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Morphological Grayscale Reconstruction: Definition, Efficient Algorithm and Applications in Image Analysis

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Abstract

Reconstruction is part of a set of morphological image transformations [4] referred to as geodesic [1]. In the binary case, it simply extracts the connected components of a binary image I which are "marked" by an image J contained in I. Reconstruction can be extended to the grayscale case, and the first goal of this paper is to provide a formal definition in this framework. Available techniques to compute reconstruction are then reviewed, and a new powerful algorithm, using both sequential [3] and queue-based [8] methods, is introduced. Finally, applications of grayscale reconstruction to image filtering and segmentation illustrate its interest in image analysis.

1 Introduction

1.1 Reconstruction for binary images

Let I and J be two binary images defined on the same discrete domain D and such that $J \subseteq I$, i.e., $\forall p \in D, \quad J(p) = 1 \Longrightarrow I(p) = 1$. J is called the *marker* image and I is the *mask*. Let $I_1, I_2, \ldots, I_n$ be the connected components of I.

Definition 1.1 *The reconstruction $\rho_I(J)$ of mask I from marker J is the union of the connected components of I which contain at least a pixel of J.*

Alternaltively, binary reconstruction can be presented using the notion of *geodesic* dilation [1]. Given a set X (the mask), the geodesic distance d_X between two pixels p and q in X is the length of the shortest paths of X joining p and q. One can then define geodesic dilations as follows:

Definition 1.2 *The geodesic dilation of size $n \geq 0$ of a set $Y \subseteq X$ within $X \subset \mathbb{Z}^2$ is the set of the pixels of X whose geodesic distance to Y is smaller or equal to n:*

$$\delta_X^{(n)}(Y) = \{p \in X \mid d_X(p, Y) \leq n\}.$$

Geodesic dilation of size n can be obtained by iterating n elementary geodesic dilations:

$$\delta_X^{(n)}(Y) = \underbrace{\delta_X^{(1)} \circ \delta_X^{(1)} \circ \ldots \circ \delta_X^{(1)}}_{n \text{ times}}(Y).$$

Fig. 1 illustrates successive geodesic dilations of a marker inside a mask. The elementary geodesic dilation can itself be obtained via an elementary dilation followed by an intersection: $\delta_X^{(1)}(Y) = (Y \oplus B) \cap X$ (B being the unit ball of the considered grid).

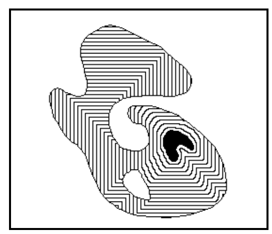

Figure 1: Successive geodesic dilations.

When performing successive elementary geodesic dilations of a set Y inside a mask X, the connected components of X whose intersection with Y is non empty are progressively flooded. We can thus state:

Proposition 1.3 *The reconstruction of X from $Y \subseteq X$ is given by: $\rho_X(Y) = \lim_{n \to +\infty} \delta_X^{(n)}(Y)$.*

1.2 Grayscale reconstruction

As stated in [4, 5], any increasing transformation ψ from $\mathbb{Z}^2$ to $\mathbb{Z}^2$ can be extended to grayscale images. By increasing, we mean that ψ is such that $\forall X, Y \subset \mathbb{Z}^2, Y \subseteq X \Longrightarrow \psi(Y) \subseteq \psi(X)$. In order to extend ψ to a grayscale image I, it suffices to consider the successive thresholds $T_k(I)$ of I: $T_k(I) = \{p \in D_I \mid I(p) \geq k\}$, which are said to constitute the *threshold decomposition* of I [2]. They obviously satisfy: $\forall k, T_k(I) \subseteq T_{k-1}(I)$. Thus, when applying ψ to each of these sets, the inclusion relations are preserved. ψ is thus extended to grayscale images as follows:

$$\forall p, \quad \psi(I)(p) = \max\{k \in [0, N-1] \mid p \in \psi(T_k(I))\}.$$

Binary reconstruction being obviously increasing, the following definition can thus be proposed:

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Definition 1.4 *Let J and I be two grayscale images defined on the same domain D, taking their values in the discrete set $\{0, 1, \ldots, N-1\}$ and such that $J \leq I$ (i.e., for each pixel $p \in D_I, J(p) \leq I(p)$). The grayscale reconstruction $\rho_I(J)$ of I from J is given by:*

$$\rho_I(J)(p) = \max\{k \in [0, N-1] \mid p \in \rho_{T_k(I)}(T_k(J))\}.$$

Fig. 2 illustrates this transformation. Just like binary reconstruction extracts the marked components of the mask, grayscale reconstruction extracts the *domes* or *peaks* of the mask which are marked by the marker-image.

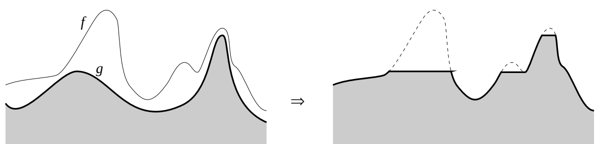


Figure 2: Grayscale reconstruction of mask f from marker g.

Unfortunately, this definition does not provide any interesting computational method to determine grayscale reconstruction in digital images. Indeed, even when a fast binary reconstruction algorithm is used, one has to apply it 2^n times to determine grayscale reconstruction for images on n bits! Therefore, we now present this transformation using geodesic dilations.

Following the threshold decomposition principle, the elementary geodesic dilation $\delta_I^{(1)}(J)$ of grayscale image $J \leq I$ "under" I is given by: $\delta_I^{(1)}(J) = (J \oplus D) \wedge I$. In this equation, $\wedge$ stands for the pointwise minimum and $J \oplus D$ is the dilation of J by flat structuring element D [4]. Grayscale geodesic dilation of any size can be defined similarly, leading to a second definition of grayscale reconstruction:

Definition 1.5 *The grayscale reconstruction $\rho_I(J)$ of I from J is obtained by iterating grayscale geodesic dilations of J "under" I until stability is reached, i.e.:*

$$\rho_I(J) = \lim_{n \to +\infty} \delta_I^{(n)}(J) = \lim_{n \to +\infty} \underbrace{\delta_I^{(1)} \circ \delta_I^{(1)} \circ \ldots \circ \delta_I^{(1)}}_{n \text{ times}}(J).$$

2 Algorithms

Many different algorithms have been proposed to compute binary reconstruction. For a review of the major ones, see [7]. According to def. 1.4, using these algorithms on the successive thresholds of grayscale images allows one to compute grayscale reconstructions, but in an extremely inefficient fashion. A better algorithm can be derived straightforwardly from def. 1.5 above, but is still very slow, since the marker and mask images have to be entirely scanned at every step of the process, until stability. This may sometimes involve several hundreds scannings [7].

A better technique, described in [6, 7], is to adapt the standard sequential [3] binary reconstruction algorithm to the grayscale case. The resulting algorithm usually only requires a few image scannings (a dozen typically) until stability is reached, and is therefore reasonably fast. However, like several other sequential algorithms [8], it does not deal well with "rolled-up structures" and may require several entire image scannings in which the value of only very few pixels is actually modified.

A grayscale reconstruction algorithm using queues of pixels [8] was proposed in [6]. It is based on the notion of *regional maxima:* A regional maximum M of image I is a connected components of pixels with a given value h, such that every pixel in the neighborhood of M has a strictly lower value. Let $R(I)$ be the image such that $R(I)(p) = I(p)$ if p belongs to a regional maximum, and 0 otherwise (for their extraction, see [6]). The following proposition holds:

Proposition 2.1 *Let I and J be two grayscale images such that $J \leq I$. Then $\rho_I(J) = \rho_I(R(J))$.*

For a proof, see [7]. The algorithm described in [6] initializes the queue with the pixels on the boundaries of the regional maxima and propagates their values under the mask image using the queue for breadth-first scanning.

Although faster than the previous techniques, this algorithm is slown down by the initial computation of the regional maxima of the marker image. Furthermore, unlike for its binary counterpart, some image regions may be scanned more than once during the breadth-first scanning step. This happens, e.g., when two regional maxima of J with different gray-level are next to each other. The sequential grayscale reconstruction algorithm does not have this drawback, but after the first two passes, it requires several additional scannings in which only few pixels are modified.

These two algorithms have therefore complementary drawbacks and advantages, and this is the motivation for the hybrid algorithm introduced now: the idea is to start with the two first scannings of the sequential algorithm. During the second one (anti-raster), every pixel p such that its current value could still be propagated during the next raster scanning, i.e. such that $\exists q \in N_G^-(p), J(q) < J(p)$ and $J(q) < I(q)$, is put into the queue. The last step is then identical to the propagation stel of the above queue-based algorithm. However, the number of pixels to be considered is considerably smaller than previously and overlaps almost bever occur:

$N_G^+(p)$ and $N_G^-(p)$ in 8-connectivity

- mask: I, marker: J,
- Scan D_I in raster order
  Let p be the current pixel;
  $J(p) \leftarrow \big(\max\{J(q), q \in N_G^+(p) \cup \{p\}\big) \wedge I(p)$;
- Scan D_I in anti-raster order

Let p be the current pixel;
$J(p) \leftarrow \big(\max\{J(q), q \in N_G^-(p) \cup \{p\}\}\big) \wedge I(p)$;
For every $q \in N_G^-(p)$
if $J(q) < J(p)$ and $J(q) < I(q)$ then
Put p in queue; BREAK;

• Until queue is empty, do
$p \leftarrow$ first pixel of queue;
For every pixel q in the neighborhood of p
If $J(q) < J(p)$ and $I(q) \neq J(q)$
$J(q) \leftarrow \min\{J(p), I(q)\}$;
Put q in queue;

This algorithm offers the best compromise for computing grayscale reconstructions. It takes advantage of the strong points of both previously described algorithms, without retaining their drawbacks. For almost any kind of input images of size 256×256, its execution time is less than $1/2$ second on a *Sun Sparc Station 1*. In addition, the algorithm works equally well for binary images and its extensions to any kind of grid and to multi-dimensional images are straightforward.

3 Applications

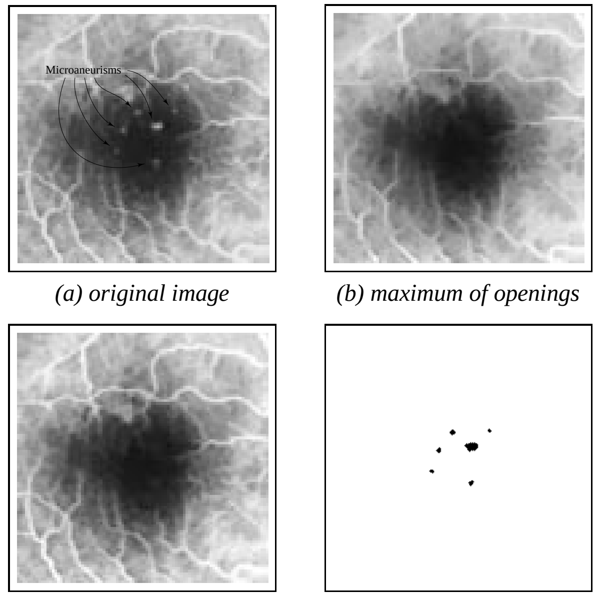
Grayscale reconstruction is of tremendous practical interest in image analysis and a fast algorithm to compute it makes it even more useful. Among other application, we may quote (for more details, refer to [6, 7, 8]):

• Filtering by opening followed by reconstruction: this has the effect of preserving the structures not entirely removed by opening, wiping out the others [5].

• Extraction of maxima, domes and crests in grayscale images: by reconstructing an image I from $I - h$, one gets the so-called h-domes [7]; unlike top-hat transformations [4] their extraction does not involve any size or shape criterion, but only a contrast parameter.

• Watershed segmentation: grayscale reconstruction is at the basis of almost every preprocessing required prior to the use of watersheds for segmentatoin (see the marker-controlled segmentation approach presented in [6, 8]).

• "Top-hat" by reconstruction: on Fig. 3(a), microaneurisms have top be extracted from a picture of eye blood vessels. They are small and compact, so that unlike the vessels, they are entirely removed by a maximum of openings with segments (Fig. 3(b)). Since they are also disconnected from the vessels network, one does not recover them after reconstruction of Fig. 3(a) from Fig. 3(b) (see Fig. 3(c)). After subtracting 3(c) from 3(a) and thresholding, we get the desired aneurisms (Fig. 3(d)).



(a) original image *(b) maximum of openings*

(c) reconstruction *(d) segmented aneurisms*

Figure 3: Extraction of microaneurisms.

4 Summary

Grayscale reconstruction has been formally defined for discrete images. A brief summary of the existing techniques to compute it has been provided, and a new "hybrid" algorithm was introduced, which is an order of magnitude faster than any other algorithm. Some of its application to image filtering and segmentation have been listed. They illustrate the interest of this transformation and it is hoped that thanks to this paper, the use of grayscale reconstruction will become increasingly popular.

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