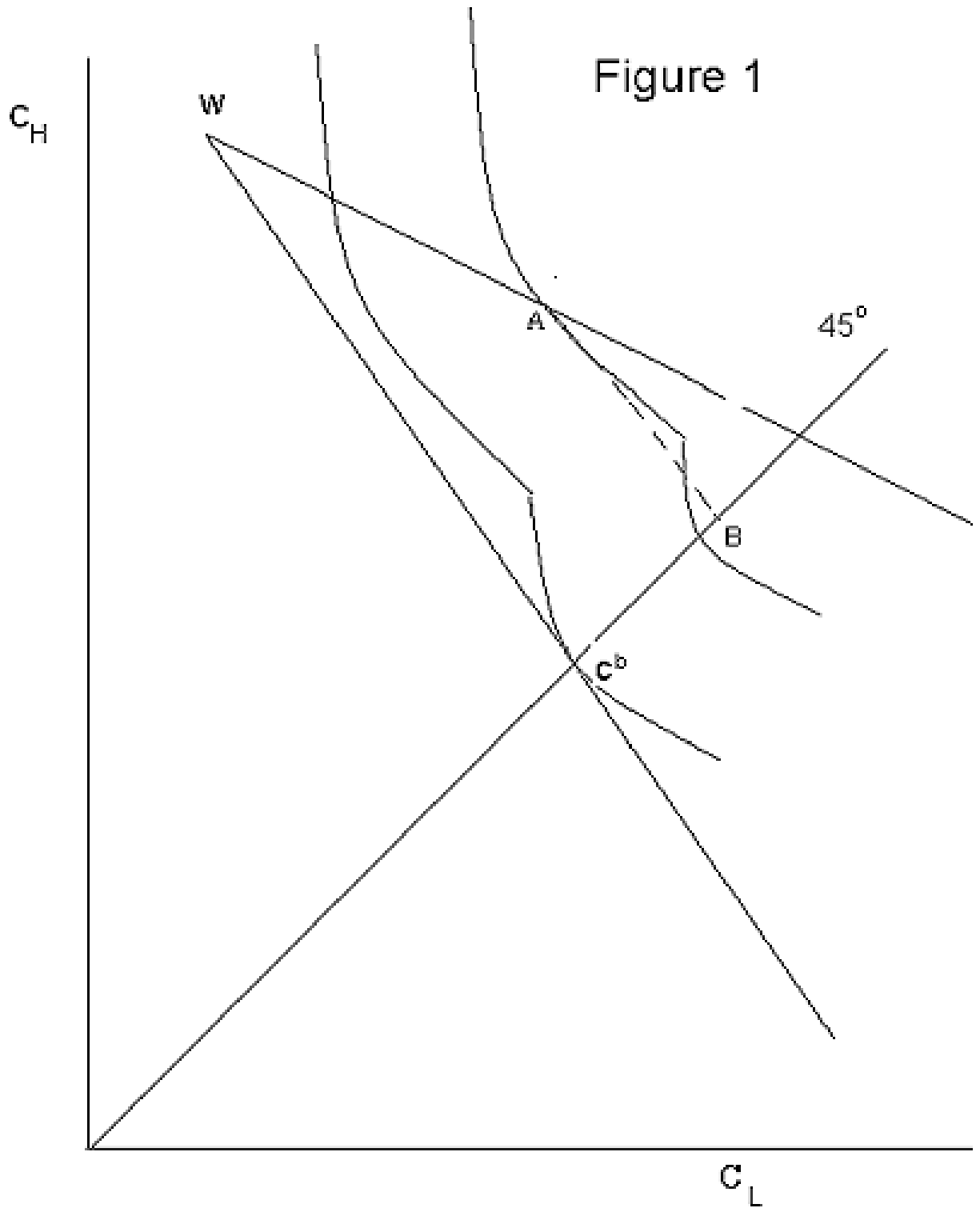
Proof of Proposition 4. Since we have already proved that high effort equilibria lie on (7) (Prop. 2), we just need to show that the incentive constraint (12) is binding at any third-best optimum and the third-best frontier coincides with (7). From Definition 3 we see that a third-best optimum is a point in the incentive compatible set (12) of maximum expected utility for the agents along a given isoprofit line. Isoprofit lines under high effort have a slope (in absolute value) $(1 - \pi_a)/\pi_a$ (parallel to the zero profit line, see (9)-(10)). Agents' marginal rate of substitution (the slope of indifference curves) is always greater than $(1 - \pi_a)/\pi_a$ in the region of underinsurance (hence at all incentive compatible points). Therefore, along any isoprofit line, the point preferred by agents is the one closest to full insurance, so the third-best optimum must lie on the boundary of the incentive compatible set (12) closer to the full insurance set. That boundary obviously coincides with (7). \diamond

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Figure 1



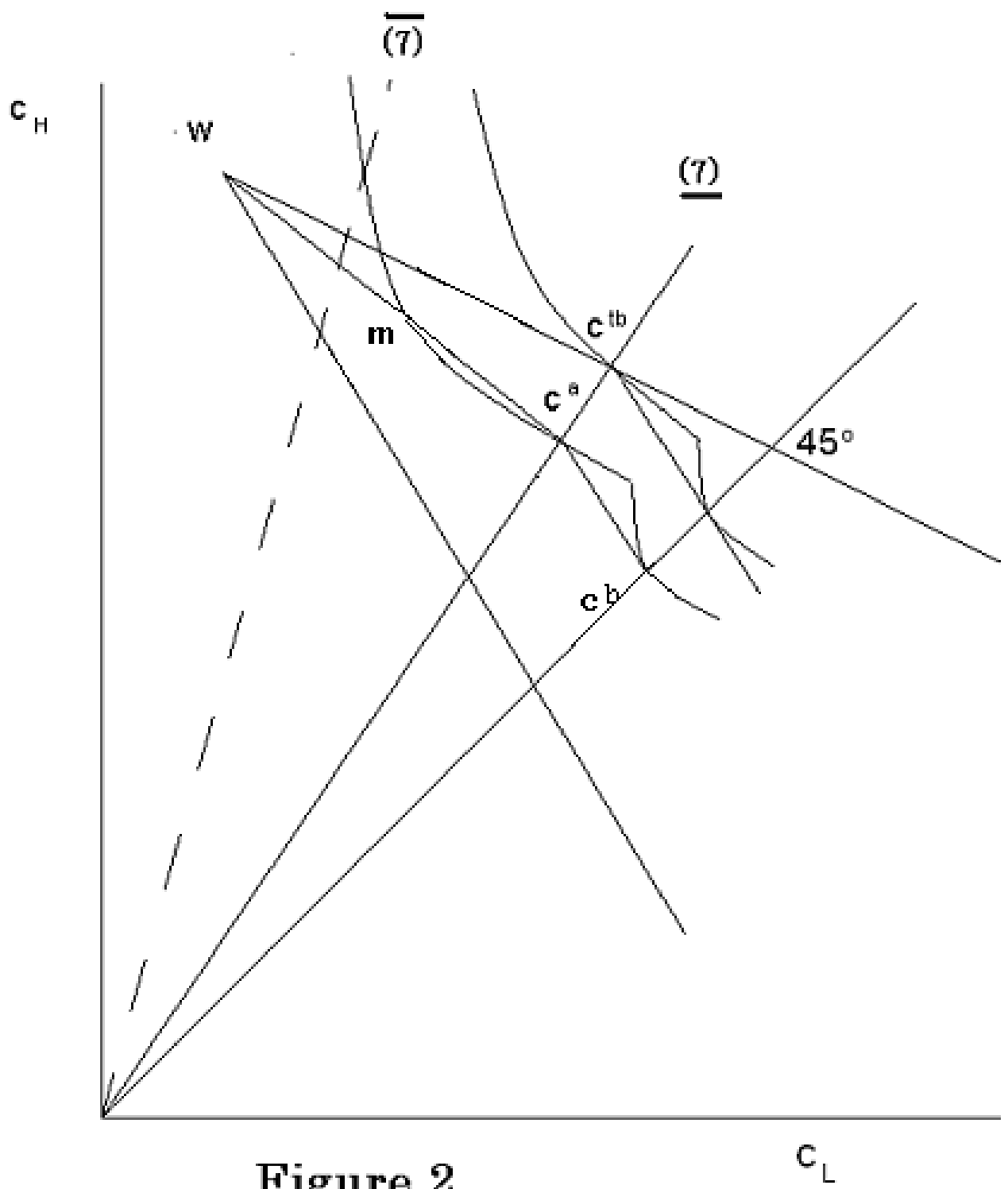


Figure 2

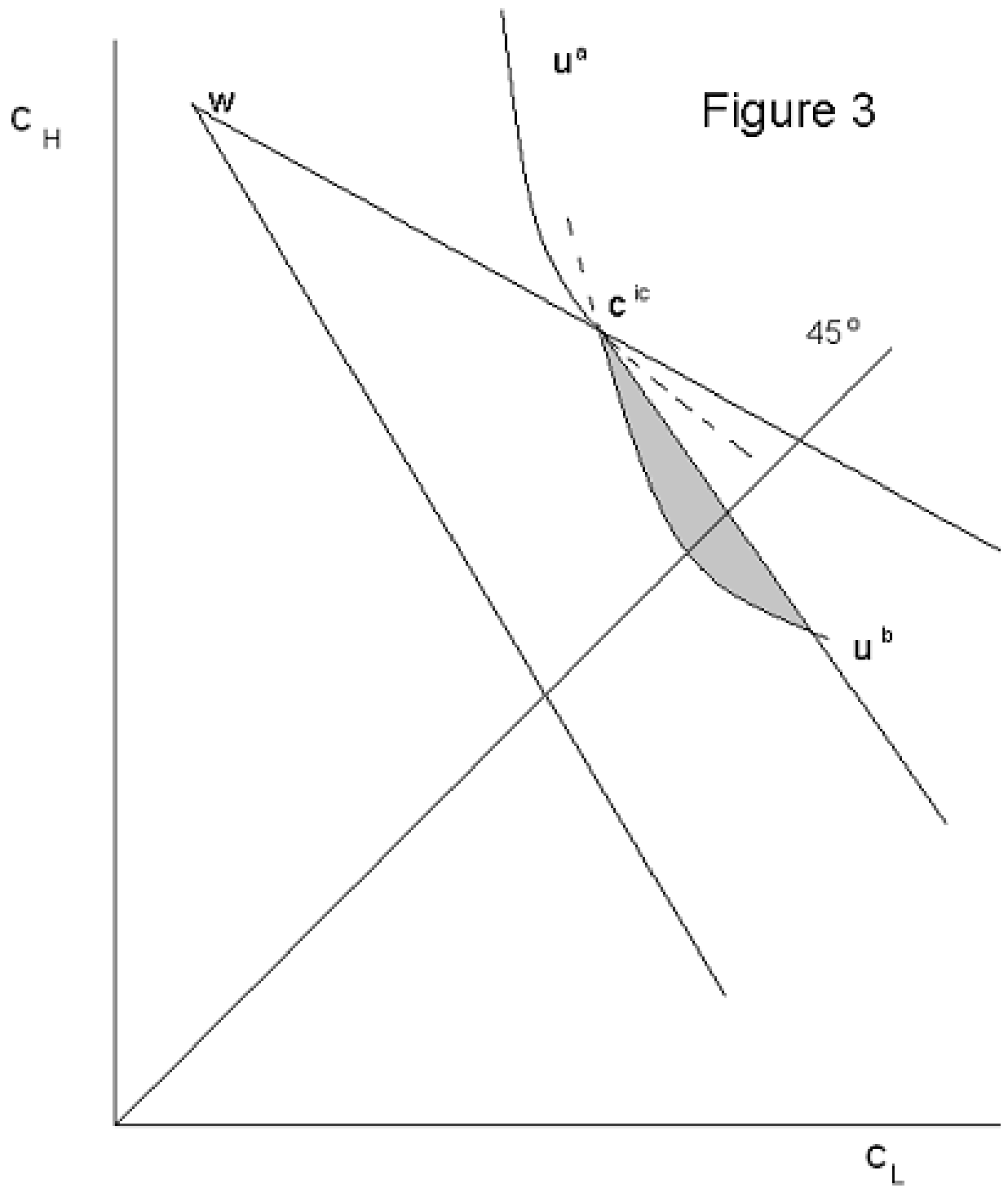
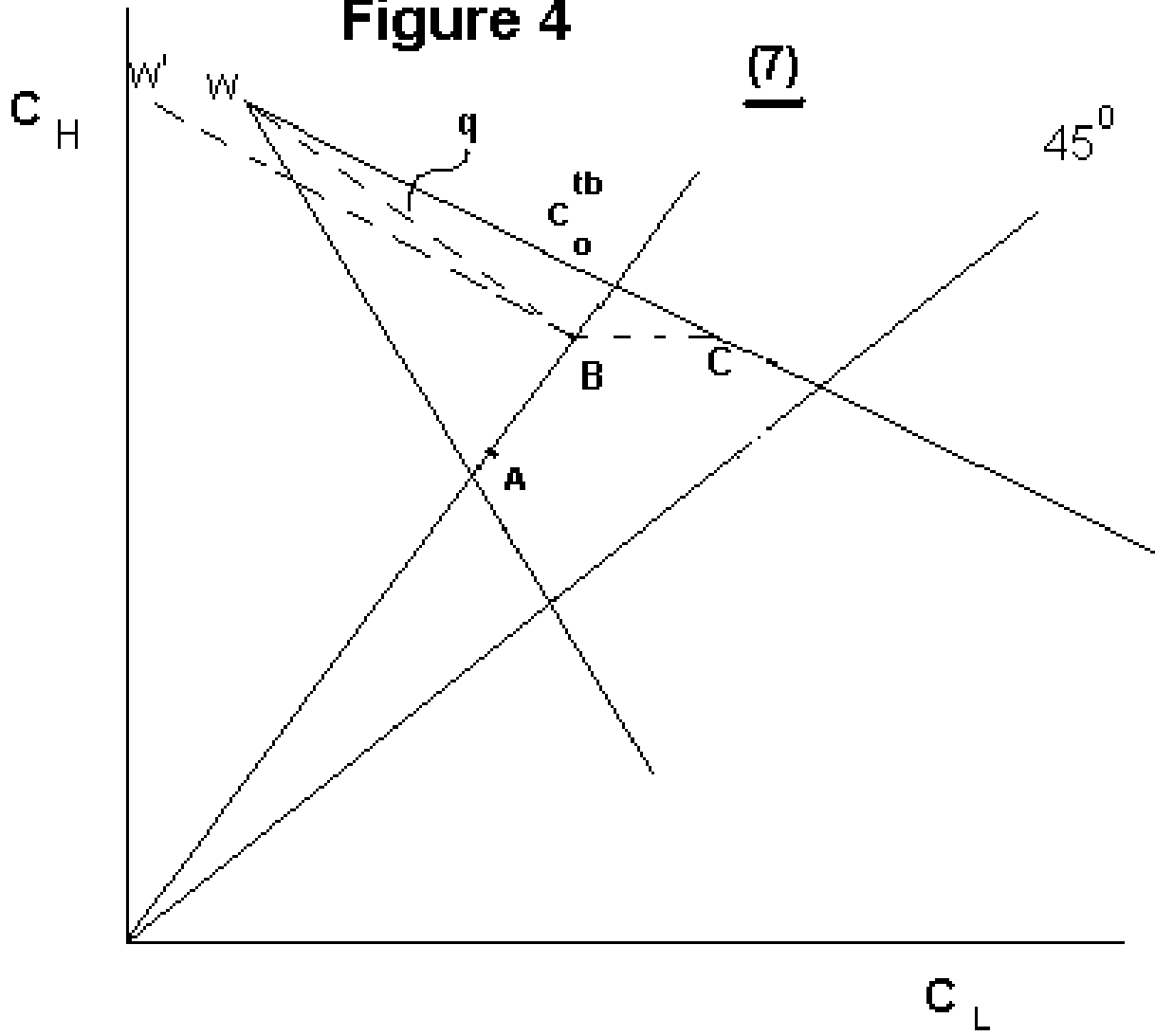


Figure 4



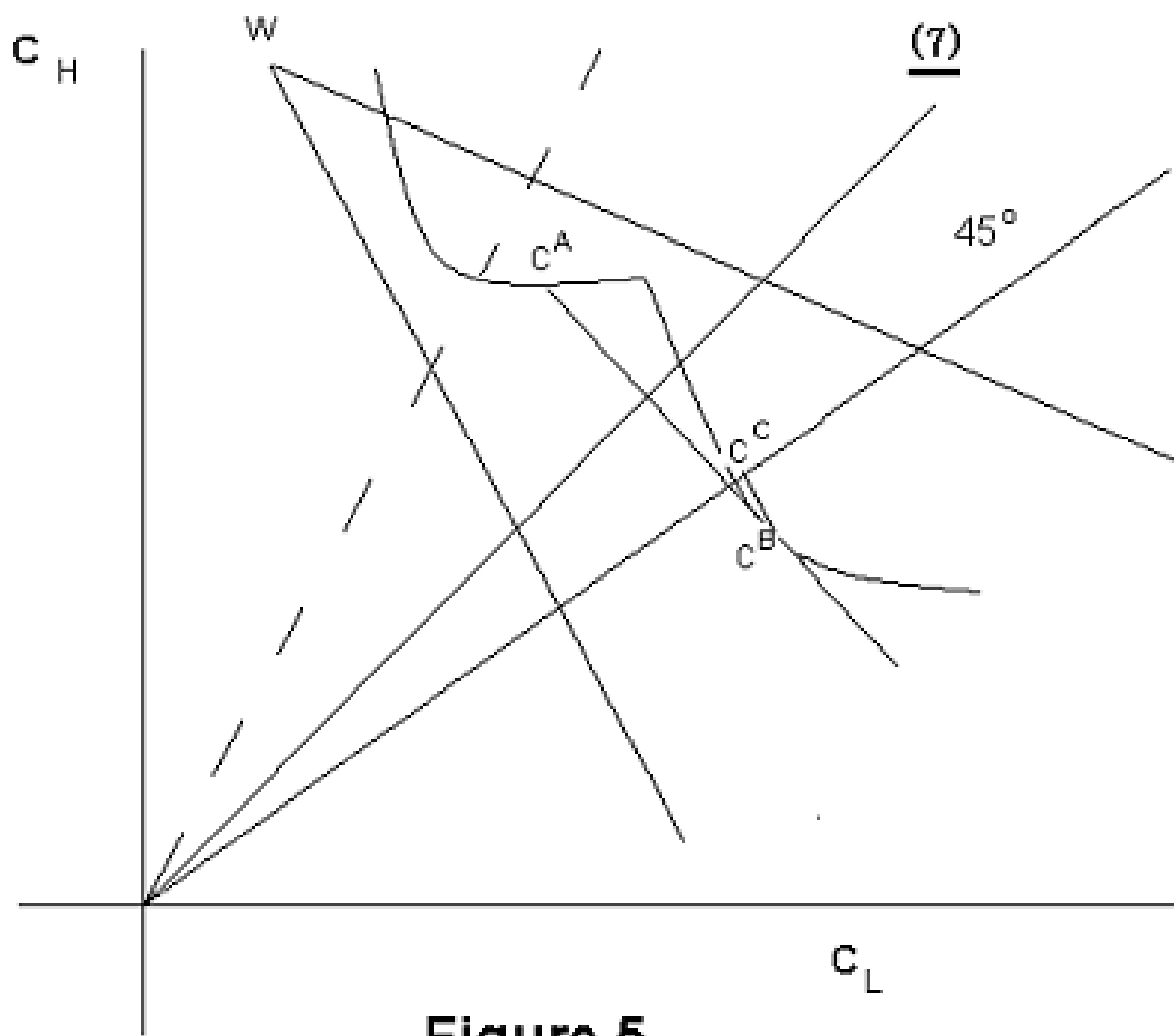


Figure 5

Figure 6

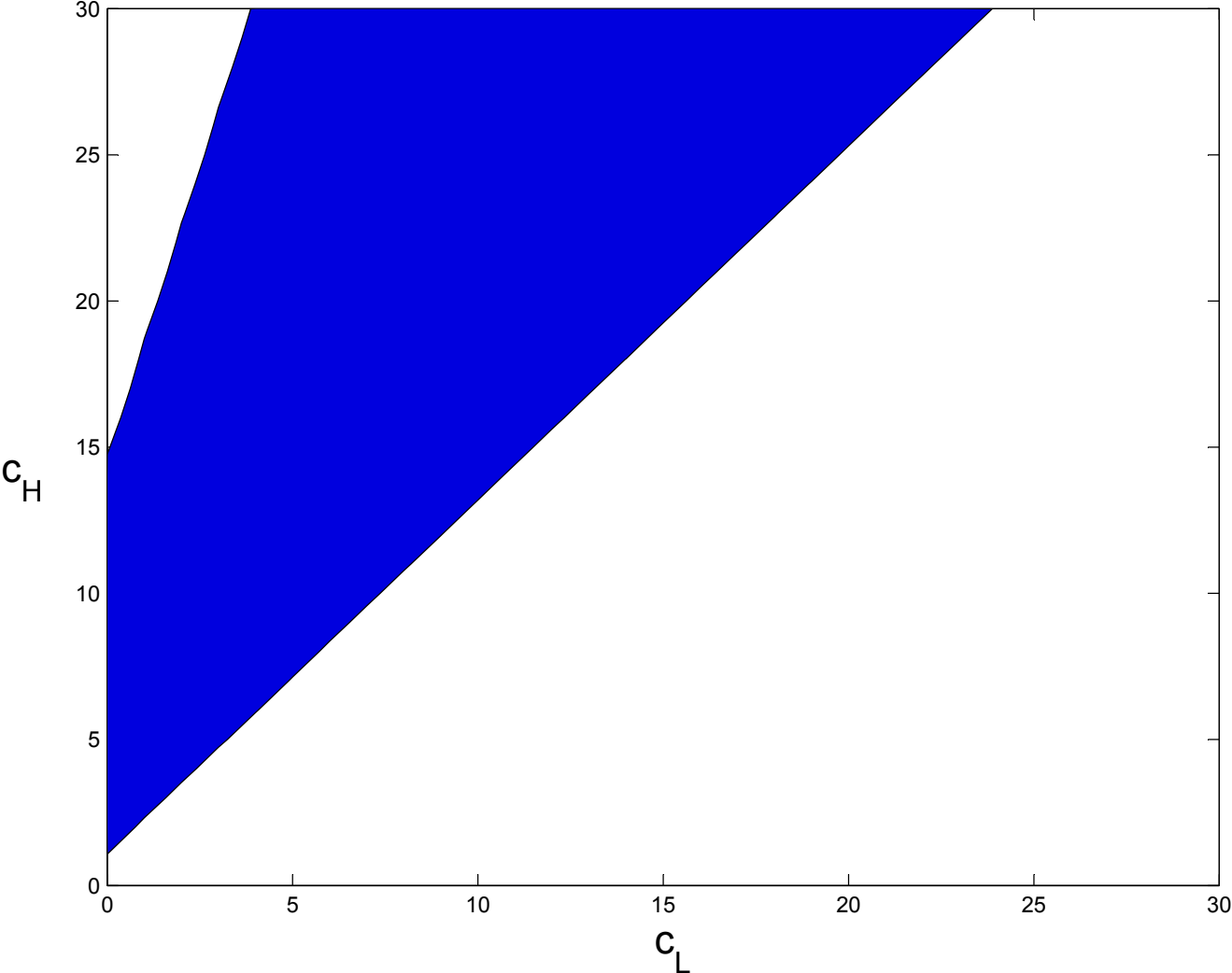


Figure 7

