# Moral Hazard and the US Stock Market: Has Mr Greenspan Created a Bubble? 

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#### Abstract

The current risk premium in the US stock market is far below its historic level and the market continues to rise.. With the long run real interest rate not much, if at all, higher than the growth rate, dividend valuations become very sensitive to variations in the premium: and we show how the latter can be reduced by one-sided intervention policy by the Fed which lulls investors into a false sense of security. We assume investors value their portfolios as if they held a put option, with an exercise price $25 \%$ below the previous market peak. Since the Fed cannot determine the real value of stocks, the resulting asset prices are not rational, so our account involves a degree of myopia and over-optimism on the part of the average investor.

We provide calibrations to demonstrate that the "sliding put" can reconcile booming stock prices with unchanged risk premia. Although there are some good reasons why risk premia may have fallen below the long run average of $7 \%$, by showing the powerful effect that changing perceptions of down-side risk can exert on asset prices, we have strengthened the case for treating current asset valuations with suspicion.


"Beaucoup d'investisseurs ont conclu que les cours boursiers ne peuvent que monter". Blanchard (1999)

## 1. Introduction

Since the stock market break of 1987, shares in the US market have appreciated at a recordbreaking pace. The S \& P 500 index, for example, has increased from about 220 in October 1987 to a little over 1,400 now, an increase of over $500 \%$ or an average annual growth rate of about $17 \%$. This asset price boom implies that relative to the past, estimated growth rates have risen, the risk premium has fallen, or there is a bubble (or some combination of the three).

With the high technology sector as market leader, there has been much discussion of faster than expected growth based on the new communications technology. This is not the focus of this paper. What we do here is to investigate the fall in the risk premium in the US stock market, taking as given the (fairly conservative) growth forecasts of around $3 \%$ made by the economists we cite. In particular, we suggest that the apparent fall in the premium could be a sophisticated asset bubble.

The idea we develop is what Blanchard alludes to in the quotation above from his discussion of market developments before the French Council of Economic Advisers: that many investors are convinced that the market can only go up! Why should this be so? And how could it affect the market?

The reason, we suggest, is a form of moral hazard. Investors in the US have become convinced that the Federal Reserve will take decisive action to prevent the market falling but not to stop it rising: and
they believe that these actions will work. So the Fed is apparently insuring them. The effect is like a put: but the reality is a bubble, because the put will not exist when it comes to be exercised.

Key pieces of evidence are the actions taken by Mr Greenspan in halting the market break of 1987 and in checking the market fall in the liquidity crunch of 1998, in both cases by cutting interest rates and pumping in liquidity. The monetary authority cannot control the real interest rate in the long run, but it can over the short run when prices and inflation expectations are sticky. So it can affect share prices, at least for a while. By correcting one crash and averting another Mr Greenspan has done enough to persuade investors that he will stabilise the market long enough for them to get out, keeping the gains they have made to date. If they all sell, the market will crash: so the logic is fallacious. But the gains are tempting and even Mr Greenspan is beginning to talk of a new paradigm for the US economy.

Historically the risk premium has been estimated to lie between $7 \%$ and $8 \%$. But consider the two estimates of the risk premium in the US in 1999 shown in Table 1. These are arrived at by simply subtracting the risk free real interest rate from the total yield (dividend yield plus growth). In his comments to the French CEA (between April and June), Blanchard put the premium at $2 \%$ in the second quarter of 1999. A few months later, in September, Wadhwani, a member of Britain's Monetary Policy Committee who had written a widely cited paper on the US bubble in 1998, reckoned the premium was down to only $1 \%$. As can be seen from the table the reason for the one point lower

| Dividend/price <br> Ratio (\%) | Dividend growth <br> Rate (\%) | Real interest <br> Rate | Risk premium <br> $(\%)$ | Author |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |


| 2 | 3 | 3 | 2 | Blanchard <br> (Q2,1999 |
| :--- | :--- | :--- | :--- | :--- |
| 2.15 | 2.85 | 4 | 1 | Wadhwani <br> (Q3, 1999) |

Table 1 Two recent estimates of equity risk premium in the US stock market

In his comments, Blanchard acknowledged that there are good reasons for the risk premium being lower than in the past, but thought $2 \%$ is too low and expressed the view that there is a bubble, arising partly from overestimating growth, partly from pure extrapolative pricing expectations. In his NIER paper, Wadhwani (1998, see also Financial Times, 18-Sep-1999) also dismissed the idea that the risk premium had virtually disappeared. He noted that if you put the premium at $3 \%$, and keep the growth figures this gives estimated over-valuation of about $100 \%$ (i.e., a fall of $50 \%$ ).

Before developing our explanation, we summarise the lively debate on the equity risk premium in the next section of the paper. Then we develop our account of the bubble, namely that the asymmetric behaviour of the monetary authorities has established a floor to market prices, but no ceiling: and that this floor ratchets up whenever the market reaches a new peak. More precisely, stocks are priced as if market participants were in possession of an undated put with an exercise price $25 \%$ below the last peak. The idea of monetary intervention having price effects like the issue of derivatives is familiar from the work of Paul Krugman (1991) on "target zones" for exchange rates: this is a one sided target zone for the stock market. While Krugman's target zone for the nominal rate depended on the authorities having enough reserves, a perceived floor on the real price of stocks requires gullibility and myopia of the part of the
average investor. Linking the exercise price to past peaks is a feature of stock market trading rules explored by Grossman and Zhou (1993); in circumstances where there is a positive trend in fundamentals it adds greatly to the value of a put.

We show theoretically how the perceived put raises prices and reduces the implied risk premium: and we prove there exists a unique cone that market prices are restricted to, suspended well above their fundamental value. Then we calibrate the model using parameters from the table above and the risk premium set at its post-war average. We findthat a sliding put allowing for a $25 \%$ fall could bring estimated risk premia down from $7 \%$ to $2 \%$, i.e., that it could account for the current low values reported above, even though the underlying parameters are at their historical values. The implication is that the market has a long way to fall when fundamentals drag prices far enough down for the average investor to try exercising. With overvaluation of over $100 \%$, the fall is bigger than $50 \%$. (These results depend on the put being completely credible, and will be less dramatic as this assumption is weakened.)

Before concluding, we suggest how to reconcile our approach with the price of puts on the market - which notoriously charges a lot for puts that are far out of the money. Those buying and selling puts do not share the popular view, but are not big enough to change it.

## 2. The Equity Premium

The essence of the equity premium puzzle identified by Mehra and Prescott (1985) is that, in a representative agent asset pricing model, it is necessary to assume an implausibly high degree of risk aversion in order to reproduce the historical level of the premium. The reason for this is that the risk premium in such a model is determined by the covariance between consumption growth and the return on
the stock market multiplied by the coefficient of relative risk aversion. Since consumption growth has an annual standard deviation of about one per cent, the covariance is small, and this translates into a large value for the measure of risk aversion if one is to match the value of the equity premium in post-war US data. Campbell (1998) reports a value of $7.85 \%$ for the period 1947-96.

There have been numerous attempts to explain the equity premium. We group them into five broad categories, which are not necessarily mutually exclusive. The first category contains models that aim to make the assumption of high risk aversion more plausible. Campbell and Cochrane (1999) construct a model with time-varying risk aversion driven by habit persistence. When consumption falls close to the level of the habit, for example in recession, individuals become highly risk averse. But in periods of expansion risk aversion falls. One of the attractive features of this model is its ability to match a number of other features of the data for which the standard model fails. Hansen, Sargent and Tallarini (1997) describe an economy in which the representative agent is assumed to be ignorant of the true model generating stock prices. They specify a form for the utility function in which high risk aversion can alternatively be interpreted as a preference for robustness to small specification errors.

The second category argues that the objective uncertainty of stock market returns is greater than is revealed in the sample data. Rietz (1988) shows that introducing a small probability of a large negative shock to consumption growth is sufficient to explain the premium. The probability can be made sufficiently small that there would be little chance of observing such a shock even in data spanning a century. Brown, Goetzmann and Ross (1995) observe that in the past history of some major markets other than the US Russia, China, Germany and Japan - there have been one or more major interruptions that lead to their being excluded from long term studies of stock returns. This form of sample selection bias may lead to a
substantial underestimation of the true risk of the stock market, a risk that is correctly perceived by investors.

The third category introduces various departures from strict or narrow rationality. In Kurz and Motolese (1999) the presence of endogenously determined differences of opinion consistent with a weaker notion of rationality can account for the premium. Cecchetti, Lam and Mark (1998) modify an otherwise standard Lucas asset pricing model by assuming that individuals misperceive the persistence of expansions and contractions. They show that if individuals believe that both expansions and contractions are less persistent than is revealed by the data, and if these beliefs exhibit random variation over time, then one can reproduce the level and volatility of the equity premium.

The fourth category introduces heterogeneity among investors. Constantinides and Duffie (1996) construct a framework in which there is cross-sectional variation in consumption. They show that if the cross-sectional variance of log consumption growth is negatively correlated with the level of aggregate consumption, so that individual consumption risk increases in recessions, this can help explain the excess returns to stocks without invoking high levels of risk aversion. The fifth category introduces frictions. Heaton and Lucas (1996) and Krusell and Smith (1997) find that it is necessary to postulate large costs of trading equities or borrowing constraints to explain the equity premium. Marshall and Parekh show that very small fixed costs of adjusting non-durable consumption are capable of explaining some, but not all of the equity premium. Their finding is explained by the fact that at the optimum the utility gains from adjusting consumption in response to changes in asset returns are small.

Some have argued that the standard model can be used to rationalise the current market valuation. In the light of the research summarised above, this seems unconvincing. Although no single
model provides a fully satisfactory explanation for the equity premium on its own, many supply persuasive arguments as to why one should not expect the standard model to match the data. In addition, for the standard model to rationalise current stock valuations it is not sufficient simply to observe that the level of the market is now consistent with a plausible level of risk aversion. The model has to explain why there has been a precipitous drop in the level of risk aversion over the space of a few years, and why this phenomenon has been largely confined to the US. This it conspicuously fails to do.

The habit persistence model of Campbell and Cochrane (1999) can produce periods during which the price-dividend ratio is high, but a necessary condition for this is high consumption growth in the recent past. This does not characterise the US experience over the last decade. Models such as that of Rietz (1988) could in principle tell a story in which agents suddenly come to believe that the risk of a serious market collapse no longer exists. However, in a world of rational learning, changes in beliefs are generally rather gradual. The sudden change is difficult to rationalise other than in an irrational world.

## 3. The Model

We consider the problem facing a representative investor who can trade an asset which pays dividends at the rate $D(t) d t$. Dividends are assumed to evolve according to:

$$
\begin{equation*}
\frac{d D}{D}=\mu_{D} d t+\sigma_{D} d z \tag{1}
\end{equation*}
$$

where $\mu_{D}$ is the trend, $z$ is a standard Brownian motion and $\sigma_{D}$ the standard deviation. The price of the asset, $V(\cdot)$, then follows a diffusion process:

$$
\begin{equation*}
\frac{d V}{V}=\mu(.) d t+\sigma(.) d z \tag{2}
\end{equation*}
$$

The notation (.) indicates that both drift and volatility can be functions of the state variable $D(t)$. The instantaneous total return on the asset is given by:

$$
\begin{equation*}
\frac{d V}{V}+\frac{D}{V} d t \tag{3}
\end{equation*}
$$

where the first term indicates capital gain and the second the dividend rate.
The utility of the investor is:

$$
\begin{equation*}
E_{0} \int_{0}^{\infty} u[c(t)] e^{-\delta t} d t \tag{4}
\end{equation*}
$$

where $E_{0}$ is the expectations operator conditional on time $0, u(\cdot)$ is the instantaneous utility function, $c(t)$ the consumption and $\delta$ the rate of time preferences. If we define a stochastic discount factor as

$$
\begin{equation*}
M(t)=e^{-\delta t} u^{\prime}[c(t)], \tag{5}
\end{equation*}
$$

then the investor's Euler equation in continuous time takes the form of

$$
\begin{equation*}
0=M D d t+E_{t}[d(M V)] . \tag{6}
\end{equation*}
$$

This can be rewritten as

$$
\begin{equation*}
0=\frac{D}{V} d t+E_{t}\left[\frac{d M}{M}+\frac{d V}{V}+\frac{d M}{M} \frac{d V}{V}\right] \tag{7}
\end{equation*}
$$

If we apply the above equation to a risk free asset, we obtain an expression for the risk free rate of interest:

$$
\begin{equation*}
r^{f} d t=-E_{t}\left[\frac{d M}{M}\right] \tag{8}
\end{equation*}
$$

Substituting (8) into (7) yields:

$$
\begin{equation*}
E_{t}\left[\frac{d V}{V}\right]+\frac{D}{V} d t=r^{f} d t-E_{t}\left[\frac{d M}{M} \frac{d V}{V}\right] . \tag{9}
\end{equation*}
$$

The last term on the right hand side represents the risk premium.
We assume that the utility function has the form:

$$
\begin{equation*}
u[c(t)]=\frac{c(t)^{1-\gamma}}{1-\gamma} \tag{10}
\end{equation*}
$$

and that consumption evolves as:

$$
\begin{equation*}
\frac{d c}{c}=\mu_{C} d t+\sigma_{C} d z \tag{11}
\end{equation*}
$$

Treating $V$ as a function of dividends and applying Ito's Lemma gives the following expression for the risk premium.

$$
\begin{equation*}
-E_{t}\left[\frac{d M}{M} \frac{d V}{V}\right]=\gamma \rho \sigma_{C} \sigma_{D} \frac{V^{\prime} D}{V} \tag{12}
\end{equation*}
$$

where $\rho$ is the correlation between consumption growth and dividend growth, and the prime indicates derivatives.

Applying Ito's Lemma to (9) and substituting in the expression in (12) gives

$$
\begin{equation*}
\frac{1}{2} \sigma_{D}^{2} D^{2} V^{\prime \prime}(D)+\left(\mu_{D}-\gamma \rho \sigma_{C} \sigma_{D}\right) D V^{\prime}(D)-r^{f} V(D)+D=0 . \tag{13}
\end{equation*}
$$

This equation has the linear solution

$$
\begin{equation*}
V(D)=\frac{D}{r^{f}-\mu_{D}+\gamma \rho \sigma_{C} \sigma_{D}} . \tag{14}
\end{equation*}
$$

This is just a continuous time version of the familiar Gordon growth model formula. In fact, one can derive equation (14) by treating the asset price, $V(D)$, as the expected present value of all current and future dividends discounted by the risk adjusted rate of $r=r^{f}+\gamma \rho \sigma_{c} \sigma_{D}$, i.e.,

$$
\begin{equation*}
V(D)=E_{0} \int_{0}^{\infty} D(t) e^{-r t} d t=\frac{D}{r^{f}-\mu_{D}+\pi}, \tag{15}
\end{equation*}
$$

where $\pi=\gamma \rho \sigma_{c} \sigma_{D}$ is the true risk premium, as distinct from the risk premium in (12) which is influenced by false investor beliefs about the effects of intervention by the Federal Reserve. However, we are interested in a class of non-linear solutions to (13) that arise as a consequence of such beliefs about intervention. Specifically, we interpret $V(\cdot)$ as the value of the market portfolio and suppose that investors believe that the Federal Reserve will adjust monetary policy to support the market whenever it has fallen to a level of ? times its previous maximum level. In what follows, we show how a belief in the effectiveness of such a policy can create stock price bubbles.

## 4. Asymmetric Monetary Policy, Moral Hazard And Stock Price Bubbles

Assume that monetary policy is conducted in such a way that real rates are unchanged when stocks rise, but that nominal (and short run real) interest rates are cut whenever stock prices fall to a fraction $\eta$ of the previous market peak. If investors expect that this will stabilise prices for long enough to exit the market, it is as if they have free put options insuring them against downside risks. With this asymmetry of monetary policy built into expectations, stock prices will be substantially inflated. In this sector we characterise these bubble solutions.

Let the maximum value of the market up to time $t$ be

$$
\begin{equation*}
\bar{S}_{t}=\left\{\operatorname{Max}\left\{V\left(D_{\tau}\right)\right\}, \tau \leq t\right\} . \tag{16}
\end{equation*}
$$

If the stock price lies in the range $\left(\eta \bar{S}_{t}, \bar{S}_{t}\right)$, then its value is determined by equation (13), with general solution as follows

$$
\begin{equation*}
V(D)=\frac{D}{r^{f}-\mu^{\prime}}+A_{+}\left(\frac{D}{D_{p}}\right)^{\xi_{+}}+A_{-}\left(\frac{D}{D_{p}}\right)^{\xi_{-}} \tag{17}
\end{equation*}
$$

where $\mu^{\prime}=\mu_{D}-\pi, D_{p}$ is the dividend level at $\bar{S}_{t}, A_{+}$and $A_{-}$are two constants to be determined, and $\xi_{+}$and $\xi_{-}$are the positive and negative roots of the quadratic equation

$$
\begin{equation*}
\frac{1}{2} \sigma_{D}^{2} \xi(\xi-1)+\mu \xi-r^{f}=0 . \tag{18}
\end{equation*}
$$

It can be shown that $\xi_{+}>1$ and $\xi_{-}<0$. Deflating dividends by $D_{p}$ in the second and third terms on the right hand side of (17) simplifies the subsequent algebra but does not affect the solution.

If stock prices reach $\eta \bar{S}_{t}$, stabilisation is assumed to occur. This implies that the following value matching and smooth pasting conditions must hold:

$$
\begin{align*}
& V\left(D_{b}\right)=\eta \bar{S},  \tag{19}\\
& V^{\prime}\left(D_{b}\right)=0, \tag{20}
\end{align*}
$$

where $D_{b}$ is the lower dividend level corresponding to the level of stock prices at which investors believe the market will be stabilised.

At a market peak, no change of policy is expected. The definition of $D_{p}$ implies that

$$
\begin{equation*}
V\left(D_{p}\right)=\bar{S} . \tag{21}
\end{equation*}
$$

If dividends move above $D_{p}$, then a new market peak is attained and the solution in (17) is revised upwards conditional on this new peak. If dividends move below $D_{p}$, then stock prices will be determined by the solution to (17). Let all peaks be given by the envelope

$$
\begin{equation*}
\bar{S}=f\left(D_{p}\right) \tag{22}
\end{equation*}
$$

The boundary condition at the peak implies that

$$
\begin{equation*}
V^{\prime}\left(D_{p}\right)=f^{\prime}\left(D_{p}\right) . \tag{23}
\end{equation*}
$$

Then we can state the following result.

Proposition For $0 \leq \eta<\eta_{\text {max }}<1$, there exists a unique solution $\alpha \geq 0$ such that

$$
\begin{equation*}
f\left(D_{p}\right)=\frac{1+\alpha}{r^{f}-\mu^{\prime}} D_{p}, \tag{24}
\end{equation*}
$$

The solution satisfies the following conditions:

$$
\begin{equation*}
\frac{\partial \alpha}{\partial \eta}>0 \tag{25}
\end{equation*}
$$

and

$$
\begin{align*}
& \lim _{\eta \downarrow 0} \alpha(\eta)=0 \\
& \lim _{\eta \uparrow \eta_{\max }} \alpha(\eta) \rightarrow+\infty \tag{26}
\end{align*}
$$

Proof: See Appendix A.

One way of understanding the linearity of the envelope established in the proposition is to introduce the transformation $y=D / D_{p}$. Then we see from (13) that $v(y)=V\left(D / D_{p}\right) / D_{p}$ satisfies the same
equation. It is natural to conjecture that the boundary condition for $v(y)$ will be homogeneous of degree zero with respect to boundary values of $y$. This suggests that the envelope must be a linear function of $D_{p}$.

Since the envelope for all peaks is given by (24), the envelope for all stabilisation points will also be a linear function. Specifically, as

$$
\begin{equation*}
\eta \bar{S}=\frac{\eta(1+\alpha) D_{p}}{r^{f}-\mu^{\prime}}=\frac{\eta}{x^{*}} \cdot \frac{(1+\alpha) D_{b}}{r^{f}-\mu^{\prime}}, \tag{27}
\end{equation*}
$$

so the envelope for all stabilisation points is

$$
\begin{equation*}
e\left(D_{b}\right)=\frac{\eta}{x^{*}} \cdot \frac{(1+\alpha) D_{b}}{r^{f}-\mu^{\prime}}, \tag{28}
\end{equation*}
$$

where $x^{*}$ is the solution to (A9) in Appendix A and $\eta / x^{*}>1$ as shown in Appendix B.

These two envelopes form a cone within which all solutions conditional on a given value of $D_{p}$ will lie. We use Figure 1 to illustrate one of these solutions. The fundamental solution as in (15) is shown as the lowest straight line from the origin. The two envelopes, which form the cone, are given by $f\left(D_{p}\right)$ and $e\left(D_{b}\right)$ for all peaks and all stabilisation points respectively. The solution for the stock price conditional on $D_{p}$ in (17) is represented by the convex curve $V$, which smooth pastes to a horizontal line at the bottom and smooth pastes to the envelope $f\left(D_{p}\right)$ at $D_{p}$. All other short run solutions will resemble $V$.

As the conditional solutions are flat at the stabilisation points and steadily rise towards peaks, the stock price volatility is lower when the stock price is lower and increases as the stock price rises. (Note that the instantaneous variance of the stock price depends on the slope of the conditional solution.)


Figure 1. Asymmetric monetary policy, moral hazard and stock price bubbles.

We see from (12) that our model predicts a risk premium which depends on $D$ and $D_{p}$. We proceed to calculate the upper and lower bounds occurring at $D_{b}$ and $D_{p}$. Using the formula in (15), one can show that the implied risk premium at peaks is given by

$$
\begin{equation*}
\pi_{p}=\frac{\pi-\alpha\left(\mu_{D}-r^{f}\right)}{1+\alpha} \tag{29}
\end{equation*}
$$

This gives the upper bound for the implied market risk premium. The lower bound is derived using the envelope for stabilisation points which gives

$$
\begin{equation*}
\pi_{b}=\frac{\pi+\left[1-(1+\alpha) \eta / x^{*}\right]\left(r^{f}-\mu_{D}\right)}{1+\alpha} \cdot \frac{x^{*}}{\eta} . \tag{30}
\end{equation*}
$$

The over-valuation at market peaks is given by $\alpha$, while at stabilisation points it is $(1+\alpha) \eta / x^{*}-1$. In the next section, we use numerical method to look at these measures.

## 5. Numerical Results

The parameter values we use for the base line case below are as follows: the real interest rate $r^{f}=0.03$, the true risk premium $\pi=0.07$, the dividend growth rate $\mu_{D}=0.03$, the volatility of stock prices $\sigma=0.2$. Stabilisation is assumed to occur when stock prices are $25 \%$ below the previous peak, so $\eta=0.75$. To examine the sensitivity of the results to our choice of parameter values we vary the real interest rate from 0.02 to 0.04 , the risk premium from 0.06 to 0.08 , and the volatility of stock prices from 0.15 to 0.25 . The numerical results below show the maximum possible value for $\eta, \eta_{\max }$; the implied upper and lower bounds of risk premia, $\pi_{p}$ and $\pi_{b}$, and the over-valuation of stock prices at peaks.

| Risk premium | Maximum <br> Stabilisation $\eta_{\max }$ | Upper bound <br> $\pi_{p}$ | Lower bound <br> $\pi_{b}$ | Over-valuation <br> $\alpha$ |
| :--- | :--- | :--- | :--- | :--- |
| $6 \%$ | 0.83 | $1.85 \%$ | $1.26 \%$ | 2.24 |
| $7 \%$ | 0.84 | $\mathbf{2 . 4 6 \%}$ | $\mathbf{1 . 6 1 \%}$ | $\mathbf{1 . 8 5}$ |
| $8 \%$ | 0.86 | $3.04 \%$ | $1.89 \%$ | 1.63 |

Table 2: The Effect of Changing Stock Price Risk Premium ( $r^{f} 0.03, \sigma=0.2, \mu=0.03$ ).

| Price <br> Volatility $\sigma$ | Maximum <br> Stabilisation $\eta_{\max }$ | Upper bound <br> $\pi_{p}$ | Lower bound <br> $\pi_{b}$ | Over-valuation <br> $\alpha$ |
| :--- | :--- | :--- | :--- | :--- |
| 0.15 | 0.9059 | $3.83 \%$ | $2.33 \%$ | 0.8298 |
| 0.20 | 0.8429 | $2.46 \%$ | $1.61 \%$ | 1.8457 |
| 0.25 | 0.7725 | $0.65 \%$ | $0.44 \%$ | 9.8043 |

Table 3. Effects of Changing Stock Price Volatility ( $r^{f}=0.03, \pi=0.07, \mu=0.03$ ).

| Interest rate <br> $r^{f}$ | Maximum <br> Stabilisation $\eta_{\max }$ | Upper bound <br> $\pi_{p}$ | Lower bound <br> $\pi_{b}$ | Over-valuation <br> $\alpha$ |
| :--- | :--- | :--- | :--- | :--- |
| $2 \%$ | 0.83 | $2.92 \%$ | $2.14 \%$ | 2.64 |
| $3 \%$ | 0.84 | $2.46 \%$ | $1.61 \%$ | 1.85 |
| $4 \%$ | 0.85 | $\mathbf{1 . 9 2 \%}$ | $\mathbf{1 . 0 2 \%}$ | $\mathbf{1 . 4 0}$ |

Table 4. Effects of Changing Real Interest Rate ( $\pi=0.07, \sigma=0.2, \mu=0.03$ ).

Tables 2-4 illustrate how changing the risk premium, the volatility of stock prices and the real interest rate affect implied risk premia and stock price over-valuations. In almost all cases, a stabilisation rule implying that intervention occurs when stock prices have fallen by $25 \%$ from their previous peak can reduce the historical risk premium by more than half.

Table 2 shows how changing the true risk premium affects the implied risk premium. The third and the fourth columns show that the implied risk premium goes up with the true risk premium, while the degree of stock price over-valuation goes down. Since higher $\pi$ means lower $\alpha$, from (29) both of these two effects increase the implied risk premium. Table 3 illustrates the impact on the results when stock price volatility is increased. Higher volatility means that, for a given stabilisation rule, the insurance value is higher. This pushes up $\alpha$ and so reduces the implied risk premium.

Table 4 shows the effect of changing the real interest rate. Increasing the real interest rate decreases the value for $\alpha$, which translates into a higher implied risk premium. But a higher interest rate has a direct negative effect on the implied risk premium. The simulation results in Table 4 show that this direct effect dominates.

Note finally that the numerical results are consistent with the risk premia estimated in Table 1. The highlighted row in Table 2 shows a case similar to that of Blanchard (1999). With the same real interest rate and dividend growth rate as in Blanchard, the average of the two implied risk premia from our simulation is $2 \%$, exactly the same as Blanchard's estimate.

## 6. Why Are Out-Of-Money Puts So Expensive?

If we look to the options market for support for our thesis, we face an obvious problem. It is well known that implied volatility for put options is higher for low values of fundamentals. At first sight, this appears to refute the model we are proposing. If the Federal Reserve is believed to be insuring asset prices for free, why should private insurance be so expensive? With fully rational investors option prices do indeed reflect true risk-neutral probabilities. But we are describing a world inhabited by a majority of
investors who have one specific irrational belief, namely that the Fed has the power to stabilise the market once it has fallen by more than a certain amount. Sophisticated investors do not believe this. Thus they believe that the market is overvalued and have an incentive to buy crash insurance. This insurance is sold by other sophisticated investors who price the puts accordingly. Of course, the group of sophisticated investors has to be in a sufficiently small minority that their trades in stocks do not have a significant impact on the market. If, as one presumes should be the case, some portfolio managers are included in this group, we can appeal to the arguments of Shleifer and Vishny (1997) for an explanation as to why their beliefs are not reflected in market prices. Fund managers are discouraged from taking large bets against the market because they know that in the short run if prices move against them i.e. further from fundamentals, they will suffer cash withdrawals and be less able to exploit what is now a more favourable investment situation. This still leaves open the question of why the irrational investors do not sell put options which they consider to be overpriced. We suggest that there are several reasons for this. Many such investors delegate the task of portfolio management to mutual fund and pension fund managers, who are tightly restricted in their trading of derivatives. These managers may also not believe in the power of the Fed. In addition, options are more complex financial instruments than stocks, whose returns are much less transparent. It should also be remembered that we are describing a situation that cannot last forever, and that is likely to be rather short-lived. By the time the average investor has realised that his belief in the power of the Fed implies unexploited profit opportunities in the options market, those opportunities will probably no longer exist.

## 7. Conclusion

Both economists cited in the Introduction assume that the long run real interest rate ( 3 to 4 percent) is not much, if at all, higher than the growth rate (say 3 percent). This means that dividend valuation is extraordinarily sensitive to the estimated risk premium: and we have shown how the risk premium can be reduced by one-sided intervention policy by the Fed which lulls people into a false sense of security. Since the Fed cannot determine the real value of stocks, the resulting asset prices are not rational, and our account admittedly involves a degree of myopia and over-optimism on the part of the average investor. Myopia is needed so that temporary changes can be treated as long-lasting, and (with short run cuts in real rates treated as persistent) the power of the monetary authorities over asset prices is exaggerated. But even if the Fed cannot hold rates down for ever, could it not stabilise prices long enough for you to get out first? Over-optimism is needed for the average investor to believe that.

The calibrations demonstrate that our account could reconcile booming stock prices with very high underlying risk premia. But we do not, in fact, want to claim that it is only mistaken beliefs about monetary policy that explain current high valuations. There are, as Blanchard remarked, some good reasons why risk premia may have fallen below the long run average of $7 \%$-- better stabilisation of the economy ("fine-tuning") and more efficient distribution of risk ("financial engineering") being two. And, if investors are myopic, they can extrapolate short run surges of growth into the long run (as seems to be true of internet stocks).

By showing the powerful effect that changing perceptions of down-side risk can exert on asset prices, we have strengthened the case for treating current asset valuations with suspicion. In their account of intrinsic bubbles, Froot and Obstfeld (1991) appealed to the idea that the market might select the
wrong stochastic solution. ${ }^{1}$ But they do not say why. Our sliding option is, like theirs, the "wrong" solution. But it is the kind of bubble you can almost believe in.

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## Appendices

## A. Proof of Proposition

Substituting boundary conditions (19) and (21) into (17) one can solve for $A_{+}$and $A_{-}$as

$$
\begin{equation*}
A_{+}=\frac{f\left(D_{p}\right)\left(x^{\xi_{+}}-\eta\right)-\left(x^{\xi_{-}-}-x\right) D_{p} /\left(r^{f}-\mu^{\prime}\right)}{x^{\xi_{-}}-x^{\xi_{+}}} \tag{A1}
\end{equation*}
$$

and

$$
\begin{equation*}
A_{-}=-\frac{f\left(D_{p}\right)\left(x^{\xi_{+}}-\eta\right)-\left(x^{\xi_{+}}-x\right) D_{p} /\left(r^{f}-\mu^{\prime}\right)}{x^{\xi_{-}}-x^{\xi_{+}}} \tag{A2}
\end{equation*}
$$

where $x=D_{b} / D_{p}$. Substituting (A1) and (A2) into (20) yields

$$
\begin{align*}
f\left(D_{p}\right) & =\frac{\xi_{+}\left(x^{\xi_{-}}-x\right) x^{\xi_{+}-1}-\xi_{-}\left(x^{\xi_{+-}}-x\right) x^{\xi_{-}-1}-\left(x^{\xi_{-}}-x^{\xi_{+-}}\right)}{\xi_{+}\left(x^{\xi_{-}}-\eta\right) x^{\xi_{+}-1}-\xi_{-}\left(x^{\xi_{+-}}-\eta\right) x^{\xi_{-}-1}} \cdot \frac{D_{p}}{r^{f}-\mu^{\prime}}  \tag{A3}\\
& =g(x) \frac{D_{p}}{r^{f}-\mu^{\prime}}
\end{align*}
$$

Substituting (A1) and (A2) into (22) and rearranging yields

$$
f^{\prime}\left(D_{p}\right)=\frac{1}{r^{f}-\mu^{\prime}}\left[1-\frac{\xi_{+}\left(x^{\left.\xi_{-}-x\right)-\xi_{-}\left(x^{\xi_{+-}}-x\right)}\right.}{x^{\xi_{-}}-x^{\xi_{+-}}}\right]+\frac{\xi_{+}\left(x^{\xi_{-}}-\eta\right)-\xi_{-}\left(x^{\xi_{+-}}-\eta\right)}{x^{\xi_{-}-x^{\xi_{+-}}} \cdot \frac{f\left(D_{p}\right)}{D_{p}},{ }^{2}}
$$

It can be shown that

$$
\begin{equation*}
f^{\prime}\left(D_{p}\right)=\frac{f\left(D_{p}\right)}{D_{p}}-x \frac{\partial}{\partial x}\left(\frac{f\left(D_{p}\right)}{D_{p}}\right) . \tag{A5}
\end{equation*}
$$

Substituting (A5) into (A4), one obtain a first order linear differential equation for $f\left(D_{p}\right) / D_{p}$ which has a solution as follows

$$
\begin{equation*}
\frac{f\left(D_{p}\right)}{D_{p}}=h(x) . \tag{A6}
\end{equation*}
$$

Combining (A6) and (A3), we have a fixed point equation for $x$ as

$$
\begin{equation*}
h(x)=\frac{g(x)}{r^{f}-\mu^{\prime}} . \tag{A7}
\end{equation*}
$$

As long as there exists a solution $x^{*}$ to (A7), $g\left(x^{*}\right)$ will be a constant. Let

$$
\begin{equation*}
g\left(x^{*}\right)=1+\alpha, \tag{A8}
\end{equation*}
$$

we have the linear envelope given in Proposition 1. (We will show later there indeed exists a fixed point to (A7).) Comparing the stock price with expected stabilisation and without (as in equation (15)), the former is always greater than or equal to the latter, so $\alpha \geq 0$.

Substituting (24) into (A3) and (A4), and eliminating $\alpha$ yields the following fixed point equation for $x$ :

$$
\begin{equation*}
\eta=\frac{\left(\xi_{+}-1\right)\left(1-\xi_{-}\right)\left(x^{\xi_{-}-}-x^{\xi_{+}}\right)}{\xi_{+}-\xi_{-}+\xi_{+}\left(\xi_{-}-1\right) x^{\xi_{+}-1}-\xi_{-}\left(\xi_{+}-1\right) x^{\xi_{-}-1}} \equiv k(x) . \tag{A9}
\end{equation*}
$$

Since $0 \leq x \leq 1$, we only have to look at the property of function $k(x)$ for $x \in[0,1]$.
It is straight forward to show that

$$
\begin{align*}
\lim _{x \downarrow 0} k(x) & =0 \\
\lim _{x \uparrow 1} k(x) & =1 \tag{A10}
\end{align*}
$$

and

$$
k^{\prime}(x)=\frac{\left(\xi_{+}-1\right)\left(1-\xi_{-}\right)\left(\xi_{-} x^{\xi_{-}-1}-\xi_{+} x^{\xi_{+}-1}\right)\left[\xi_{+}-\xi_{-}+\left(\xi_{-}-1\right) x^{\xi_{+}-1}-\left(\xi_{+}-1\right) x^{\xi_{-}-1}\right]}{\left[\xi_{+}-\xi_{-}+\xi_{+}\left(\xi_{-}-1\right) x^{\xi_{+}-1}-\xi_{-}\left(\xi_{+}-1\right) x^{\xi_{-}-1}\right]^{2}}>0 .
$$

So for $0 \leq \eta<1$ there exists a unique solution $x^{*}$ to (A9). In particular, if $\eta=0$ then $x^{*}=0$.

Substituting (24) into (A3) and using (A9) to replace $\eta$ yields

$$
\begin{equation*}
\alpha=\frac{\xi_{+}-\xi_{-}}{\xi_{+}\left(\xi_{-}-1\right) x^{\xi_{+}-1}-\xi_{-}\left(\xi_{+}-1\right) x^{\xi_{-}-1}} \equiv \frac{\xi_{+}-\xi_{-}}{n(x)} . \tag{A12}
\end{equation*}
$$

As

$$
\begin{equation*}
n^{\prime}(x)=\left(\xi_{+}-1\right)\left(\xi_{-}-1\right)\left(\xi_{+} x^{\xi_{+}}-\xi_{-} x^{\xi_{-}-}\right) x^{-2}<0 \tag{A13}
\end{equation*}
$$

if $n\left(x_{\max }\right)=0$ then

$$
\begin{align*}
& n(x) \geq 0, \quad \text { if } \quad 0 \leq x \leq x_{\max } ; \\
& n(x)<0, \quad \text { if } \quad \mathrm{x}_{\max }<x \leq 1 . \tag{A14}
\end{align*}
$$

Since $\alpha \geq 0$, so $0 \leq x \leq x_{\max }$, where

$$
\begin{equation*}
x_{\max }=\left(\frac{\xi_{-}}{\xi_{-}-1} \cdot \frac{\xi_{+}-1}{\xi_{+}}\right)^{1 /\left(\xi_{+}-\xi_{-}\right)}<1 \tag{A15}
\end{equation*}
$$

From (A9), this implies an upper bound for $\eta$ such that a solution to $\alpha$ exists. Substituting (A15) into (A9) yields

$$
\begin{equation*}
\eta_{\max }=\frac{\xi_{+}-1}{\xi_{+}} x_{\max }^{\xi_{-}}<1 . \tag{A16}
\end{equation*}
$$

From (A11) and (A12), it can be shown that

$$
\begin{equation*}
\frac{\partial \alpha}{\partial \eta}=-\frac{\xi_{+}-\xi_{-}}{n^{2}(x)} n^{\prime}(x) \frac{d x}{d \eta}>0 . \tag{A17}
\end{equation*}
$$

## And

$$
\begin{align*}
& \lim _{\eta \downarrow 0} \alpha=\lim _{x \downarrow 0} \alpha=0 \\
& \lim _{\eta \uparrow \eta_{\max }} \alpha=\lim _{x \uparrow x_{\max }} \alpha \rightarrow+\infty . \tag{A18}
\end{align*}
$$

## B. The Envelope for Stabilisation Points

To prove $\eta / x^{*}>1$, from (A9) we only need to show

$$
\begin{equation*}
\frac{\eta}{x}=\frac{\left(\xi_{+}-1\right)\left(1-\xi_{-}\right)\left(x^{\xi_{-}-1}-x^{\xi_{+}-1}\right)}{\xi_{+}-\xi_{-}+\xi_{+}\left(\xi_{-}-1\right) x^{\xi_{+}-1}-\xi_{-}\left(\xi_{+}-1\right) x^{\xi_{-}-1}}>1, \tag{B1}
\end{equation*}
$$

or equivalently to show

$$
\begin{equation*}
m(x) \equiv\left(\xi_{+}-1\right) x^{\xi_{-}-1}-\left(\xi_{-}-1\right) x^{\xi_{+}-1}>\xi_{+}-\xi_{-} . \tag{B2}
\end{equation*}
$$

As $m(x)$ is strictly decreasing and

$$
\begin{equation*}
m\left(x_{\max }\right) \geq \xi_{+}-\xi_{-} \tag{B3}
\end{equation*}
$$

then $\eta / x^{*}>1$.


[^0]:    ${ }^{1}$ The same idea was also explored in Miller and Weller (1990).

