More on the Tensor Response of the QCD Vacuum to an External Magnetic Field

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P. N. Kopnin[1]
with A. Gorsky[1] A. Krikun[1] A. Vainshtein[2]

[1]Institute for Theoretical and Experimental Physics, Moscow, Russia

[2]University of Minnesota, USA

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Outline

Magnetic Susceptibility of the Chiral Condensate

- Introduced in the framework of QCD Sum Rules [B. L. Ioffe, A. V. Smilga]
- $\langle \bar{q}\sigma_{\mu\nu}q\rangle_F = \chi \langle \bar{q}q\rangle F_{\mu\nu}$, where $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$.
- Measures induced tensor current in the QCD vacuum

Outline

- $\chi = -\frac{N_c}{4\pi^2 f_\pi^2} = -8.9\,\mathrm{GeV}^{-2}$ - OPE of the $\langle VVA \rangle$ correlator and pion dominance [A. Vainshtein]
- Sum rule fit $\chi = -3.15 \pm 0.30\,\mathrm{GeV}^{-2}$
- Vector dominance $\chi = -(3.38 \div 5.67)\,\mathrm{GeV}^{-2}$ [Balitsky, Yung, Kogan ...]

Outline

There is a significant discrepancy in the numerical results. It is natural to start looking elsewhere, e.g. holography, to identify the missing ingredients

- Holographic calculation of $\langle VVA \rangle$: $\chi \sim -11.5\,\mathrm{GeV}^{-2}$ [A. Gorsky, A. Krikun]. Vainshtein relation isn't exact, but fulfilled to good accuracy.
- Holographic Son–Yamamoto relations: χ agrees with Vainshtein. Assumed to be valid at any momentum transfer. No field-theoretical derivation.
- A direct holographic calculation - motivated by enhanced AdS/QCD models [Cappiello, Cata, D'Ambrosio; Domokos, Harvey, Royston; Alvares, Hoyos, Karch] that take into account the 1^{+-} mesons.
- Check them for self-consistency.

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Outline

The Setup

According to the holographic prescription:

Quantum Field Theory	Classical Gravity in 5D
Source $J(x_\mu)$ of an operator $\mathcal{O}$	Boundary value $\Phi(x_\mu, 0)$ of a 5D field $\Phi(x_\mu, z)$
Effective action with sources	Action on classical trajectories
Asymptotic freedom at small distances and confinement at large distances	AdS near the boundary, an IR hard wall far from the boundary

- KK-decompose all the fields and integrate out the z-axis, get an effective action for mesons - a chiral Lagrangian.
- An "expansion" of χPT

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The Action

$$\begin{aligned} \mathcal{S}_{5D} &= \int d^5x\sqrt{-g}\,\mathrm{Tr}\left\{-\frac{1}{4g_5^2}\left(F_L^2+F_R^2\right)+g_X^2\left(|DX|^2-m_X^2|X|^2\right)\right. \\ &+ \frac{\lambda}{2}\left(X^+F_LB+BF_RX^+ + \mathrm{c.c.}\right) \\ &- \left. 2g_B\left(\frac{i}{6}\frac{\epsilon^{MNPQR}}{\sqrt{-g}}\left(B_{MN}H^+_{PQR}-B^+_{MN}H_{PQR}\right)+m_B|B|^2\right)\right\}. \end{aligned}$$

$H = DB = dB - iL\wedge B + iB\wedge R,$

$\bar{q}_{R\bar{f}}\,q_L^f \leftrightarrow X_{\bar{f}}^f\,,\qquad \bar{q}_{R\bar{g}}\gamma_\mu q_R^{\bar{f}} \leftrightarrow R^{\bar{f}}_{\mu\,\bar{g}}\,,$

$\bar{q}_{R\bar{f}}\,\sigma_{\mu\nu}q_L^f \leftrightarrow B^f_{\mu\nu\,\bar{f}}\,,\qquad \bar{q}_{L\,g}\gamma_\mu q_L^f \leftrightarrow L^f_{\mu\,g}\,.$

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Rearranging the Degrees of Freedom

- In 4D, $\bar{q}\sigma^{\mu\nu}\gamma_5 q = \frac{i}{2}\epsilon^{\mu\nu}{}_{\lambda\rho}\bar{q}\sigma^{\lambda\rho}q$
- From the holographic point of view, this condition is ensured by the fact that the kinetic term for $B_{\mu\nu}$ is of the first order in derivatives, which leads to its complex self-duality.
- The "double counting" of the degrees of freedom that arises after we have introduced a complex tensor field is compensated by constraints imposed on half of them.

$$\bar{q}q \leftrightarrow X_+ \,, \qquad \frac{1}{\sqrt{2}}\bar{q}\sigma_{\mu\nu}q \leftrightarrow B_{+\mu\nu} \,,$$

$$i\bar{q}\gamma_5 q \leftrightarrow X_- \,, \qquad \frac{i}{\sqrt{2}}\bar{q}\gamma_5\sigma_{\mu\nu}q \leftrightarrow B_{-\mu\nu} \,,$$

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Outline

Equations of Motion

According to the prescription, we have to solve the classical E.o.M.'s

$$\left(\partial_z^2 + \frac{1}{z}\partial_z - \frac{1}{z^2} - \partial_\mu\partial^\mu\right)(B_\pm)_{12} = -\frac{\lambda}{8g_B}\frac{1}{z^2}X_\pm\,(F_V)_{12}\,,$$

$$\left(\partial_z^2 - \frac{3}{z}\partial_z + \frac{3}{z^2} - \partial_\mu\partial^\mu\right)X_\pm = -\frac{2\lambda}{g_X^2}z^2\,(F_V)_{12}\,(B_\pm)_{12}\,.$$

and calculate

$$\langle\bar{q}q\rangle \propto \left.\frac{\delta\mathcal{S}}{\delta X_+}\right|_{z=0} \qquad \langle\bar{q}\sigma_{\mu\nu}q\rangle \propto \left.\frac{\delta\mathcal{S}}{\delta B_{+\mu\nu}}\right|_{z=0}$$

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The Solution

- A most general property – the scalar and tensor degrees of freedom X_+, B_{+12} decouple from the pseudoscalar and pseudotensor X_-, B_{-12}, thus forming two independent sectors.
- The solutions for X and B are expressed in terms of Bessel and Neumann functions of $|\mathbf{B}|/q^2$ and qz (where $q = \sqrt{q_\mu q^\mu}$ is the momentum and $|\mathbf{B}| = (F_V)_{12}$ is the magnetic field) or their analytical continuations into the complex plane.

Outline

Magnetization

We are able to determine the magnetization $\mu(\mathbf{B}) = \dfrac{\langle \bar{q}\sigma_{12}q \rangle}{\langle \bar{q}q \rangle}$:

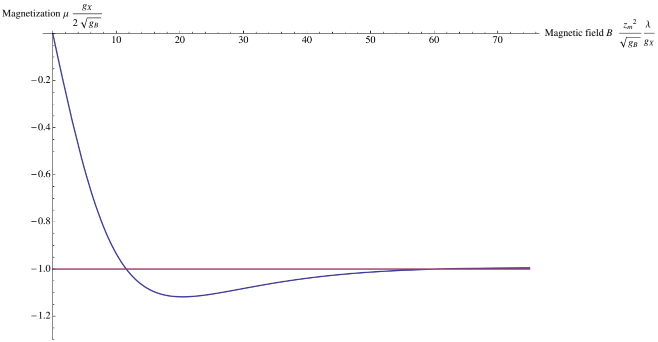

Figure: Magnetization of the chiral condensate $\mu(\mathbf{B})$ as a function of the magnetic field (blue) vs its strong field asymptotics (red).

Magnetization:

- is linear in $|\mathbf{B}|$ when the field is weak,
- becomes a negative constant at $\mathbf{B} \sim z_m^{-2} \sim \Lambda_{QCD}^2$.
- Constant asymptotic is to be expected. In large magnetic fields 4D reduces to 2D and the tensor chiral condensate is kinematically reduced to a scalar one.
- The result points to the fact that not only the lowest Landau level plays a significant role: $\lim_{\mathbf{B}\to\infty} \mu(\mathbf{B}) \neq -1$.

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Magnetic Susceptibility

... and the magnetic susceptibility $\chi(\mathbf{B}) = \frac{d}{d\mathbf{B}}\mu(\mathbf{B})$:

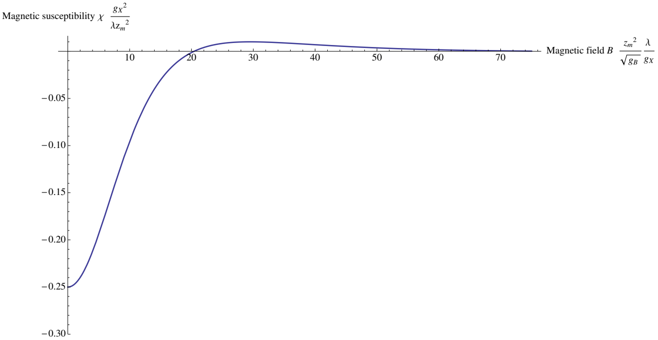

Figure: Magnetic susceptibility of the chiral condensate $\mu(\mathbf{B})$ as a function of the magnetic field.

Susceptibility:

- possesses a quadratic behavior $\chi \sim -(const - \mathcal{O}(|\mathbf{B}|^2))$ when the field is weak,
- tends to 0 at $\mathbf{B} \sim z_m^{-2} \sim \Lambda_{QCD}^2$.
- Parametrically $\chi(|\mathbf{B}| = 0)$ is reasonable ($\sim m_\rho^{-2}$), but numerically drastically differs from previous results.
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Summary

- A non-perturbative calculation of $\mu(\mathbf{B})$ and $\chi(\mathbf{B})$ to all orders in the magnetic field has been carried out.
- It has been performed in a holographic model enhanced by the inclusion of a tensor field – allows for a direct calculation.
- Our results reproduce the general properties both of the susceptibility and of the magnetization – the weak-field expansion of the former and the negative constant asymptotic of the latter.
- The numerical discrepancy may indicate the incompleteness of the model.

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