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ABSTRACT

Policy discussions on financial market regulation tend to assume that whenever a corrective policy is used ex post to ameliorate the effects of a crisis, there are negative side effects in terms of moral hazard ex ante. This paper shows that this is not a general theoretical prediction, focusing on the case of monetary policy interventions ex post. In particular, we show that if the central bank does *not* intervene by monetary easing following a crisis, this creates an aggregate demand externality that makes borrowing ex ante inefficient. If instead the central bank follows the optimal discretionary policy and intervenes to stabilize asset prices and real activity, we show examples in which the aggregate demand externality disappears, reducing the need for ex ante intervention.

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1 Introduction

Many economists and commentators have remarked that the conduct of interest policy by the central bank can affect the incentives of the financial sector. In particular, a common complaint is that the so called "Greenspan put"encourages excessive leverage and risk taking by banks. The argument goes as follows. Suppose that when an adverse shock drives down asset prices the central bank intervenes systematically by lowering interest rates. This becomes an implicit commitment to prop up asset prices in times of distress and encourages banks and other financial players to borrow more and take on more risk ex ante. In turns, this increases the risk of systemic crises, with possible harmful repercussions on aggregate activity. Therefore, countercyclical interest rate policy may end up increasing, rather than reducing macroeconomic volatility. Countercyclical monetary policy generates a form of moral hazard, where financial firms do not receive help directly, but are subsidized indirectly by the central bank's low interest rate policy.

A number of recent papers have formalized this idea in different ways, including Lorenzoni (2001), Diamond and Rajan (2012), Farhi and Tirole (2012), and Chari and Kehoe (2016). This paper attacks the problem from a different perspective. While the existing literature emphasizes the distortionary role of an overly active monetary policy, here we emphasize the distortions that arise when monetary policy is *too passive*. In other words, we show that the lack of countercyclical interventions, in the face of negative aggregate shocks, can also worsen the problem of excessive leverage in the financial sector. While the existing literature emphasizes a form of pecuniary externality, which travels through asset prices, this paper emphasizes the presence of an aggregate demand externality, which travels through the level of real output. In this respect, our model builds on the recent literature on macroprudential policy and aggregate demand externalities started by Farhi and Werning (2016), Korinek and Simsek (2016), and Schmitt-Grohé and Uribe (2106).

We present first a simple example in which a countercyclical interest rate policy can completely eliminate the overborrowing distortion. We consider first a monetary policy regime in which the interest rate is completely unresponsive to real activity and asset prices. In that regime, the equilibrium is constrained inefficient and there is overborrowing. We then show that in our example the interest rate policy can be designed so as to completely eliminate both output volatility and asset price volatility. Such a policy does indeed induce higher borrowing ex ante. However, under this interest rate policy the overborrowing problem disappears. So even though the level of borrowing goes up, the distance between the lassez faire level of borrowing and the socially (constrained) efficient level goes down.

Two closely related papers are Benigno et al. (2013) and Korinek and Jeanne (2016), who analyze the relative benefits of ex ante and ex post policies to deal with financial instability in different models.

Korinek and Jeanne (2016) look at a model in which the ex post policy is a form of bailout in which resources are transferred from lenders to borrowers. Proposition 6 in their papers shows that when more resources are available for an ex post bailout, this can reduce the need for ex ante macroprudential policy, that is ex ante and ex post policies can be substitutes rather than complements. That result is close in spirit to the message of this paper. However, their model is a purely real model with an exogenous interest rate pinned down by a storage technology, while here we focus on the use of interest rate policy as a tool to stabilize asset prices and deal with a crisis ex post.

Benigno et al. (2013) is closer to our model, in that the ex post intervention is also captured by monetary policy.[1] There are two main differences with our paper. First, our paper features an explicit role for asset prices, and thus can be used to discuss asset price stabilization by the central bank. Second, our paper focuses on a simple model where analytic results can be derived. In particular, we focus on understanding whether monetary tools and macroprudential tools are substitutes or complements, provide numerical examples of both cases, and show how the interplay of pecuniary externalities and aggregate demand externalities leads to one case or the other. An additional novelty of this paper, is the extension of the analysis to unconventional monetary policy in Section 6.

Finally, our paper is related to a classic debate on the benefits of asset price stabilization as a monetary policy objective. Bernanke and Gertler (1999) made the point that monetary policy should respond to asset price movements insofar as they affect aggregate demand. In this paper, we look indeed at monetary responses to asset prices that are motivated by aggregate demand management, and ask the question whether these responses ex post encourage instability and excess leverage ex ante.

2 The model

Consider a three period economy, with $t = 0, 1, 2$. In periods 0 and 2 the economy is an endowment economy. In period 1 output is produced with a linear technology that uses only labor.

There are two groups of agents of equal size, labeled A and B. The preferences of

[1] See also Benigno et al. (2016) in an open economy context.

agent A are represented by the utility function

$$E\left[c_0^A + u(c_1^A) - v\left(n_1^A\right) + u\left(c_2^A\right)\right], \tag{1}$$

where c_t^A denotes consumption, n_1^A denotes labor effort in period 1. The functions u and v have the isoelastic forms

$$u\left(c\right) = \frac{1}{1-\gamma}c^{1-\gamma}, \quad v\left(n\right) = \frac{\psi}{1+\phi}n^{1+\phi}.$$

The preferences of agents B are represented by the utility function

$$E\left[c_0^B + \beta u\left(c_1^B\right) + \beta^2 u\left(c_2^B\right)\right], \tag{2}$$

where c_t^B denotes consumption and $0 < \beta < 1$. The assumption of linear utility for both agents in period 0 simplifies the welfare analysis, by making utility transferable ex ante.

Both agents receive a large endowment of the single consumption good in period 0. In period 1, agent A receives labor income by supplying labor on a competitive labor market at the wage rate w_1.

There is a discrete set of states of the world $s \in S$, with probability distribution $\pi\left(s\right)$. The state s is revealed in period 1. There is a risky asset in fixed supply in the economy, which pays $\delta_1\left(s\right)$ units of consumption good in period 1 and $\delta_2\left(s\right)$ units in period 2. The risky asset can only be held by B agents. In periods 0 and 1, agents trade a real non-state contingent bond that pays 1 unit of consumption goods in the following period. Assuming that only B agents can hold the risky asset and that they are more impatient than A agents is a simple way of obtaining levered agents, whose balance sheets are exposed to shocks and affect aggregate spending in the economy. The channels captured in this simple model can be extended to richer models of borrowing and lending as, for example, models with levered intermediaries exposed to aggregate risk who make lending decisions that affect aggregate spending.

In period 1, there is a continuum of monopolistic firms on the interval $[0, 1]$ that produce intermediate goods. Each intermediate good is produced with a linear technology that uses only labor $x_j = n_{j1}$. The goods are then combined to produce the final consumption good using the production function

$$Y = \left[\int_0^1 x_{j1}^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}},$$

with $\epsilon > 1$. The firms selling the differentiated goods set the price of their good p_{j1} one period in advance, in period 0. The firms are fully owned by A agents. The aggregate nominal price level in period t is denoted by p_t.[2] Each firm j sets its price to maximize expected profits

$$\mathbb{E}_0 \left[\frac{u'(c_1^A)}{p_1} \left((1+\sigma) p_{j1} x_{j1} - w_1 x_{j1} \right) \right]$$

taking as given the demand function for good j, which is $x_{j1} = \left(p_{j1}/p_1 \right)^{-\epsilon} Y$. In the expressions above $m_1 =$ is the nominal stochastic discount factor of A agents, w_1 is the nominal wage in period 1, and σ is a subsidy on the production of intermediate goods. We assume the government finances the subsidy σ by levying a lump-sum tax on A agents, so that the real income of A agents in period 1 is simply Y. We introduce the subsidy to give the government a tool to correct for the monopolistic distortion.

Policy is captured by three instruments. The nominal interest rate i_t, set by the central bank in periods 0 and 1. The subsidy σ introduced above. And a macroprudential linear tax τ on borrowing at $t = 0$. The tax revenues from the macroprudential tax are rebated as a lump sum transfer to the B agents, who pay the tax. Our assumptions on how the subsidy σ and the tax τ are financed imply that all interventions are wealth neutral, so we do not allow the government to directly reallocate resources across agents.

3 Equilibrium under different monetary regimes

Our goal is to study the interaction between monetary policy and macroprudential policy. In this section, we start by characterizing the equilibrium under three different monetary policy regimes, setting the macroprudential tax to zero. In the next sections, we investigate the benefits of adding macroprudential policy under different monetary regimes.

3.1 Preliminary steps

Before introducing the different regimes, we need a few preliminary steps that characterize equilibrium allocations independently of the regime.

All price setters face the same strictly concave problem, so they all choose the same nominal price $p_{j1} = p_1$ and they all produce the same amount of goods Y with the same amount of labor $n = Y$. Since prices are set one period in advance, monetary policy can

[2]The aggregate price in the economy in period 1 is equal to $\left(\int_0^1 p_{j1}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$.

determine the real interest rate between periods 1 and 2, which we denote by

$$r \equiv (1 + i_1)\frac{p_1}{p_2} - 1.$$

We can then focus on characterizing the equilibrium allocations that can be achieved by a social planner who chooses r at date 1 and the subsidy σ at date 0.

To characterize the equilibrium, we go backwards in time and solve first for the equilibrium at dates $t = 1, 2$. In this way, we obtain the output level Y that arises in period 1, in state s, when agents A hold D units of real debt issued by B agents and the real interest rate is r. We use the function $Y(D, r, s)$ to denote this output level and leave its explicit derivation to Appendix 8.1.

We now introduce two value functions which will be useful in the rest of the analysis. Let $V^A(b, D, r, s)$ denote the expected utility in state s of an agent A who enters period 1 with b units of bonds in an economy in which all other A agents hold D units of bonds, all other B agents have D units of debt, and the real interest rate is r. Let $V^B(d, D, r, s)$ denote the analogous value for the B agents, with d denoting the individual level of debt. In equilibrium we have $b = d = D$, but for the analysis it is convenient to separate individual decisions from aggregates. The formal definitions of V^A and V^B are:

$$\begin{aligned}
V^A(b, D, r, s) &= \max_{c_1^A, c_2^A} \quad u(c_1^A) - v\left(Y(D, r, s)\right) + u(c_2^A) \\
&\qquad\qquad \text{s.t.} \quad c_1^A + \frac{1}{1+r}c_2^A = Y(D, r, s) + b, \\
V^B(d, D, r, s) &= \max_{c_1^B, c_2^B} \quad u(c_1^B) + \beta u(c_2^B) \\
&\qquad\qquad \text{s.t.} \quad c_1^B + \frac{1}{1+r}c_2^B = \delta_1(s) + \frac{1}{1+r}\delta_2(s) - d.
\end{aligned}$$

The equations above show that the expected utility of the two agents depend on three key variables: the individual financial positions, the aggregate level of debt in the economy, and the real interest rate. The following lemma characterizes these relations. The proof is in the appendix.

Lemma 1. *At $b = d = D$ the partial derivatives of the value functions are*

$$
\begin{aligned}
&V_b^A(b, D, r, s) = u'(c_1^A), \\
&V_d^B(d, D, r, s) = -u'(c_1^B), \\
&V_D^A(b, D, r, s) = \left(u'(c_1^A) - v'(Y)\right) \frac{\partial Y}{\partial D}, \\
&V_D^B(d, D, r, s) = 0, \\
&V_r^A(b, D, r, s) = u'(c_1^A) \left(Y + D - c_1^A\right) \frac{1}{1+r} + \left(u'(c_1^A) - v'(Y)\right) \frac{\partial Y}{\partial r}, \\
&V_r^B(d, D, r, s) = -u'(c_1^B) \left(Y + D - c_1^A\right) \frac{1}{1+r}.
\end{aligned}
$$

The values of c_1^A and c_1^B in the expressions above are the values that solve the problems in the value functions of A and B, at $b = d = D$; Y is short for $Y(D, r, s)$.

Let us provide intuition for the results in the lemma. The first two partial derivatives in Lemma 1 are completely standard and capture the private marginal benefit of lending for agent A and the private marginal cost of borrowing for agent B.

The derivative V_D^A captures an aggregate demand externality that plays a crucial role in the rest of the analysis. The effect of debt on output is captured by the derivative $\partial Y/\partial D$. This derivative is negative because a debt payment is a transfer of resources from B agents to A agents, and the former have a higher propensity to consume, due to higher impatience.[3] In this way, the model captures in a nutshell the real effects of debt overhang. A marginal change in output increases or decreases welfare depending on the sign of $u'(c_1^A) - v'(Y)$. With flexible prices—after removing the monopolistic distortion—this difference would always be zero. But in a new Keynesian environment this term can be non zero. In particular, when it is positive, increasing output and hours worked yields marginal benefits greater than marginal costs. This is the efficiency loss typically associated to the notion of "output gap." We call $u'(c_1^A) - v'(Y)$ the labor wedge and use interchangeably the language "the labor wedge is positive/negative" and "the output gap is positive/negative."[4] When there is a positive output gap, a higher level of debt is welfare reducing as it reduces output at a point where output increases would be welfare improving.

The next derivative, V_D^B, is zero because the B agents' decision problem is not directly

[3]The formal proof that $\partial Y/\partial D < 0$ is in the appendix.

[4]Here we are thinking of the welfare benefits of producing more goods and assigning them to agent A. Given that there are two agents and imperfect insurance, the notion would be different if we looked at the welfare benefits of assigning the extra goods to agent B. This would change slightly the decomposition and interpretation of our two main forces, but it would change nothing in the substance of the analysis.

affected by the level of Y.

Finally, the derivatives V_r^A and V_r^B describe how agents' utilities depend on the real interest rate. The real interest rate affects agents in two ways. First, increasing the interest rate reallocates resources from borrowers to lenders, in proportion to the amount of borrowing and lending taking place between periods 1 and 2. Since in equilibrium agents A save $Y + D - c_1^A$ and this is equal to the amount borrowed by agents B, in this way we obtain the first term in V_r^A and the only term in V_r^B. The real interest rate also affects A agents through the aggregate demand channel. In particular, raising the interest rate changes the equilibrium level of output according to $\partial Y/\partial r$ and this affects welfare in proportion to the labor wedge.

To complete the equilibrium characterization, we need three additional steps: determine how monetary policy chooses r in each state of the world; determine the level of D which arises in equilibrium at $t = 0$; ensure that the price setters' optimality condition is satisfied at $t = 0$. The solution of these steps depends on the monetary policy regime. In the next section we consider three possible regimes and complete the equilibrium characterization.

3.2 Inertial regime

To define our first two regimes, we assume that policy is set by a planner who maximizes the social welfare function

$$W = E\left[V^A(D, D, r, s) + \beta V^B(D, D, r, s)\right], \tag{3}$$

with side transfers taking place at date 0 to reallocate utility between A and B agents.[5]

In the first regime, the interest rate at $t = 1$ is set by the planner *before* the realization of the state s. We think of this regime as capturing inertia in the response of the central bank to information about aggregate shocks, so we call it the "inertial" regime.[6]

The timing is as follows. First, the equilibrium debt level $d = b = D$ is determined in the bond market at date 0.[7] Next, at the beginning of period 1, before the realization of the state s, the planner chooses a real interest rate $\bar{r}$ to maximize (3). After the realization

[5]The presence of heterogeneous discount factors introduces a source of time inconsistency in optimal policy. By focusing on the welfare function (3) we leave aside that source of time inconsistency. Other interesting sources are present, as we shall see.

[6]An alternative source of real interest rate rigidity that would lead to related results would be to consider an open economy with a fixed exchange rate regime or a monetary union.

[7]Since prices are flexible in period 0 the central bank can choose the nominal interest rate i_0 but has no power to affect the real interest rate between dates 0 and 1, $(1 + i_0)p_0/p_1$, which is determined at the level that clears the bond market.

of s, the central bank cannot revisit its choice of the interest rate. Agents choose their spending in period 1 and that determines output. Finally, in period 2, agents consume their endowment net of their final bonds positions. The subsidy σ is set to ensure that the price setting condition of the producers at $t = 0$ is satisfied.[8]

The crucial restriction in this regime is that the central bank cannot use the interest rate policy to mitigate the drop in the asset price Q for low realizations of δ. As we shall see, this sluggish response of the central bank may reduce the incentive of B agents to borrow in period 0 as they cannot rely on the implicit bailout coming from looser policy.

An equilibrium under this regime is given by a pair $D, \bar{r}$ that satisfies the following two conditions: (i) optimal monetary policy, characterized by the first-order condition

$$E\left[V_r^A\left(D, D, \bar{r}, s\right) + \beta V_r^B\left(D, D, \bar{r}, s\right)\right] = 0; \tag{4}$$

(ii) bond market equilibrium at date 0, characterized by the condition

$$E\left[V_b^A\left(D, D, \bar{r}, s\right)\right] + \beta E\left[V_d^B\left(D, D, \bar{r}, s\right)\right] = 0. \tag{5}$$

Since the central bank chooses the interest rate $\bar{r}$ after the bonds market in period 0 has cleared, the central bank takes D as given when choosing $\bar{r}$. At the same time, consumers do not internalize the effect that the equilibrium level of D has on the central bank's choice of $\bar{r}$, simply because they are atomistic. This explains why strategic considerations do not appear in conditions (4) and (5).

Using Lemma 1, condition (4) can be rewritten as

$$E\left[\left(u'(c_1^A) - v'\left(Y\right)\right)\frac{\partial Y}{\partial r}\right] + E\left[\left(u'(c_1^A) - \beta u'(c_1^B)\right)\left(Y + D - c_1^A\right)\frac{1}{1+r}\right] = 0. \tag{6}$$

This condition shows that the central bank is balancing two effects when choosing the interest rate. The first is a standard new Keynesian effect: changing the interest rate affects equilibrium output and this increases or decreases welfare depending on the sign

[8]Using the optimality condition for labor supply we obtain the equilibrium real wage

$$\frac{w_1}{p_1}u'(c_1^A) = v'\left(Y\right).$$

Substituting in the optimality condition of price setters we obtain

$$E\left[Y\left(\left(1+\sigma\right)u'(c_1^A) - \frac{\epsilon}{\epsilon - 1}v'\left(Y\right)\right)\right] = 0.$$

The value of σ is chosen to satisfy this condition.

of the output gap. The second effect is a pecuniary externality associated to incomplete markets: changing the interest rate reallocates resources from borrowers to lenders and if the marginal utilities of borrowers and lenders are different ex post this can have welfare benefits. Davila and Korinek (2017) call this a distributive externality. Here, given that r is not allowed to be state contingent, the monetary authority chooses a level of the interest rate that balances these two welfare effects in expectation.

Using Lemma 1 we can also rewrite (5) as follows

$$Eu'(c_1^A) = \beta Eu'(c_1^B), \tag{7}$$

which shows that the debt level is chosen to equalize expected marginal utilities.

Conditions (6) and (7) will be used to characterize the equilibrium in the inertial regime in the rest of the paper.

3.3 Proactive regime

In our second regime, the planner also aims to maximize the social welfare function (3), but can now choose a fully state contingent interest rate. The timing is as in the previous regime, but r is set after observing s. We call this regime the proactive regime.

An equilibrium under this regime is given by a debt level D and interest rates $\{r(s)\}_{s \in S}$ that satisfy the following two conditions: (i) optimal monetary policy, characterized by the first-order condition

$$V_r^A(D, D, r(s), s) + \beta V_r^B(D, D, r(s), s) = 0 \text{ for all } s \in S; \tag{8}$$

(ii) bond market equilibrium at date 0, characterized by the condition

$$E\left[V_b^A(D, D, r(s), s)\right] + \beta E\left[V_d^B(D, D, r(s), s)\right] = 0. \tag{9}$$

As in the previous regime, Lemma 1 can be used to rewrite these conditions. In particular, condition (6) now holds state by state and takes the form

$$\left(u'(c_1^A) - v'(Y)\right)\frac{\partial Y}{\partial r} + \left(u'(c_1^A) - \beta u'(c_1^B)\right)\left(Y + D - c_1^A\right)\frac{1}{1+r} = 0. \tag{10}$$

Condition (9) can be written as (7), as in the inertial regime.

As in the previous regime, the subsidy σ is chosen to ensure that price-setters' optimality is satisfied.

3.4 Output targeting regime

The third regime is one in which the planner responds to the state s, but its only objective is to replicate the equilibrium that would arise under flexible prices and perfect competition (i.e., with no monopolistic distortion). So we are looking at a regime in which σ and $r(s)$ are set so that the following condition holds state by state

$$u'(c_1^A) - v'(Y) = 0.$$

We call this regime the output targeting regime, because it keeps the output gap at zero. The ex ante optimality condition that ensures equilibrium in the debt market at $t = 0$ is (7) as in the previous regimes.

The conditions just described, together with the conditions characterizing a continuation equilibrium, are sufficient to pin down the equilibrium allocation under this regime. However, for the analysis to follow we need to be precise on the timing. We assume that the central bank commits at $t = 0$ to set at $t = 1$ the real interest rate $r(s)$ that implements the flexible price allocation. Therefore this regime is different from the previous two not only because of the planner's objective, but also for its commitment to future interest rates. As we shall see, this commitment assumption will deliver a clean characterization of this regime in terms of macroprudential policy.

4 More borrowing, less overborrowing

We now turn to our main questions: How does the monetary regime affect the level of borrowing? How does it affect the benefits of macroprudential intervention? In a simple example with log preferences, the answers to these two questions go in opposite directions: the proactive regime yields more debt than the inertial regime, but the benefits of macroprudential policy are larger in the inertial regime. This example makes the main point of the paper. In this section, we present the log example and provide some intuition for it. In the next section, we go beyond the example to investigate more general implications of our analysis.

Let us specialize our model by assuming log preferences, $u(c) = \log c$, and no dividend at $t = 1$, $\delta_1(s) = 0$. Consumption levels at $t = 1$ are now

$$c_1^A = \frac{1}{2}(Y + D),$$

and

$$c_1^B = \frac{1}{1+\beta}\left(\frac{\delta_2}{1+r} - D\right). \tag{11}$$

Adding them up and solving for Y gives aggregate output

$$Y = \frac{2}{1+\beta}\frac{\delta_2}{1+r} - \frac{1-\beta}{1+\beta}D.$$

The following proposition characterizes the equilibrium in the proactive regime, which, in this case, is identical to the output targeting regime. The proof is in the appendix.

Proposition 1. *With log preferences and $\delta_1 = 0$, the proactive regime and the output targeting regime coincide and they are characterized by a real interest rate that exactly offsets changes in δ_2, so the asset price $\delta_2/(1+r)$, output, and the consumption levels of both agents are constant across states.*

In this example, optimal discretionary monetary policy achieves perfect stabilization of asset prices through interest rate policy—an especially stark example of the Greenspan put at work. This policy is optimal because it achieves, at the same time, a zero output gap and perfect insurance between agents, thus making both terms of (10) zero. The insurance effect of the policy can be seen immediately from (11), that shows that the wealth of B agents is perfectly stabilized by keeping $\delta_2/(1+r)$ constant.

Turning to the inertial regime, it is easy to see that we no longer obtain perfect insurance for B agents, as now the interest rate is constant and asset prices move with δ_2, making consumption more volatile. We can then ask whether this increased volatility induces B agents to be more cautious and borrow less ex ante. Substituting the consumption values just derived, equation (7) takes the form

$$E\left[\frac{2}{Y+D}\right] = \beta E\left[\frac{1+\beta}{\delta_2/\left(1+r\right) - D}\right].$$

Since the utility function displays prudence, this equation suggests that higher volatility of the asset price $\delta_2/(1+r)$ leads to increased savings, that is, less borrowing, by B agents. This intuition is confirmed in the next proposition. To prove the proposition requires some additional steps, because in the inertial regime the variable Y is also volatile, and not only the volatility, but also the average levels of r and Y are different in the two regimes. To deal with these complications, we restrict attention to the case of small shocks to δ_2. The proof is in the appendix.

Proposition 2. *With log preferences, $\delta_1 = 0$, and small shocks to δ_2, the proactive regime features a higher level of borrowing D in equilibrium than the inertial regime.*

This result seems to offer a perfect example of the evil incentive effects of the Greenspan put. However, this is just a positive result about the levels of D in the two regimes, not about their efficiency properties. The next proposition shows that looking at efficiency leads to different conclusions.

To evaluate the benefits of macroprudential policy, we look at the effect of a marginal change in D on welfare.

Proposition 3. *With log preferences and $\delta_1 = 0$, in the inertial regime there is excessive borrowing ex ante, that is, social welfare is locally decreasing in D:*

$$\frac{dW}{dD} = \frac{d}{dD} E\left[V^A\left(D, D, \bar{r}, s\right) + \beta V^B\left(D, D, \bar{r}, s\right)\right] < 0.$$

In the proactive regime the level of borrowing is socially efficient:

$$\frac{dW}{dD} = \frac{d}{dD} E\left[V^A\left(D, D, r\left(s\right), s\right) + \beta V^B\left(D, D, r\left(s\right), s\right)\right] = 0.$$

In order to build intuition for this result, let us first provide some derivations that characterize the marginal social benefit of a change in D. Using the results in Lemma 1 and the private optimality condition for debt (7), we can characterize the marginal effect of debt on social welfare as follows.

Lemma 2. *In all the monetary regimes considered, the marginal welfare effects of a change in D is equal to*

$$\frac{dW}{dD} = E\left[\left(u'(c_1^A) - v'\left(Y\right)\right)\frac{\partial Y}{\partial D}\right]. \tag{12}$$

This result is independent of the functional form assumptions made for u and v, and it does not rely on the special assumptions made in this section. Furthermore, it holds in all the monetary regimes considered. One may wonder why there is no term capturing the fact that changing D ex ante will affect the choice of r ex post. In the first two regimes, where monetary policy is chosen under discretion, this happens because the ex post choice of r is optimal, so an envelope argument implies that this effect can be ignored. In the output targeting regime, this happens because the central bank commits to r ex ante and does not change it off the equilibrium path if D is changed. This is where the assumption of commitment in the third regime helps to simplify the analysis.

The effect of debt on output is given by

$$\frac{\partial Y}{\partial D} = -\frac{1-\beta}{1+\beta} < 0,$$

which is constant and independent of the state of the world. So we can focus on establishing the sign of the average labor wedge

$$E\left[u'(c_1^A) - v'(Y)\right].$$

In the inertial regime, it is possible to show that the output gap is positive in low δ_2 states and negative in high δ_2 states. This is due to the fact that in that regime the risk δ_2 is not insured and a reduction in δ_2 reduces the wealth of the B agents who have a relatively larger marginal propensity to consume. Therefore the sign of $E\left[u'(c_1^A) - v'(Y)\right]$ depends on whether the positive wedge in the bad states dominates the negative wedge in the good states. In economic terms, a larger debt stock D worsens recessions in bad states, which is bad for welfare, and dampens booms in good states, which is good for welfare. The proof of Proposition 3 in the appendix shows that the first effect dominates, so, overall, more borrowing reduces welfare.

In the proactive regime, we have already established that the output gap is always zero. Therefore, there is no welfare gain from changing the level of borrowing ex ante, as the level of output is already at its socially efficient level state by state.

Summing up, using interest rate policy to fight a recession leads to higher borrowing ex ante, but this is not a symptom of inefficient borrowing ex ante. In fact, the presence of optimal discretionary monetary policy ex post eliminates the distance between equilibrium borrowing and its socially efficient level, thus making macroprudential policy unnecessary.

It is useful to connect our results here with results in Farhi and Werning (2016) and Korinek and Simsek (2016) on macroprudential policy at the zero lower bound. So far we have ignored the possibility of the zero lower bound binding, which is fine in our context because the economy has flexible prices at $t = 2$ so the monetary authority can choose an inflation rate between $t = 1$ and $t = 2$ to achieve any real interest rate desired. However, it is easy to add an ad hoc form of ZLB by imposing $r \geq \underline{r}$. Consider now a central bank that tries to achieve a zero output gap but is constrained by the ZLB. Then the output gap can never be positive, as increasing rates is always possible. Therefore, if the ZLB is binding with positive probability the expression $E\left[u'(c_1^A) - v'\left(n_1^A\right)\right]$ is positive and the aggregate demand externality yields excessive borrowing in line with results in Farhi and Werning (2016) and Korinek and Simsek (2016). Our results show that ex post monetary policy inertia provides a rationale for macroprudential policy even if the ZLB is not binding. We will return to the ZLB in Section 6.

5 Tradeoffs, complementarity and substitutability

In the previous section we established that, under certain assumptions, ex post monetary intervention is a substitute for macroprudential policy: more intervention ex post reduces the marginal benefit of macroprudential policy. In this section we move beyond the special assumptions of the previous section and ask more generally in what cases is ex post monetary intervention a substitute for macroprudential policy and in what cases there can be complementarity between the two policies.

In the analysis of the previous section, the proactive regime achieves the same allocation as the output targeting regime. As argued above, this happens because the proactive regime is able to set both terms of (10) to zero at the same time. By choosing the state contingent interest rate r the central bank can take care of both frictions present in our environment: price rigidity and lack of insurance against δ shocks.

This is a knife-edge result that only holds with log utility. With log utility changing the interest rate has no direct effect on the consumption decisions of A agents, as income and substitution effects cancel each other. For B agents, the interest rate affects their consumption decisions only by changing the discounted value of dividends in period 2. So, by moving interest rates proportionally to dividends, the central bank can reach constant asset prices and constant ouptut without distorting the agents' intertemporal margin. This is no longer true when income and substitutions effect do not cancel each other, i.e., when the elasticity of intertemporal substitution is different than 1.

Once we move away from log utility, the central bank faces a tradeoff when setting interest rates and needs to balance a traditional macro objective—stabilizing aggregate output—and a financial stability objective—using the interest rate to redistribute in favor of B agents hit by a negative shock. In this section we want to investigate the role of macroprudential policy when monetary policy has to deal with this tradeoff.

We will make our main points using numerical examples. Our main example uses the parameters

$$\beta = 0.5, \quad \gamma = 0.5, \quad \psi = 1, \quad \phi = 1, \quad \delta_1 = 0$$

and a binary distribution for the δ_2 shock with values 1.05 and 1.5 that occur with equal probabilities 1/2. The reason why we assume $\gamma < 1$, is that in this simple setup this assumption ensures that in the output targeting regime or, equivalently, under flexible prices, asset prices and the consumption of B agents are monotone increasing in δ_2. The reason why we choose a low discount factor for B agents is to give them a strong incentive to borrow, so we can better visualize the potential inefficiencies associated to overborrowing. In the appendix, we show that the qualitative results identified here also hold

Table 1: Interest rates in the output targeting regime and in the proactive regime

	Output gap targeting	Proactive regime
$\delta_2 = 1.05$	6.04%	4.33%
$\delta_2 = 1.5$	29.74%	32.14%

in examples with higher β.

5.1 Output stability and financial stability

First, let us understand better the nature of the tradeoff faced by the monetary authority ex post. To do so it is useful to concentrate on the equilibrium in the output targeting regime, where, by definition, the first term in (10) is set to zero. Equilibrium in the bonds market at $t = 1$ requires

$$E\left[u'(c_1^A) - \beta u'(c_1^B)\right] = 0.$$

As long as c_1^A and c_1^B are, respectively, monotone decreasing and monotone increasing in δ_2, we have

$$u'(c_1^A) - \beta u'(c_1^B) < 0$$

in the low δ_2 states and

$$u'(c_1^A) - \beta u'(c_1^B) > 0$$

in the high δ_2 states. This means that, starting from pure output gap stabilization, the central bank's incentive is to further lower interest rates in the bad state and to further increase them in the good state. That is, the central bank is driven to stabilize asset prices over and above what is required to stabilize output. This happens because of the insurance motive: moving one dollar from lenders to borrowers in the bad state means transferring it to agents with higher marginal utility. Reducing the interest rate achieves this transfer.

In Table 1 we show how the interest rate r responds to δ in the output targeting regime and in the proactive regime. In the second case, the central bank uses the interest rate more aggressively in response to bad shocks.

Now we are in a situation in which the central bank is tempted to ease more—in a recession caused by weak borrowers' balance sheets—than what it would be warranted under a pure output stabilization objective. In this situation, we ask again whether macroprudential policy may be of help.

5.2 An example of complementarity

As we did above, let us first focus on the marginal benefit of changing D starting from no borrowing tax. Equation (12) continues to hold in this case. In the output targeting regime it gives $dW/dD = 0$. So in that regime a macroprudential tax gives no welfare benefit. In the proactive regime, on the other hand, this expression is in general different from zero once $\gamma \neq 1$. In the numerical example above the value of this expression is negative.

Turning to optimal policy, we illustrate graphically the optimal choice of macroprudential and monetary policy in Figure 1, where we plot the contours of the social welfare function in the space spanned by the borrowing tax τ and the interest rate in the bad state, denoted by r_B. To represent the policy space in two dimensions, for each point (r_B, τ) we set the interest rate in the good state, denoted by r_G, as a linear function

$$r_G = \alpha_0 + \alpha_1 r_B,$$

choosing α_0, α_1 so that we go trough the pairs r_B, r_G corresponding to the output targeting regime and to the proactive regime with an optimal macroprudential tax, depicted by the two red points in the figure.

In the example considered here, monetary policy and macroprudential policy are complements, because, along the line that connects the two regimes considered, the cross derivative $\partial^2 W / \partial r_B \partial \tau$ is negative.[9] That is, if we compare the output targeting regime to the proactive regime the choice of a lower rate in the second regime increases the marginal benefit of the borrowing tax. This leads to an optimal tax in the proactive regime that is larger than the optimal tax in the output targeting regime (which is zero).

We derive two conclusions from this analysis. One is that once we move away from the knife-edge of Section 4, optimal policy requires a combination of monetary tools and macroprudential tools. So monetary policy is no longer a perfect substitute for macroprudential policy. The second conclusion is that we finally have a result more in line with conventional wisdom on moral hazard effects. Comparing a regime of pure output targeting to a fully discretionary regime, in the latter regime the central bank increases its interest rate response, stabilizes asset prices more, and there is a stronger motive to impose a borrowing tax ex ante.

Digging a bit more in the intuition, shows that this result relies on the central bank exceeding its output objectives. In the bad state of the world the central bank is choosing to overstimulate the economy, so we are reaching an allocation with $u'(c_1^A) - v'\left(n_1^A\right) < 0$.

[9] Complementarity/substitutability is defined in this way in this policy space, because lowering the rate more in the bad state is a more aggressive intervention.

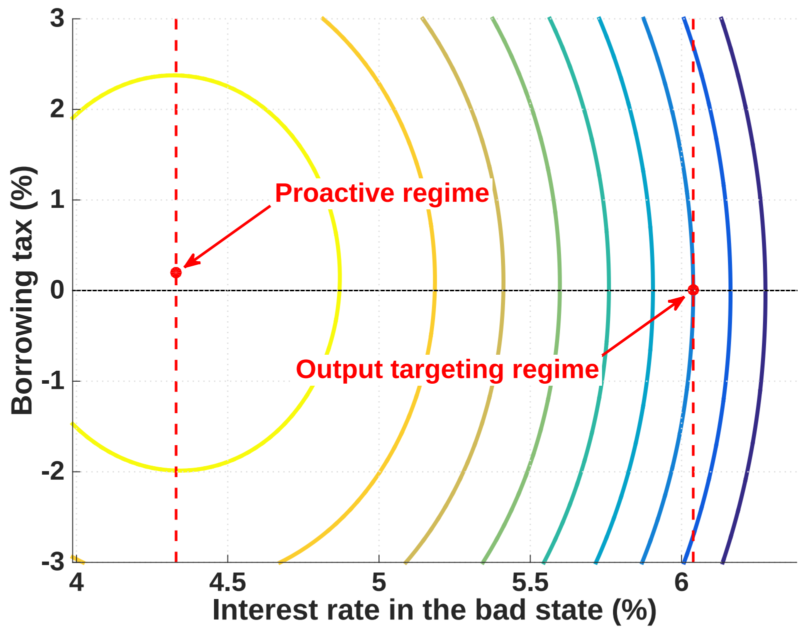

Figure 1: Monetary policy, macroprudential tax and welfare: complementarity

In the good state of the world the opposite is true. Inducing agents to borrow less ex ante leads to more output ex post. This is bad for welfare in the bad state of the world where the economy is already overstimulated. But it is good in the good state of the world, in which output is below potential. The latter effect dominates in our example, so overall a borrowing tax is welfare improving. This explains why we get complementarity between monetary policy and macroprudential policy in this example.

5.3 An example of substitutability

In Figure 1 we compared the proactive regime and the output targeting regime. What happens when we compare them to the inertial regime with rigid interest rates? In that regime, the sign of the macroprudential policy can be derived analytically, extending the first part of Proposition 3.

Proposition 4. *With CRRA preferences and $\delta_1 = 0$, in the inertial regime there is always overborrowing*

$$\frac{d}{dD} E\left[V^A(D, D, \bar{r}, s) + \beta V^B(D, D, \bar{r}, s)\right] < 0.$$

Figure 2 illustrates this result using our numerical example. The figure is constructed as Figure 1, but choosing the parameters α_0, α_1 so as to go through the output targeting regime and the inertial regime with an optimal macroprudential tax. As the figure shows,

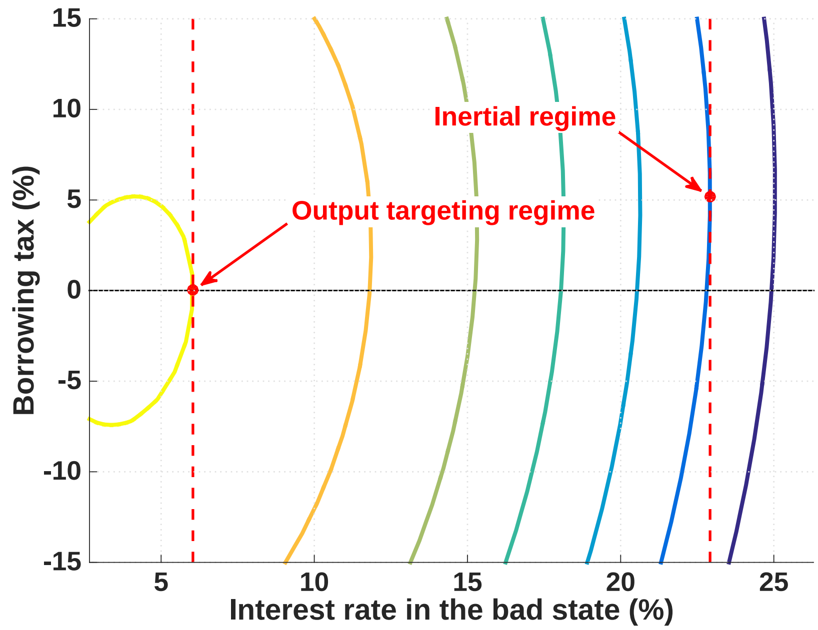

Figure 2: Monetary policy, macroprudential tax and welfare: substitutability

in this region of the policy space monetary policy and macroprudential policy are substitutes. That is, along the line that connects the two regimes considered, the cross derivative $\partial^2 W / \partial r_B \partial \tau$ is positive. As we compare the inertial regime to the output targeting regime, a more aggressive reduction of the interest rate in the second regime leads to a lower macroprudential tax.

To complete the analysis of our example, it remains to compare the two extreme regimes, the inertial regime and the proactive regime. We know that in both regimes the borrowing tax is positive. But in what regime is the tax larger? And in what regime are the welfare benefits of the tax larger? In Table 2 we answer this questions in the context of our example. The table reports the value of the optimal tax, the difference between D at $\tau = 0$ and D at the optimal tax, and the welfare gain due to the introduction of the optimal borrowing tax, in equivalent consumption terms. In the inertial regime all these measures of overborrowing are larger than in the proactive regime. Therefore, over the range of monetary policies that connect the two regimes, the forces of substitutability dominate the forces of complementariety identified in the previous subsection, so a more interventionist monetary policy ends up requiring less macroprudential intervention.

The table also shows how the results change when we choose a lower value of γ. All measures of overborrowing are larger in this case, but the relative comparison of the two regimes is qualitatively the same. In the appendix, we experiment with different parameter combinations and our qualitative results do not seem to depend on the choice

Table 2: Overborrowing in the inertial and proactive regimes

Value of γ	0.5		0.3	
Monetary regime	Inertial	Proactive	Inertial	Proactive
Optimal borrowing tax	5.2%	0.2%	8.1%	1.0%
Reduction in D due to the tax	0.025	0.0010	0.019	0.003
Welfare gain of tax (in %)	0.014	0.00002	0.023	0.0005

of β, γ, ψ, and ϕ.

We can then give a more nuanced answer the question: does a more aggressive monetary regime increase or decrease the need for ex ante regulation? The answer turns out to be sensitive not only to the parameters of the model but to the points we region of the policy space we are analyzing. If we compare a monetary policy that does not respond at all to negative shocks to a more aggressive policy, the need for macroprudential policy is lower, as the more aggressive monetary policy mitigates the aggregate demand externality. If we compare a monetary policy that is already responding strongly enough so as to eliminate the output gap to an even more aggressive policy, then the need for macroprudential policy increases in the latter regime.

The conclusions above are of course partly driven by the special nature of our example. In all the numerical examples we have explored the benefit of macroprudential policy are lower in the proactive regime as compared to the inertial regime, but we do not know if that can be proved in general. The general message we can draw from these examples is that moral hazard concerns should not be necessarily stronger in presence of a more aggressive monetary policy, and that whether macroprudential policy needs to be tighter or looser depends on where we are in the policy space.

5.4 Another example of substitutability

The examples considered so far shows that once the central bank goes over and above its output stability objective to achieve a financial stability objective, this calls for a corrective macroprudential tax ex ante. One may wonder whether this is a general result. We now look at an example that shows that it is not. We maintain our benchmark parametrization but no longer assume zero dividends in period 1. Instead we assume that the dividends in the two periods are perfectly collinear

$$\delta_1 = \xi \delta_2,$$

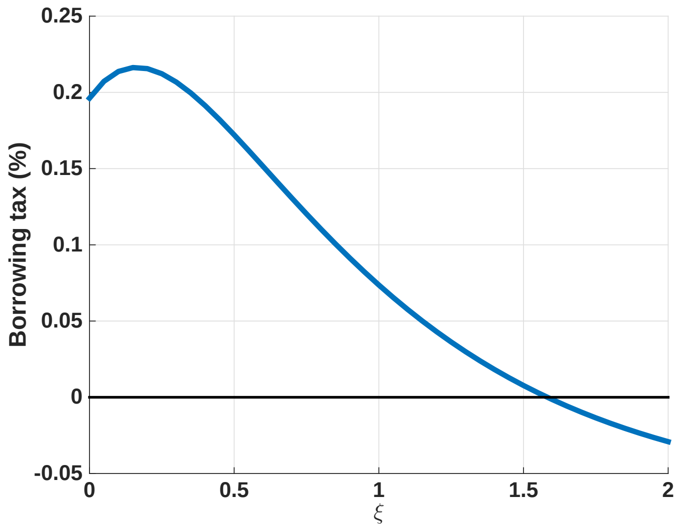

Figure 3: Optimal macroprudential tax with positive dividends in period 1

where $\xi \geq 0$. We continue to assume δ_2 has a binary distribution with values $\{1.05, 1.5\}$ and equal probabilities.

Figure 3 shows the optimal macroprudential tax in the proactive regime for different levels of ξ. The output targeting regime features a zero tax, as in all previous examples. When $\xi = 0$ we replicate the result in the proactive column of Table 2: the optimal tax is 0.2%. However, as ξ increases the optimal tax is lower and when ξ is high enough the optimal tax becomes a subsidy, so we have underborrowing in the proactive regime.

Recall the intuition for the complementarity result in Subsection 5.2. Optimal monetary policy generates a negative output gap in the bad state and a positive output gap in the good state. Increasing debt ex ante improves welfare in the bad state and reduces welfare in the good state. In the example of Subsection 5.2 the latter effect was larger. In the example here, with ξ high enough, the first effect is larger, so subsidizing borrowing is welfare improving. To get some intuition for why the magnitudes of the two effects are flipped with high ξ, notice that when δ_1 increases relative to δ_2 it gets harder for monetary policy to provide insurance to the B agents, as their wealth becomes less sensitive to r. This leads monetary policy in the bad state to lower rates more and produce a larger negative output gap.

6 Unconventional monetary policy

So far we have focused on a monetary policy response that uses the traditional tool of setting nominal interest rates. Recent experience has seen central banks use non-conventional tools, such as asset purchases, to fight recessions, after nominal rates hit the zero lower bound. In this section, we extend the model to ask how the use of non-conventional tools affects the message of the previous sections. We consider here a simple variation on the case of log preferences and zero dividends at $t = 1$ studied in Section 4. As argued at the end of that section, it is easy to add a ZLB constraint to the policy problem in the form

$$r \geq \underline{r}.$$

We will do so and explore what happens when the ZLB is binding and the central bank is considering unconventional interventions. Our question is now: Do ex post unconventional interventions that prop up asset prices increase the need of ex ante macroprudential policy? Before addressing that question, we need to characterize the equilibrium and introduce unconventional monetary policy.

6.1 Equilibrium with collateral constraints

Assume B agents are not allowed to pledge the entire future value of the asset when they borrow in period $t = 1$, but only a fraction θ of it. This introduces a form of market segmentation that will make unconventional interventions have some bite. Then we need to rewrite the budget constraints of the B agents in periods 1 and 2 as follows

$$xQ + c_1^B \leq Q - D + d_1,$$

$$c_2^B \leq \delta_2 x - (1+r)\, d_1,$$

where Q denotes the asset price, x is the amount of the asset kept by B agents at the end of period 1, and d_1 is borrowing in period 1. The collateral constraint is

$$(1+r)\, d_1 \leq \theta \delta_2 x.$$

Here we allow the agent to sell some of the asset and keep x of it. We introduce this possibility because we want to model unconventional monetary policy as asset purchases by the central bank. In particular, we assume that the central bank raises a lump sum tax T on A agents and uses it to purchase x^G units of the asset, so

$$T = x^G Q.$$

The asset then produces goods in the government sector according to the strictly concave production function $F\left(x^G\right)$, and these goods are paid to the A agents. The intertemporal constraint of the A agents then takes the form

$$c_1^A + \frac{1}{1+r} c_2^A \leq n_1^A + D - T + \frac{1}{1+r} F\left(x^G\right). \tag{13}$$

We assume $F'(0) < \delta_2$ for all realizations of δ_2, so it will be inefficient to purchase the asset in normal circumstances. This policy features a tradeoff between increased purchases, which push up asset prices and improve the balance sheet of B agents, and the inefficiency cost of the government technology.[10]

We now want to characterize the optimization problem of the B agents, imposing equilibrium in the asset market for a given value of government purchases x^G and a given interest rate r. Two cases are possible. First, if the collateral constraint is not binding the asset price is

$$Q = \frac{\delta_2}{1+r},$$

and the consumption levels are

$$c_1^B = \frac{1}{1+\beta}\left(\frac{\delta_2}{1+r} - D\right), \quad c_2^B = \frac{\beta(1+r)}{1+\beta}\left(\frac{\delta_2}{1+r} - D\right),$$

exactly as in Section 4. After some algebra, the condition that ensures that the collateral constraint is not binding can be written as

$$\frac{\beta}{1+\beta}\left(\frac{\delta_2}{1+r} - D\right) \geq (1-\theta)\, x \frac{\delta_2}{1+r}. \tag{14}$$

So, as long as the B agents are solvent, meaning that their assets valued at the first best price are sufficient to repay their debt D, there is always a value of x low enough that ensures an unconstrained equilibrium. That is, given that $x^G = 1 - x$, if the government purchases enough of the asset at the first best price, the B agents are unconstrained.

The second case is when the collateral constraint is binding, which arises when inequality (14) is violated. With a binding collateral constraint the values of c_1^B, c_2^B and Q must satisfy:

$$c_1^B = (1-x)\, Q + \frac{\theta \delta_2 x}{1+r} - D,$$

[10] This is different from a policy that subsidizes directly asset prices. If such policy could be financed by lump sum taxes on A agents, it would effectively transfer resources between the agents with no efficiency cost, thus allowing the planner to reach the first best.

$$c_2^B = (1-\theta)\,\delta_2 x, \tag{15}$$

$$Q = \theta \frac{1}{1+r}\delta_2 + (1-\theta)\,\beta \frac{u'\left(c_2^B\right)}{u'\left(c_1^B\right)}\delta_2, \tag{16}$$

$$u'\left(c_1^B\right) > \beta\,(1+r)\,u'\left(c_2^B\right).$$

The first equation says that consumption in period 1 plus the debt payment D are financed by selling some of the asset and borrowing up to the limit. The second equation has consumption in period 2 given by the dividends on the unsold asset minus the maximum debt payment $\theta\delta_2 x$. The third equation shows that the asset price is a weighted average of two valuations: the pledgeable fraction θ of the asset is valued using the market discount factor $1/\left(1+r\right)$, the unpledgeable fraction $1-\theta$ is valued using the discount factor of the constrained B agent $\beta u'\left(c_2^B\right)/u'\left(c_1^B\right)$. This equation comes from deriving first order conditions of the problem and solving for lagrange multipliers. The last inequality ensures that the borrowing constraint is binding, i.e., that the Lagrange multiplier on the collateral constraint is positive. The last two conditions imply that when the collateral constraint is binding the asset price is depressed below its first best value

$$Q < \frac{\delta_2}{1+r}.$$

Using log preferences it is possible to manipulate the conditions just presented and obtain consumption in the first period as a function of x and of the interest rate alone:

$$c_1^B = \frac{x}{(1+\beta)\,x - \beta}\left[\frac{\theta\delta_2}{1+r} - D\right]. \tag{17}$$

This implies that as the government purchases more of the asset, i.e. as x is reduced, the consumption c_1^B increases, the consumption c_2^B decreases (directly from (15)), and the asset price Q increases (from (16) and the results for c_1^B, c_2^B).

Summing up, if the government purchases enough of the asset the equilibrium is unconstrained and consumption of the B agents and the asset price are unaffected by further asset purchases. If instead purchases are low enough and

$$\frac{\beta}{1+\beta}\left(\frac{\delta_2}{1+r} - D\right) < (1-\theta)\,\frac{\delta_2}{1+r}$$

then the B agents are constrained and government purchases have the effect of raising asset prices and of tilting the consumption profile towards date 1.

6.2 Aggregate demand and asset price effects of asset purchases

Suppose now that in a given state of the world the B agents' collateral constraint is binding and the government is considering the welfare benefits of an asset purchase. What is the effect on aggregate demand and thus on real activity for a given level of r? What are the additional welfare effects of the intervention?

Consider the marginal effects of an increase $dx^G > 0$ from a given x^G. The effect on the consumption of B agents can be obtained from (17) and is positive:

$$\frac{\partial c_1^B}{\partial x^G} = \frac{\beta}{(1+\beta)\,x - \beta} c_1^B > 0.$$

The effect on the consumption of the A agents can be derived from the intertemporal budget constraint (13) and optimality of the A agents and is

$$\frac{\partial c_1^A}{\partial x^G} = \frac{1}{2}\left(\frac{F'\left(x^G\right)}{1+r} - Q - \frac{\partial Q}{\partial x^G} x^G\right).$$

The sign of this effect depends on how depressed are asset prices, how inefficient is the government technology and how large are asset purchases. The total effect on output is the sum of these two effects

$$\frac{\partial Y}{\partial x^G} = \frac{\partial c_1^B}{\partial x^G} + \frac{\partial c_1^A}{\partial x^G}.$$

In this economy asset prices are not determined one for one by interest rates as it was the case in previous sections. This adds an interesting dimension to the problem as it introduces a pecuniary externality that is not directly controlled by the choice of the interest rate. In particular, as argued above

$$\frac{\partial Q}{\partial x^G} > 0.$$

We can then look at the total welfare effects of an increase in asset purchases x^G, using envelope arguments similar to the ones used in previous sections, and obtain the following marginal effects on social welfare

$$\left(u'(c_1^A) - v'\left(n_1^A\right)\right)\frac{\partial Y}{\partial x^G} + \left(\beta u'\left(c_1^B\right) - u'(c_1^A)\right) x^G \frac{\partial Q}{\partial x^G} + u'(c_1^A)\left(\frac{F'\left(x^G\right)}{1+r} - Q\right). \quad (18)$$

The first effect is the aggregate demand effect, the second is a redistributive effect associated to an increase in asset prices, which is good for the B agents selling the asset and bad for the A agents who, through government intervention, are purchasing the asset. Finally, the third effect is the direct benefit of the intervention in terms of employing the asset in the government technology.

6.3 Complementarity and substitutability

We now want to analyze how the introduction of unconventional policy ex post alters the benefits of macroprudential policy ex ante. To do so our starting point is an equilibrium in which asset purchases are zero, $x^G = 0$ and $x = 1$ in all states.

It is easy to construct examples with two states of the world where the equilibrium has the following features. In the high δ_2 state, the good state, neither the collateral constraint nor the zero lower bound are binding and the central bank sets $r > \underline{r}$ so as to satisfy the optimality condition

$$\left(u'(c_1^A) - v'\left(n_2^A\right)\right)\frac{\partial Y}{\partial r} + \left(u'(c_1^A) - \beta u'(c_1^B)\right) d_1 \frac{1}{1+r} = 0,$$

derived in Section 3.3. In the low δ_2 state, the bad state, both the zero lower bound and the collateral constraint are binding and we have

$$\left(u'(c_1^A) - v'\left(n_2^A\right)\right)\frac{\partial Y}{\partial r} + \left(u'(c_1^A) - \beta u'(c_1^B)\right) d_1 \frac{1}{1+r} < 0.$$

In the good state we have $u'(c_1^A) > v'\left(n_2^A\right)$ and $u'(c_1^A) > \beta u'(c_1^B)$: it would be beneficial to raise rates to redistribute in favor of A agents, since B agents are relatively rich, but raising rates is costly since there is a positive output gap. In the bad state we have $u'(c_1^A) > v'\left(n_2^A\right)$ and $u'(c_1^A) < \beta u'(c_1^B)$: it would be beneficial to lower rates both in terms of output effects and in terms of distributive effects, as B agents are relatively poor, but it is not possible because we have reached the ZLB.[11]

Let us look first at the ex post incentives of the central bank to engage in asset purchases. In the good state, the marginal effect on social welfare (18) boils down to

$$\left(u'(c_1^A) - v'\left(n_1^A\right)\right)\frac{\partial c_1^A}{\partial x^G} + u'(c_1^A)\left(\frac{F'(0) - \delta_2}{1+r}\right) < 0,$$

because the effects on asset prices and the consumption of the B agents are zero and the effect on the consumption of the A agents and the direct effect are both negative because

[11] The presence of the collateral constraints does not alter the fact that the effect of a change in r on the utility of the B agents is captured by $u'\left(c_1^B\right) d_1 / (1+r)$.

we assume $F'(0) < \delta_2$. This implies that the optimal policy is at the corner $x^G = 0$. In the bad state on the other hand, all three terms in (18) can be positive if it is the case that $F'(0) - Q \geq 0$ in equilibrium. The aggregate demand effect is positive because the consumption of both A and B agents is increasing in x^G: for the B agents because their constrained gets relaxed, for the A agents because the government is passing them the profits made purchasing assets at distressed prices. The redistributive effect is positive because higher asset prices benefit the B agents with higher marginal utility. And the direct fiscal benefit is positive because, again, the government is making profits on the asset purchases. So on net asset purchases are beneficial ex post.

Now let us look at the motive for macroprudential policy ex ante. The marginal effect of D on social welfare in the continuation equilibrium is given by

$$E\left[\left(u'(c_1^A) - v'\left(n_1^A\right)\right)\frac{\partial Y}{\partial D}\right] + E\left[\left(\beta u'(c_1^B) - u'(c_1^A)\right)x^G\frac{\partial Q}{\partial D}\right].$$

The first is the aggregate demand externality discussed in previous sections. In the configuration discussed here this effect is negative. The second is a pecuniary externality due to the fact that higher asset prices reallocate resources from A to B agents. In the configuration discussed here $\partial Q/\partial D = 0$ in the good state as the collateral constraint is slack. In the bad state, conditions (15)-(17) can be used to show that $\partial Q/\partial D < 0$. In the configuration discussed here with $\beta u'(c_1^B) > u'(c_1^A)$ in the bad state the second effect is negative as soon as x^G is not zero. Therefore, given our configuration at $x^G = 0$ and, by continuity, for x^G near zero, the total externality ex ante is negative and a tax on borrowing is welfare improving.

Now we can return to the question addressed of this paper: how does the use of policy ex post (asset purchases now) affect the desirability of borrowing tax ex ante? In general it seems to depend on the relative strength of the two externalities identified. The aggregate demand externality gets smaller as asset purchases increase, as an increase in x^G leads to an increase in output which reduces the output gap. On the other hand, the pecuniary externality gets larger as we move from $x^G = 0$ to $x^G > 0$. However, in the numerical examples we have explored the aggregate demand externality dominates and the economy features a smaller borrowing tax when ex post asset purchases take place. So the example we have solved with the features above suggest that also when unconventional monetary tools are used, ex post interventions seem to be substitutes for ex ante macroprudential policy.

7 Concluding remarks

In this paper we have shown that the presence of aggregate demand externalities tends to reverse the conventional wisdom on the need of corrective macroprudential policy to balance ex post monetary interventions that prop up asset prices, either by interest rate policy or by unconventional asset purchases.

Our model is stylized and we have proceeded by examples and counterexamples, so our take away is not that moral hazard concerns regarding central bank interventions are necessarily unfounded. Rather our model is a cautionary tale on how looking at the positive predictions of the models (lower rates ex post lead to more borrowing) can lead to incorrect conclusions on complementarity or substitutability between ex ante macroprudential tools and ex post monetary policy.

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8 Appendix

8.1 Characterization of continuation equilibrium

8.1.1 General utility function

Recall the value functions for A and B agents are given by[12]

$$
\begin{aligned}
V^A(b,D,r,s) &= \max_{c_1^A,c_2^A} \quad u(c_1^A) - v\left(Y(D,r,s)\right) + u\left(c_2^A\right) \\
& \qquad\qquad \text{s.t.} \quad c_1^A + \frac{1}{1+r}c_2^A = Y(D,r,s) + b, \\
V^B(d,D,r,s) &= \max_{c_1^B,c_2^B} \quad u\left(c_1^B\right) + \beta u\left(c_2^B\right) \\
& \qquad\qquad \text{s.t.} \quad c_1^B + \frac{1}{1+r}c_2^B = \delta_1\left(s\right) + \frac{1}{1+r}\delta_2(s) - d.
\end{aligned}
$$

The optimality conditions of the two types of agents are

$$
u'\left(c_1^A\right) = (1+r)u'\left(c_2^A\right), \qquad\qquad u'\left(c_1^B\right) = \beta(1+r)u'\left(c_2^B\right),
$$

which together with their resource constraint characterize the continuation equilibrium. That is, given D and r, a continuation equilibrium is given by the quantities c_1^A, c_2^A, c_1^B, c_2^B, Y that satisfy:

$$
\begin{aligned}
u'\left(c_1^A\right) &= (1+r)u'\left(c_2^A\right), & u'\left(c_1^B\right) &= \beta(1+r)u'\left(c_2^B\right), \\
c_1^A + \frac{1}{1+r}c_2^A &= Y + D, & c_1^B + \frac{1}{1+r}c_2^B &= \delta_1 + \frac{1}{1+r}\delta_2 - D,
\end{aligned}
$$

and the goods market clearing conditions,

$$
c_1^A + c_1^B = Y + \delta_1, \qquad\qquad c_2^A + c_2^B = \delta_2.
$$

We use Y to denote produced output in period 1. Then the definition of continuation equilibrium above defines a mapping $Y\left(D,r,s\right)$, which corresponds to the equilibrium produced output in period 1 as a function of the debt level and the level of interest rate. In the next section we derive an explicit formula for $Y\left(D,r,s\right)$ by considering a specific functional form for the utility of consumption, the CRRA form.

[12]For ease of notation, whenever possible, we omit the state s for the different endogenous variables. Note that these variables do vary with the state of the economy.

8.1.2 CRRA utility

Here we provide a characterization of a continuation equilibrium that will be used for the proofs to follow and for the numerical examples. The utility function u has the CRRA form

$$u(c) = \frac{1}{1-\gamma} c^{1-\gamma}.$$

From consumer optimization, we obtain

$$c_1^A = \frac{1}{1 + (1+r)^{\frac{1}{\gamma}-1}} (Y + D), \quad c_1^B = \frac{1}{1 + \beta^{\frac{1}{\gamma}} (1+r)^{\frac{1}{\gamma}-1}} \left(\delta_1 + \frac{\delta_2}{1+r} - D\right).$$

Aggregating and using goods market clearing we have

$$Y = \frac{1}{1 + (1+r)^{\frac{1}{\gamma}-1}} (Y + D) + \frac{1}{1 + \beta^{\frac{1}{\gamma}} (1+r)^{\frac{1}{\gamma}-1}} \left(\delta_1 + \frac{\delta_2}{1+r} - D\right) - \delta_1.$$

After rearranging and defining $\tilde{R} = (1+r)^{\frac{1-\gamma}{\gamma}}$ and $\tilde{\beta} = \beta^{\frac{1}{\gamma}}$, the continuation equilibrium quantities in period 1 can be written as follows:

$$c_1^A = \frac{1}{1+\tilde{R}}(Y + D), \tag{19}$$

$$c_1^B = \frac{1}{1+\tilde{\beta}\tilde{R}} \left(\delta_1 + \frac{\delta_2}{1+r} - D\right), \tag{20}$$

$$Y = -\frac{1-\tilde{\beta}}{1+\tilde{\beta}\tilde{R}} D - \left(1 - \frac{1-\tilde{\beta}}{1+\tilde{\beta}\tilde{R}}\right) \delta_1 + \frac{1+\tilde{R}}{\tilde{R}\left(1+\tilde{\beta}\tilde{R}\right)} \frac{\delta_2}{1+r}. \tag{21}$$

Taking derivatives with respect to D and and r we obtain:

$$\frac{\partial Y}{\partial D} = -\frac{1-\tilde{\beta}}{1+\tilde{\beta}\tilde{R}}, \tag{22}$$

$$\frac{\partial Y}{\partial r} = \theta_1(r)(\delta_1 - D) + \theta_2(r)\delta_2, \tag{23}$$

where

$$\theta_1(r) = -\frac{1}{1+r} \frac{1-\gamma}{\gamma} \frac{\left(1-\tilde{\beta}\right)\tilde{\beta}\tilde{R}}{\left(1+\tilde{\beta}\tilde{R}\right)^2},$$

$$\theta_2(r) = -\frac{1}{(1+r)^2} \frac{1}{\tilde{R}\left(1+\tilde{\beta}\tilde{R}\right)^2} \frac{1}{\gamma} \left[1 + \tilde{\beta}\tilde{R} + \left(\gamma(1-\tilde{\beta}) + \tilde{\beta}(1+\tilde{R})\right)\tilde{R}\right].$$

Notice that $\theta_2(r)$ is negative, while the sign of $\theta_1(r)$ depends on whether γ is greater or lower than 1. The intuition is as follows. An increase in the interest rate decreases the value of endowment of B agents in period 2. So the relative discounted income of A agents goes up. As A agents are more patient and output is demand-determined, this channel decreases the level of output in period 1. However, a change in the interest rate also affects B agents through their net endowment, $\delta_1 - D$, in period 1. On the one hand, the substitution effect incentivizes households to increase their demand for savings and lower consumption in period 1. On the other hand, there is an income effect incentivizing households to increase consumption in period 1. With log preferences ($\gamma = 1$) the substitution and income effect cancel each other and the expressions above become:

$$c_1^A = \frac{1}{2}\left(Y + D\right), \quad c_1^B = \frac{1}{1+\beta}\left(\delta_1 + \frac{\delta_2}{1+r} - D\right),$$

$$Y = -\frac{1-\beta}{1+\beta}D - \frac{2\beta}{1+\beta}\delta_1 + \frac{2}{1+\beta}\frac{\delta_2}{1+r}.$$

8.2 Proofs for Section 4

8.2.1 Proof of Proposition 1

Proof. Denote the asset price by Q, where

$$Q = \frac{\delta}{1+r}.$$

We guess and verify that an equilibrium with constant asset prices, $\bar{Q}$, exists. Given the derivations in Section 8.1.2, it is immediate that c_1^B is constant across states when $\delta/1+r$ is constant

$$c_1^B = \frac{1}{1+\beta}\left(\frac{\delta}{1+r} - D\right).$$

Equilibrium in the bonds market at $t = 0$ and equilibrium in the goods market at $t = 1$ then give the two equations

$$u'(c_1^A) = \beta u'\left(c_1^B\right),$$

$$Y = c_1^A + c_1^B.$$

With log preferences, these yield

$$c_1^A = \frac{1}{1+\beta}Y, \quad c_1^B = \frac{\beta}{1+\beta}Y.$$

The zero-output-gap condition then becomes

$$u'\left(\frac{1}{1+\beta}Y\right) = v'(Y),$$

which has a unique solution Y. The value of D can be found using the condition

$$c_1^A = \frac{1}{1+\beta}Y = \frac{1}{2}(Y+D),$$

and the value for $\bar{Q}$ using the condition

$$c_1^B = \frac{\beta}{1+\beta}Y = \frac{1}{1+\beta}\left(\frac{\delta}{1+r} - D\right).$$

The interest rates are

$$1 + r(s) = (1 + \delta(s)) / \bar{Q}.$$

It is immediate to check that all conditions for a zero-output-gap equilibrium are satisfied. It is also easy to check that the optimal monetary policy condition (10) is satisfied, since both of its terms are equal to zero in all states. □

Note that when there is no uncertainty, i.e., $\sigma = 0$, we have that the inertial regime is equivalent to the proactive regime.

8.2.2 Proof of Proposition 2

Proof. Without loss of generality we let

$$\delta_2 = \bar{\delta} + \sigma z,$$

where $\bar{\delta}$ is the mean of δ_2, z is a random variable with mean 0 and variance of 1, and σ is a positive scalar. In this proof we show that there exists a $\underline{\sigma}$, such that if $\sigma \in (0, \underline{\sigma})$ then the level of borrowing in the inertial regime is lower than in the proactive regime.

In this proof we use a change of variables: $m = \frac{1}{1+\beta}\frac{1}{1+r}$, and $b = \frac{1}{1+\beta}D$. Since the proof uses only quantities in period 1, we use the notation $c_a \equiv c_1^A$ and $c_b \equiv c_1^B$. Using the change of variables we have that

$$\begin{aligned} c_A &= m(\bar{\delta} + \sigma z) + \beta b \,, \\ c_B &= m(\bar{\delta} + \sigma z) - b \,, \\ Y &= 2m(\bar{\delta} + \sigma z) - (1-\beta) b \,. \end{aligned}$$

The two equilibrium conditions are

$$\mathbb{E}\left[\frac{1}{m(\bar{\delta}+\sigma z)+\beta b} - \beta \frac{1}{m(\bar{\delta}+\sigma z)-b}\right] = 0 \,, \tag{24}$$

$$\begin{aligned} &\mathbb{E}\left[\left(\frac{1}{m(\bar{\delta}+\sigma z)+\beta b} - \beta \frac{1}{m(\bar{\delta}+\sigma z)-b}\right)\left(m(\bar{\delta}+\sigma z)+\beta b\right)\right] = \\ &\qquad \mathbb{E}\left[\left(\frac{1}{m(\bar{\delta}+\sigma z)+\beta b} - \psi\left(2m(\bar{\delta}+\sigma z)-(1-\beta)b\right)^{\phi}\right) 2m(\bar{\delta}+\sigma z)\right] \,, \end{aligned}$$

where the first one is the equation which pins down the level of borrowing, and the second one is the optimality condition for the social planner's choice of interest rate. We can rearrange the second equation as follows,

$$\begin{aligned} &\mathbb{E}\left[\frac{m(\bar{\delta}+\sigma z)+\beta b}{m(\bar{\delta}+\sigma z)+\beta b} - \beta \frac{m(\bar{\delta}+\sigma z)+\beta b}{m(\bar{\delta}+\sigma z)-b}\right] = \\ &\qquad 2\mathbb{E}\left[\frac{m(\bar{\delta}+\sigma z)}{m(\bar{\delta}+\sigma z)+\beta b} - \psi\left(2m(\bar{\delta}+\sigma z)-(1-\beta)b\right)^{\phi} m(\bar{\delta}+\sigma z)\right] \end{aligned}$$

So that

$$\begin{aligned} &1-\beta-(1+\beta)b\mathbb{E}\left[\frac{\beta}{m(\bar{\delta}+\sigma z)-b}\right] = \\ &\qquad 2-2\beta b\mathbb{E}\left[\frac{1}{m(\bar{\delta}+\sigma z)+\beta b}\right] - \mathbb{E}\left[2\psi\left(2m(\bar{\delta}+\sigma z)-(1-\beta)b\right)^{\phi} m(\bar{\delta}+\sigma z)\right] \end{aligned}$$

Using equation (24) to substitute for $\mathbb{E}\left[\frac{\beta}{m(\bar{\delta}+\sigma z)-b}\right]$ and rearranging, we obtain

$$\mathbb{E}\left[1+\beta+(1-\beta)\frac{b}{m(\bar{\delta}+\sigma z)+\beta b} - 2\psi\left(2m(\bar{\delta}+\sigma z)-(1-\beta)b\right)^{\phi} m(\bar{\delta}+\sigma z)\right] = 0$$

Let's define the following two functions,

$$G(m,b,\sigma) = \mathbb{E}\left[\frac{1}{m(\bar{\delta}+\sigma z)+\beta b} - \beta\frac{1}{m(\bar{\delta}+\sigma z)-b}\right], \tag{25}$$

$$H(m,b,\sigma) = \mathbb{E}\left[1+\beta+(1-\beta)\frac{b}{m(\bar{\delta}+\sigma z)+\beta b} - 2\psi\left(2m(\bar{\delta}+\sigma z)-(1-\beta)b\right)^{\phi} m(\bar{\delta}+\sigma z)\right]. \tag{26}$$

In equilibrium both $G(m,b,\sigma)$ and $H(m,b,\sigma)$ are equal to 0. So these two equations define an implicit equilibrium levels of m and b given σ, $m(\sigma)$ and $b(\sigma)$:

$$G\left(m(\sigma),b(\sigma),\sigma\right) = 0\,, \tag{27}$$

$$H\left(m(\sigma),b(\sigma),\sigma\right) = 0\,. \tag{28}$$

A corollary of proposition 1 is that the levels of m and b in the proactive regime are equal to $m(0)$ and $b(0)$, i.e., the inertial regime equilibrium when there is no uncertainty about δ_2. We want to show that $b(\sigma) < b(0)$ for σ in the neighborhood of 0. As a byproduct, we also show that $m(\sigma) < m(0)$ for small enough σ. To do so, we prove that $m'(0) = b'(0) = 0$ and that both $m''(0)$ and $b''(0)$ are negative. This, in turn, implies that around a neighborhood of $\sigma = 0$ both $m(\sigma)$ and $b(\sigma)$ are lower than $m(0)$ and $b(0)$.

Step I - showing $m'(0) = b'(0) = 0$. We take total derivative of equations (27) and (28):

$$G_m(\sigma)m'(\sigma) + G_b(\sigma)b'(\sigma) + G_\sigma(\sigma) = 0\,,$$
$$H_m(\sigma)m'(\sigma) + H_b(\sigma)b'(\sigma) + H_\sigma(\sigma) = 0\,,$$

where $G_m(\sigma)$ is a short notation for $G_m(m(\sigma),b(\sigma),\sigma)$, and similarly for the other partial derivatives. We can rearrange the following equations at $\sigma = 0$ to obtain

$$\begin{bmatrix} m'(0) \\ b'(0) \end{bmatrix} = -\begin{bmatrix} G_m(0) & G_b(0) \\ H_m(0) & H_b(0) \end{bmatrix}^{-1} \begin{bmatrix} G_\sigma(0) \\ H_\sigma(0) \end{bmatrix}.$$

The partial derivatives of $G(\cdot)$ and $H(\cdot)$ are presented below. To save on notation we use

c_a and c_b when appropriate.

$$
\begin{aligned}
G_m(m,b,\sigma) &= \mathbb{E}\left[\left(-\frac{1}{c_a^2}+\beta\frac{1}{c_b^2}\right)(\bar{\delta}+\sigma z)\right] ,\\
G_b(m,b,\sigma) &= \mathbb{E}\left[-\beta\left(\frac{1}{c_a^2}+\frac{1}{c_b^2}\right)\right] ,\\
G_\sigma(m,b,\sigma) &= \mathbb{E}\left[\left(-\frac{1}{c_a^2}+\beta\frac{1}{c_b^2}\right)mz\right] ,\\
H_m(m,b,\sigma) &= \mathbb{E}\left[-\left((1-\beta)\frac{b}{c_a^2}+2\psi Y^\phi+2\psi\phi Y^{\phi-1}m(\bar{\delta}+\sigma z)\right)(\bar{\delta}+\sigma z)\right] ,\\
H_b(m,b,\sigma) &= \mathbb{E}\left[(1-\beta)\left(\frac{1}{c_a^2}+2\psi\phi Y^{\phi-1}\right)m(\bar{\delta}+\sigma z)\right] ,\\
H_\sigma(m,b,\sigma) &= \mathbb{E}\left[-\left((1-\beta)\frac{b}{c_a^2}+2\psi Y^\phi+4\psi\phi Y^{\phi-1}m(\bar{\delta}+\sigma z)\right)mz\right] .
\end{aligned}
$$

Evaluating at $\sigma=0$ using $\frac{c_b(0)}{c_a(0)}=\beta$, together with $\mathbb{E}(z)=0$ and $\mathbb{E}\left(z^2\right)=1$,

$$
\begin{aligned}
&G_m(0) = -\left(\frac{1}{c_b(0)}\right)^2\left(\beta^2-\beta\right)\bar{\delta}>0 ,\\
&G_b(0) = -\left(\frac{1}{c_b(0)}\right)^2\beta\left(\beta^2+1\right)<0 ,\\
&G_\sigma(0) = 0 ,\\
&H_m(0) = -\bar{\delta}\left(\frac{(1-\beta)b}{(c_a(0))^2}+2\psi\left(Y(0)\right)^\phi+2\psi\phi\left(Y(0)\right)^{\phi-1}m\bar{\delta}\right)<0 ,\\
&H_b(0) = (1-\beta)\left(\frac{1}{(c_a(0))^2}+2\psi\phi\left(Y(0)\right)^{\phi-1}\right)m\bar{\delta}>0 ,\\
&H_\sigma(0) = 0 .
\end{aligned}
$$

Define the matrix A as follows,

$$
A=\begin{bmatrix} G_m(0) & G_b(0) \\ H_m(0) & H_b(0)\end{bmatrix}\equiv\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4\end{bmatrix} .
$$

We showed above that $a_1>0$, $a_2<0$, $a_3<0$, and $a_4>0$. So the determinant of A is positive and A is non-singular. In particular, the inverse of A is given by

$$A^{-1} = \frac{1}{a_1 a_4 - a_2 a_3} \begin{bmatrix} a_4 & -a_2 \\ -a_3 & a_1 \end{bmatrix} .$$

So all the elements of A^{-1} are positive. For this step of the proof it is enough to know that A is non-singular, but in the next step we will use the fact that A^{-1} is a positive matrix. Since $G_\sigma(0) = H_\sigma(0) = 0$, we have that

$$\begin{bmatrix} m'(0) \\ b'(0) \end{bmatrix} = -A^{-1} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} .$$

Step II - showing $m''(0)$ and $b''(0)$ are negative. Taking a second total derivative of equations (27) and (28), together with $G_\sigma(0) = H_\sigma(0) = m'(0) = b'(0) = 0$, we obtain

$$\begin{aligned} G_m(0)m''(0) + G_b(0)b''(0) + G_{\sigma\sigma} = 0 \, , \\ H_m(0)m''(0) + H_b(0)b''(0) + H_{\sigma\sigma} = 0 \, . \end{aligned}$$

So that

$$\begin{bmatrix} m'(0) \\ b'(0) \end{bmatrix} = -A^{-1} \begin{bmatrix} G_{\sigma\sigma}(0) \\ H_{\sigma\sigma}(0) \end{bmatrix} .$$

$G_{\sigma\sigma}$ and $H_{\sigma\sigma}$ are equal to

$$\begin{aligned} G_{\sigma\sigma}(m, b, \sigma) &= \mathbb{E}\left[\left(\frac{2}{c_a^3} - \beta \frac{2}{c_b^3} \right) m^2 z^2 \right] , \\ H_{\sigma\sigma}(m, b, \sigma) &= \mathbb{E}\left[\left((1-\beta)\frac{2b}{c_a^3} + 8\psi\phi Y^{\phi-2} \left(Y + (\phi - 1)m(\bar{\delta} + \sigma z) \right) \right) m^2 z^2 \right] . \end{aligned}$$

At $\sigma = 0$ we have,

$$\begin{aligned} G_{\sigma\sigma}(0) &= -\left(\frac{1}{c_b(0)} \right)^3 2 \left(\beta^3 - \beta \right) m^2 > 0 \, , \\ H_{\sigma\sigma}(0) &= \left((1-\beta)\frac{2b}{(c_a(0))^3} + 8\psi\phi \left(Y(0) \right)^{\phi-2} \left(c_b(0) + \phi m \bar{\delta} + \beta b \right) \right) m^2 > 0 \, . \end{aligned}$$

Since A^{-1} is a positive matrix and both $G_{\sigma\sigma}(0)$ and $H_{\sigma\sigma}(0)$ are positive, we conclude that

$m''(0)$ and $b''(0)$ are negative:

$$\begin{bmatrix} m''(0) \\ b''(0) \end{bmatrix} = -A^{-1} \begin{bmatrix} G_{\sigma\sigma}(0) \\ H_{\sigma\sigma}(0) \end{bmatrix} < \begin{bmatrix} 0 \\ 0 \end{bmatrix} .$$

□

8.2.3 Proof of Proposition 3

Proof. Proposition 1 and equation (12) imply that there is no benefit from changing the level of borrowing, D, in the proactive regime. In what follows we prove that in the inertial regime a decrease in the level of borrowing has welfare benefits.

Step I. Show that

$$E\left[\left[u'(c_1^A) - \beta u'\left(c_1^B\right)\right]\left(c_1^B + D\right)\right] > 0.$$

Notice that given CRRA and a constant interest rate the ratio $c_1^A / (Y + D)$ is constant. Since $Y = c_1^A + c_1^B$, this implies that there is a constant ξ such that

$$c_1^A = \xi\left(c_1^B + D\right).$$

We then want to evaluate

$$E\left[\left[\left(\xi\left(c_1^B + D\right)\right)^{-\gamma} - \beta\left(c_1^B\right)^{-\gamma}\right]\left(c_1^B + D\right)\right].$$

Notice that there is a unique cutoff $\hat{c}_2^B$ such that

$$\left(\xi\left(c_1^B + D\right)\right)^{-\gamma} - \beta\left(c_1^B\right)^{-\gamma} \gtrless 0,$$

iff $c_1^B \gtrless \hat{c}_1^B$. Therefore

$$E\left[\left[\left(\xi\left(c_1^B + D\right)\right)^{-\gamma} - \beta\left(c_1^B\right)^{-\gamma}\right]\left(c_1^B - \hat{c}_1^B\right)\right] > 0.$$

Moreover, from consumers optimality at $t = 0$ we have

$$E\left[\left[\left(\xi\left(c_1^B + D\right)\right)^{-\gamma} - \beta\left(c_1^B\right)^{-\gamma}\right]\right] = 0.$$

Combining the last two equations we have

$$E\left[\left[\left(\xi\left(c_1^B+D\right)\right)^{-\gamma}-\beta\left(c_1^B\right)^{-\gamma}\right]\left(c_1^B+D\right)\right]>0.$$

Step II. From Step I and optimality of monetary policy we deduce that

$$E\left[\left(u'(c_1^A)-v'\left(Y\right)\right)\frac{\partial Y}{\partial r}\right]<0.$$

With log preferences and a constant interest rate we deduce that

$$\frac{\partial Y}{\partial r}=-\Xi_1\frac{\delta}{(1+\bar{r})^2}\text{ and }\frac{\partial Y}{\partial D}=-\Xi_2, \tag{29}$$

for some positive constant terms Ξ_1,Ξ_2. Therefore, we have the inequality

$$E\left[\left(u'(c_1^A)-v'\left(Y\right)\right)\delta\right]>0.$$

The expression $u'(c_1^A)-v'\left(Y\right)$ is a monotone decreasing function of δ, so we have the chain of inequalities

$$E\left[\left(u'(c_1^A)-v'\left(Y\right)\right)\right]E\left[\delta\right]>E\left[\left(u'(c_1^A)-v'\left(Y\right)\right)\delta\right]>0.$$

Using (29) we then conclude that

$$E\left[\frac{\partial V^A}{\partial D}+\beta\frac{\partial V^B}{\partial D}\right]=E\left[\left[u'(c_1^A)-v'\left(Y\right)\right]\frac{\partial Y}{\partial D}\right]<0.$$

$\square$

8.2.4 Proof of Proposition 4

Proof. Throughout the proof all prices and quantities are in the inertial regime equilibrium. We want to prove

$$E\left[\left(u'(c_1^A)-v'(Y)\right)\frac{\partial Y}{\partial D}\right]<0,$$

and equation (22) shows that $\partial Y/\partial D$ is negative and constant across states in the inertial regime. So we need to prove

$$E\left[u'(c_1^A)-v'\left(Y\right)\right]>0. \tag{30}$$

Since interest rate $\bar{r}$ is chosen optimally, the following condition holds

$$E\left[\frac{1}{1+r}\left(u'(c_1^A)-\beta u'(c_1^B)\right)\left(c_1^B+D\right)+\left(u'(c_1^A)-v'\left(Y\right)\right)\frac{\partial Y}{\partial r}\right]=0.$$

Using the optimal debt choice condition and the fact that D and r are independent of the state we can rearrange this into

$$\frac{1}{1+r}E\left[\left(u'(c_1^A)-\beta u'\left(c_1^B\right)\right)c_1^B\right]+E\left[\left(u'(c_1^A)-v'\left(Y\right)\right)\frac{\partial Y}{\partial r}\right]=0. \tag{31}$$

Rewrite (19) as

$$c_1^A=\frac{1}{1+\tilde{R}}(c_1^A+c_1^B+D),$$

to get

$$c_1^A=\frac{c_1^B+D}{\tilde{R}}.$$

This implies

$$u'(c_1^A)-\beta u'\left(c_1^B\right)=\tilde{R}^{\gamma}\left(c_1^B+D\right)^{-\gamma}-\beta\left(c_1^B\right)^{-\gamma}.$$

This expression satisfies single crossing in c_1^B, that is, there exists a level $\hat{c}_1^B$ such that the expression is zero at $c_1^B=\hat{c}_1^B$ and is positive if and only if $c_1^B>\hat{c}_1^B$. This, together with $E\left(u'(c_1^A)-\beta u'\left(c_1^B\right)\right)=0$, implies that

$$E\left[\left(u'(c_1^A)-\beta u'(c_1^B)\right)c_1^B\right]=E\left[\left(u'(c_1^A)-\beta u'(c_1^B)\right)\left(c_1^B-\hat{c}_1^B\right)\right]>0.$$

Equation (31) then implies

$$E\left[\left(u'(c_1^A)-v'\left(Y\right)\right)\frac{\partial Y}{\partial r}\right]<0.$$

Substituting in the derivative of output with respect to the interest rate (23), we obtain

$$E\left[\left(u'(c_1^A)-v'\left(Y\right)\right)\left(\theta_1(r)D+\theta_2(r)\delta\right)\right]<0.$$

Rearranging the expression above, using $\theta_2(r)<0$, we have

$$\frac{\theta_1(r)D+\theta_2(r)E\left(\delta\right)}{\theta_2(r)}E\left[u'(c_1^A)-v'\left(Y\right)\right]>-cov\left[u'(c_1^A)-v'\left(Y\right),\delta\right]. \tag{32}$$

Since Y and c_1^A are increasing in δ, together with concavity of u and convexity of v, we have that $cov\left[u'(c_1^A) - v'(Y), \delta\right] < 0$. Therefore, a necessary and sufficient condition for equation (30) to hold is

$$\theta_1(r)D + \theta_2(r)E(\delta) < 0.$$

Substituting in the expressions for $\theta_1(r)$ and $\theta_2(r)$ and rearranging we have

$$\left(\frac{1+\tilde{\beta}\tilde{R}}{\tilde{R}} + \gamma(1-\tilde{\beta}) + \tilde{\beta}(1+\tilde{R})\right) E\left(\frac{\delta}{1+r}\right) > (1-\gamma)\left(1-\tilde{\beta}\right)\tilde{\beta}\tilde{R}D. \tag{33}$$

This holds immediately if $\gamma \geq 1$. Let us show it also holds for $\gamma < 1$. Recall that D is implicitly defined by the following condition

$$\tilde{R}E\left[\left(c_1^B + D\right)^{-\gamma}\right]^{\frac{1}{\gamma}} = \tilde{\beta}E\left[\left(c_1^B\right)^{-\gamma}\right]^{\frac{1}{\gamma}}.$$

Substituting in the expression for c_1^B from equation (20) and rearranging we get

$$\tilde{R}E\left[\left(\frac{\delta}{1+r} + \tilde{\beta}\tilde{R}D\right)^{-\gamma}\right]^{\frac{1}{\gamma}} = \tilde{\beta}E\left[\left(\frac{\delta}{1+r} - D\right)^{-\gamma}\right]^{\frac{1}{\gamma}}.$$

Notice this equation (looking at the right-hand side) implies that $D < \frac{\delta}{1+r}$ for any realization of δ, so that

$$D < E\left(\frac{\delta}{1+r}\right).$$

Therefore, a sufficient condition for equation (33) to hold when $\gamma < 1$ is that

$$\frac{1+\tilde{\beta}\tilde{R}}{\tilde{R}} + \gamma(1-\tilde{\beta}) + \tilde{\beta}(1+\tilde{R}) > (1-\gamma)\left(1-\tilde{\beta}\right)\tilde{\beta}\tilde{R}.$$

Rearranging this equation we have

$$(1+\tilde{\beta}\tilde{R})\left[\frac{1}{\tilde{R}} + \gamma\left(1-\tilde{\beta}\right) + \tilde{\beta}\right] > 0,$$

which holds as all terms on the left-hand side are positive. This completes the argument.

$\square$

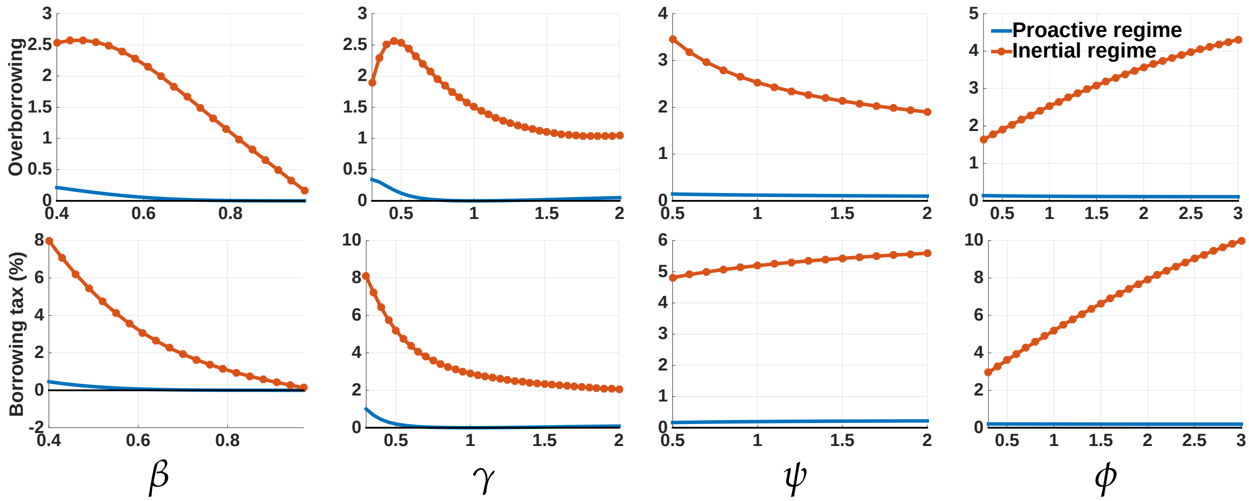

Figure 4: Sensitivity analysis

8.3 Sensitivity analysis for numerical results

Our baseline numerical example features the following parameterization:

$$\beta = 0.5, \quad \gamma = 0.5, \quad \psi = 1, \quad \phi = 1, \quad \delta_1 = 0.$$

Figure 4 displays the results of the model for different parameter values. The plots in the top row present the level of overborrowing: the difference between socially optimal D and D under no tax, both for the inertial and for the proactive regime. The plots in the bottom row present the optimal macroprudential tax τ under both regimes. In each column, we vary one parameter at a time, holding constant all the other parameters.

The fact that the red lines are consistently above the blue lines show that in all the examples considered the inertial regime displays more severe overborrowing.