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UNIVERSITY OF CALIFORNIA, SAN DIEGO

Mostly Digital ADCs for Highly-Scaled CMOS Processes

A dissertation submitted in partial satisfaction of the requirements for the degree
Doctor of Philosophy

in

Electrical Engineering (Electronic Circuits and Systems)

by

Gerard E. Taylor

Committee in charge:

Professor Ian Galton, Chair
Professor James Buckwalter
Professor William S. Hodgkiss
Professor Lawrence E. Larson
Professor Thomas T. Liu

2011

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The dissertation of Gerard E. Taylor is approved, and it is acceptable in quality and form for publication on micro-film and electronically:

Chair

University of California, San Diego

2011

DEDICATION

*To my wife Kathleen, who supported and encouraged me throughout this
long and difficult undertaking .*

TABLE OF CONTENTS

Signature Page	iii
Dedication	iv
Table of Contents.....	v
List of Figures	vii
List of Tables	ix
Acknowledgements	x
Vita	xii
Abstract of the Dissertation.....	xiii
I. Introduction.....	1
<i>I.A. Motivation for the New $\Delta\Sigma$ Modulator Architecture.....</i>	<i>1</i>
<i>I.B. VCO-Based $\Delta\Sigma$ Modulator</i>	<i>2</i>
<i>I.C. Dissertation Organization.....</i>	<i>3</i>
II. VCO-Based $\Delta\Sigma$ Modulator Overview.....	5
<i>II.A. Ideal Operation.....</i>	<i>5</i>
<i>II.B. A ring VCO Implementation</i>	<i>7</i>
<i>II.C. The Nonlinearity Problem.....</i>	<i>9</i>
III. Signal Processing Details.....	12
<i>III.A. Digital Background Calibration</i>	<i>12</i>
<i>III.B. Pseudo-Differential Topology</i>	<i>17</i>
<i>III.C. Self-Cancelling Dither Technique</i>	<i>19</i>
<i>III.D. The Implemented $\Delta\Sigma$ Modulator Architecture</i>	<i>21</i>
<i>III.E. Quantization Noise, No-overload Range, and the Number of Ring Elements..</i>	<i>22</i>
IV. Circuit Details	25
<i>IV.A. ICRO, Ring Sampler, and Phase Decoder</i>	<i>25</i>
<i>IV.B. V/I converter</i>	<i>26</i>
<i>IV.C. Dither DACs.....</i>	<i>29</i>
<i>IV.D. Nonlinearity Correction Block.....</i>	<i>30</i>

IV.E. Circuit Noise Sources.....	31
V. First Prototype IC	33
VA. Measurement Results	33
VB. Conclusions.....	37
VI. Architectural Enhancements.....	40
VI.A. Quantization Noise Reduction in a VCO-Based ADC	40
VI.B. Quantization Noise Reduction by Extended No-Overload Range	42
VI.C. Dither	48
VI.D. Low Voltage V/I Converter.....	55
VI.E. Digital Background Calibration	59
VII. Circuit Level Enhancements	66
VII.A. Achieving Theoretical Maximum SQNR	66
VII.B. Signal Path AC Nonlinearity	67
VII.C. Ring Sampler Hysteresis Problem	69
VII.D. Process Scaling	70
VIII. Second Prototype IC	72
VIII.A. Measurement Results	72
VIII.B. Conclusions.....	77
IX. Third Prototype IC	80
IX.A. Measurement Results	80
IX.B. Conclusions.....	83
X. CONCLUSIONS.....	84
Figures	85
Tables	110
References	113

LIST OF FIGURES

Figure 1: Equivalent systems: (a) a generic VCO-based $\Delta\Sigma$ modulator, (b) the cascade of a continuous-time lowpass filter, sampler, quantizer, and digital differentiator, and (c) the cascade of a continuous-time lowpass filter, sampler and first-order $\Delta\Sigma$ modulator.....	85
Figure 2: Example of a ring VCO and phase-to-digital converter.....	86
Figure 3: The prototype IC's on-chip calibration unit shown with a single VCO-based $\Delta\Sigma$ modulator signal path for simplicity.	86
Figure 4: A pseudo-differential signal path and the calibration unit.....	87
Figure 5: High-level block diagram of the implemented VCO-based $\Delta\Sigma$ modulator.	87
Figure 6: Example of the signal-dependent non-uniform quantization problem.	88
Figure 7: Example of the solution used to solve the signal-dependent non-uniform quantization problem.....	88
Figure 8: Circuit diagrams of the V/I converter and ICRO.	89
Figure 9: The dither DAC swapping technique which causes the PSD of the error component in the $\Delta\Sigma$ modulator output arising from mismatches between the dither DACs to have a first-order highpass shape.....	90
Figure 10: Nonlinearity correction block details.	90
Figure 11: Die photograph.....	91
Figure 12: Representative measured PSD plots of the first prototype IC $\Delta\Sigma$ modulator output before and after digital background calibration (initial convergence time of digital calibration unit is 233ms).....	92
Figure 13: Plots of the first prototype IC's measured output PSD for a two-tone out-of-band input signal (top) and inter-modulation distortion (bottom) for the $\Delta\Sigma$ modulator run with $f_s = 1.152\text{GHz}$. The top and bottom plots indicate how the inter-modulation values were measured.....	93
Figure 14: Plots of the first prototype IC's measured SNR and SNDR for an 18 MHz signal band (top) bandwidth 9MHz signal-band (bottom) for the $\Delta\Sigma$ modulator run with $f_s = 1.152\text{GHz}$.	94
Figure 15: Representative measured PSD plots of the first prototype IC $\Delta\Sigma$ modulator output with and without dither.....	95
Figure 16: Circuit diagrams of various ICROs: (a) 15-element ICRO used in the first prototype IC, (b) example 7-element ICRO, (c) dual, 7-element injection-locked ICRO used in the second prototype IC.....	96
Figure 17: Over-Range Correction block (ORC): (a) signal path with the addition of over-range correction, (b) details of the ORC block, (c) overflow logic truth table, (d) example overload waveform before ORC, (e) example waveform after ORC, (f) example waveform after ORC demonstrating clipping behavior.	97
Figure 18: High-level block diagram of two VCO-based $\Delta\Sigma$ modulators: (a) single pseudo-differential modulator ADC with dither injected as a	

common-mode signal, (b) dual pseudo-differential modulator ADC with dither injected as a differential signal.....	98
Figure 19: V/I circuit diagrams: (a) first prototype IC V/I converter, (b) new open-loop V/I converter.....	99
Figure 20: The second prototype IC's on-chip calibration unit shown with a single VCO-based $\Delta\Sigma$ modulator signal path for simplicity.....	100
Figure 21: Calibration unit details: (a) example transformer-based input circuit for the ADC, (b) example active-circuit based input circuit for the ADC, (c) detailed circuit diagram of the calibration DAC and signal path replica.	101
Figure 22: Circuit diagrams of the ring sampler D-type flip-flops, (a) transmission-gate flip-flop used in the first prototype that produces data-dependent hysteresis, (b) non-transmission-gate flip-flop that is largely hysteresis free.....	102
Figure 23: Die photograph of second prototype IC.....	103
Figure 24: Representative measured PSD plots of the second prototype IC $\Delta\Sigma$ modulator output before and after digital background calibration.....	104
Figure 25: Plots of the second prototype IC's measured output PSD for a single-tone input signal (top) and the harmonic distortion values (bottom) for the $\Delta\Sigma$ modulator run with $f_s=2.4\text{GHz}$. The top and bottom plots indicate how the harmonic distortion values were measured.	105
Figure 26: Plots of the second prototype IC's measured SNR and SNDR for a 37.5MHz signal bandwidth (top) and an 18MHz signal bandwidth (bottom) for the second prototype $\Delta\Sigma$ modulator run with $f_s=2.4\text{GHz}$	106
Figure 27: Representative measured PSD plots of the third prototype IC $\Delta\Sigma$ modulator output before and after digital background calibration.....	107
Figure 28: Plots of the third prototype IC's measured output PSD for a two- tone out-of-band input signal (top) and inter-modulation distortion (bottom) for the $\Delta\Sigma$ modulator run with $f_s = 1.6\text{GHz}$. The top and bottom plots indicate how the inter-modulation values were measured.....	108
Figure 29: Plots of the third prototype IC's measured SNR and SNDR for a 25MHz signal bandwidth (top) and an 12.5MHz signal bandwidth (bottom) for the third prototype $\Delta\Sigma$ modulator run with $f_s=1.6\text{GHz}$	109

LIST OF TABLES

Table 1: Performance table and comparison of first prototype IC to prior state-of-the-art $\Delta\Sigma$ modulators.	110
Table 2: Performance table and comparison of second prototype IC to prior state-of-the-art $\Delta\Sigma$ modulators.	111
Table 3: Performance table and comparison of third prototype IC fabricated in 65nm LP process to first IC fabricated in 65nm G+ process.....	112

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ABSTRACT OF THE DISSERTATION

Mostly Digital ADCs for Highly-Scaled CMOS Processes

by

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Doctor of Philosophy in Electrical Engineering (Electronic Circuits and Systems)

University of California, San Diego, 2011

Professor Ian Galton, Chair

Delta-Sigma ($\Delta\Sigma$) modulator ADCs are used extensively in applications where the analog signal bandwidth is narrow compared to practical ADC sample-rates because these ADCs are very efficient and the oversampling relaxes the analog filtering requirements prior to digitization. Conventional continuous-time $\Delta\Sigma$ modulator ADCs require

high accuracy building block including low-leakage analog integrators, high-linearity feedback DACs, high-accuracy reference voltages, high-speed comparators, and low-jitter clocks. Unfortunately, as process technologies scale and supply voltages are reduced it becomes increasingly difficult to build these circuits. Fortunately however, highly scaled CMOS processes offer very fast, very dense and very low-power digital logic gates.

This dissertation presents continuous-time $\Delta\Sigma$ modulator ADCs that consist mostly of digital logic gates. The ADCs are a voltage-controlled ring oscillator based design with new digital background calibration and self-cancelling dither techniques applied to enhance performance. Unlike conventional delta-sigma modulators, they do not contain analog integrators, feedback DACs, comparators, or reference voltages, and do not require a low-jitter clock. Therefore, they use less area than comparable conventional delta-sigma modulators, and the architecture is well-suited to IC processes optimized for fast digital circuitry.

Prototype ICs were fabricated in both the 65nm LP and 65nm G+ CMOS processes. The performance of the prototype ICs is comparable to the state-of-the-art in terms of power figure-of-merit but this new architecture uses significantly less circuit area.

I. INTRODUCTION

I.A. Motivation for the New $\Delta\Sigma$ Modulator Architecture

In many analog-to-digital converter (ADC) applications such as wireless receiver handsets, the bandwidth of the analog signal of interest is narrow relative to practical ADC sample-rates. Delta-sigma ($\Delta\Sigma$) modulator ADCs are used almost exclusively in such applications because they offer exceptional efficiency and relax the analog filtering required prior to digitization [1]. Continuous-time $\Delta\Sigma$ modulator ADCs with clock rates above several hundred MHz have been shown to be particularly good in these respects [2],[3],[4],[5].

Unfortunately, conventional analog $\Delta\Sigma$ modulators present significant design challenges when implemented in highly-scaled CMOS IC technology optimized for digital circuitry. They require analog comparators, high-accuracy analog integrators, high-linearity feedback DACs, and low-noise, low-impedance reference voltage sources. Continuous-time $\Delta\Sigma$ modulators with continuous-time feedback DACs additionally require low-jitter clock sources. These circuit blocks are increasingly difficult to design as CMOS technology is scaled below the 90 nm node because the scaling tends to worsen supply voltage limitations, device leakage, device nonlinearity, signal isolation, and $1/f$ noise.

I.B. VCO-Based $\Delta\Sigma$ Modulator

An alternate type of $\Delta\Sigma$ modulator which does not require the above-mentioned analog blocks consists of a voltage-controlled ring oscillator (ring VCO) with its inverters sampled at the desired output sample-rate followed by digital circuitry [6],[7],[8],[9], [10], [11]. Although the ring VCO inevitably introduces severe nonlinearity, the structure otherwise has the same functionality as a first-order continuous-time $\Delta\Sigma$ modulator. Unfortunately, the nonlinearity problem and the high spurious tone content of first-order $\Delta\Sigma$ modulator quantization noise has limited the deployment of such VCO-based $\Delta\Sigma$ modulators to date. The only previously published method of circumventing these problems is to use the VCO-based $\Delta\Sigma$ modulator as the last stage of an otherwise conventional analog $\Delta\Sigma$ modulator, but this solution requires all the high-performance analog blocks of a conventional analog $\Delta\Sigma$ modulator except comparators [12].

This work presents a VCO-based $\Delta\Sigma$ modulator that incorporates two new techniques with which it avoids these problems: digital background correction of VCO nonlinearity, and self-cancelling dither [13]. The digital background calibration technique is an extension of a technique originally used to correct nonlinear distortion in pipelined ADCs [14], [15]. The self-cancelling dither technique eliminates the spurious tone problem by adding dither sequences prior to quantization and then cancelling them in the digital domain. Additionally, the $\Delta\Sigma$ modulator uses a new digital calibration technique that enables reconfigurability by automatically retuning the VCO's center frequency whenever the $\Delta\Sigma$ modulator's sample-rate is changed.

The new techniques enable the $\Delta\Sigma$ modulator to achieve high-performance data conversion without analog integrators, feedback DACs, comparators, reference voltages, or a low-jitter clock. Therefore, it uses less area than comparable conventional analog $\Delta\Sigma$ modulators, and the architecture is well-suited to highly-scaled CMOS technology optimized for fast digital circuitry.

I.C. Dissertation Organization

The dissertation presents the theory, analysis and circuit results for three different versions of a VCO-based $\Delta\Sigma$ modulator. The dissertation consists of ten chapters.

Chapter II describes the VCO-based $\Delta\Sigma$ modulator concept, and quantifies the VCO nonlinearity problem. Chapter III presents the signal processing enhancements including digital background calibration, pseudo-differential topology, and self-cancelling dither technique. Chapter IV presents the $\Delta\Sigma$ modulator's key circuits. Chapter V presents measurement results for the first prototype IC, fabricated in the 65nm LP process, and draws conclusions about the ADCs performance and shortcomings. Chapter VI describes a number of architectural enhancements to improve the $\Delta\Sigma$ modulator's performance and usability. Chapter VII describes a number of circuit-level enhancements to boost the $\Delta\Sigma$ modulator's performance. Chapter VIII presents measurement results for a second prototype IC, fabricated in the 65nm G+ process, which incorporates all of the enhancements described in Chapters VI and VII. Chapter IX presents measurement results for a third prototype IC, also fabricated in the 65nm G+ process, which is similar to the first prototype IC and is intended to prove the benefits of process scaling for the

VCO-based $\Delta\Sigma$ modulator architecture. And chapter X presents conclusions of this research.

Chapter I is largely taken from Section I of the paper entitled “A Mostly-Digital Variable-Rate Continuous-Time Delta-Sigma Modulator ADC” published in the IEEE Journal of Solid-State Circuits, volume 45, number 12, pages 2634-2646, December 2010. The dissertation author is the primary investigator and author of this paper. Professor Ian Galton supervised the research which forms the basis for this paper

II. VCO-BASED $\Delta\Sigma$ MODULATOR OVERVIEW

II.A. Ideal Operation

An idealized VCO-based $\Delta\Sigma$ modulator with a continuous-time input voltage, $v(t)$, and a digital output signal, $y[n]$, is shown in Figure 1(a). It consists of a VCO, a phase-to-digital converter, and a digital differentiator block with a transfer function of $1-z^{-1}$. Ideally, the instantaneous frequency of the VCO is

$$f_{VCO}(t) = f_s + \frac{K_{VCO}}{2\pi} v(t) \quad (1)$$

where f_s is the center frequency of the VCO in Hz, and K_{VCO} is the VCO gain in radians per second per volt. The phase-to-digital converter quantizes the VCO phase, i.e., the time integral of the instantaneous frequency, and generates output samples of the result at times nT_s , $n = 0, 1, 2, \dots$, where $T_s = 1/f_s$.

In a practical implementation the phase-to-digital converter would typically generate its output samples modulo one-cycle. It can be verified that provided

$$0.5f_s < f_{VCO}(t) < 1.5f_s \quad (2)$$

for all t and another modulo one-cycle operation is performed after the digital differentiator, then the digital output signal is not affected by the modulo operations. Therefore, the

modulo operations are not considered in the following to simplify the explanation.

Aside from an integer multiple of a cycle (which ultimately has no effect on $y[n]$ because of the modulo operations), the n th output sample of the phase to digital converter in radians is a quantized version of

$$\phi[n] = \int_0^{nT_s} K_{VCO} v(\tau) d\tau. \quad (3)$$

Equivalently, (3) can be written as

$$\phi[n] = \sum_{k=1}^n \omega[k], \quad (4)$$

where

$$\omega[n] = \int_{(n-1)T_s}^{nT_s} K_{VCO} v(\tau) d\tau. \quad (5)$$

It follows that $\omega[n]$ could have been obtained by passing $v(t)$ through a lowpass continuous-time filter with transfer function

$$H_c(f) = K_{VCO} e^{-j\pi T_s f} \frac{\sin(\pi T_s f)}{\pi f} \quad (6)$$

and sampling the output of the filter at a rate of f_s .

The system of Figure 1(b) is, therefore, equivalent to that of Figure 1(a). It obtains $\omega[n]$ by sampling a filtered version of the input signal as described above and implements (4) as a discrete-time integrator. The discrete-time integrator is followed by the same quantizer and digital differentiator as in Figure 1(a) to obtain $y[n]$.

Given that the discrete-time integrator and differentiator both have integer-valued impulse responses, it can be verified that the system of Figure 1(b), and, hence, the system of Figure 1(a), is equivalent to the system of Figure 1(c) [16]. Thus, the VCO-based $\Delta\Sigma$ modulator is equivalent to a conventional first-order continuous-time $\Delta\Sigma$ modulator, so it can be analyzed by applying well-known properties of the first-order $\Delta\Sigma$ modulator to the system of Figure 1(c) [1]. In particular

$$y[n] = \omega[n] + e_{\Delta\Sigma}[n], \quad (7)$$

where $e_{\Delta\Sigma}[n]$ is first-order highpass shaped quantization noise.

II.B. A ring VCO Implementation

A practical topology with which to implement the VCO and phase-to-digital converter is shown in Figure 2. In this example, the VCO is a ring oscillator that consists of five inverters, each with a transition delay that depends on the VCO input voltage, $v(t)$. The ring sampler consists of five flip-flops clocked at a rate of f_s , where the D input of each flip-flop is driven by the output of one of the VCO's inverters. At each rising edge of the clock signal, i.e., at times nT_s , the output of each flip-flop is set high if the corresponding VCO inverter output signal at that time is above the flip-flop's digital logic threshold of approximately half the supply voltage, and is set low otherwise.

A well known property of ring oscillators is that at any given time during oscillation, exactly one of the VCO's inverters is in a state of either *positive transition* or *negative transition*, i.e., a state in which the inverter's input and output are both below or both

above the digital logic thresholds of the flip-flops to which they are connected, respectively. For example, suppose Inverter 1 in Figure 2 enters positive transition at time t_0 . The inverter remains in positive transition until a time t_1 at which its output rises above the digital logic threshold of the flip-flop to which it is connected. At this same instant, Inverter 2 enters negative transition. This process continues in a clockwise direction around the VCO such that Inverter $(1+(i \bmod 5))$ is in positive transition from time t_i to time t_{i+1} if i is even, and is in negative transition from time t_i to time t_{i+1} if i is odd for $i = 0, 1, 2, \dots$, where $t_{i+1} > t_i$.

Therefore, each inverter goes once into positive transition and once into negative transition during each VCO period, and there are only 10 possible 5-bit values that the ring sampler can generate regardless of when it is sampled. The phase decoder maps each of the 10 values into a phase number, $\tilde{\phi}[n]$, in the range $\{0, 1, 2, \dots, 9\}$ (the corresponding phase in radians is given by $2\pi\tilde{\phi}[n]/10$). Since each phase number corresponds to one of the inverters being in a state of transition and there are 10 such states per VCO period, $\tilde{\phi}[n]$ represents the phase of the VCO modulo one-cycle quantized to the nearest 10th of a cycle as depicted in Figure 2.

Ideally, the VCO inverters are such that the i th transition delay is given by

$$t_{i+1} - t_i = \frac{1}{10} \left[T_s - K_d \bar{v}(t_i, t_{i+1}) \right]. \quad (8)$$

where

$$\bar{v}(t_i, t_{i+1}) = \frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} v(t) dt \quad (9)$$

is the average value of $v(t)$ over the time interval from t_i to t_{i+1} . This time interval represents a 10th of the corresponding VCO cycle as described above, so (8) implies that the VCO's *average* frequency during this time interval, i.e.,

$$\frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} f_{VCO}(t) dt \quad (10)$$

where $f_{VCO}(t)$ is the VCO's *instantaneous* frequency at time t , can be written as

$$\frac{1}{10(t_{i+1} - t_i)}. \quad (11)$$

Substituting (8) into (11) and expanding the result as a power series yields

$$\frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} f_{VCO}(t) dt = \frac{1}{T_s} \sum_{n=0}^{\infty} \left(\frac{K_d}{T_s} \bar{v}(t_i, t_{i+1}) \right)^n. \quad (12)$$

Provided that $v(t)$ does not change significantly between t_i and t_{i+1} , it follows that the VCO can be modeled as having an instantaneous frequency given by

$$f_{VCO}(t) = f_s + \frac{K_{VCO}}{2\pi} v(t) + \frac{1}{T_s} \sum_{n=2}^{\infty} \left(\frac{T_s K_{VCO}}{2\pi} v(t) \right)^n \quad (13)$$

where $K_{VCO} \equiv 2\pi K_d / T_s^2$.

II.C. The Nonlinearity Problem

A comparison of the instantaneous frequency of the ring VCO given by (13) to the

ideal instantaneous frequency given by (1) indicates that the ring VCO introduces nonlinear distortion. Applying the reasoning of Section II.A leads to the conclusion that the distortion causes the input to the first-order $\Delta\Sigma$ modulator in the equivalent system of Figure 1(c) to be

$$\omega[n] + \int_{(n-1)T_s}^{nT_s} \left[\frac{2\pi}{T_s} \sum_{i=2}^{\infty} \left(\frac{T_s K_{VCO}}{2\pi} v(\tau) \right)^i \right] d\tau \quad (14)$$

instead of just $\omega[n]$. It follows from (5) and (7) that provided $v(t)$ does not change significantly over each sample interval, the output of the $\Delta\Sigma$ modulator is

$$y[n] = \omega[n] + e_{\Delta\Sigma}[n] + \sum_{i=2}^{\infty} \alpha_i (\omega[n])^i, \quad (15)$$

where

$$\alpha_i \equiv \left(\frac{1}{2\pi} \right)^{i-1}, \quad (16)$$

for $i = 2, 3, \dots$, are nonlinear distortion coefficients.

It should be stressed that the nonlinearity is not the result of non-ideal circuit behavior. It is a systematic nonlinearity that occurs even with ideal circuit behavior. The problem is that the VCO's *period* changes linearly with $v(t)$, but to eliminate the nonlinear terms in (14) it would be necessary for the VCO's *frequency* to change linearity with $v(t)$. It is the reciprocal relationship between VCO's period and frequency that give rise to the nonlinear terms in (14). Of course, in practice the relationship between the inverter delays and the input voltage is not perfectly linear as assumed by (8). While this intro-

duces additional significant nonlinearity it tends to be less severe than the reciprocal nonlinearity described above.

Transistor-level simulations of the VCO-based $\Delta\Sigma$ modulator described above with the 15-element VCO designed for the IC prototype presented in this paper support these findings and demonstrate the severity of the problem. For instance, the output of the simulated $\Delta\Sigma$ modulator with $f_s = 1.152$ GHz and a full-scale 250 KHz sinusoidal input signal has second, third, and fourth harmonics at -26 dBc, -47 dBc, and -64 dBc, respectively. When the simulated output sequence is corrected in the digital domain to cancel just the second-, third-, and fourth-order distortion terms using the techniques presented in the next chapter, the largest harmonic in the corrected sequence is less than -90 dBc¹. This suggests that for the target specifications of the IC prototype presented in this paper it is only necessary to cancel the second-, third-, and fourth-order distortion terms.

Chapter II is largely taken from Section II of a paper entitled “A Mostly-Digital Variable-Rate Continuous-Time Delta-Sigma Modulator ADC” published in the IEEE Journal of Solid-State Circuits, volume 45, number 12, pages 2634-2646, December 2010. The dissertation author is the primary investigator and author of this paper. Professor Ian Galton supervised the research which forms the basis for this paper.

¹ It can be verified that the technique used to cancel the α_3 term in (15) introduces a fifth-order term that happens to largely cancel the α_5 term in (15) as a side-effect.

III. SIGNAL PROCESSING DETAILS

The prototype IC contains two identical $\Delta\Sigma$ modulators that each incorporate four of the basic VCO-based $\Delta\Sigma$ modulators described above as separate signal paths. They also contain additional components that implement the digital background calibration and self-cancelling dither techniques. The signal processing details of the $\Delta\Sigma$ modulator design and the reasons for using four such signal paths in a single $\Delta\Sigma$ modulator are presented in this chapter.

III.A. Digital Background Calibration

Two types of digital background calibration are implemented in each $\Delta\Sigma$ modulator: 1) digital background cancellation of VCO-induced second-order and third-order distortion, and 2) digital background tuning of the VCO's center frequency to the $\Delta\Sigma$ modulator's sample rate, f_s . The former in combination with a pseudo-differential architecture to be explained shortly addresses the nonlinearity problem described in the previous chapter. The latter centers the input range of the $\Delta\Sigma$ modulator about the midscale input voltage. This maximizes the dynamic range, and enables reconfigurability by automatically retuning the VCO's center frequency whenever f_s is changed.

Figure 3 shows a block diagram of a single VCO-based $\Delta\Sigma$ modulator signal path and the on-chip *calibration unit* shared by all the signal paths in both $\Delta\Sigma$ modulators. The signal path is similar to the VCO-based $\Delta\Sigma$ modulator described in Section II.B, except

that its VCO is implemented as a voltage-to-current (*V/I*) converter followed by a 15-element current-controlled ring oscillator (ICRO), and it contains a *nonlinearity correction block* that cancels the distortion terms in (15). The calibration unit measures the VCO center frequency and nonlinear distortion of a *signal path replica*, and generates digital data used by the actual signal path to properly tune the VCO's center frequency and cancel nonlinear distortion. The calibration unit operates continuously in background, and periodically updates its output data with new measurement results.

The calibration unit's signal path replica is identical to the actual signal path except that it does not have a nonlinearity correction block, its differential input voltage is zero (i.e., it has a constant, midscale input signal), and a four-level current steering $f_s/64$ -rate DAC adds a calibration sequence to the input of its ICRO. The calibration sequence is $t_1[n]+t_2[n]+t_3[n]$ where the $t_i[n]$ sequences are 2-level, independent, zero-mean, pseudo-random sequences.

III.A.1 VCO Center Frequency Calibration

The calibration unit's *VCO center frequency calculator* block adds each successive set of 2^{28} output samples from the signal path replica and scales the result by a constant, K , to create an $f_s/2^{28}$ -rate digital sequence given by

$$\Delta I[m] = K \sum_{i=0}^{P-1} r[mP+i] \quad (17)$$

where $P = 2^{28}$, and $r[n]$ is the output of the signal path replica. The eight most significant bits (MSBs) of this sequence are used to adjust the output current of the *V/I* converter in

the signal path replica. This forms a negative feedback loop with a bandwidth that depends on K . The feedback drives the VCO's output frequency to the point at which $r[n]$ has zero mean. The frequency to which the VCO converges is f_s , because the VCO's input voltage is zero and the calibration sequence has a mean of zero. The V/I converter in the signal path is also adjusted by the $\Delta I[m]$ sequence. To the extent that the signal path and signal path replica match, this causes the signal path's VCO to have a frequency very close to f_s when $v(t) = 0$.

The choice of K is not critical because settling error in the loop introduces only a small common-mode error in the $\Delta\Sigma$ modulator. In the prototype IC, K was chosen to achieve one-step settling.

III.A.2 Nonlinearity Correction

The nonlinearity correction block in the signal path is a high-speed look-up table with mapping data updated periodically by the *nonlinearity coefficient calculator block* of the calibration unit. The look-up table maps each 5-bit input sample, $y[n]$, into an output sample, $y[n]|_{\text{corrected}}$, such that

$$y[n]|_{\text{corrected}} = y[n] - \tilde{\alpha}_2 (y[n])^2 - (\tilde{\alpha}_3 - 2\tilde{\alpha}_2^2) (y[n] - \tilde{\alpha}_2 (y[n])^2)^3 \quad (18)$$

where $\tilde{\alpha}_2$, and $\tilde{\alpha}_3$ are measurements of the α_2 and α_3 coefficients in (15), respectively. It can be verified that if $\tilde{\alpha}_i = \alpha_i$, for $i = 2$ and 3 , then $y[n]|_{\text{corrected}}$ does not contain any VCO-induced second-order or third-order distortion terms.

Applying (18) to obtain $y[n]_{\text{corrected}}$ also has some side effects. A positive side effect is that it adds a fifth-order term that happens to nearly cancel the portion of the fifth-order distortion corresponding to α_5 given by (16). Negative side effects are that it adds higher-order distortion terms and cross terms that include $(e_{\Delta\Sigma}[n])^i$ for $i = 2, 3, 4, 5$, and 6. Fortunately, these terms are sufficiently small that they do not significantly degrade the simulated or measured performance of the $\Delta\Sigma$ modulator. The cross terms containing $(e_{\Delta\Sigma}[n])^i$ fold some of the $\Delta\Sigma$ quantization noise into the signal band but the folded noise is well below the overall signal band noise floor of the $\Delta\Sigma$ modulator. This is because the 15-element ring oscillator quantizes each phase estimate to within 1/30 of a VCO period so $e_{\Delta\Sigma}[n]$ is small relative to $\omega[n]$. Had a VCO with fewer ring elements been used, the folding of $\Delta\Sigma$ quantization noise into the signal band would not necessarily have been negligible.

III.A.3 Nonlinearity Coefficient Measurement

The purpose of the *nonlinearity coefficient calculator block* is to generate the 30 values of (18) that correspond to the 30 possible values of $y[n]$. While using the values of α_2 and α_3 given by (16) for $\tilde{\alpha}_2$ and $\tilde{\alpha}_3$, respectively, in (18) would result in cancellation of much of the nonlinear distortion, it would not address nonlinear distortion arising from non-ideal circuit behavior, and simulations suggest that this would limit the ADC's signal-to-noise-and-distortion-ratio (SNDR) to between 60 dB and 65 dB.

Therefore, the calibration unit continuously measures α_2 and α_3 by correlating the output of the signal path replica against the three 2-level sequences: $t_1[n]$, $t_1[n] \times t_2[n]$,

and $t_1[n] \times t_2[n] \times t_3[n]$, to obtain the three $f_s/2^{28}$ -rate sequences given by

$$\gamma_1[m] = \frac{1}{P} \sum_{i=0}^{P-1} r[mP+i] t_1[mP+i], \quad (19)$$

$$\gamma_2[m] = \frac{1}{P} \sum_{i=0}^{P-1} r[mP+i] t_1[mP+i] t_2[mP+i], \quad (20)$$

and

$$\gamma_3[m] = \frac{1}{P} \sum_{i=0}^{P-1} r[mP+i] t_1[mP+i] t_2[mP+i] t_3[mP+i]. \quad (21)$$

where $P = 2^{28}$. It can be verified that when the signal path replica's VCO frequency is f_s ,

$$\frac{\gamma_2}{2\gamma_1^2} \approx \alpha_2 \quad \text{and} \quad \frac{\gamma_3}{6\gamma_1^3} \approx \alpha_3. \quad (22)$$

Therefore, the nonlinearity coefficient calculator block calculates the 30 values of (18)

with

$$\tilde{\alpha}_2 \triangleq \frac{\gamma_2}{2\gamma_1^2} \quad \text{and} \quad \tilde{\alpha}_3 \triangleq \frac{\gamma_3}{6\gamma_1^3}. \quad (23)$$

It does this and loads the 30 values into the nonlinearity correction block's look-up table once every $2^{28}T_s$ seconds.

The nonlinearity calibration technique described above is based on the same principle as that presented in [15], but one of its differences is that it measures the nonlinear distortion coefficients of a signal path replica instead of the actual signal path. The nonlinearity coefficients could have been measured directly from the output of the actual

signal path, but if this had been done there would have been unwanted terms corresponding to $v(t)$ in the correlator output sequences, $\gamma_i[n]$. The variance of each such term is proportional to $1/P$, so for large enough values of P the terms can be neglected. However, P would have had to be much larger than 2^{28} for the terms to be negligible, so the time required to measure the nonlinear distortion coefficients would have been much longer than the $2^{28}T_s$ seconds required by the system described above. For example, when f_s is set to its maximum value of 1.152GHz, the system described above requires 233 ms to measure the nonlinear distortion coefficients, whereas several tens of seconds would have been required had a signal path replica not been used.

The peak amplitude of the calibration signal also affects the time required to measure the nonlinear distortion coefficients. Each time the amplitude is doubled, P can be divided by four without reducing the variances of the measured nonlinear coefficient values. Therefore, it is desirable to have as large of a calibration sequence as possible in the signal path replica that does not cause the path to overload.

III.B. Pseudo-Differential Topology

The accuracy with which the nonlinear distortion terms can be cancelled depends on how well the actual signal path matches the signal path replica and also on bandwidth limitations of the signal path itself. For example, transistor-level simulations of the system shown in Figure 3 indicate that the nonlinearity correction block only reduces the worst-case second-order distortion term from -28 dBc to -65 dBc, which is well below the target specifications for this project.

This limitation is addressed in the $\Delta\Sigma$ modulator by combining two signal paths to form a single *pseudo-differential signal path* as shown in Figure 4. The two signal paths differ from the signal path shown in Figure 3 in that they share a single fully-differential V/I converter. Otherwise, the signal path blocks shown in Figure 4 are the same as those shown in Figure 3. The outputs of the two signal paths are differenced to form the output of the pseudo-differential signal path. The differencing operation causes the residual even-order distortion components in the outputs of the two nonlinearity correction blocks to cancel up to the matching accuracy of the two signal paths.

Both differential and pseudo-differential architectures have been used previously in VCO-based $\Delta\Sigma$ modulators [7], [9], [10], [11]. Each approach offers the benefit of cancelling much of the even-order nonlinearity. Unfortunately, simulation and measurement results indicate that the expected matching accuracy of the two signal paths is not sufficient to cancel the worst-case second-order distortion term below about -65 dBc. Furthermore, while the pseudo-differential architecture is better for low voltage operation than the differential architecture, it has the disadvantage that the strong second-order distortion introduced by each ICRO introduces a large error component proportional to the product of the difference and sum of the two ICRO input currents. Therefore, in the absence of second-order nonlinearity correction prior to differencing the two signal paths, any common-mode error on the two ICRO input lines would be converted to a differential-mode error signal. These problems are addressed by having the nonlinearity correction blocks in each signal path correct second-order distortion prior to the differencing operation.

The signal components in the output of the two signal paths have the same magnitudes and opposite signs, whereas the quantization noise and much of the circuit noise in the two outputs are uncorrelated. Therefore, the differencing operation increases the signal by 6dB and increases the noise by approximately 3dB, so the SNR of the pseudo-differential signal path is approximately 3dB higher than that of each individual path.

III.C. Self-Cancelling Dither Technique

The quantization noise from first-order $\Delta\Sigma$ modulators is notoriously poorly behaved, particularly for low-amplitude input signals [1]. It often contains large spurious tones and can be strongly correlated to the input signal. In theory this problem can be solved by adding a dither sequence to the input of the $\Delta\Sigma$ modulator's quantizer. If the dither sequence is white and uniformly distributed over the quantization step size, it causes the quantizer to be well modeled as an additive source of white noise that is uncorrelated with the input signal [17]. The dither has the same variance and is subjected to the same noise transfer function as the quantization noise so it increases the noise floor of the $\Delta\Sigma$ modulator by no more than 3 dB.

Unfortunately, in a VCO-based $\Delta\Sigma$ modulator there is no physical node at which to add such a dither sequence, because the integration and quantization are implemented simultaneously by the VCO. Another option is to add the dither to the input of the $\Delta\Sigma$ modulator. This has the desired effect on the quantization noise, but severely degrades the signal-band SNR because the dither is not subjected to the $\Delta\Sigma$ modulator's highpass noise transfer function. While highpass shaping the dither prior to adding it to the input of the

$\Delta\Sigma$ modulator would solve this problem, doing so tends to negate the positive effects of the dither on the quantization noise.

A self-cancelling dither technique is used in this work to circumvent these problems. The idea is to construct the $\Delta\Sigma$ modulator as the sum of two pseudo-differential signal paths each of the form shown in Figure 4, but with a dither signal added to the input of one of the paths and subtracted from the input of the other path. The overall $\Delta\Sigma$ modulator output is the sum of the two pseudo-differential signal path outputs. The dither causes the quantization noise from each pseudo-differential signal path to be free of spurious tones and uncorrelated with the input signal and it also degrades the signal-band SNR of each pseudo-differential signal path output as described above. However, the dither components that cause the SNR degradation in the output sequences of the two pseudo-differential signal paths have equal magnitudes and opposite polarities, whereas the signal components in the two output sequences are identical, and the noise components in the two output sequences are uncorrelated. Therefore when the two output sequences are added, the unwanted dither components cancel, the signal components add in amplitude, and the noise components add in power. This results in an SNR that is 3dB higher than would be achieved by a single pseudo-differential signal path in which the unwanted dither component were somehow subtracted directly. It also doubles the circuit area and power dissipation, the implications of which are discussed shortly.

An advantage of the fine quantization performed by the 15-element ring oscillators is that low-amplitude dither sequences are effective. In this design, approximately 1

dB of dynamic range is used to accommodate the dither sequences.

An alternate approach to the self-cancelling dither technique described above is to add a common-mode dither signal to a single pseudo-differential signal path. The dither would then be cancelled by the pseudo-differential signal path's final differencing operation. The reason this approach was not used is that the second-order distortion correction performed by the nonlinearity correction blocks is not perfect, particularly at frequencies well above the signal band, so the residual second-order error would cause a small but potentially significant differential error term proportional to the product of the input and dither signals.

III.D. The Implemented $\Delta\Sigma$ Modulator Architecture

Figure 5 shows a block diagram of the full $\Delta\Sigma$ modulator architecture incorporating the features described above. It consists of two of the pseudo-differential signal paths shown in Figure 4, the calibration unit shown in Figure 3, and a pair of 4-level DACs that add and subtract a pseudo-random dither sequence to and from the top and bottom pseudo-differential signal paths, respectively. The outputs of the two pseudo-differential signal paths are added to form the $\Delta\Sigma$ modulator output sequence.

The input to each dither DAC is a 4-level white pseudo-random sequence with a sample-rate of $f_s/8$. Each dither DAC converts this sequence into a differential current signal with a peak-to-peak range approximately equal to the quantization step-size referred to the inputs of the ICROs. Extensive system-level and circuit-level simulations

and measurement results indicate that the dither whitens the noise injected by each ICRO's quantization process sufficiently to meet the target specifications of the $\Delta\Sigma$ modulator, despite having only four levels and an update rate of only $f_s/8$.

As described above, each pseudo-differential signal path has an SNR that is 3 dB higher than that of its two non-differential signal paths, and adding the outputs of the two pseudo-differential signal paths results in a 3dB improvement in SNR relative to that which could be achieved by a single pseudo-differential signal path. Therefore, compared to a single non-differential signal path, the four signal paths in the $\Delta\Sigma$ modulator consume four times the power and circuit area, but they also result in an SNR improvement of 6 dB. A commonly-used figure of merit for $\Delta\Sigma$ modulators is

$$FOM = SNDR + 10\log_{10}\left(\frac{\text{signal bandwidth}}{\text{power dissipation}}\right) \quad (24)$$

with $SNDR$ in dB. To the extent that the $SNDR$ is noise-limited it follows that the use of multiple signal paths does not degrade the FOM .

III.E. Quantization Noise, No-overload Range, and the Number of Ring Elements

As described in Section II.A, well-known results for the first-order $\Delta\Sigma$ modulator can be applied to the VCO-based $\Delta\Sigma$ modulator [1]. The theoretical maximum signal-to-quantization-noise-ratio, $SQNR_{max}$, is that of a conventional first-order $\Delta\Sigma$ modulator plus 6 dB to account for the four signal paths and minus 1 dB to account for the reduction in dynamic range required for dither. Hence,

$$SQNR_{max} = 20 \log_{10}(2M) + 30 \log_{10} \left(\frac{f_s}{2B_s} \right) + 1.59, \quad (25)$$

where M is the number inverters in each ring oscillator (so the number of quantization steps is $2M$), and B_s is the signal bandwidth. The oversampling ratio is defined as $OSR = f_s/(2B_s)$. The no-overload range $\Delta\Sigma$ modulator is the range of input voltages for which (2) is satisfied, so it follows from (1) that the no-overload range is

$$|v(t)| < \frac{\pi f_s}{K_{VCO}}. \quad (26)$$

Unlike a conventional $\Delta\Sigma$ modulator, f_s and M in (25) cannot be chosen independently because $f_s = 1/(M\tau_{inv})$ where τ_{inv} is the nominal delay of each VCO inverter when $v(t) = 0$. For a given inverter topology, τ_{inv} is determined by the speed of the CMOS process. Therefore, to increase f_s for a given design, it is necessary to reduce M proportionally. It follows from (25) that $SQNR_{max}$ increases by 3 dB each time f_s is doubled for any given τ_{inv} and B_s . However, increasing f_s has two negative side effects. First, it increases the quantization noise folding described in Section III.A because reducing M causes coarser quantization. Second, it increases the clock rate at which the digital circuitry following the ring oscillators must operate, which increases power consumption. The choice of 15-element ring oscillators for the IC presented in this paper represent was made on the basis of these considerations.

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2010. The dissertation author is the primary investigator and author of this paper. Professor Ian Galton supervised the research which forms the basis for this paper.

IV. CIRCUIT DETAILS

IV.A. ICRO, Ring Sampler, and Phase Decoder

If the ring oscillator inverters have mismatched rise and fall times or signal-dependent amplitudes, the result is non-uniform quantization that can cause significant nonlinear distortion which is not corrected by the background calibration technique. The problem is illustrated in Figure 6 for the case of a 5-element ring oscillator implemented as a V/I converter that drives five current-starved inverters. The output waveform from each inverter is shown for the case of a constant VCO input voltage, i.e., a constant VCO frequency. The transition times and values that the phase decoder output would have if the ring sampler were bypassed are also shown. Each inverter waveform oscillates between a minimum voltage of zero and a maximum voltage that depends on the VCO input voltage. This causes the duration of each inverter's positive transition state to be much shorter than that of its negative transition state. The effect is evident in the non-uniform transition times of the phase decoder output. Since the amount of non-uniformity depends on the VCO's input signal, this phenomenon causes the $\Delta\Sigma$ modulator to introduce strong nonlinear distortion.

The implemented $\Delta\Sigma$ modulator avoids this problem with differential inverters and a modified ring sampler and phase decoder. The concept is illustrated in Figure 7, again for a 5-element ring oscillator. In this case, each inverter is defined to be in *positive transition* when its positive input voltage and positive output voltage are less than and

greater than the digital logic threshold (e.g., half the supply voltage), respectively. Similarly, each inverter is defined to be in *negative transition* when its negative input voltage and negative output voltage are less than and greater than the digital logic threshold, respectively. With these definitions all the conclusions of Section II.B apply to this example. However, unlike the example shown in Figure 6, the duration of each inverter's positive transition state is the same as that of its negative transition state because each of the times, t_i , occur only when a *falling* output from one of the inverters crosses the logic threshold. Therefore the transition times of the phase decoder output are uniformly spaced for any given VCO frequency.

This idea can be applied to any ring oscillator with an odd number of elements. In particular, each ICRO in the prototype IC is a ring of 15 current-starved pseudo-differential inverters as shown in Figure 8. The ring sampler latches the 30 inverter outputs on the rising edge of each f_s -rate clock, and the phase decoder calculates a corresponding instantaneous phase number by identifying which inverter was either in positive or negative transition at the last sample time as described above and in Section II.B.

IV.B. V/I converter

The V/I converter is shown in Figure 8. The outputs are from a pair of pMOS cascode current sources in which the gates of the cascode transistors are regulated by the outputs of a fully-differential op-amp, and current proportional to the differential input voltage is injected into the sources of the cascode transistors. To the extent that the op-amp input terminals present a differential virtual ground, the output current variation

about the bias current into the top and bottom ICROs is $\frac{1}{2}(V_{in+} - V_{in-})/R$ and $-\frac{1}{2}(V_{in+} - V_{in-})/R$, respectively.

The V/I converter operates from a 2.5 V supply, so it consists of all thick-oxide transistors. The op-amp has a telescopic cascode structure with common-mode feedback achieved by sensing the common-mode input voltage. The simulated differential-mode open-loop gain and unity-gain bandwidth of the op-amp are 50dB and 2.3GHz, respectively, and the phase margin of the feedback loop is 55 degrees over worst-case process and temperature corners. Two-tone simulations across the 0 to $f_s/2$ frequency band with layout-extracted parasitics indicate that nonlinear distortion from the V/I converter is at least 20dB less than that of the overall $\Delta\Sigma$ modulator regardless of input signal frequency.

The closed-loop bandwidth of the V/I converter is approximately g_m/C_C , where g_m is the transconductance of the op-amp's differential pair nMOS transistors and C_C is the value of the compensation capacitors. For any given phase margin, C_C depends on the magnitude of the two non-dominant poles at the sources of the pMOS cascode transistors in the op-amp and in the output current sources. These poles are inversely proportional to the intrinsic capacitances of the devices, which ultimately depends on the f_T of the CMOS process. Since g_m is relatively independent of f_T , the closed-loop bandwidth increases as f_T is increased. This implies that if the V/I converter were implemented in a more highly-scaled CMOS process, it could be designed to have a larger closed-loop bandwidth without increasing the current consumption.

The ICRO bias current is controlled by the calibration unit as described in Section

III.A. The gate voltage of the pMOS current source, $V_{cal}[n]$, is the drain voltage of a diode connected pMOS transistor connected to an nMOS current-steering DAC driven by the 8-bit output of the VCO center frequency calculator in the calibration unit.

A side benefit of the pseudo-differential architecture is that its cancellation of common-mode circuit noise eliminates the need to filter the ICRO bias voltages. Otherwise large bypass capacitors would have been required as they are in conventional continuous-time $\Delta\Sigma$ modulators that use current steering DACs.

As shown in Figure 3, the calibration signal bypasses the V/I converter so the digital background nonlinearity correction technique does not cancel nonlinear distortion introduced by the V/I converter. As described above, the V/I converter is sufficiently linear that this is not a problem. Alternatively, an open loop V/I converter without an op-amp could have been used. This would have introduced significant nonlinear distortion, so it would have been necessary to modify the calibration unit to inject the calibration signal into the input of a V/I converter replica. In this case, the V/I converter distortion would be cancelled along with ICRO distortion by the digital background nonlinearity correction technique. One side effect of this approach is that the dither would have to be added prior to the V/I converters in the actual signal paths. Otherwise they would be subject to distortion that the digital background nonlinearity correction technique would not properly cancel. While this alternative approach is viable, it was not implemented because it would have dictated more complicated DACs for the calibration and dither sequences.

IV.C. Dither DACs

The accuracy of the self-cancelling dither technique described in Section III.C depends on how well the two pseudo-differential signal paths match and how well the two dither DACs match. Mismatches between the pseudo-differential signal paths occur mainly among the ICROs, and simulations predict that such mismatches are so small as to have a negligible effect on the $\Delta\Sigma$ modulator's performance. The dither DACs generate current outputs, so their matching depends on how well multiple switched current sources can be matched, which, in turn, depends on device sizing. Unfortunately, conventional current-steering DACs with sufficient matching accuracy to meet the target specifications would occupy almost half of the total circuit area of the $\Delta\Sigma$ modulator.

A solution to this problem is shown in Figure 9. The idea is to use a pair of very small current-steering DACs but suppress the effect of their mismatch error by alternately swapping their roles at twice their update-rate. Therefore, the outputs of each DAC are connected to the ICRO inputs in one of the pseudo-differential signal paths for the first half the DAC's update period, and to the ICRO inputs in the other pseudo-differential signal path for the second half of the DAC's update period. It can be verified that this causes the residual dither component in the $\Delta\Sigma$ modulator output sequence arising from DAC mismatches to have a first-order highpass power spectral density. This suppresses the error sufficiently over the $\Delta\Sigma$ modulator's signal band so as to have a negligible effect on the SNR.

A potential problem with non-return-to-zero (NRZ) current steering DACs is that

parasitic capacitance at the source coupled node of the current steering cell can cause nonlinear inter-symbol interference. The DACs used in this work avoid this problem via the dual return-to-zero (RZ) technique in which a pair of RZ DACs offset from each other by half an update period are interlaced to achieve the combined effect of an NRZ DAC [18].

The architecture described above can be implemented directly as shown in Figure 9 with the 4-level DACs implemented as RZ DACs. Alternatively, the switches in the swapper cells shown in Figure 9 can be built into the current steering cells of the RZ DACs. The latter approach is taken in this work. The two implementation methods are equivalent from a signal processing point of view, but the latter results in a more compact circuit with less degradation from non-ideal circuit behavior.

IV.D. Nonlinearity Correction Block

As described in Section III.A, each nonlinearity correction block is a high-speed look-up table (LUT). It maps a 5-bit input sequence to a 14-bit output sequence at a rate of f_s , where f_s can be as high as 1.152 GHz. The details of the block are shown in Figure 10. The calibration unit loads the 32 14-bit registers with mapping data via the LUT write address and LUT write value lines during the first 32 T_s clock periods once every $2^{28}T_s$. The 5-bit input sequence is used as a LUT read address. Each 5-bit value routes the 14-bit output from the corresponding register to the output.

IV.E. Circuit Noise Sources

The lowpass ring oscillator phase noise is subjected to the highpass transfer function of the $1-z^{-1}$ blocks, so the resulting contribution to the output sequence in the signal band is nearly white noise. Simulations indicate that in each $\Delta\Sigma$ modulator the V/I converter resistors, V/I converter op-amps, VCO bias current sources, and ICROs together contribute $10 \text{ nV}/\sqrt{\text{Hz}}$, $9 \text{ nV}/\sqrt{\text{Hz}}$, $10 \text{ nV}/\sqrt{\text{Hz}}$, and $9 \text{ nV}/\sqrt{\text{Hz}}$, respectively, of noise referred to the input. For a full-scale sinusoidal input signal (800 mV differential peak-to-peak) and a signal bandwidth of 18 MHz, the resulting SNR from thermal noise only is 77 dB. It follows from (25) that for this signal bandwidth $SQNR_{max} = 76 \text{ dB}$, so the expected peak SNR from thermal and quantization noise together is 73 dB.

The $\Delta\Sigma$ modulator is much less sensitive to clock jitter than conventional $\Delta\Sigma$ modulators with continuous-time feedback DACs because it does not contain feedback DACs. Jitter-induced ring sampler error is suppressed in the signal band because it is subjected to first-order highpass shaping by the subsequent $1-z^{-1}$ blocks, and jitter-induced errors from the dither DACs largely cancel along with the dither when the outputs of the pseudo-differential signal paths are added. In contrast, jitter-induced error from the feedback DACs in the first stage of a conventional continuous-time $\Delta\Sigma$ modulator is neither highpass shaped nor cancelled. Most of the published wideband continuous-time $\Delta\Sigma$ modulators use current-steering feedback DACs whose pulse widths and pulse positions are both subject to clock jitter. The jitter mixes high-frequency quantization noise into the signal band, so a very low-jitter clock is necessary so as not to degrade the noise floor of

the signal band [3].

Chapter IV is largely taken from Section IV of a paper entitled “A Mostly-Digital Variable-Rate Continuous-Time Delta-Sigma Modulator ADC” published in the IEEE Journal of Solid-State Circuits, volume 45, number 12, pages 2634-2646, December 2010. The dissertation author is the primary investigator and author of this paper. Professor Ian Galton supervised the research which forms the basis for this paper.

V. FIRST PROTOTYPE IC

V.A. Measurement Results

The IC was fabricated in the TSMC 65nm LP process with the deep nWell option and both 1.2V single-oxide devices and 2.5V dual-oxide devices, but without the MiM capacitor option. All pads have ESD protection circuitry. The IC was packaged in a 64-pin LFCSP package.

Each IC contains two $\Delta\Sigma$ modulators with a combined active area of 0.14 mm². A die photograph of one of the $\Delta\Sigma$ modulators is shown in Figure 11. The calibration unit area is 0.06 mm². The *signal converter*, i.e., the portion of each $\Delta\Sigma$ modulator not including the calibration unit, has an area of 0.04 mm². A single calibration unit is shared by the two $\Delta\Sigma$ modulators, so the area per $\Delta\Sigma$ modulator is 0.07 mm².

All components of both $\Delta\Sigma$ modulators are implemented on-chip except for the $f_s/2^{28}$ -rate coefficient calculation block within the calibration unit's nonlinearity coefficient calculator block. A schedule problem just prior to tapeout prevented on-time completion of this block so it is implemented off-chip. It has since been laid out for a new version of the IC and found to increase the overall area by 0.004mm² with negligible incremental power consumption because of its low rate of operation.

A printed circuit test board was used to evaluate the IC mounted on a socket. The test board includes input signal conditioning circuitry, clock conditioning circuitry, and

an FPGA for ADC data capture and serial port communication. The input conditioning circuitry uses a transformer to convert the single-ended output of a laboratory signal generator into a differential input signal for the IC. The clock conditioning circuitry also uses a transformer. It converts the single-ended output of a laboratory signal generator to a differential clock signal for the IC. Two power supplies provide the 1.2 and 2.5 V power supplies for the IC. The V/I converters operate from the 2.5 V supply, and all other blocks on the IC operate from the 1.2 V supply.

Measurements were performed with a clock frequency, f_s , ranging from 500MHz to 1.152GHz. Single-tone and two-tone input signals were generated by high-quality laboratory signal generators and were passed through passive narrow-band band-pass filters to suppress noise and distortion from the signal generators. Each output spectrum presented below was obtained by averaging 4 length-16384 periodograms from non-overlapping segments of $\Delta\Sigma$ modulator output data, and the SNR and SNDR values were calculated from the resulting spectra via the technique presented in [19]. Both $\Delta\Sigma$ modulators on five copies of the IC were tested with no noticeable performance differences.

Figure 12 shows representative measured output spectra of the $\Delta\Sigma$ modulator for a 0 dBFS, 1 MHz single-tone input signal with $f_s = 1.152$ GHz, both with and without digital background calibration enabled. Without calibration, the SNDR over the 18MHz signal band is only 48.5dB because of harmonic distortion and a high noise floor. The high noise floor is the result of common-mode to differential-mode conversion of common-mode thermal noise via the strong second-order distortion introduced by the VCOs as de-

scribed in Section III.B. With calibration enabled, the SNDR improves to 69 dB. In particular, the second-order term cancels extremely well.

The measured inter-modulation performance of the $\Delta\Sigma$ modulator with $f_s = 1.152$ GHz is shown in Figure 13. The top plot shows the measured spectrum of the $\Delta\Sigma$ modulator output for a two-tone out-of-band input signal, and shows the corresponding signal to third-order and fifth-order inter-modulation distortion ratios, denoted as IM3 and IM5, respectively. Measurements indicate that the IM3 and IM5 values depend mainly on the difference in frequency between the two input tones, but not on where in the 576 MHz Nyquist band the two input tones are placed.

The bottom plot in Figure 13 shows the measured IM3 and IM5 values as a function of the frequencies at which they occur within the signal band. Each value was measured by injecting a full-scale, out-of-band, two-tone input signal into the $\Delta\Sigma$ modulator and measuring the IM3 and IM5 values corresponding to inter-modulation terms within the 18 MHz signal band. For example, the IM3 value measured from the top plot corresponds to the circled data point in the bottom plot of Figure 13. The IM3 values before and after digital calibration are shown. The IM5 values were not measurably affected by digital calibration, so only the IM5 values after calibration are shown.

The low-frequency IM3 of better than 83dB suggests that the calibration unit does a very good job of measuring third-order distortion for low-frequency inter-modulation products (even when the input tones are well above the signal bandwidth). However, the reduction in IM3 values for inter-modulation products near the high end of the 18 MHz

signal band indicate that the third-order distortion coefficient is somewhat frequency dependent. Simulations suggest that this frequency dependence is caused by nonlinear phase shift at the output nodes of the V/I converters. Nevertheless, throughout the maximum signal bandwidth of 18MHz, the IM3 product is greater than 69dB.

Figure 14 shows plots of the SNR and SNDR versus input amplitude for the $\Delta\Sigma$ modulator measured over an 18 MHz signal bandwidth and a 9 MHz signal bandwidth with $f_s = 1.152\text{GHz}$. These signal bandwidths correspond to oversampling ratios of 32 and 64, respectively. The SNR and SNDR for a peak input signal with an oversampling ratio 32 are 70 dB and 69 dB, respectively, and those for an oversampling ratio of 64 are 76 dB and 73 dB. This suggests that quantization noise as opposed to thermal and $1/f$ noise limits performance at the lower oversampling ratio.

As described in Section IV.E a peak SNR of 73 dB was expected over a signal bandwidth of 18 MHz, but as mentioned above the measured SNR over this bandwidth is 70 dB. The authors believe that this discrepancy is caused by non-uniform quantization effects arising from an asymmetric layout of the ICROs. Simulations with parasitics extracted from the layout indicate that this increases the quantization noise by roughly 3dB and reduces the no-overload range of the $\Delta\Sigma$ modulator by roughly 0.5dB.

Figure 15 shows representative measured output spectra of the $\Delta\Sigma$ modulator with f_s reduced to 500 MHz for a large input signal with the dither DACs enabled, and for a zero input signal both with and without the dither DACs enabled. The spectrum corresponding to the zero input signal with the dither DACs disabled has significant spurious

content, as expected. The spectrum corresponding to the zero input signal with the dither DACs enabled indicates that the quantization noise is well-behaved and the dither cancellation process is effective because the noise floor over the signal band does not change as a result of enabling the dither DACs. Clock feed-through from the dither DACs is visible at $f_s/8$, but it lies well outside the signal bandwidth. Similar results to those shown in Figure 13 occur when f_s is varied between 500 MHz and 1.152 GHz.

Measured results from the prototype IC are summarized relative to comparable state-of-the-art $\Delta\Sigma$ modulators in Table 1. As indicated in the table, the performance of the $\Delta\Sigma$ modulator is comparable to the state-of-the-art, but uses significantly less circuit area.

The $\Delta\Sigma$ modulator's performance depends mainly on the digital circuit speed of the CMOS process. As described above, quantization noise, which limits the implemented $\Delta\Sigma$ modulator's performance at low oversampling ratios, scales with the minimum delay through a ring VCO inverter. The V/I converter accounts for less than a third of the total power dissipation, and as described in Section IV.B its bandwidth should increase as f_T increases. Therefore, unlike conventional analog $\Delta\Sigma$ modulators, the $\Delta\Sigma$ modulator architecture described in this paper is likely to yield even better results when implemented in more highly scaled CMOS technology.

V.B. Conclusions

The ADC described in Sections I through V is the first high-performance, stand-

alone VCO-based $\Delta\Sigma$ modulator ADC presented in the literature and it successfully demonstrates the feasibility and capabilities of this architecture. The first prototype IC achieves near state-of-the-art power figure-of-merit and beyond state-of-the-art die-size figure-of-merit. However, the first prototype IC has a number of shortcomings that limit the performance and usability of this ADC. First, quantization noise limits the ADC's performance for low oversample ratios. Second, signal channel bandwidth limitations cause a roll-off in linearity at higher bandwidths which reduces achievable SNDR. Third, the theoretical maximum SQNR is not achieved in the first silicon due to a handful of implementation errors. Fourth, the input V/I converter requires a 2.5V supply which may be undesirable for some applications. And last, the overall power consumption is dominated by digital power used to reduce quantization noise whereas in a highly efficient ADC most of the power should be used to reduce thermal noise. In the following sections a number of modifications and improvements to the original design are proposed to enhance the VCO ADC's performance and usability. Section VI describes a number of architectural enhancements made to improve performance. Section VII describes a number of circuit-level enhancements to help the ADC achieve performance closer to the theoretical maximum. Section VIII presents silicon measurement results for a second prototype IC fabricated in the 65nm G+ process which incorporates all of the enhancements described in Sections VI and VII. Section IX presents silicon measurement results for a third prototype IC fabricated in the 65nm G+ process which is similar to the first prototype IC..

Chapters V is largely taken from Section V of a paper entitled "A Mostly-Digital

Variable-Rate Continuous-Time Delta-Sigma Modulator ADC” published in the IEEE Journal of Solid-State Circuits, volume 45, number 12, pages 2634-2646, December 2010. The dissertation author is the primary investigator and author of this paper. Professor Ian Galton supervised the research which forms the basis for this paper.

VI. ARCHITECTURAL ENHANCEMENTS

VI.A. Quantization Noise Reduction in a VCO-Based ADC

In the first prototype IC, quantization noise limits the overall SNR performance for oversample ratios less than 64. A method is desired that can reduce quantization noise in the VCO-based ADC architecture.

The first prototype IC used four parallel signal paths to solve a number of issues as described in Sections III.B and III.C. Since the quantization noise in each of the four signal paths is largely uncorrelated, adding the four signal paths in parallel reduced the overall SNR by 6dB as described in section III.D. The tradeoff is roughly a 3dB increase in SQNR for each doubling of power and die size. SQNR could be further reduced by increasing the number of parallel channels, however a more efficient method is desired and presented in this section.

Recall that equation (25) of Section III.E gives the theoretical maximum achievable signal-to-quantization noise ratio for the dual pseudo-differential ADC of Figure 5. It can be inferred from (25) that doubling the number of ring elements, M , increases the SQNR by 6dB and doubling the sample rate, f_s , increases the SQNR by 9dB. Unfortunately, unlike a conventional $\Delta\Sigma$ modulator, f_s and M in cannot be chosen independently because the nominal ICRO oscillating frequency is equal to the sampling frequency, f_s , and the following relationship must be satisfied: $f_s = 1/(2M\tau_{inv})$ where τ_{inv} is the nominal delay of

each VCO inverter when $v(t)=0$. Therefore, a VCO ADC's quantization step size is limited by the minimum inverter delay, τ_{inv} which is determined by the process and the supply voltage available.

The first prototype IC uses a 15-element ring oscillator sampled at 1.152GHz shown in Figure 16(a) which gives an $SQNR_{MAX}$ of 75dB for 18MHz of signal bandwidth and an OSR of 32. The minimum inverter delay is limited by the process parameters and supply voltage. One method to improve SQNR is to reduce the number of delay elements and increase the sample rate. A 7-element ring oscillator with a 2.4GHz sample rate is shown in Figure 16(b). An ADC that uses the ring oscillator of Figure 16(b) will have a SQNR advantage of only 3dB when compared to an ADC that uses the ring oscillator of Figure 16(a). This is a modest improvement for doubling the clock rate. Increasing the sample rate further is not desirable due to speed limitations of the digital circuitry in the digital signal processing path. Additionally, reducing M causes coarser quantization resulting in increased quantization noise folding when correcting signal path nonlinearity as described in Section III.A.

A solution exists that avoids these limitations and it is shown in Figure 16(c). By injection locking two ring oscillators with an offset of $\tau_{inv}/2$, the quantization step size can be effectively cut in half which is equivalent to increasing M by a factor of 2 and therefore the quantization noise is reduced by 6dB. Resistor interpolation is used to keep the pseudo-differential inverters 180 degrees out of phase and to keep the two ring oscillators phase-locked with $\tau_{inv}/2$ offset. A modulator employing the two injection

locked ring oscillators of Figure 16(c) with $f_s=2.4\text{GHz}$ has a 9dB increase in SQNR relative to a modulator using the oscillator of Figure 16(a) for the same signal bandwidth, B_s . Additionally, a delay cell using resistor feedback instead of weak inverter feedback is inherently faster and produces less thermal noise.

Alternately, a modulator employing two 7-element injection locked ring oscillators of Figure 16(c) with $f_s=2.4\text{GHz}$ can use the higher sampling rate to increase the signal bandwidth. For example, a modulator using a dual, 7-element injection-locked ring oscillator with $f_s=2.4\text{GHz}$ and a 32x OSR will have a nearly identical SQNR but twice the signal bandwidth when compared to a modulator using a single, 15-element ring oscillator with $f_s=1.152\text{GHz}$.

The injection locking idea can be extended beyond two rings. For example, four 7-element rings can be injection-locked to further increase M by a factor of 2 and therefore reduce quantization noise by another 6dB. However, a dual, 7-element ring oscillator was chosen for the second prototype IC because it has 28 possible output codes values which meet the SQNR target specification and allows reuse of the first prototype IC's digital processing blocks since its 5-bit signal processing path can process a 28 code ring oscillator output.

VI.B. Quantization Noise Reduction by Extended No-Overload Range

The output of the $\Delta\Sigma$ modulator, as described in Section II.A, is equal to the difference in VCO phase value from one sample to the next. This phase difference is lim-

ited to a range of 0-360 degrees. The measured phase difference value, as a function of VCO frequency, sampled at the sample rate, f_s , is periodic. In other words, multiple values of VCO frequency correspond to the same phase difference value. For example, if the VCO frequency is equal to the sample frequency, then the sampled phase state does not change from one phase sample to the next. But this is also true for

$$f_{VCO} = M \cdot f_s \quad (27)$$

where $M=\{0,1,2,\dots\}$ and the sample-to-sample measured phase difference is zero for all values of M . If the sample-to-sample measured phase difference was plotted versus VCO frequency, then the result would be a periodic triangle waveform with an amplitude range of 0 to 360 degrees and a period of f_s . The output of the $\Delta\Sigma$ modulator, which is equal to this sample-to-sample measured phase difference, has a unique value only over a frequency range of f_s . Therefore, the VCO frequency range of operation must be limited to

$$f_{VCO} = f_{NOMINAL} \pm 0.5f_s \quad (28)$$

where $f_{NOMINAL}$ is the VCO frequency value when $v(t)=0$.

The ADC described in this paper is designed to operate with a nominal VCO frequency equal to the sample rate, f_s , which corresponds to differentiator output equal to zero. Maximum and minimum differentiator output values occur at $f_s = f_s \pm 0.5f_s$ therefore the no-overload range of operation corresponds to an instantaneous VCO frequency of

$$0.5f_s < f_{VCO}(t) < 1.5f_s. \quad (29)$$

When the magnitude of the input voltage, $v(t)$, causes $f_{VCO}(t)$ to be outside the value specified in (29), then the VCO frequency of oscillation exceeds the full-scale range. When the ring sampler samples the ring phase at the sample rate, f_s , aliasing prevents the ADC from determining correctly whether $f_{vco}(t)$ is above or below the range specified in (29). An input signal that causes $f_{vco}(t)$ to exceed this range will result in an overloaded and distorted output signal waveform. An example output waveform for the overloaded ADC is shown in Figure 17(d). The output does not saturate like a flash converter instead the output is severely distorted.

Fortunately, advantage can be taken of the large OSR of the converter. If the input signal is changing slowly compared to f_s , the previous oscillator sample value can be used to determine the current sample value. This technique allows $f_{vco}(t)$ to extend beyond the range specified in (29).

A new block is added to the signal path called an Over-Range Corrector (ORC) shown in Figure 17(a). Output data from the digital differentiator is fed into the ORC block. A more detailed view of the ORC is shown in Figure 17(b). It consists of a lookup-table based state-machine that compares the current output of the digital differentiator, labeled $a[n]$, with the past output of the ORC, labeled $b[n-1]$. A truth table is shown in Figure 17(c) which describes the operation of the Overflow Logic block. A simple algorithm is desirable to minimize die size and power dissipation in this block.

The Overflow Logic block uses simple logic to determine if the current value of $a[n]$ is within the range specified in (29) or above or below this range. If the current value of $a[n]$ is greater than 7 and the previous Overflow Logic output value, $b[n-1]$, is less than zero, then an over-range condition has occurred and 32 is subtracted from the current value. In other words, the logic assumes that the current value must be negative because the previous value was negative and an underflow situation has occurred. Conversely, if the current value of $a[n]$ is less than -8 and the previous Overflow Logic output value, $b[n-1]$ is greater than zero, then an over-range condition has occurred and 32 is added to the current value of $a[n]$. In other words, the logic assumes that that current value must be positive because the previous value was positive and therefore an overflow situation has occurred. An example waveform at the output of the digital differentiator block is shown in Figure 17(d) where the frequency of the ICRO, f_{VCO} , swings above and below the limits set by (29). Notice that aliasing causes the digital differentiator to decode $f_{VCO} > 1.5f_s$ as a value slightly greater than $0.5f_s$ instead. The opposite occurs when $f_{VCO} < 0.5f_s$ and this condition is decoded as an f_{VCO} value slightly less than $1.5f_s$. The corrected output waveform at the output of the ORC block, without distortion, is shown in Figure 17(e).

The ORC logic is only valid if the input signal is slow moving compared to the sample rate. For the ORC to function properly, the input signal must not change more than 8 digital differentiator output code values (out of a possible 32 values) per clock period, T_s , and therefore the input signal must be band-limited. If the digital differentiator

output code value changes by more than 8 code values per clock period, then an ambiguous output is possible. For a sinusoidal input signal with a frequency of $f_s/2$, its peak-to-peak amplitude, A_{MAX-PP} , must be less than $\frac{8}{32}FS_{PP}$, where FS_{PP} is the peak-to-peak full-scale input, to limit the differentiator output delta to less than 8 codes per sample period, T_S . For a sinusoidal input signal with a frequency of $f_s/4$, the maximum peak-to-peak amplitude must be less than $(\sqrt{2}/8)FS_{PP}$ to guarantee that the differentiator output does not travel more than 8 codes per sample period, T_S . For all input signal frequencies an input amplitude upper-bound limit can be established by limiting the slope of the input signal to be less than $\frac{8}{32}FS_{PP}$ per sample period, T_S . The slope of a sinusoidal input signal is equal to $2\pi f_{IN}FS_{PP}$, where f_{IN} is the input sinusoid frequency. Since the input signal voltage cannot travel more than $\frac{8}{32}FS_{PP}$ then it follows that the upper bound limit for input amplitude is

$$A_{MAX-PP} = \frac{f_s}{f_{IN}} \frac{FS_{PP}}{4\pi}, \quad (30)$$

where A_{MAX-PP} is the maximum allowable peak-to-peak input signal for a given sample rate, f_s , and input sinusoid frequency, f_{IN} . It follows from equation (30) that when f_{IN}/f_s is less than $1/(4\pi)$, then no input signal band-limiting is required. The maximum attenuation required is -15dBFS at $f_s/4$ and -12dBFS at $f_s/2$. A single-pole filter at $f_s/4\pi$ provides sufficient filtering to meet the requirements of (30) because the roll-off of a sin-

gle pole filter is equal to the change in sinewave slope versus input frequency, f_{IN} . For example, with a 2.4GHz sample rate, f_s , a single pole at 190MHz is sufficient to band-limit the input signal.

The ORC function improves the performance of the $\Delta\Sigma$ modulator in two ways. First, it can be used to extend the dynamic range of the converter. In theory, the over-range corrector can extend the VCO frequency range well beyond that shown in (29). However, for this test chip, the over-range corrector was limited to extending the differentiator output range from 28 to 32 possible levels which corresponds to a 1dB increase in dynamic range. The ADC was not allowed to extend beyond that range for two reasons. First, exhaustive simulation and silicon data shows that the nonlinearity of the VCO in the $0.5f_s$ to $1.5f_s$ range tends to be well modeled as a weakly nonlinear function that can in turn be modeled as a Taylor series as mentioned previously in Section III.A. If the VCO is to be used beyond the $0.5f_s$ to $1.5f_s$ range, then care must be taken to ensure that the VCO continues to exhibit a weakly nonlinear function. At some point as the VCO is driven farther beyond the range specified in (29), the current-to-frequency transfer function will transition from a “weak” to a “hard” nonlinearity where higher order terms become significant. Extending the usable range further would have increased the achievable overall SNR but at the expense of reduced SNDR with a full scale input. Second, re-use of the original 5-bit signal process path was desirable. The 5-bit signal processing allows for 32 possible output codes where a maximum positive output code is +15 and a maximum negative output code is -16.

Additionally, the ORC allows for more usable dynamic range in the converter because, practically speaking, the maximum input signal can be allowed to exist closer to full-scale since an input signal excursion slightly beyond full-scale results in a much smaller error with the over-range corrector enabled.

Second, the ORC function improves the overload performance of the VCO-based $\Delta\Sigma$ modulator. A traditional $\Delta\Sigma$ ADCs behave poorly when the input exceeds full scale resulting in modulator instability poor recovery time and unusable output codes. This new design behaves very differently – it saturates like a flash converter. This behavior is important for many applications. When the input signal exceeds full scale the output of the over-range corrector saturates at codes +15 or -16 as shown in Figure 17(f). This feature also allows for over-range detection.

VI.C. Dither

As discussed in Section III.C, the quantization noise from a first-order $\Delta\Sigma$ modulator is very poorly behaved and for low-amplitude input signals the output may contain large spurious tones. The self-cancelling dither technique presented in Section III.C is used to eliminate the spurious tone problem by adding dither sequences prior to quantization and then cancelling them in the digital domain.

VI.C.1 Case #1: Extremely Linear V/I Converter

From a practical circuit point of view, it is extremely difficult to add a dither signal as a voltage prior to the V/I converter. However, it is trivial to add dither as a current

with a simple current steering DAC at the output of the V/I converter. One method to create self-cancelling dither is shown in Figure 18(a). Dither is added as a common-mode signal to the signal path through two 4-level DACs. The digitized outputs of the two converters are subtracted resulting in a doubling of the signal level and a cancelling of the dither signal.

Problems arise with this self-cancelling dither scheme when nonlinearity exists in the signal path. The first prototype IC corrects for the 2nd and 3rd order distortion of the ICRO. However, as mentioned in Section III.B, the 2nd order distortion is large and cancelling it perfectly is difficult due to channel-to-channel mismatch as well as bandwidth limitations in the system. Recall that the pseudo-differential architecture was used to remove the resulting residual uncorrected even-order distortion. Assuming the distortion can be modeled with a small number of Taylor series coefficients, the nonlinearity distortion coefficients for the ICRO are α_2 and α_3 . The output of each channel's differentiator is

$$y_{CH}[n] = \omega_{CH}[n] + \alpha_2(\omega_{CH}[n])^2 + \alpha_3(\omega_{CH}[n])^3 + e_{\Delta\Sigma}[n], \quad (31)$$

where CH is the channel number, either 1 or 2 for this first example, and the channel's input signal is

$$\begin{aligned} \omega_1[n] &= +\omega_l[n] + d[n] \\ \omega_2[n] &= -\omega_l[n] + d[n] \end{aligned} \quad (32)$$

where $d[n]$ is the discrete-time dither signal integrated over one sample interval and $\omega_l[n]$

is the input signal, $v_1(t)$, integrated over one sample interval and multiplied by K_{VCO} . The nonlinearity coefficient estimator calculates the correction terms $\tilde{\alpha}_2$ and $\tilde{\alpha}_3$. Consider the case mention above, where the third order estimated term is exactly equal to the actual distortion coefficient ($\tilde{\alpha}_3 = \alpha_3$) but the second-order estimate is not perfect ($\tilde{\alpha}_2 \neq \alpha_2$). Substituting (32) into (31), then (31) into (18) gives the corrected output of channel 1 and channel 2 of Figure 18(a). Taking the difference of the channel 1 and channel 2 corrected outputs gives the following

$$y_{OUT} = y_1[n] \Big|_{corrected} - y_2[n] \Big|_{corrected} = 2\omega_l[n] + 4 \underbrace{(\alpha_2 - \tilde{\alpha}_2)}_{\neq 0} \omega_l[n]d[n] + \dots \quad (33)$$

The un-cancelled second-order distortion causes common-mode dither signal to be converted to differential signal through the residual uncorrected 2nd order distortion shown in the second term of (33). Because the second order distortion is very large, the residual, uncorrected 2nd order distortion can be quite large and therefore the conversion of common-mode dither signal to differential-signal can be quite large.

For this reason, the dual pseudo-differential architecture with differential dither DACs shown in Figure 18(b) was chosen for the first prototype IC because it eliminates the common-mode to differential conversion seen in (34). The input signal is injected differentially into the two pseudo-differential channels with the same polarity but the dither signal is injected differentially into the two pseudo-differential channels with the opposite polarity. Therefore, the input signals for the four channels are

$$\begin{aligned}
\omega_1[n] &= +\omega_l[n] + d[n] \\
\omega_2[n] &= -\omega_l[n] - d[n] \\
\omega_3[n] &= +\omega_l[n] - d[n] \\
\omega_4[n] &= -\omega_l[n] + d[n]
\end{aligned} \tag{34}$$

Substituting (34) into (31), then (31) into (18) gives the corrected outputs of the four channels of Figure 18(b). Summing the four corrected outputs give the following

$$\begin{aligned}
y_{OUT}[n] &= y_1[n] \Big|_{corrected} - y_2[n] \Big|_{corrected} + y_3[n] \Big|_{corrected} - y_4[n] \Big|_{corrected} \\
&= 4\omega_l[n] + 8 \underbrace{[(\alpha_2 - \tilde{\alpha}_2) - (\alpha_2 - \tilde{\alpha}_2)]}_{=0} \omega_l[n] d[n] + \dots
\end{aligned} \tag{35}$$

The result is that the residual un-cancelled 2nd order distortion, self-cancels with this architecture and there is no common-mode to differential conversion with the dual pseudo-differential architecture.

VI.C.2 Case #2: Nonlinear V/I Converter

Consider a signal path where the input V/I converter is not sufficiently linear as to be ignored. With such a system, the input signal, $v_I(t)$, is subject to V/I converter nonlinearity as well as ICRO nonlinearity, but the dither signal is subject only to the ICRO nonlinearity. The nonlinearity correction coefficients can be generated to correct for nonlinearity from the V/I converter input to the $\Delta\Sigma$ modulator output or from the ICRO input to the $\Delta\Sigma$ modulator output but not both. If the nonlinearity of the V/I converter is significant and the nonlinearity correction block corrects for it then the dither will not completely cancel and inter-modulation products will be present at the output of the converter. The nonlinear distortion is modeled with a 3rd order Taylor series where the

nonlinearity distortion coefficients for the V/I converter are α'_2 and α'_3 and the nonlinearity distortion coefficients for the ICRO are α''_2 and α''_3 .

A problem occurs because the input signal is passed through the V/I converter nonlinearity before the sinc filtering and sampling described in (6) in Section II.A. This poses a problem when high-frequency interfering signals are present at the input because they cause distortion that can fall in-band but then these high-frequency interferers are filtered by (6) before the nonlinearity correction. If interferer signal is attenuated by the sinc transfer function, then it becomes impossible for the nonlinearity correction block to remove the distortion products created by these interferers. Fortunately, the input signal must be lightly filtered for both anti-aliasing requirements as well as the over-range correction block requirements. This light filtering reduces the amplitude of out-of-band interferers that are affected by the sinc transfer function so that they create negligible distortion in-band. Extensive simulations indicate that the light filtering mentioned here is enough to make the system immune to any out-of-band interferer.

Recall from (5) that $\omega[n]$ is a phase increment and it is proportional to the integral of the VCO input over one sample interval, T_s , where the VCO input is the input signal $v_I(t)$ plus the dither signal $v_D(t)$. Therefore, when the input signal is passed through the V/I converter nonlinearity, the phase increment values for channel 1 and channel 2 are

$$\begin{aligned}\omega_1[n] &= \int_{(n-1)T_s}^{nT_s} \left[K_{VCO} \left[+v_I(\tau) + \alpha'_2 (v_I(\tau))^2 + \alpha'_3 (v_I(\tau))^3 \right] + K_{VCO} v_D(\tau) \right] d\tau \\ \omega_2[n] &= \int_{(n-1)T_s}^{nT_s} \left[K_{VCO} \left[-v_I(\tau) + \alpha'_2 (v_I(\tau))^2 - \alpha'_3 (v_I(\tau))^3 \right] + K_{VCO} v_D(\tau) \right] d\tau.\end{aligned}\quad (36)$$

The V/I converter nonlinearity happens inside the integral. To simplify the following analysis, the assumption is made that the input signal varies slowly compared to the sample rate, f_s , and therefore the input signal changes very little over one sample interval and the following simplifications can be made

$$\begin{aligned}\alpha'_2 K_{VCO} \int_{(n-1)T_s}^{nT_s} (v_I(\tau))^2 d\tau &\approx \alpha'_2 K_{VCO} \left(\int_{(n-1)T_s}^{nT_s} v_I(\tau) d\tau \right)^2 = \alpha'_2 (v_I[n])^2 \\ \alpha'_3 K_{VCO} \int_{(n-1)T_s}^{nT_s} (v_I(\tau))^3 d\tau &\approx \alpha'_3 K_{VCO} \left(\int_{(n-1)T_s}^{nT_s} v_I(\tau) d\tau \right)^3 = \alpha'_3 (v_I[n])^3\end{aligned}\quad (37)$$

where $v_I[n]$ is the integral of the input signal over one sample interval multiplied by K_{VCO} . With the simplifications of (37), then (36) reduces to the following

$$\begin{aligned}\omega_1[n] &\approx +v_I[n] + \alpha'_2 (v_I[n])^2 + \alpha'_3 (v_I[n])^3 + d[n] \\ \omega_2[n] &\approx -v_I[n] + \alpha'_2 (v_I[n])^2 - \alpha'_3 (v_I[n])^3 + d[n]\end{aligned}\quad (38)$$

where $d[n]$ is the integral of the dither signal over one sample interval multiplied by K_{VCO} . It follows from (15) that the output of each channel's differentiator is

$$\begin{aligned}y_1[n] &= \omega_1[n] + \alpha''_2 \omega_1^2[n] + \alpha''_3 \omega_1^3[n] + e_{\Delta\Sigma}[n] \\ y_2[n] &= \omega_2[n] + \alpha''_2 \omega_2^2[n] + \alpha''_3 \omega_2^3[n] + e_{\Delta\Sigma}[n]\end{aligned}\quad (39)$$

Substituting (38) into (39) and then (39) into (18) gives the corrected results at the output of each channel, then taking the difference of channel 1 and channel 2 outputs results in the following

$$y_{OUT} = y_1[n] \Big|_{corrected} - y_2[n] \Big|_{corrected} = 2v_I[n] + 4 \underbrace{(\alpha''_2 - \tilde{\alpha}_2)}_{\neq 0} v_I[n] d[n] + \dots \quad (40)$$

Perfect estimation of the combined V/I converter and ICRO nonlinearity results in second and third order correction terms equal to $\tilde{\alpha}_2 = \alpha'_2 + \alpha''_2$ and $\tilde{\alpha}_3 = \alpha'_3 + \alpha''_3$, respectively. If the V/I converter distortion is large then the term $(\alpha''_2 - \tilde{\alpha}_2)$ is very large and therefore the dither and input signal inter-modulation at the output of the ADC shown as the second term in (40) is very large. Given a full-scale input signal, a typical dither signal that is roughly equal to 1dB of the input full scale range, and V/I converter and ICRO distortion components of similar magnitude as described in II.C, extensive simulations show that the un-cancelled dither signal present at the output will be significant and severely reduce the performance of the ADC.

However, a solution exists that greatly reduces the severity of this problem: the dual pseudo-differential converter shown in Figure 18(b) described in the previous section. Recall that the input signal, $v(t)$, is added differentially to each pseudo-differential converter channel and the dither signal is added as a differential signal to each pseudo-differential signal path but with the opposite polarity. The outputs of the two pseudo-differential channels are added with the result that the digitized input signal adds at the output but the digitized dither signal largely cancels at the output. As in the case of the single pseudo-differential converter in Figure 18(a), the dual pseudo-differential converter gives undesired inter-modulation terms at the output. However, in the dual pseudo-differential architecture all the even-order undesired terms cancel leaving only the odd-order terms

$$\begin{aligned}
y_{OUT}[n] &= y_1[n] \Big|_{corrected} - y_2[n] \Big|_{corrected} + y_3[n] \Big|_{corrected} - y_4[n] \Big|_{corrected} \\
&= 4\omega_l[n] + 8 \underbrace{[(\alpha_2'' - \tilde{\alpha}_2) - (\alpha_2'' - \tilde{\alpha}_2)]}_{=0} \omega_l[n] d[n] \\
&\quad + 12 \underbrace{[2(\tilde{\alpha}_2^2 - \tilde{\alpha}_2 \alpha_2'') + (\alpha_3'' - \tilde{\alpha}_3)]}_{\neq 0} \omega_l[n] d[n]^2 + \dots
\end{aligned} \tag{41}$$

Fortunately, the second term in (41) completely cancels and the third term in (41) is very small because the dither signal and the second-order distortion terms are both squared and the third-order term is small. Given a full-scale input signal, a 1dBFS dither signal and V/I converter and ICRO distortion components of similar magnitude as described in II.C, extensive simulations show that the inter-modulation product falls well below the noise floor and has no effect on the overall SNR.

It can be shown that a converter with 4 psuedo-differential channels can be configured so that the third-order terms also cancel at the output of the converter. However, this approach was not taken because the added complexity outweighs any potential benefit for this application.

These results make possible the use of a moderately nonlinear V/I converter. This approach will be investigated in the next section.

VI.D. Low Voltage V/I Converter

The closed-loop V/I converter used in the first prototype IC required a 2.5V supply and a feedback amplifier to create a highly linear voltage-to-current conversion as shown in Figure 19(a). This ADC front-end is perfectly suited for applications requiring

a low-bandwidth, moderately high dynamic range AFE signal path that uses a 2.5V supply. However, the first prototype IC's V/I converter would not interface well in applications using a 1.2V signal path.

Fortunately, the dual pseudo-differential architecture described in the previous section allows for the use of a moderately nonlinear V/I converter. A much simpler V/I converter can be constructed which uses a 1.2V supply and consumes much less power and produces less noise. The V/I converter chosen is a simple, open-loop, resistor-degenerated pMOS common-source amplifier shown in Figure 19(b). The V/I converter operates from a 1.2V supply, so it uses thin-oxide transistors. The V/I converter input common-mode is chosen to be roughly mid-supply. Headroom limitations at the output of the V/I converter limit the amount of resistor degeneration resulting in a fairly small amount of linearization improvement in the voltage-to-current conversion relative to a non-degenerated common-source amplifier.

Headroom allowed for a degeneration resistance that corresponded to roughly a 200mV drop across this resistor with a zero differential input signal. The degeneration resistance is typically 310 ohms which is roughly 3 times greater than the transistor transimpedance, g_M^{-1} . The full-scale input swing for $f_S=2.5\text{GHz}$ is 800mV differential peak-to-peak with a K_{VTI} approximately equal to 2.2mS. Simulations indicate that for each $\Delta\Sigma$ modulator, the input referred noise voltage for the V/I converter degeneration resistors and V/I converter pMOS devices are $5.4\text{nV}/\sqrt{\text{Hz}}$ and $2.9\text{nV}/\sqrt{\text{Hz}}$, respectively. Simulations indicate that the V/I converter 1/f noise corner occurs at roughly 400 kHz. This

noise corner is fairly high due to the small area of the V/I converter pMOS devices and the limited degeneration resistance used. If a lower $1/f$ noise corner is required, then either higher degeneration or larger pMOS devices are required – the penalty for this is greater headroom requirements for the V/I converter or greater parasitic capacitance at the current starved node of the ICRO which will increase AC distortion in the signal path (more about this in section VII.C). The low-pass ring oscillator phase noise is subjected to the high-pass transfer function of the $1-z^{-1}$ blocks, so the resulting contribution to the output sequence in the signal band is nearly white noise. Simulations indicate that the ICRO noise is very small relative to the V/I converter noise referred to the input of the converter.

The open-loop V/I converter's transfer function is close to that of a resistor degenerated square law device with a fairly strong second-order distortion term but a fairly small third-order distortion term. It is interesting to note that the second, third, and fourth order distortion terms of the open-loop V/I converter have similar amplitudes but opposite signs to that of the ICRO. These distortion terms tend to negate each other which results in lower overall distortion through the system. For example, the output of the simulated open-loop V/I converter with a near full-scale 250KHz sinusoidal input signal has second, third, and fourth harmonics at -29 dBc, -43 dBc, -60dBc, respectively. These numbers closely match the distortion term reported for the ICRO of section II.B which has second, third, and fourth harmonics at -26dBc, -47dBc, and -64dBc, respectively. The overall converter distortion including the open-loop V/I converter and the ICRO with a 250KHz sinusoid input signal had second, third, and fourth harmonics at -

35dBc, -49dBc, and -66dBc, respectively.

Recall from Section VI.D that V/I converter third order distortion can cause un-cancelled dither signal to fall into the signal band. Exhaustive simulations indicate that for a V/I converter with a third harmonic term of -43dBc and a dither signal amplitude of -26dBFS (which corresponds to a peak-to-peak dither signal roughly equal to the quantization step size) the resulting un-cancelled dither inter-modulation products are below the noise floor.

Care must be exercised to ensure that the V/I converter's nonlinearity can be well modeled as a weakly nonlinear function that, in turn, can be modeled as a Taylor series as mentioned previously in section III.A. This requirement is ultimately what sets the required degeneration resistor value. The minimum power supply voltage minus the maximum output swing of the ICRO sets the minimum headroom for the V/I converter. It is necessary to keep the degenerated pMOS device well in saturation at all times in order to keep the V/I converter transfer function equivalent to a weak nonlinearity. Unfortunately, the small headroom budget limited the amount of resistor degeneration. Due to this limited resistor degeneration, the V/I converter nonlinearity transitions from a weak to a strong nonlinearity at roughly -2.5dBFS. For input signals with amplitudes greater than -3dBFS, the weak linearity model begins to break down and the nonlinearity correction block fails to adequately correct the nonlinearity which results in a reduction in SNDR for input signals greater than -3dBFS. (This is another reason why the overload range extension mentioned in Section VI.B was not pushed much beyond the nominal

range.)

Two open-loop V/I converters are configured in a pseudo-differential fashion as shown in Figure 19(b) but unfortunately this configuration provides no input-signal common-mode rejection. In other words, the input common-mode dc bias point sets the dc bias current flowing into the ICRO which therefore sets the center frequency of the ICRO. A calibration unit is required to set the correct ADC input common-mode voltage which will align the ICRO center frequency with the ADC sample rate, f_s .

VI.E. Digital Background Calibration

As mentioned in Section III.A, two types of digital background calibration are required for each ADC. The first is digital background calibration of the V/I converter and ICRO induced second-order and third-order distortion which addresses the signal path nonlinearity problem described previously. The second is digital background tuning of the VCO's center frequency to match the ADC's sample rate, f_s , which centers the input range of the ADC about the midscale input voltage and therefore maximizes dynamic range as well as enables reconfigurability by automatically retuning the VCO's center frequency whenever f_s is changed.

Using a nonlinear V/I converter increases the implementation difficulty of the calibration unit substantially. In the first prototype IC's calibration unit, the V/I converter is assumed to be sufficiently linear and only the ICRO nonlinearity is estimated. Since the ICRO has a current input, it is a simple matter to inject the calibration sequence

as a current via a simple current steering DAC. However, a signal path employing a nonlinear V/I converter requires that the calibration signal be injected as a voltage at the V/I converter input so that both the nonlinearity of the V/I converter and the ICRO can be estimated. This requires the use of a voltage DAC which is significantly more difficult to implement than a current DAC. Figure 20 shows a detailed description of the new calibration unit including the voltage mode calibration DAC. The operation of this new calibration unit will be described in the following three sub-sections.

VI.E.1 Nonlinearity Correction

Recall that the nonlinearity correction block in the signal path is a high speed look-up table with mapping data updated periodically by the nonlinearity coefficient calculator block of the calibration unit. The look-up table maps each 5-bit input sample, $y[n]$, into an output sample, $y[n]_{\text{corrected}}$, such that

$$y[n]_{\text{corrected}} = G_1 \left[y[n] - \tilde{\alpha}_2 (y[n])^2 - (\tilde{\alpha}_3 - 2\tilde{\alpha}_2^2) (y[n] - \tilde{\alpha}_2 (y[n])^2)^3 \right] \quad (42)$$

where $\tilde{\alpha}_2$ and $\tilde{\alpha}_3$ are estimates of the α_2 and α_3 signal path distortion coefficients and G_1 is a gain scaling factor.

VI.E.2 Nonlinearity Coefficient Measurement

The Nonlinearity Correction Block needs to first measure the nonlinearity introduced by the V/I converter and ICRO in the signal converter shown at the top of Figure 20. This is accomplished by measuring the nonlinearity of a signal converter replica. A voltage-mode DAC consisting of a current-steering DAC and load resistors are used to

generate a calibration sequence at the input to the signal converter replica. Since the reference current is proportional to a precision reference divided by the on-chip poly resistor value, it follows that output of the voltage DAC is proportion to the reference voltage which results in a very precisely controlled amplitude. The amplitude of the calibration DAC is proportional to an 8-bit programmable value, $d_{DAC}[7:0]$. The calibration sequence is therefore passed through the V/I converter and the ICRO. Signal path nonlinearity produces intermodulation products that can be correlated against to estimate the nonlinear coefficients of the signal path replica.

The level of the calibration signal is important - it should be between 0.4FS and 1.0FS. It must be greater than 0.4FS because the calibration signal itself is used to dither the replica converter signal path. Exhaustive simulation and silicon measurements show that this minimum level of signal is required to get a good estimate of the 3rd order coefficient. When the calibration signal falls below this level then spurious content can corrupt the coefficient estimates. It is easy to verify the amplitude of the calibration sequence by viewing the output of the correlators. The calibration unit continuously measures the gain of the signal path replica by correlating its output against one of the three 2-level sequences, $t_1[n]$, to obtain the $fs/2^{28}$ -rate sequence given by

$$\gamma_1[m] = \frac{1}{P} \sum_{i=0}^{P-1} r[mP+i] t_1[mP+i] \quad (43)$$

where $P = 2^{28}$. Since the calibration output has 4 levels and there are 32 possible output codes, a full-scale calibration sequence should have a step size of $\frac{31}{3}$ output codes. It

follows that a single calibration sequence at the output of the replica signal path has a full-scale peak amplitude of $\frac{1}{2} \cdot \frac{31}{3}$ and when this output is multiplied by its corresponding ± 1 calibration sequence and summed over $P=2^{28}$ clock cycles, then the following sum is expected at the first correlator output

$$\gamma_{1-FULL-SCALE} = \frac{1}{2} \cdot \frac{31}{3} 2^{28} = 1.39 \times 10^9. \quad (44)$$

The calibration state machine can servo the amplitude of the calibration sequence until the γ_1 value is the desired percentage of full-scale. This can be accomplished in one step after the first value of γ_1 is measured by changing the amplitude of the calibration DAC with the following equation

$$d_{DAC-DESIRED}[7:0] = d_{DAC-INITIAL}[7:0] \cdot \frac{1.39 \times 10^9}{\gamma_{1-INITIAL}} \cdot K_{CAL} \quad (45)$$

where $d_{DAC-INITIAL}[7:0]$ is the initial value of the calibration DAC, $\gamma_{1-INITIAL}$ is the initial measured value of γ_1 , and K_{CAL} is a constant representing the target percentage of full scale for the calibration sequence, a value between 0.4 and 1.0.

The value of γ_1 also provides the user with a precise measurement of the gain of the signal converter. There are two ways to reference the output full-scale of the converter. First, the full-scale output of the converter can be referenced to the output code of the digital differentiator by setting $G_1=K_1$, where K_1 is a constant that allows use of the full 16-bit output range of the converter. A typical value of K_1 is 1024 which roughly cor-

responds to +/-16384 for a full-scale input signal. Second, the full-scale output of the converter can be referenced to a known voltage by setting $G_1=K_2 d_{DAC}[7:0] / \gamma_1[m]$ where K_2 is a constant that allow for full use of the 16-bit output range of the converter and $d_{DAC}[7:0]$ is the 8-bit digital amplitude value of the calibration DAC. A typical value for K_2 is 3.2×10^9 which gives an output code of +/-16384 for an input of +/-200mV for all process corners and temperatures.

The calibration unit continuously measures α_2 and α_3 as described in Section III.A.3 and the estimates $\tilde{\alpha}_2$ and $\tilde{\alpha}_3$ are given in (23). As before, it does this and loads the 32 values into the nonlinearity correction block's look-up table at a maximum rate of once every $2^{28}T_S$ seconds.

V.I.E.3 VCO Center Frequency Calibration

To maximize dynamic range of the signal converter, the center frequency of the ICRO is tuned to the sampling frequency, f_s . The calibration sequence injected into the replica signal converter is zero-mean. Therefore it is a simple matter to calculate the offset of the signal converter in terms of LSBs from the ICRO

$$\gamma_0[m] = \frac{1}{P} \sum_{i=0}^{P-1} r[mP+i]. \quad (46)$$

A feedback loop is implemented to drive γ_0 to be zero as described in section III.A. When γ_0 is zero the replica ICRO center frequency is very close to the sample frequency, f_s .

This concept is very simple and the implementation is quite simple for the calibration unit of the first prototype IC. However, the implementation is much more difficult when the calibration sequence drives the open-loop V/I converter. A problem arises in that the V/I converter has no inherent common-mode rejection. In fact the V/I converter input common-mode voltage sets the V/I converter DC bias current.

The calibration source shown in Figure 21(c) consists of two parts. First, the calibration DAC, I_{DAC} , sets the calibration signal voltage swing at the replica V/I converter input. Second, the calibration DAC, I_{DAC} , plus the common-mode current sources, I_{CM} , set the DC bias point for the replica V/I converter. The calibration DAC load network, R_{CAL} plus M_{CAL} , was intentionally designed to mimic stack-up of the V/I converter. The sizes of M_{DAC} and R_{DAC} are chosen so that they mimic the V/I converter transistor, M_{VTI} , and degeneration resistor, R_{VTI} , with the result that the two circuits act like a crude current mirror. In this way, the nominal current in the V/I converter is roughly equal to

$$I_{VTI} \approx \frac{3}{2} I_{DAC} + I_{CM} = K_{CAL} d_{CENTER}[7:0] \quad (47)$$

where I_{DAC} is the tail current in each of the 3 DAC elements, I_{CM} is the current in the common-mode current sources, K_{CAL} is a constant and $d_{CENTER}[7:0]$ is the 8-bit control value from the VCO center frequency calculator block. This circuit has the desirable property that the initial guess at the center current is fairly accurate since the ICRO frequency versus input current can be estimate quite accurately and does not vary significantly over process, temperature, and supply voltage.

The single-ended peak-to-peak output swing of the calibration DAC is $3I_{DAC}(R_{DAC} \parallel R_{CM})$ where $R_{CM} \gg R_{DAC}$ and I_{DAC} is equal to $K_{CAL} d_{DAC}[7:0]$.

Additionally, some simple logic was added at the bottom of Figure 21 to ensure that the value of $d_{DAC}[7:0]$ is never larger than the centering control word, $d_{CENTER}[7:0]$.

The ADC signal path must be configured to have the same bias point as the replica V/I converter input. This is accomplished with the two common-mode sensing resistors, R_{CM} , that create a voltage, V_{CM} , which is equal to the calibration signal common-mode voltage – in other words, its nominal voltage. This voltage, V_{CM} , is used to set the input common-mode of the signal converter. It does so by controlling the common-mode output of the circuit driving the signal converter. For example, in Figure 21(a), the ADC is driven by an off chip transformer where V_{CM} sets the center tap voltage of the transformer. Alternately, in Figure 21(b), the ADC is driven by an active circuit with the output common-mode set by the V_{CM} voltage. Care must be taken to minimize the driving circuit common-mode error – any deviation from the value supplied by the calibration unit, V_{CM} , will result in lost dynamic range in the signal converter.

VII. CIRCUIT LEVEL ENHANCEMENTS

VII.A. Achieving Theoretical Maximum SQNR

The SQNR measured in the first prototype IC in section V was roughly 2-3dB below the theoretical maximum achievable. Three primary factors were involved in this reduction of achieved SQNR.

Firstly, random device mismatch in each delay cell transistor and also in the ring sampling flip-flops leads to a mismatch in the unit delay size. This is equivalent to quantizer INL error which leads to distortion and therefore an increase in quantization noise. Not much can be done here except to ensure fast rise and fall times for phase output signal.

Secondly, offsets in the closed-loop V/I converters led to small mismatches in the ICRO center frequency current generated by the V/I converter. This led to a slight mismatch in center frequency value for each of the four rings oscillators. This is equivalent to an input referred voltage offset at the input of each of the four ICROs which slightly reduces the maximum swing available to all the ICROs. In the first prototype IC, the two V/I converters in each ADC had all bias voltages common except for V_{BIAS3} and V_{BIAS4} as shown in Figure 19(a). Separate bias networks were used to generate the V_{BIAS3} and V_{BIAS4} voltages and device mismatch caused random error in the center frequency current value that was as large as $\pm 1\text{LSB}$, which resulted in a maximum reduction of

usable headroom of around 0.5dB. This error can be reduced significantly by making common all of the DC bias voltages between the two V/I converters.

Lastly, and most importantly, non-uniform layout of each delay element led to unequal unit delay size among all the ICRO delay elements. This led to distortion and reduced quantization noise performance of approximately 1dB. Care was taken in the new layout to keep the layout of each delay cell very uniform and also to keep the cell-to-cell routing very uniform in order that the unit delay is kept very uniform.

VII.B. Signal Path AC Nonlinearity

The frequency dependent nonlinearity in the first prototype IC limited its performance for larger input frequencies and bandwidths. This frequency dependent distortion can be seen in Figure 13(b). Excessive parasitic capacitance at the current starved node of the ring oscillator was the primary cause of this roll-off in performance at higher input frequencies. Capacitance at the current starved node of the ring oscillator results in frequency dependent phase shift between input signal and the distortion components which lie at different frequencies. The nonlinearity correction block only corrects for DC nonlinearity. Nonlinear phase shift at the current starved node of the ring oscillator keeps the DC nonlinearity correction from being effective. As the frequency increases the phase shift increases and the DC correction become less and less effective resulting in less and less effective distortion correction. Two approaches were taken to reduce the parasitic capacitance.

Firstly, parasitic metallization capacitance on each of the ICRO current starved nodes must be minimized. Each of the four current starved nodes must be routed to the dither DAC. Care must be taken to minimize the parasitic routing capacitance on this node. The first prototype IC used poor layout resulting in excessive routing capacitance on all four current starved nodes. This routing capacitance was primarily responsible for the AC distortion.

Secondly, diffusion capacitance at the ICRO current starved node must be minimized. The current starved node connects to the source node of each and every delay cell pMOS device. In the first prototype IC, the delay cell pMOS devices, labeled MP1 and MP2 in Figure 16(b), were each constructed of two parallel devices which shared a diffusion node at their drains. Doing so minimized capacitance on the output node of the delay cell resulting in a slightly faster delay element. Unfortunately doing so caused the pMOS transistor to have twice as much diffusion area on its source which is connected to the current starved node of the ring oscillator. For the second prototype IC, the pMOS diffusion sharing was swapped. In this new configuration, the delay cell output node has the non-shared diffusion node and thus more diffusion capacitance resulting in a slightly slower delay cell but a slightly faster ICRO current starved node due to the reduction in diffusion capacitance on the pMOS source node and therefore less capacitance on the ICRO current starved node.

These two improvements significantly increased AC linearity as will be shown in the following measurement results section.

VII.C. Ring Sampler Hysteresis Problem

The first prototype IC exhibits some high, odd-order distortion terms that can be seen in the output spectrum of Figure 12. For a full-scale, low frequency input signal, with the nonlinearity correction enabled, distortion terms greater than 3rd order are expected to be insignificantly small. However, distortion terms at frequencies equal to 5, 7, 9, and 11 times the fundamental frequency are present in the output spectrum at levels slightly below the 80dBFS. The value of distortion is fairly small but it is desirable understand the cause of this distortion and find a simple fix that removes these distortion components.

The first prototype IC's ring sampler uses a transmission-gate based D-type flip-flop to sample the outputs of the ICRO. This flip-flop is shown in Figure 22(a). The previous ICRO phase sample is stored on the internal node of the flip-flop at the node labeled c. When the flip-flop transitions from hold to sample mode, the transmission gate across nodes b and c becomes low impedance and the inverter driving node b must charge or discharge the capacitance on both nodes b and c. This can result in a data-dependent sampling error which leads to distortion and to the high-order harmonic distortion components seen in Figure 12. This problem is easily solved by using a non-transmission-gate based flip-flop in the ring sampler as shown in Figure 22(b). With this new flip-flop, the sampling decision is not effected by the previous sample. The resulting improved distortion performance will be demonstrated in the following measurement results

section.

VII.D. Process Scaling

The 65nm G+ process is superior to the 65nm LP process for the VCO-based ADC because the transistors are faster and lower power. The 65nm G+ process transistors have lower threshold voltages and smaller effective gate lengths than their counterparts in the LP process. This combination allows digital logic to run much faster with the same supply voltage in the G+ process compared to the LP process. This extra speed allows the VCO ADC to operate at a faster sample rate, f_s , which can be used to increase signal bandwidth or increase the OSR and therefore reduce quantization noise. The G+ logic gates can dissipate less power than LP logic gates by running the G+ logic off a lower power supply voltage. For example, simulations indicate that logic cells in the G+ process with a power supply of 0.9V are faster than identical logic cells in the LP process with a 1.2V power supply. Digital logic power dissipation is proportional to $C_P V_{DD}^2$ where C_P is the parasitic capacitance driven by the logic circuitry and V_{DD} is the supply voltage. Since power dissipation is roughly proportional to the square of the power supply voltage, digital circuit power dissipation drops quickly with a reduction in power supply voltage. Simulations indicate that the power dissipation of identical digital logic blocks is 40% less in the G+ process compared to the LP process when the G+ logic operates with a 0.9V supply and the LP logic operates with a 1.2V supply. Because the VCO ADC performance is extremely reliant on digital gate speed and power, the G+ process gives this architecture a large boost in performance. The advantages of the 65nm

G+ process versus the 65nm LP process will be apparent in the following two sections that show measurement results for two prototype ICs manufactured in the 65nm G+ process.

VIII. SECOND PROTOTYPE IC

VIII.A. Measurement Results

The second prototype IC was fabricated in the TSMC 65nm G+ process with the deep nWell option and both single-oxide and dual-oxide devices, but without the MiM capacitor option. All pads have ESD protection circuitry. The IC was packaged in a 64-pin LFCSP package.

The second prototype IC contains two $\Delta\Sigma$ modulators that incorporate all of the architectural and circuit enhancements described in Sections VI and VII. The combined area of the two $\Delta\Sigma$ modulators, the calibration unit, and the ADC bias circuitry totaled 0.15mm^2 . A die photograph of one of the $\Delta\Sigma$ modulators is shown in Figure 23. The calibration unit area is 0.07mm^2 . The signal converter, i.e., the portion of each $\Delta\Sigma$ modulator not including the calibration unit, has an area of 0.04mm^2 . A single calibration unit is shared by the two $\Delta\Sigma$ modulators, so the area per $\Delta\Sigma$ modulator is 0.075mm^2 . All components of both $\Delta\Sigma$ modulators and the calibration unit are implemented on-chip including the $f_s/2^{28}$ -rate coefficient calculation block. The calibration unit area on the second prototype is 0.01mm^2 larger than the first prototype IC due to the addition of the $f_s/2^{28}$ -rate coefficient calculation block on the silicon. Additionally, a digital decimator was included on chip to reduce the output data rate by a factor of 8. As is typical for $\Delta\Sigma$ modulator ADC figure-of-merit calculations, decimator die size was not included in the

$\Delta\Sigma$ modulator area calculation and the decimator power consumption is not included in the $\Delta\Sigma$ modulator power consumption values.

A printed circuit test board similar to that described in Section V was used to evaluate the socket mounted IC. The test board includes input signal conditioning circuitry, clock conditioning circuitry, and an FPGA for ADC data capture and serial port communication. The input conditioning circuitry uses a transformer to convert the single-ended output of a laboratory signal generator into a differential input signal for the IC. The center tap of the transformer secondary coil is driven to a voltage equal to the calibration unit's calibration signal common-mode voltage, V_{CM} , as described in Section VI.E and as shown in Figure 21(a). The clock conditioning circuitry also uses a transformer. It converts the single-ended output of a laboratory signal generator to a differential clock signal for the IC. A single power supply provides the supply voltage for all the blocks on the second prototype IC and this voltage can be varied between 0.9V to 1.2V for optimum performance depending on the sample rate, f_s . The IC has 3 power domains that connect to the single supply voltage - one set of power and ground pins for V/I converter, another set of power and ground pins for the ring sampler and sample clock buffers, and a final set of power and ground pins for all other ADC digital logic including all the calibration circuitry.

Measurements were performed with a clock frequency, f_s , ranging from 1.3GHz to 2.4GHz. Single-tone and two-tone input signals were generated by high-quality laboratory signal generators and were passed through passive narrow-band band-pass filters

to suppress noise and distortion from the signal generators. Each output spectrum presented below was obtained by averaging 4 length-16384 periodograms from non-overlapping segments of $\Delta\Sigma$ modulator output data, and the SNR and SNDR values were calculated from the resulting spectra via the technique presented in [19]. Both $\Delta\Sigma$ modulators on four copies of the IC were tested.

Figure 24 shows representative measured output spectra of the $\Delta\Sigma$ modulator for a -3dBFS, 3.5MHz single-tone input signal with $f_s=2.4\text{GHz}$, both with and without digital background calibration enabled. Without calibration, the SNDR over the 18.75MHz signal band is only 50 dB because of harmonic distortion. For the second prototype ADC the noise floor without calibration is not elevated because there is little common-mode noise from the open-loop V/I converter which is unlike the closed-loop V/I converter which produces a lot of common-mode thermal noise. A transformer on the test board drives the ADC input pins and this transformer produces no common-mode circuit noise. If an active circuit was used to drive the ADC then common-mode thermal noise from this circuit would be converted to differential noise via the second-order distortion introduced by the VCOs as described in Section III.B. However, the second prototype overall second order nonlinearity is roughly 6dB better than the first prototype as mentioned in Section VI-D and the common-mode to differential conversion is proportionally reduced. With calibration enabled the overall SNDR improves to 74dB. Notice that the second order distortion term is very good before calibration but slightly worse after calibration. The overall signal converter second-order distortion is much less than that of the first pro-

tototype ADC so the second-order distortion self-cancels quite well without the second-order nonlinearity correction enabled. Simulations indicate that channel-to-channel V/I converter mismatch causes un-cancelled second order distortion terms, however, at the time of writing this dissertation, it is not clear why the second order distortion is worse with the second order nonlinearity correction enabled. The $1/f$ noise corner occurs at a frequency of approximately 800 KHz which is roughly twice as large as the simulation results of Section VI.D.

The measured harmonic distortion performance of the $\Delta\Sigma$ modulator with $f_s=2.4\text{GHz}$ is shown in Figure 25. The top plot shows the measured spectrum of the $\Delta\Sigma$ modulator output for a single-tone in-band -3dBFS input signal, and shows the corresponding signal to third-order and fifth-order distortion ratios, denoted as HD3 and HD5, respectively. An input signal level of -3dBFS is used because it corresponds to the input level that produces the maximum SNDR. The bottom plot in Figure 25 shows the measured HD3 and HD5 values as a function of the frequencies at which they occur within the signal band. Each value was measured by injecting a -3dBFS, in-band, single-tone input signal into the $\Delta\Sigma$ modulator and measuring the HD3 and HD5 values corresponding to the harmonic distortion components that reside in the signal bandwidth. For example, the HD5 value measured from the top plot corresponds to the circled data point in the bottom plot of Figure 25. The input signal tone is then swept in frequency and the resulting harmonic distortion components are then plotted in the bottom plot. The HD3 values before and after digital calibration are shown. The HD5 values were not measura-

bly affected by digital calibration, so only the HD5 values after calibration are shown.

The low-frequency HD3 of better than 81dB indicates that the calibration unit effectively measures the third-order distortion of the signal path. The reduction in HD3 with frequency is very small indicating that the AC distortion reduction techniques described in Section VII.B deliver the desired effect. Within a signal bandwidth of 18.75MHz, the HD3 and HD5 terms are greater than 81dBc. Above this bandwidth, the HD3 term starts to roll off at a 20dB per decade slope, although the HD3 is greater than 77.5dBc within the maximum signal bandwidth of 37.5MHz.

Figure 26 shows plots of the SNR and SNDR versus input amplitude for the $\Delta\Sigma$ modulator measured over a 37.5MHz signal bandwidth and a 18.75MHz signal bandwidth with $f_s=2.4$ GHz. These signal bandwidths correspond to oversampling ratios of 32 and 64, respectively. The peak SNDR occurs at roughly -3dBFS due to V/I converter nonlinearity as described in section VI.D. Maximum SNR in this paper is defined as the value of SNR at the input signal amplitude corresponding to the peak value of SNDR. The dynamic range, DR, is defined as the range where the SNR is greater than zero. For an oversampling ratio of 32 and an input signal at 7.49MHz, which is a worst case value since the first 5 harmonic distortion components fall in-band, the SNR, SNDR, and DR are 70dB, 69dB, and 73dB, respectively. For an oversampling ratio of 64 and an input signal at 3.5MHz, which is also a worst case value for the given bandwidth, the SNR, SNDR, and DR are 77dB, 74dB and 79dB, respectively.

VIII.B. Conclusions

Measured results from the second prototype IC are summarized relative to comparable state-of-the-art $\Delta\Sigma$ modulators and the first prototype IC in Table 2. The second prototype measured SNR, SNDR, DR, THD, SFDR, and power dissipation shown in Table 2 represent typical measured values of the eight ADC channels evaluated with second- and third order distortion calibration enabled. Due to V/I converter transistor mismatch the second order distortion component is only greater than 80dBc with a -3dBFS input signal for four of the eight evaluated parts and as low as 73dBc for one of the parts. The measured SNDR, THD, and SFDR values of four of the eight evaluated parts are equal to or better than the values shown in Table 2. However, if the second order correction is disabled and the third order correction is enabled, the second order distortion term is improved (as described in Section VIII.A and seen in Figure 24) and is greater than 80dBc for seven of the eight ADC channels evaluated. The measured SNR, SNDR, DR, THD, and SFDR in Table 2 represent the worst case values for seven of the eight ADC channels evaluated when only the third-order distortion calibration is enabled. The eighth part shows only a slight degradation in performance of around 1dB worst case depending on the OSR. A number of simple remedies to this second order distortion problem can be implemented to improve ADC performance. First, simulations indicate that increasing the V/I converter's pMOS device gate length can reduce the size of the mismatch induced second order distortion component. Second, the second prototype IC dissipates only 1/5th of its total power in the V/I converter and ICRO and simulations indicate that doubling the size and current of the V/I converter and ICRO delay elements would reduce the noise

floor by 3dB and also significantly reduce the second order distortion component due to reduced mismatch but only increase the overall ADC power dissipation by 20%.

As indicated in the table, the performance of the second prototype $\Delta\Sigma$ modulator is comparable to or better than state-of-the-art in power figure of merit, but uses significantly less circuit area [2,3,4,5,20,21]. Also, the second prototype IC's performance shows a substantial improvement versus the first prototype IC's performance due to the architectural and circuit-level enhancements mentioned in Chapters VI and VII.

The benefits of the 65nm process are evident in the results of Table 2. First, the maximum clock rates are much higher in the second prototype IC relative to the first prototype IC. Second, the power dissipation for similar clock rates is much lower in the G+ version of the chip. The signal processing paths are virtually identical in the two version of the chip yet the digital power of the G+ version is much less than the LP version at the same clock rate. As predicted by circuit simulation described in Section VII.D, the power dissipation of digital logic circuitry in the 65nm G+ process running at 0.9V is 40% less than the power dissipation of the same digital logic circuitry in the 65nm LP process running at 1.2V as shown in the following equation

$$\frac{P_{G+}/f_{S-G+}}{P_{LP}/f_{S-LP}} = \frac{8mW/1.3GHz}{12mW/1.152GHz} = 0.59. \quad (48)$$

The VCO-based $\Delta\Sigma$ modulator's performance depends mainly on the digital circuit speed of the CMOS process. The measurement results show that quantization noise and digital power still limits the implemented $\Delta\Sigma$ modulator's performance. The V/I

converter and ring oscillator account for less than $1/5^{\text{th}}$ of the total power dissipation. Therefore the $\Delta\Sigma$ modulator described in this paper is likely to yield even better results when implemented in more highly scaled CMOS processes.

IX. THIRD PROTOTYPE IC

IX.A. Measurement Results

A third prototype IC was fabricated in the TSMC 65nm G+ and it contains a $\Delta\Sigma$ modulator that is very similar to the original prototype IC. It uses the same closed-loop style V/I converter and the single 15-element ring oscillator as the original prototype IC. The purpose of the third prototype IC is to compare architectural tradeoffs in the 65nm G+ process and also to compare the LP versus G+ process and verify the improvements that process scaling offers to this VCO-based ADC architecture in terms of power and performance figure-of-merit. The 65nm G+ process is superior to the 65nm LP process for the VCO-based ADC because the transistors are faster and lower power as mentioned in section VII.D.

The third prototype IC was evaluated on the same test board as the first prototype IC and used all the same input conditioning circuitry. Two power supplies were used to power the IC. The V/I converters operate from a 2.5V supply, while all the other blocks operate from a 1.0V supply. Four $\Delta\Sigma$ modulators on four copies of the IC were tested with no noticeable performance differences.

Measurements were performed with a clock frequency, f_s , ranging from 1.2GHz to 1.8GHz. This range of sample rates provides a range of 1.5:1 which allows the ADC to decimate down to any baseband clock rate by decimating by a factor of $2^N 3^M$, where N

and M are programmable integer values where $N=4,5,\dots$ and $M=0,1,\dots$

Figure 27 shows a representative measured output spectra of the $\Delta\Sigma$ modulator for a 0dBFS, 3.5 MHz single-tone input signal with $f_s=1.6\text{GHz}$, both with and without digital background calibration enabled. Without calibration, the SNDR over the 12.5 MHz signal band, which corresponds to an OSR of 64, is only 44 dB because of harmonic distortion and a high noise floor. The high noise floor is the result of common-mode to differential-mode conversion of the common-mode thermal noise via the strong second-order distortion introduced by the ICROs as described in Section III.B. With calibration enabled, the SNDR improves to 74 dB. In particular, the second-order term cancels very well. The measured SNR and SNDR numbers exceed those of the first prototype IC because of the circuit level enhancements described in Section VII.

Also note in Figure 27 that the higher-order distortion terms, those terms greater than 5th order, are below the noise floor. This contrasts with the visible higher-order terms of the first prototype IC shown in Figure 12. The non-transmission-gate flip-flop of the second and third prototype IC indeed works well in suppressing the data-dependent hysteresis effect on signal path distortion.

The measured inter-modulation performance of the $\Delta\Sigma$ modulator with $f_s=1.6\text{GHz}$ is shown in Figure 28. The top plot shows the measured spectrum of the $\Delta\Sigma$ modulator output for a two-tone out-of-band input signal and shows the corresponding signal to third-order inter-modulation distortion ratio, denoted as IM3. As in the first IC prototype,

measurements indicate that the IM3 value depends mainly on the difference in frequency between the two input tones, but not on where in the 800 MHz Nyquist band the two input tones are placed.

The bottom plot in Figure 28 shows the measured IM3 value as a function of the frequencies at which it occurs within the signal band. Each value was measured by injecting a full-scale, out-of-band, two-tone input signal into the $\Delta\Sigma$ modulator and measuring the IM3 value corresponding to inter-modulation terms within the signal band. For example, the IM3 value measured from the top plot corresponds to the circled point in the bottom plot of Figure 28. The IM3 values before and after digital calibration are shown.

The low-frequency IM3 of better than 80dB suggests that the calibration unit effectively measures the third-order distortion for low-frequency inter-modulation products. The first prototype IC has a reduction in IM3 values for inter-modulation products greater than about 2MHz with a roll-off in performance of 20dB per decade after this point. However, the third prototype IC shows much better AC performance due to the circuit enhancements mentioned in Section VII.B with the IM3 values greater than 80dB up to 15 MHz and greater than 72 dB at the maximum signal bandwidth of 37.5MHz.

Figure 29 shows plots of the SNR and SNDR versus input amplitude for the third prototype IC $\Delta\Sigma$ modulator measured over a 25MHz signal bandwidth and a 12.5MHz signal bandwidth with $f_s=1.6$ GHz. These signal bandwidths correspond to oversampling ratios of 32 and 64, respectively. The SNR and SNDR for a peak input signal with an

oversampling ratio of 32 and an input signal frequency of 2.3MHz are 71.5 dB and 70.5 dB, respectively. The SNR and SNDR for a peak input signal with an oversampling ratio of 64 are 77 dB and 74 dB, respectively.

IX.B. Conclusions

Measured results from the third prototype are summarized relative to the first prototype in Table 3. As indicated in the table the performance of the third prototype IC is comparable or better than state-of-the art in terms of power figure-of-merit, but uses significantly less circuit area. Also, the power figure-of-merits for the third prototype IC are much better than the first prototype IC which demonstrates the improvements in the $\Delta\Sigma$ modulators performance with process scaling and also the circuit-level enhancements described in Section VII.

X. CONCLUSIONS

This dissertation presents several high-performance stand-alone VCO-based $\Delta\Sigma$ ADCs enabled by digital background correction of VCO and V/I converter nonlinearity and by self-cancelling dither. Unlike conventional ADCs they do not require the use of high performance analog building block such as analog integrators, feedback DACs, reference voltages, comparators, or low-jitter clocks. Its performance is limited mainly by the speed of digital circuitry, so unlike conventional ADCs its performance improves as CMOS technology scales.

FIGURES

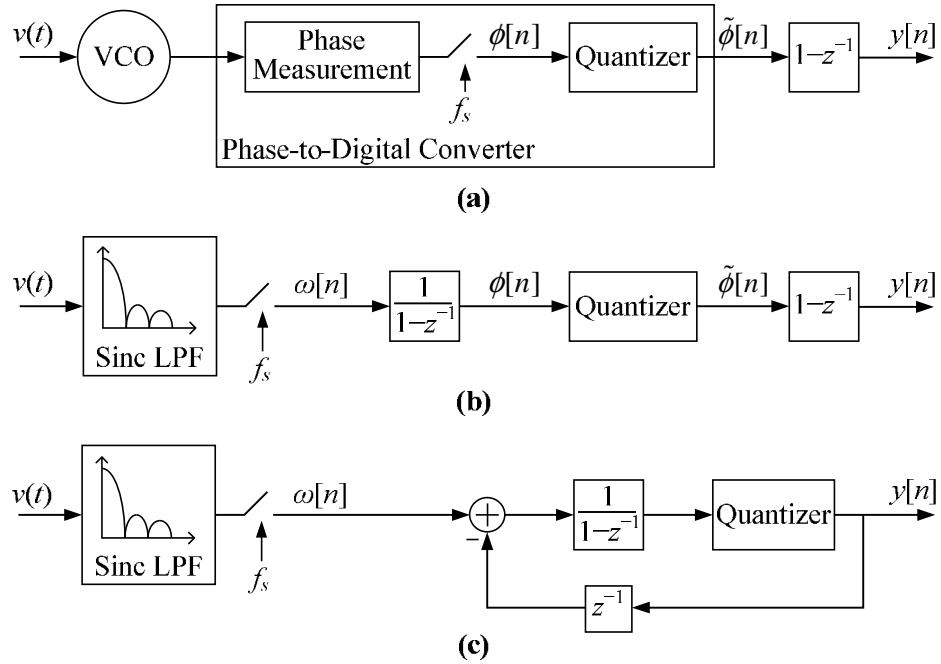


Figure 1: Equivalent systems: (a) a generic VCO-based $\Delta\Sigma$ modulator, (b) the cascade of a continuous-time lowpass filter, sampler, quantizer, and digital differentiator, and (c) the cascade of a continuous-time lowpass filter, sampler and first-order $\Delta\Sigma$ modulator.

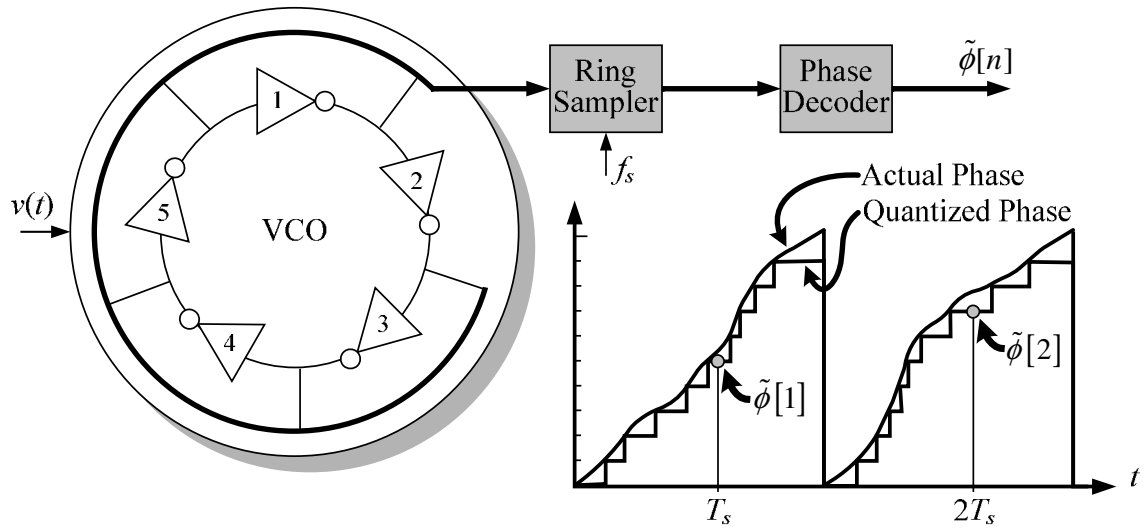


Figure 2: Example of a ring VCO and phase-to-digital converter

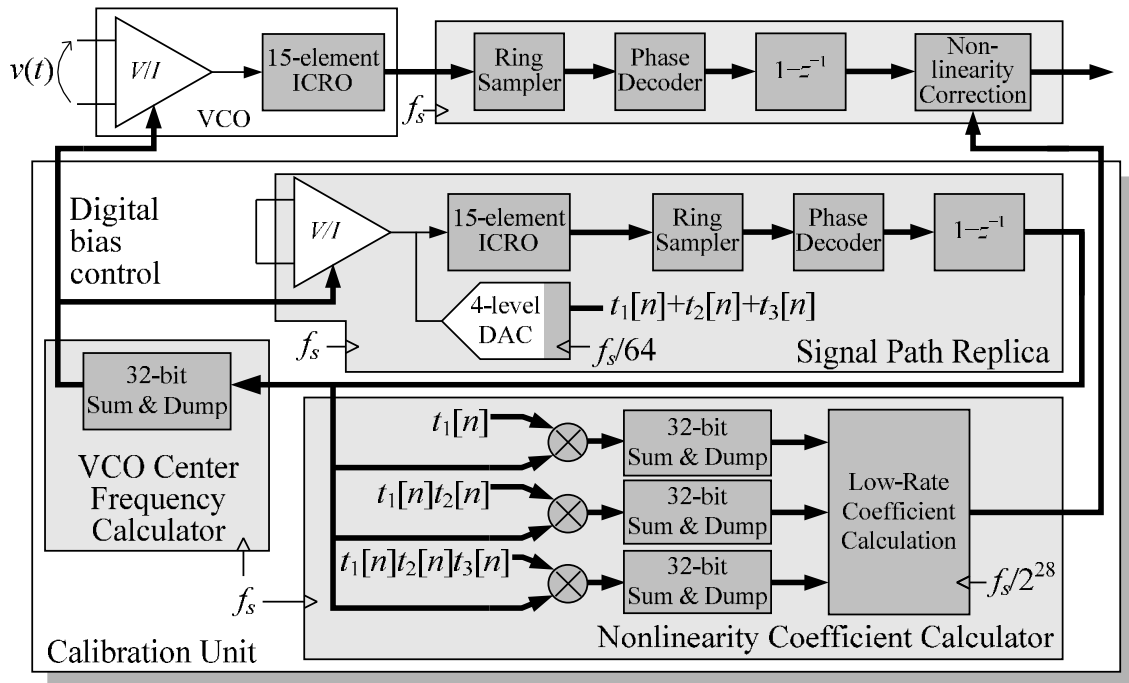


Figure 3: The prototype IC's on-chip calibration unit shown with a single VCO-based $\Delta\Sigma$ modulator signal path for simplicity.

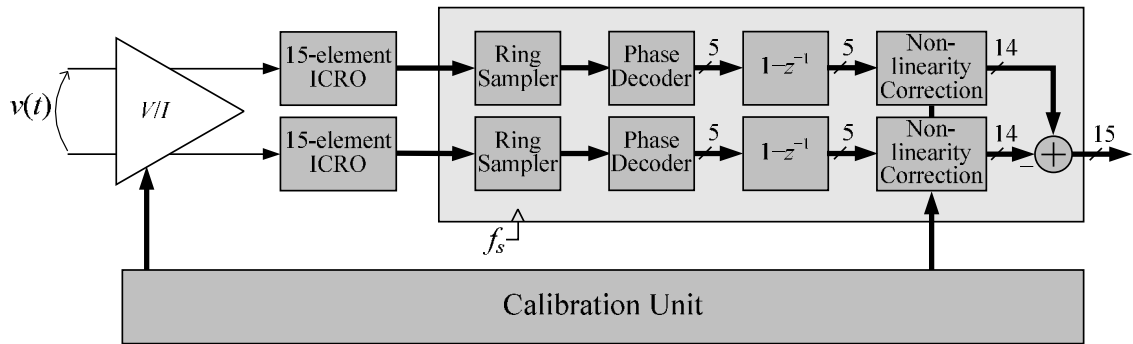


Figure 4: A pseudo-differential signal path and the calibration unit.

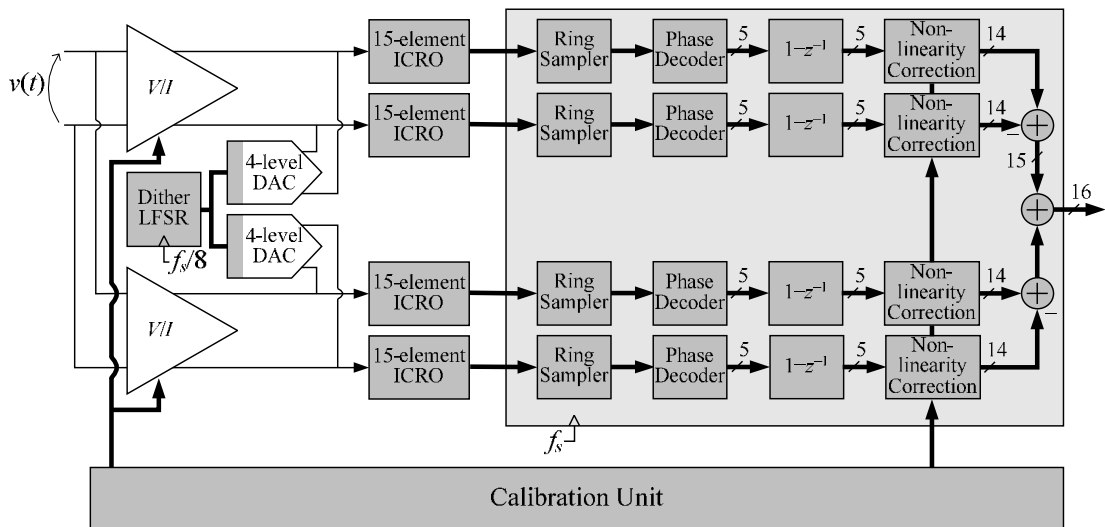


Figure 5: High-level block diagram of the implemented VCO-based $\Delta\Sigma$ modulator.

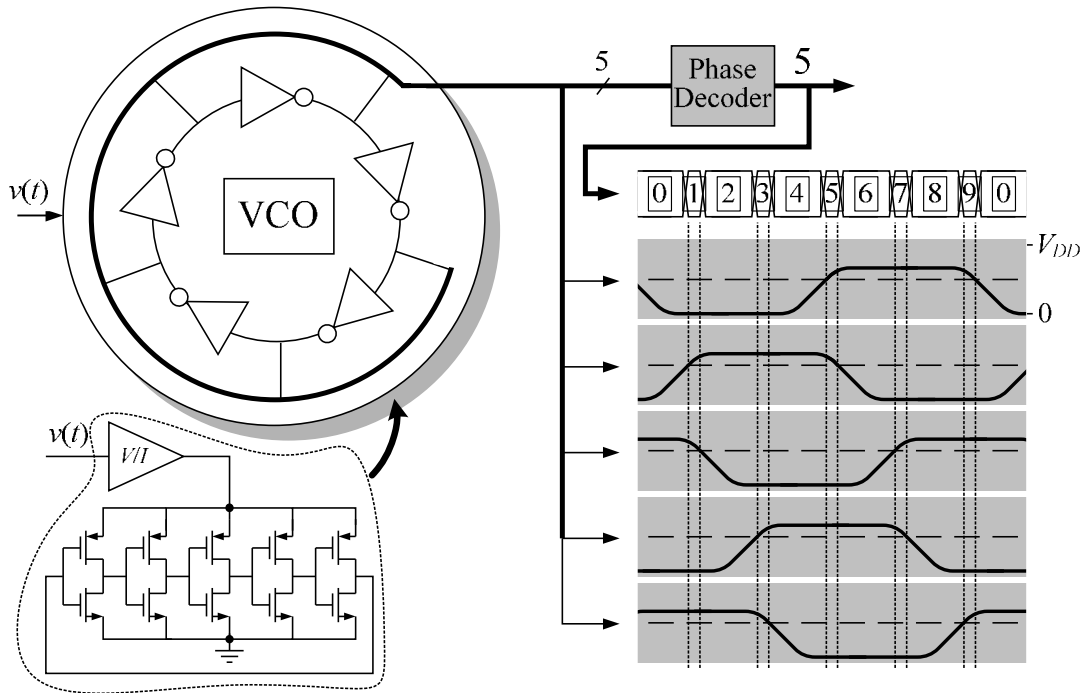


Figure 6: Example of the signal-dependent non-uniform quantization problem.

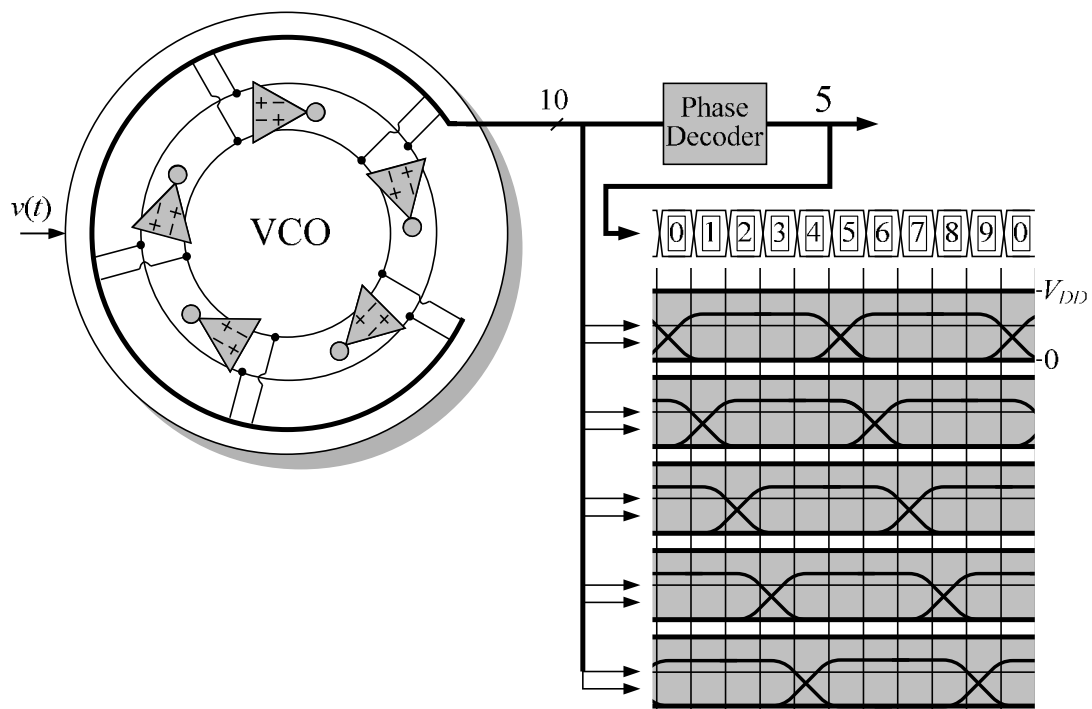


Figure 7: Example of the solution used to solve the signal-dependent non-uniform quantization problem.

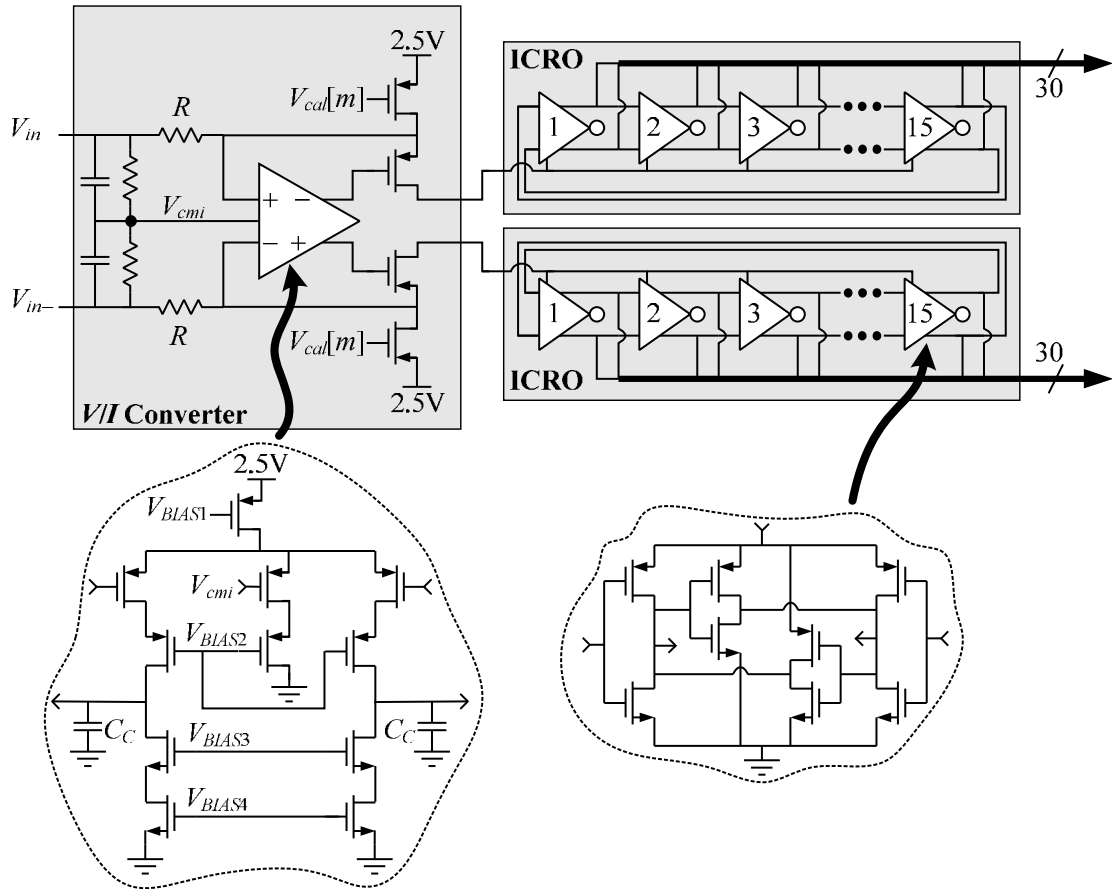


Figure 8: Circuit diagrams of the V/I converter and ICRO.

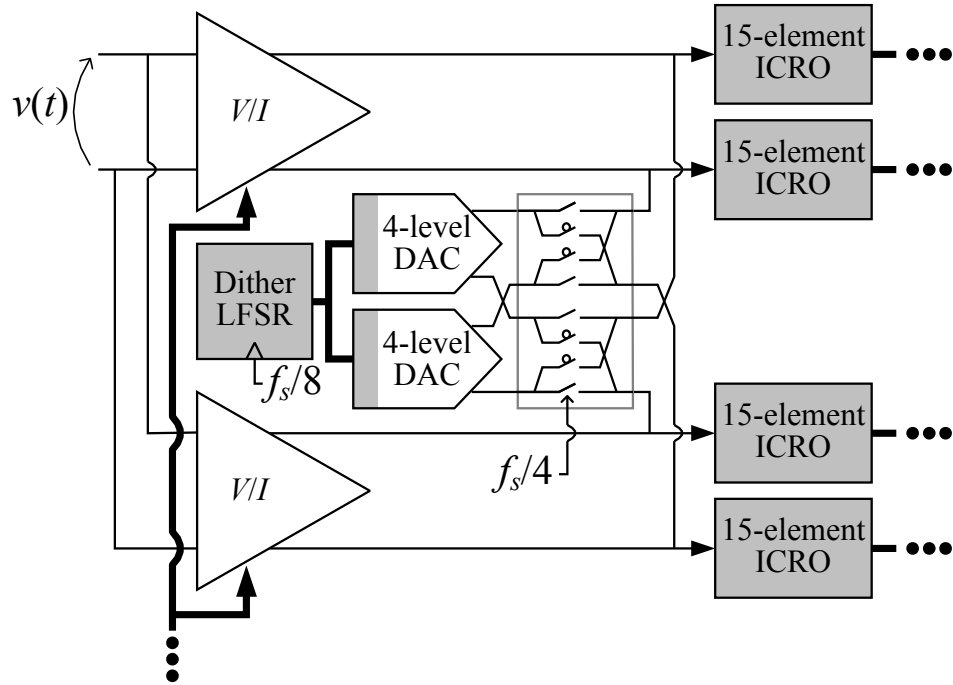


Figure 9: The dither DAC swapping technique which causes the PSD of the error component in the $\Delta\Sigma$ modulator output arising from mismatches between the dither DACs to have a first-order high-pass shape.

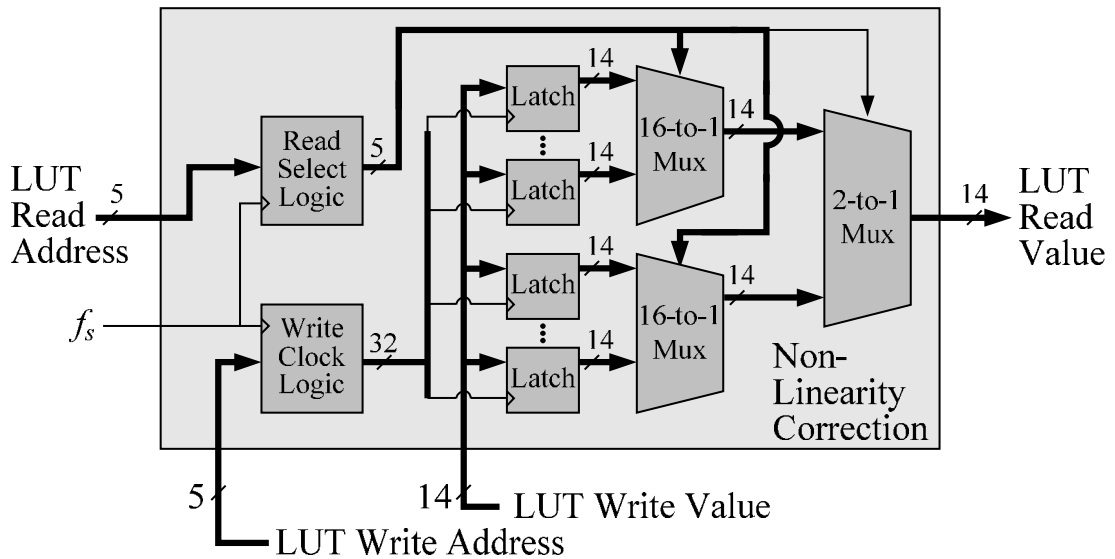


Figure 10: Nonlinearity correction block details.

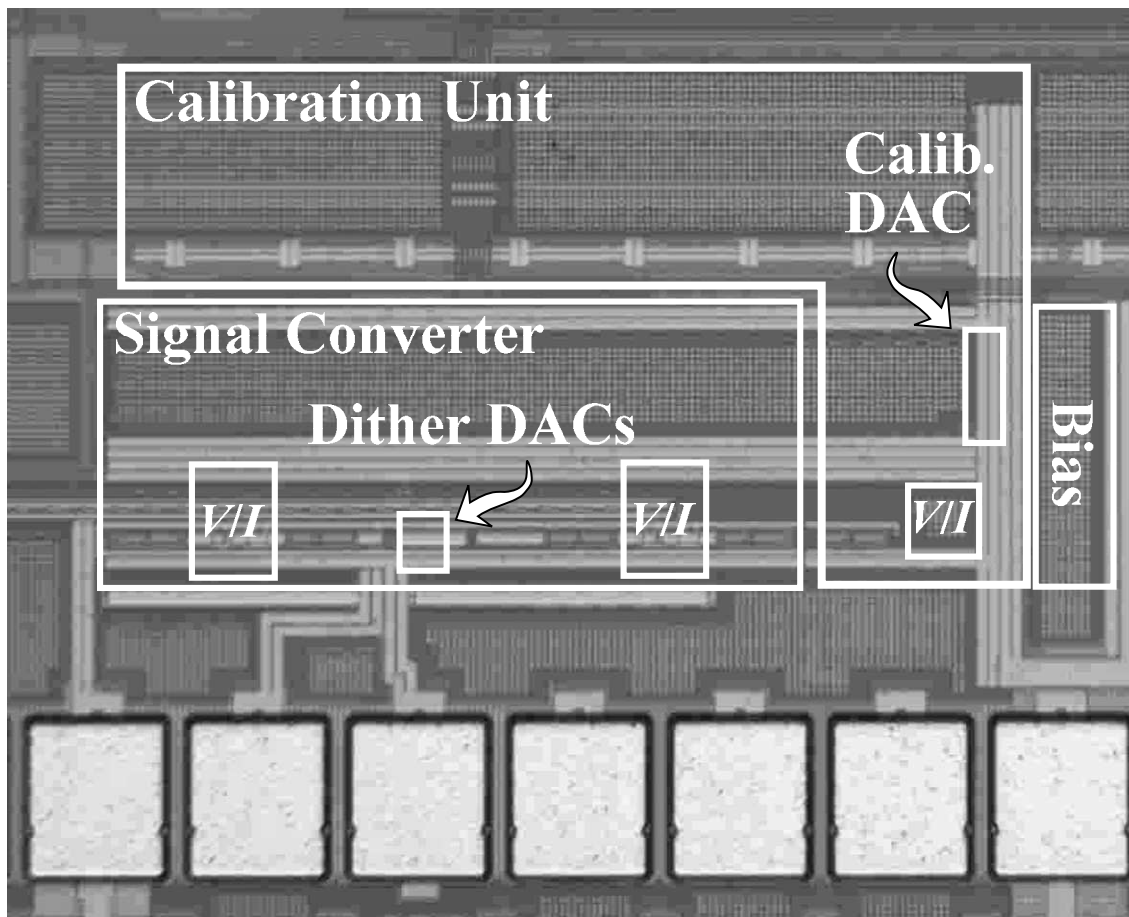


Figure 11: Die photograph.

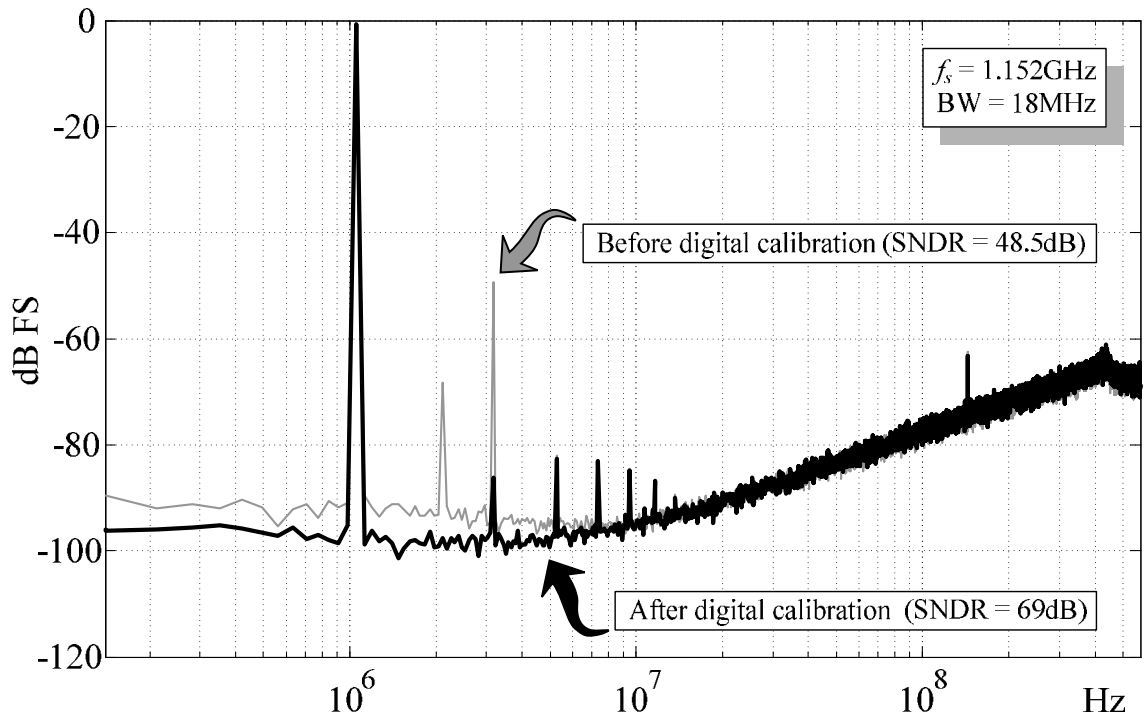


Figure 12: Representative measured PSD plots of the first prototype IC $\Delta\Sigma$ modulator output before and after digital background calibration (initial convergence time of digital calibration unit is 233ms).

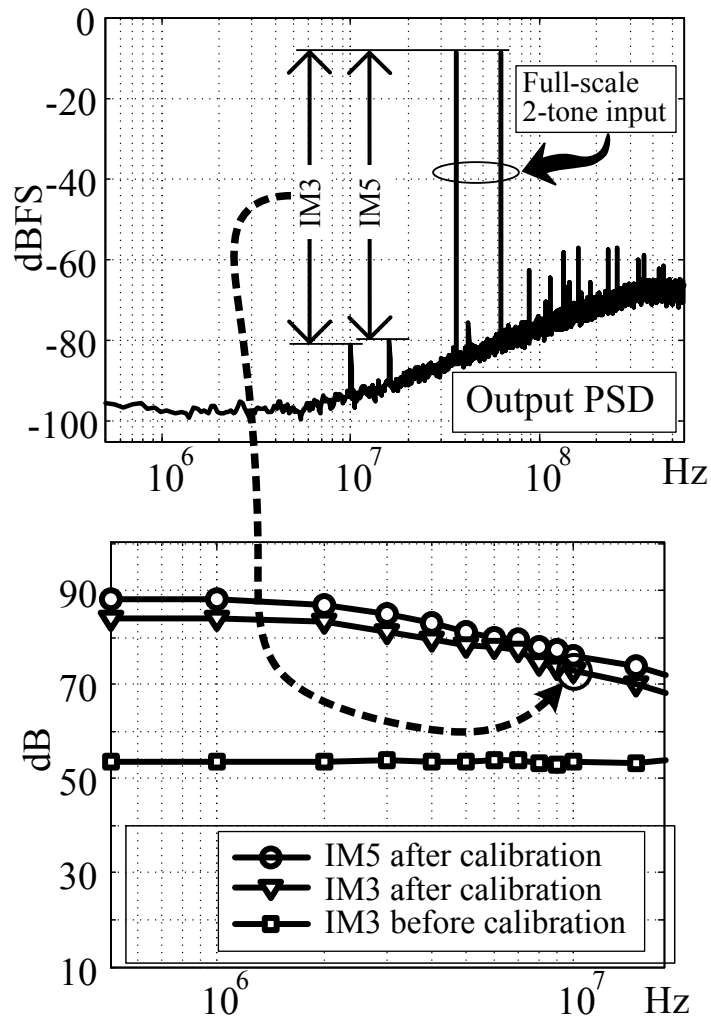


Figure 13: Plots of the first prototype IC's measured output PSD for a two-tone out-of-band input signal (top) and inter-modulation distortion (bottom) for the $\Delta\Sigma$ modulator run with $f_s = 1.152\text{GHz}$. The top and bottom plots indicate how the inter-modulation values were measured.

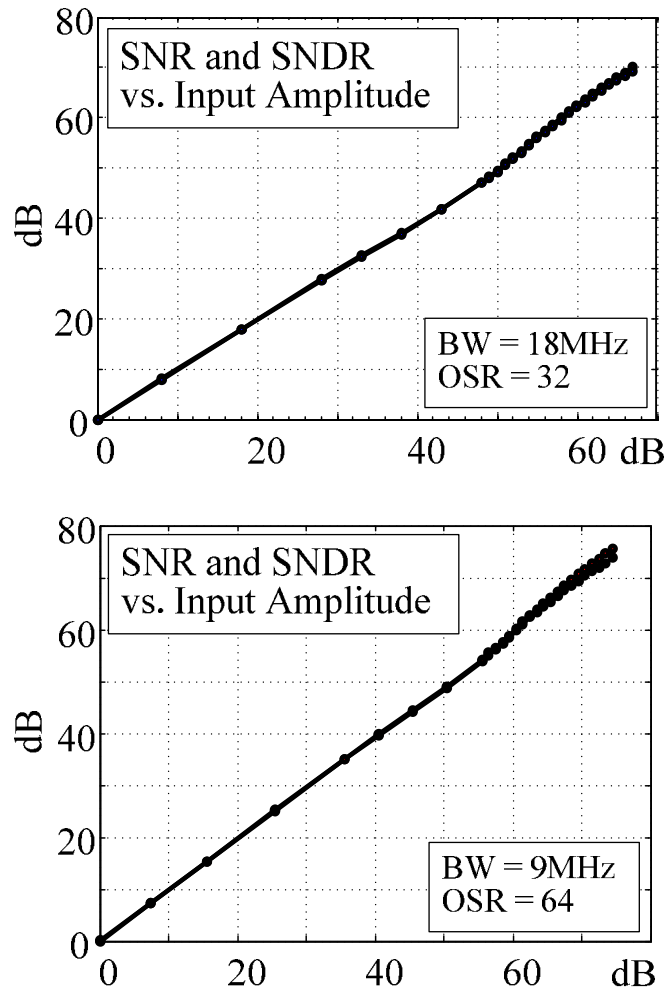


Figure 14: Plots of the first prototype IC's measured SNR and SNDR for an 18 MHz signal band (top) bandwidth 9MHz signal-band (bottom) for the $\Delta\Sigma$ modulator run with $f_s = 1.152\text{GHz}$.

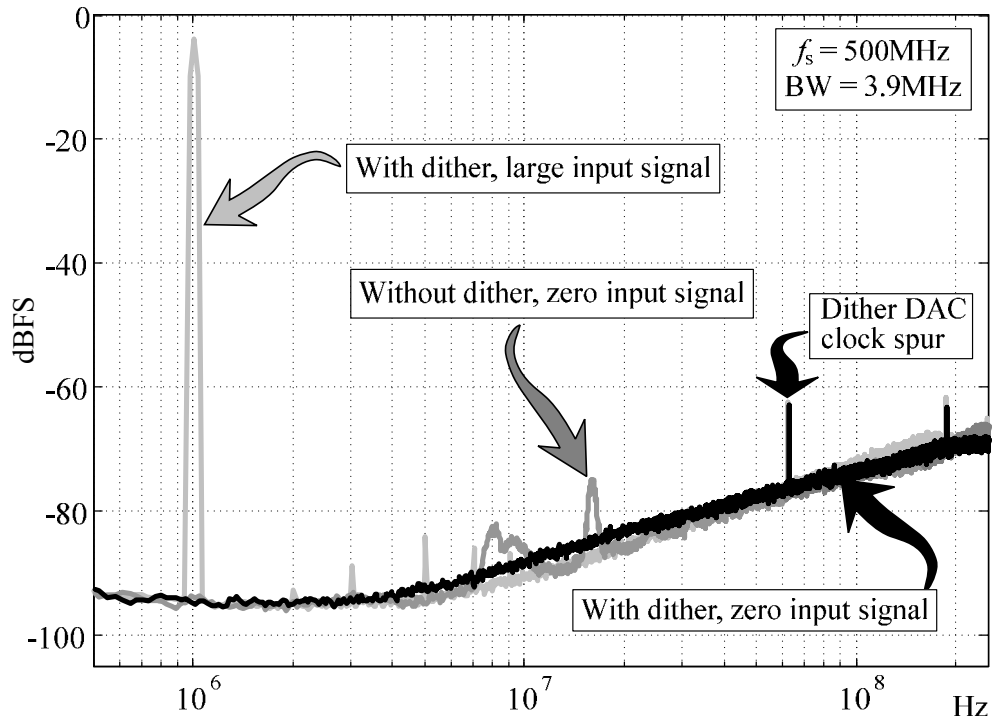


Figure 15: Representative measured PSD plots of the first prototype IC $\Delta\Sigma$ modulator output with and without dither

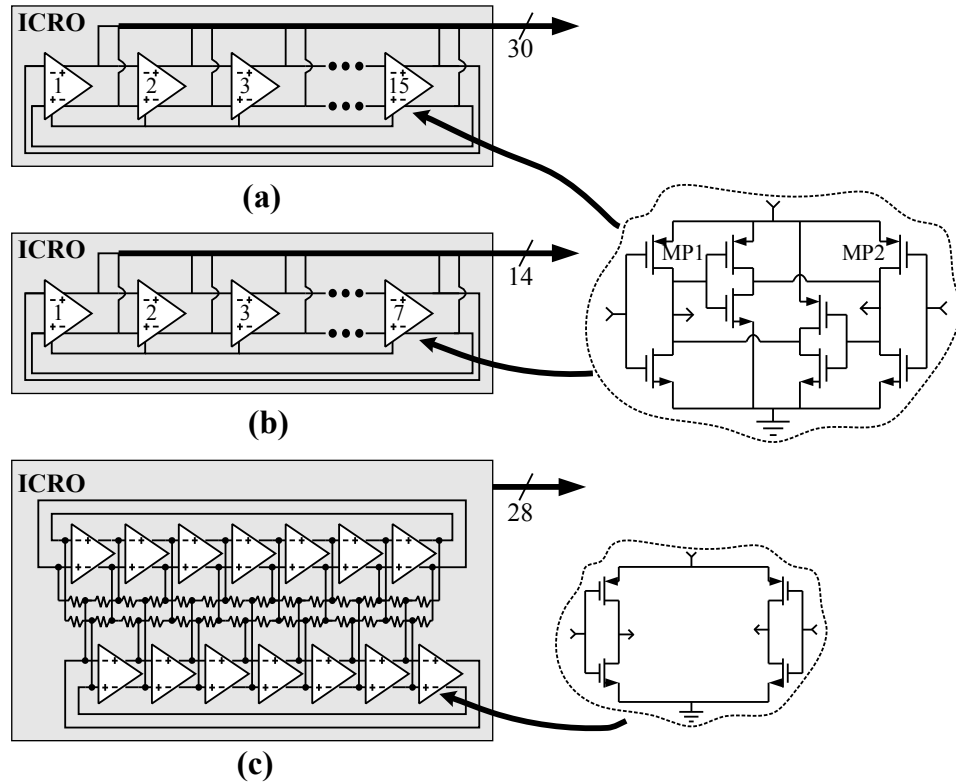


Figure 16: Circuit diagrams of various ICROs: (a) 15-element ICRO used in the first prototype IC, (b) example 7-element ICRO, (c) dual, 7-element injection-locked ICRO used in the second prototype IC

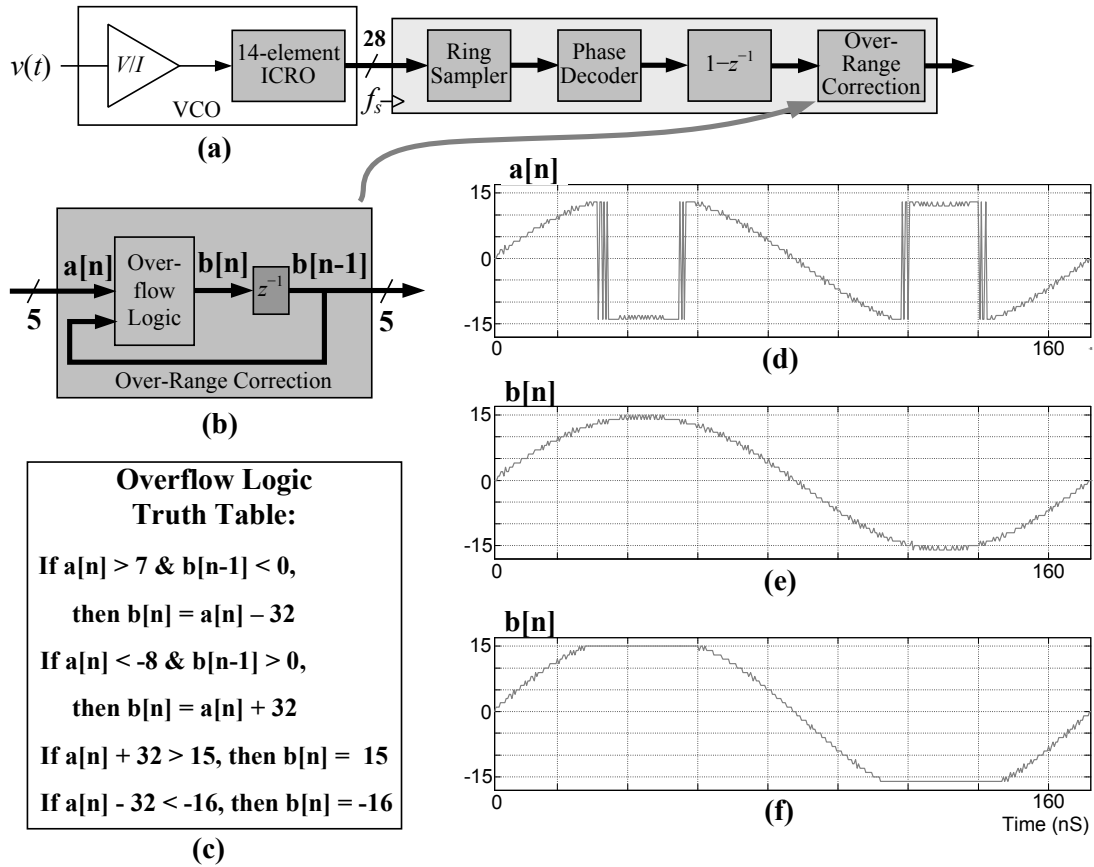


Figure 17: Over-Range Correction block (ORC): (a) signal path with the addition of over-range correction, (b) details of the ORC block, (c) overflow logic truth table, (d) example overload waveform before ORC, (e) example waveform after ORC, (f) example waveform after ORC demonstrating clipping behavior.

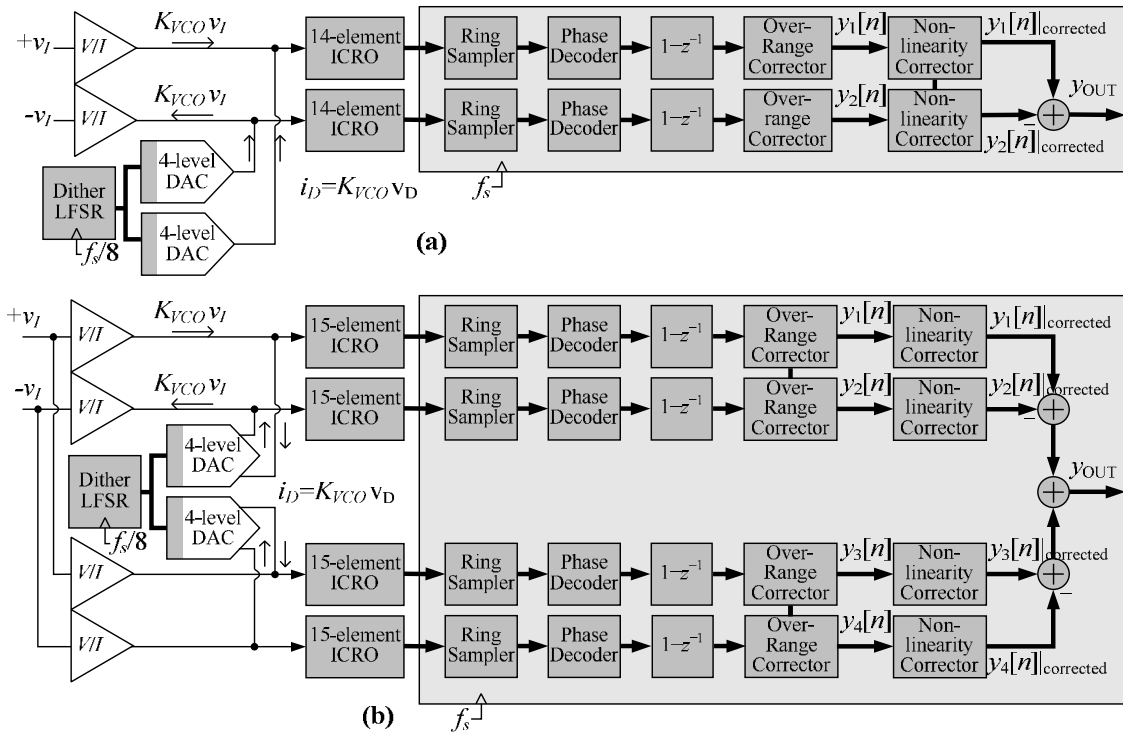


Figure 18: High-level block diagram of two VCO-based $\Delta\Sigma$ modulators: (a) single pseudo-differential modulator ADC with dither injected as a common-mode signal, (b) dual pseudo-differential modulator ADC with dither injected as a differential signal.

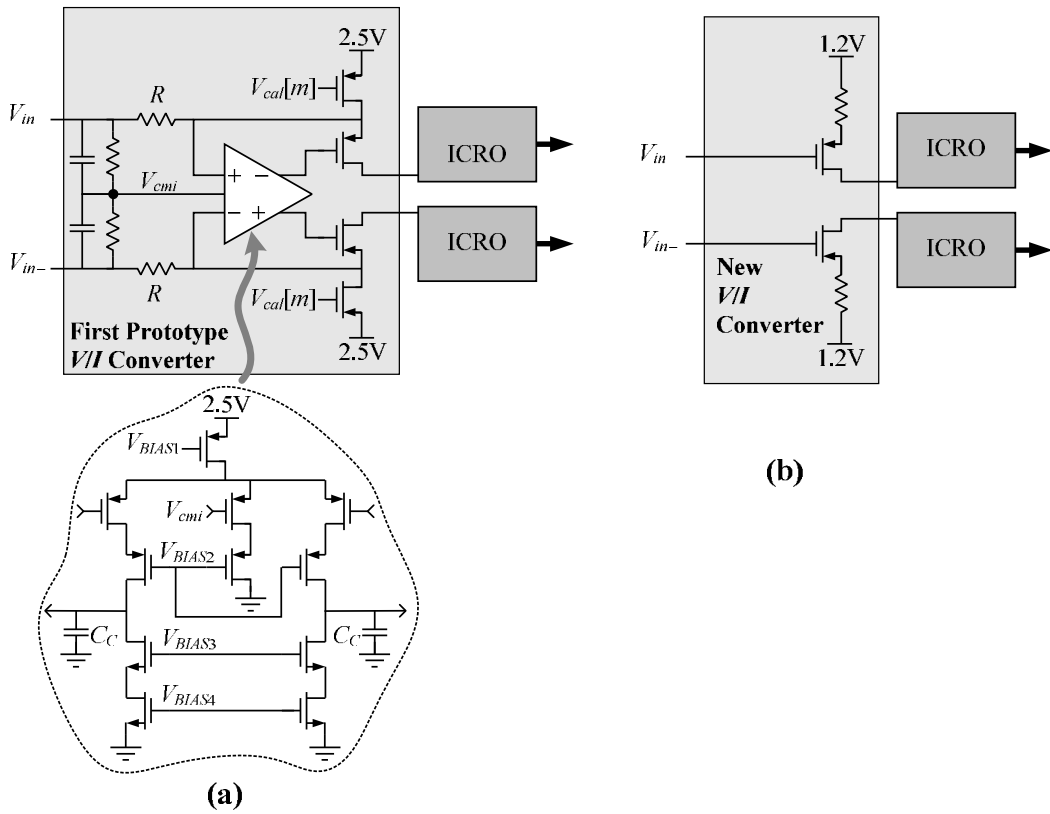


Figure 19: V/I converter circuit diagrams: (a) first prototype IC V/I converter, (b) new open-loop V/I converter

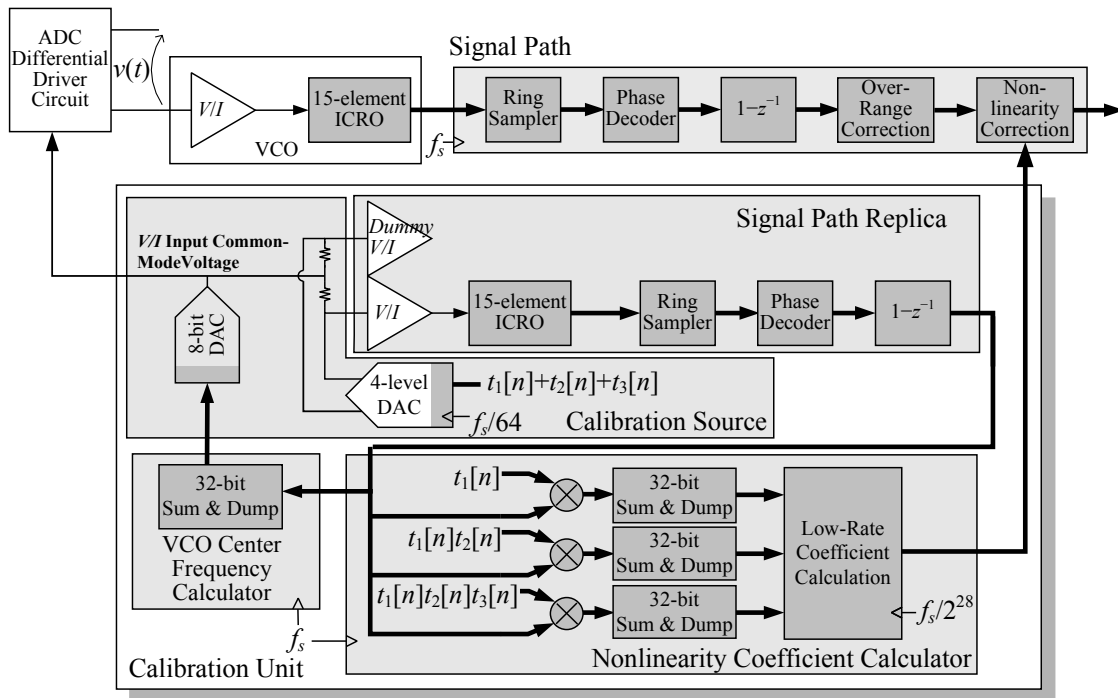


Figure 20: The second prototype IC's on-chip calibration unit shown with a single VCO-based $\Delta\Sigma$ modulator signal path for simplicity

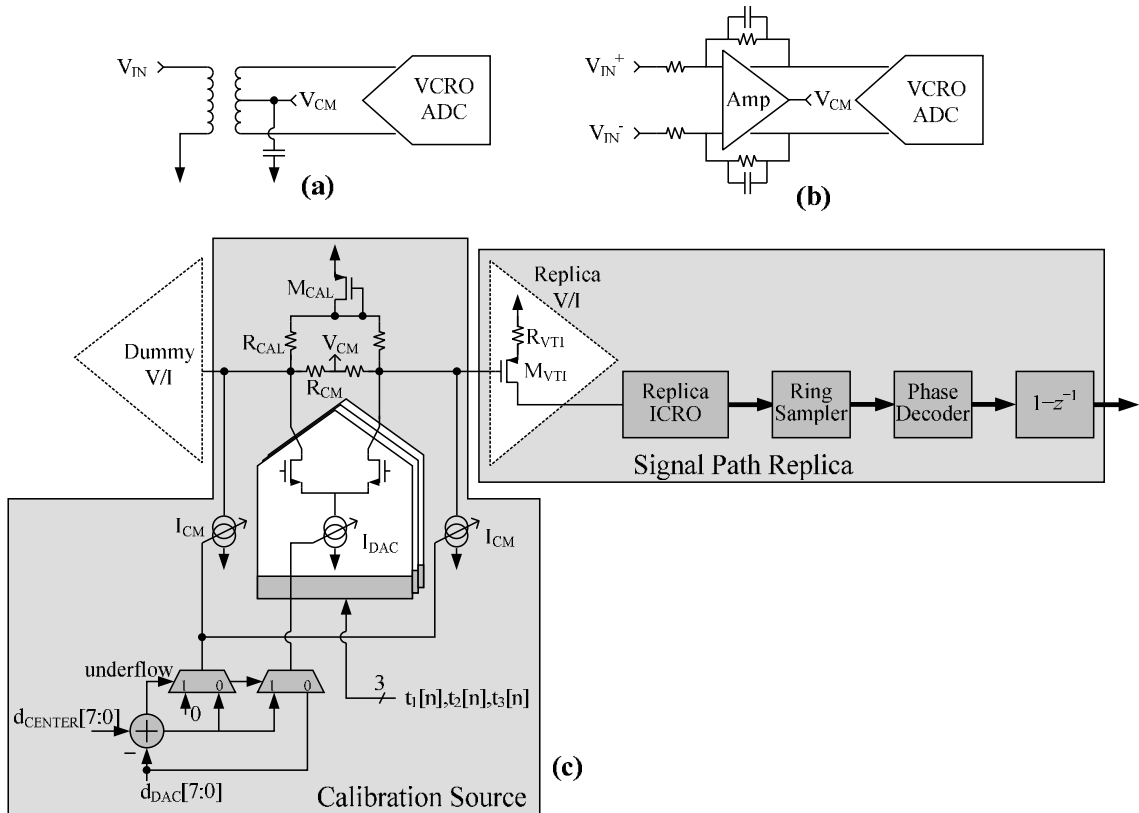


Figure 21: Calibration unit details: (a) example transformer-based input circuit for the ADC, (b) example active-circuit based input circuit for the ADC, (c) detailed circuit diagram of the calibration DAC and signal path replica.

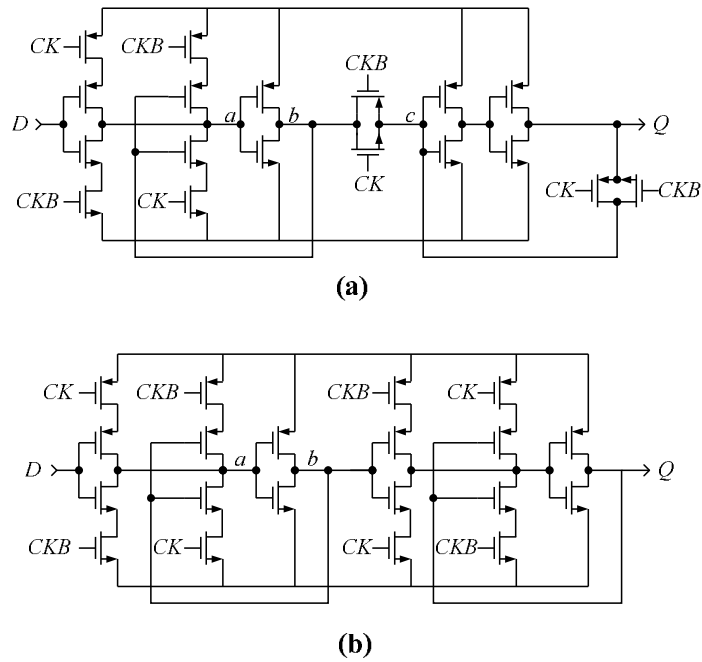


Figure 22: Circuit diagrams of the ring sampler D-type flip-flops, (a) transmission-gate flip-flop used in the first prototype that produces data-dependent hysteresis, (b) non-transmission-gate flip-flop that is largely hysteresis free.

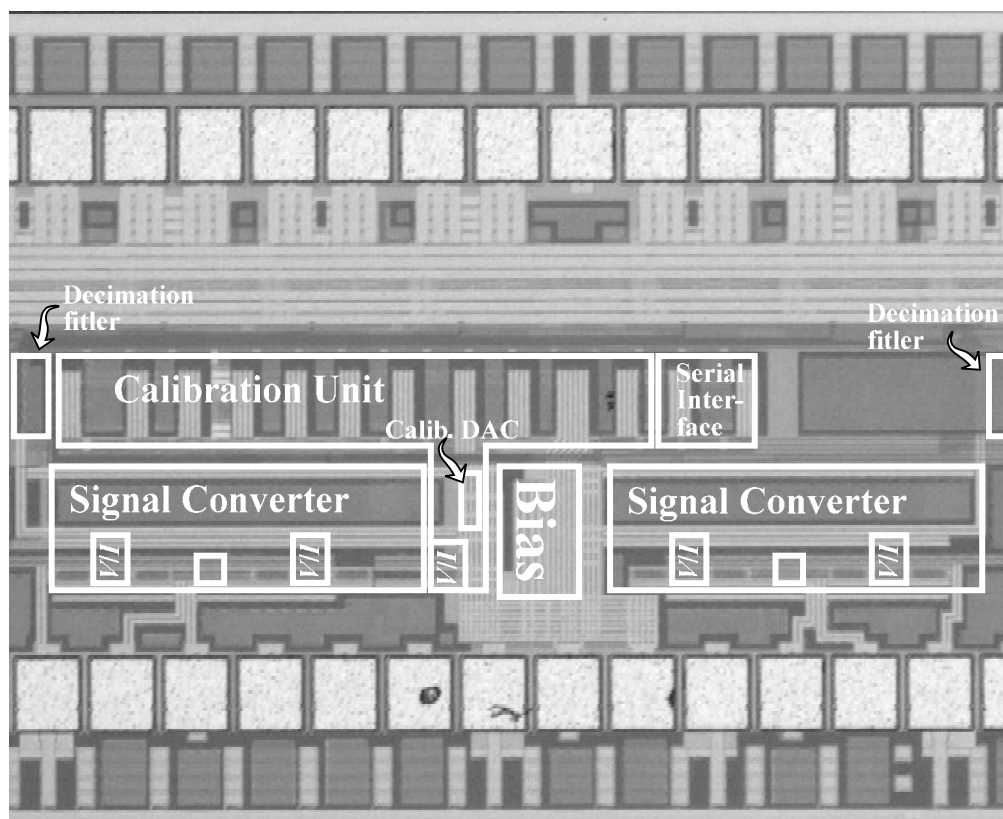


Figure 23: Die photograph of second prototype IC

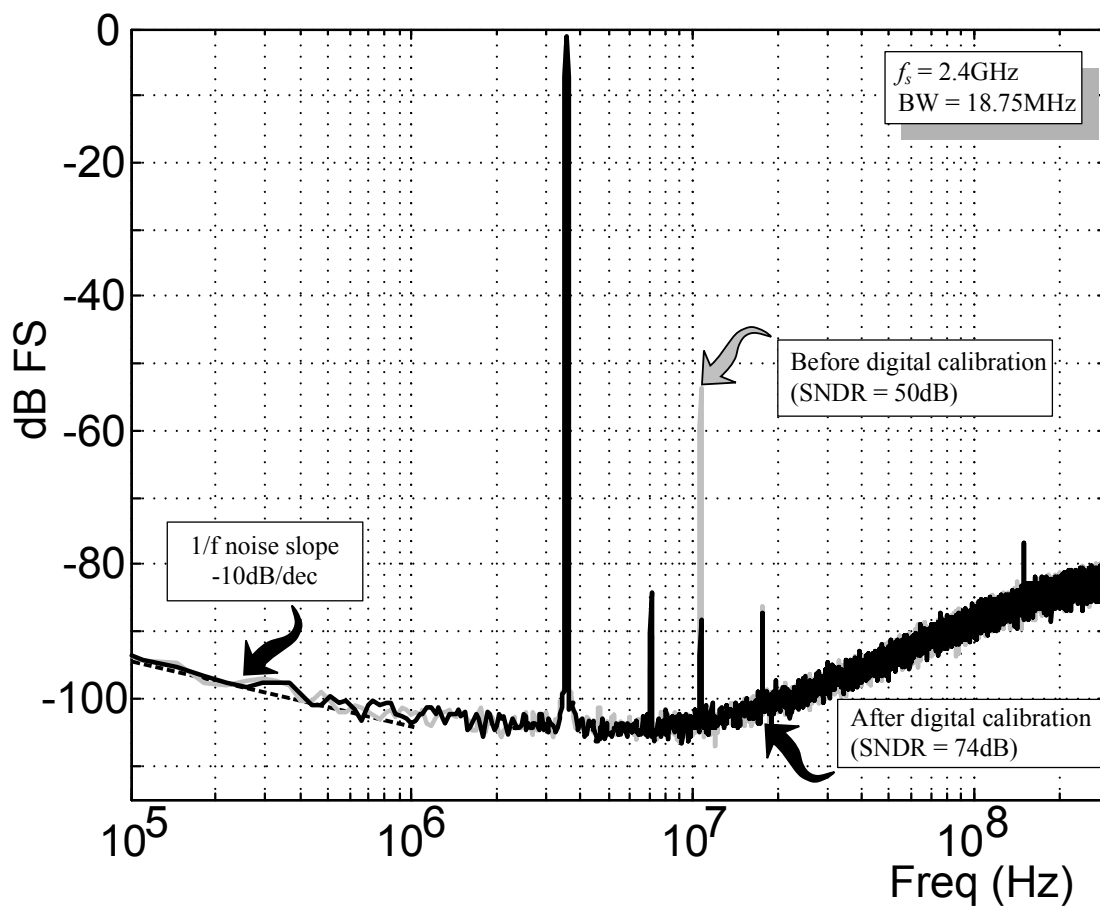


Figure 24: Representative measured PSD plots of the second prototype IC $\Delta\Sigma$ modulator output before and after digital background calibration

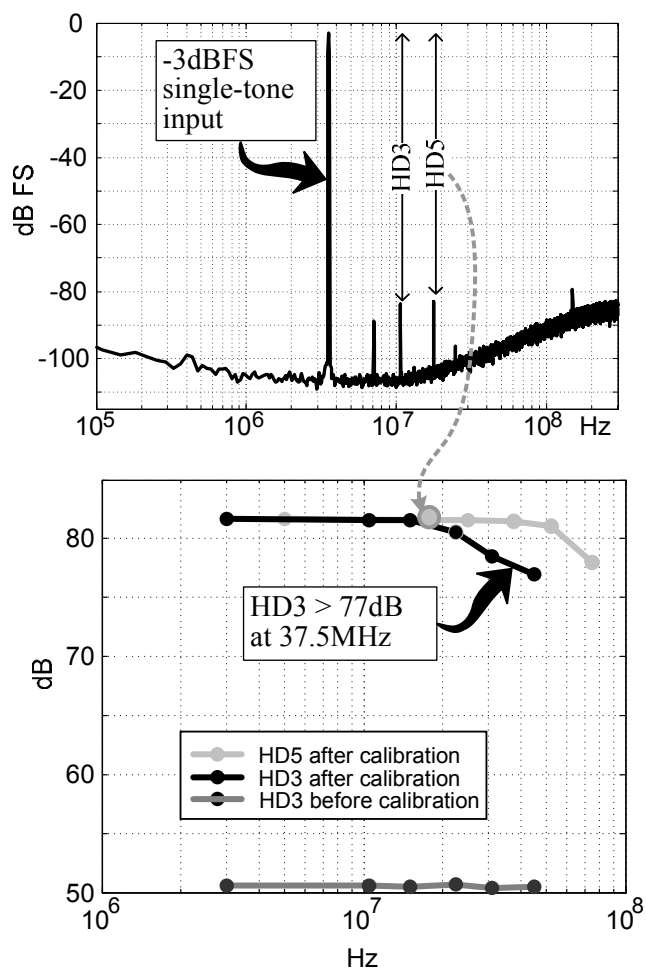


Figure 25: Plots of the second prototype IC's measured output PSD for a single-tone input signal (top) and the harmonic distortion values (bottom) for the $\Delta\Sigma$ modulator run with $f_s=2.4\text{GHz}$. The top and bottom plots indicate how the harmonic distortion values were measured.

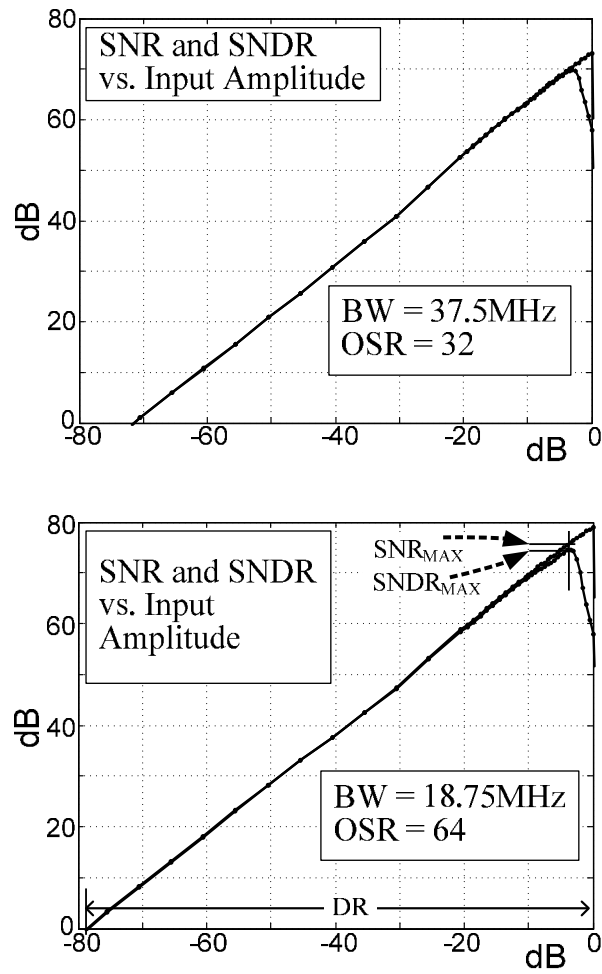


Figure 26: Plots of the second prototype IC's measured SNR and SNDR for a 37.5MHz signal bandwidth (top) and an 18MHz signal bandwidth (bottom) for the second prototype $\Delta\Sigma$ modulator run with $f_s=2.4\text{GHz}$.

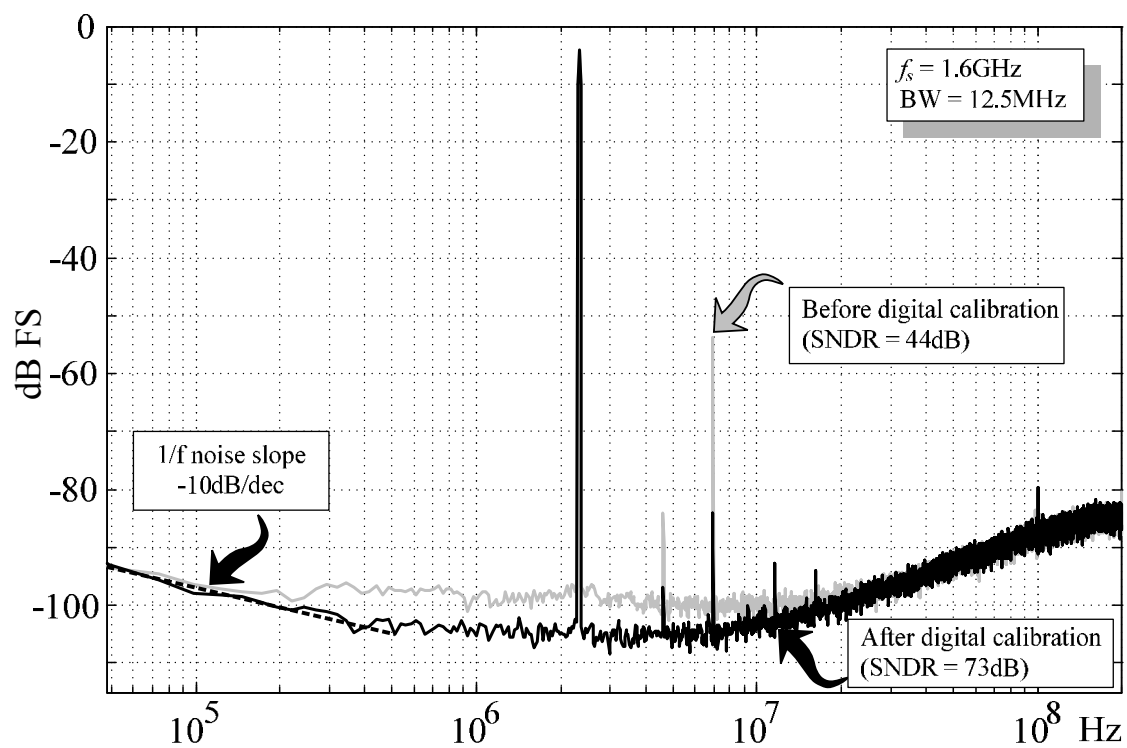


Figure 27: Representative measured PSD plots of the third prototype IC $\Delta\Sigma$ modulator output before and after digital background calibration

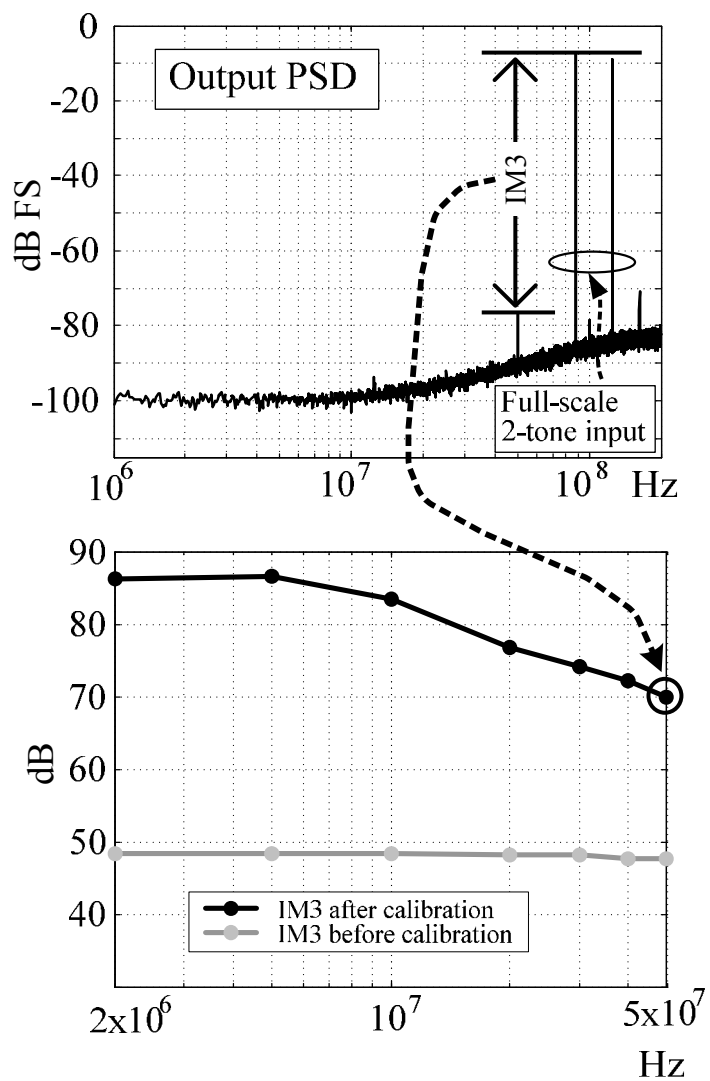


Figure 28: Plots of the third prototype IC's measured output PSD for a two-tone out-of-band input signal (top) and inter-modulation distortion (bottom) for the $\Delta\Sigma$ modulator run with $f_s = 1.6\text{GHz}$. The top and bottom plots indicate how the inter-modulation values were measured.

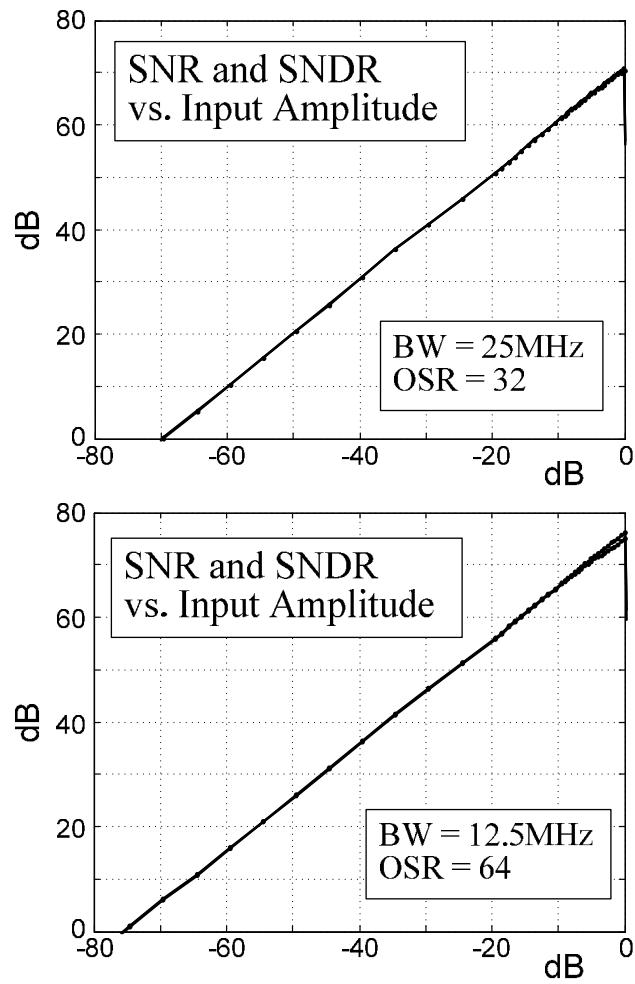


Figure 29: Plots of the third prototype IC's measured SNR and SNDR for a 25MHz signal bandwidth (top) and an 12.5MHz signal bandwidth (bottom) for the third prototype $\Delta\Sigma$ modulator run with $f_s=1.6\text{GHz}$.

TABLES

Table 1: Performance table and comparison of first prototype IC to prior state-of-the-art $\Delta\Sigma$ modulators.

	First Prototype IC							Reference [2]	Reference [3]	Reference [4]	Reference [5]
Area (mm ²)	0.07							0.7	1.5	0.45	0.15
Process	65nm LP							180nm	130nm	130nm	65nm
f _s (MHz)	1152 ⁺						500	640	640	900	250
OSR	32			64		128	64	32	16	22.5	12.5
BW (MHz)	18			9		4.5	3.9	10	20	20	20
f _{IN} (MHz)*	1	2.3	5	1	2.3	1	1	2.4	3.68	2	3.9
SNR (dB)	70	70	70	76	76	80	71.5	84	76	81.2	62
SNDR (dB)	69	67.3	67	73	72	77.8	71	82	74	78.1	60
Power (mW)	17 ⁺⁺	17	17	17	17	17	8	100	20	87	10.5
FOM (SNR)**	160.2	160.2	160.2	163.2	163.2	164.2	158.4	164.0	166.0	164.8	154.8
FOM (SNDR)***	159.2	157.5	157.2	160.2	159.2	162.0	157.9	162.0	164.0	161.7	152.8

⁺ Maximum frequency limited by test board FPGA used for data acquisition

⁺⁺ Analog (V/I circuits and DACs): 5mW, Digital: 12mW

* Worst-case input frequency value over stated BW (SNDR remains unchanged or improves at higher f_{IN} values)

** FOM (SNR) = SNR + 10 log₁₀(BW/Power)

*** FOM (SNDR) = SNDR + 10 log₁₀(BW/Power)

Table 2: Performance table and comparison of second prototype IC to prior state-of-the-art $\Delta\Sigma$ modulators.

	Second Prototype IC												First Prototype IC		[2]	[3]	[4]
Area (mm ²)	0.075												0.07		0.7	1.5	0.45
Process	65nm G+												65nm LP		180nm	130nm	130nm
f _s (MHz)	1300			1600			1920			2400			500	1152	640	640	900
OSR	128	64	32	128	64	32	64	48	32	128	64	32	64	32	32	16	22.5
BW (MHz)	5.08	10.2	20.3	6.25	12.5	25	15	20	30	9.38	18.8	37.5	3.9	18	10	20	20
f _N (MHz)*	1	1	3.5	1	2.3	4.9	2	3.5	5	1	3.5	7.49	1	2.3	2.4	3.68	2
SNR (dB)	77	74.5	69.9	78.1	75.8	70.3	75.7	74.1	71.1	79.8	76	70.4	71.5	70	84	76	81.2
SNDR (dB)	75	72.7	68.5	76.8	74.3	69.8	74.6	73.1	70	76.2	74.1	69.8	71	67.3	82	74	78.1
DR (dB)	78	76	71	80	77	72	78	76	72	82	78	73	70	68	84	80	81.2
THD (dB)	79.6	79	79.2	82.8	82	79	81	80	77	78.7	78.6	76					
SFDR (dB)	81.8	82	82	85	85	80	84	81	78	81.3	81.4	77					
Power Supply (V)	0.9			1.0			1.1			1.2			2.5/1.2	2.5/1.2	1.8	1.2	1.2
Power Total (mW)	10			17			27.5			37.5			8	17	100	20	87
Power Analog (mW)	2			3			4.5			7.2			2.5	5			
Power Digital (mW)	8			14			23			30.3			5.5	12			
FOM (SNR)**	164.1	164.6	163.0	163.8	164.5	162.0	163.1	162.7	161.5	163.8	163.0	160	158.4	160.2	164.0	166.0	164.8
FOM (SNDR)***	162.1	162.8	161.6	162.5	163.0	161.5	162.0	161.7	160.4	160.2	161.1	160	157.9	157.5	162.0	164.0	161.7
FOM (DR)****	165.1	166.1	164.1	165.7	165.7	163.7	165.4	164.6	162.4	166.0	165.0	163	156.9	158.2	164.0	170.0	164.8

* Worst-case input frequency value over stated BW (SNDR remains unchanged or improves at higher f_N values)

** FOM (SNR) = SNR + 10 log₁₀(BW/Power)

*** FOM (SNDR) = SNDR + 10 log₁₀(BW/Power)

**** FOM (DR) = DR + 10 log₁₀(BW/Power)

Table 3: Performance table and comparison of third prototype IC fabricated in 65nm LP process to first IC fabricated in 65nm G+ process

	First Prototype IC							Third Prototype IC		
Area (mm ²)	0.07							0.075		
Process	65nm LP							65nm G+		
f _s (MHz)	1152						500	1600		
OSR	32			64		128	64	32		64
BW (MHz)	18			9		4.5	3.9	25		12.5
f _{IN} (MHz)*	1	2.3	5 *	1	2.3 *	1	1	2.3	4.9 *	2.3 *
SNR (dB)	70	70	70	76	76	80	71.5	71.5	71.5	77
SNDR (dB)	69	67.3	67	73	72	77.8	71	70.5	69.9	74
Power Supply (V)	2.5/1.2	2.5/1.2	2.5/1.2	2.5/1.2	2.5/1.2	2.5/1.2	2.5/1.2	1.0	1.0	1.0
Power (mW)	17	17	17	17	17	17	8	20	20	20
Analog Power(mW)	5	5	5	5	5	5	2.5	6	6	6
Digital Power (mW)	12	12	12	12	12	12	5.5	14	14	14
FOM (SNR)**	160.2	160.2	160.2	163.2	163.2	164.2	158.4	162.5	162.5	165.0
FOM (SNDR)***	159.2	157.5	157.2	160.2	159.2	162.0	157.9	161.5	160.9	162.0

* Worst-case input frequency value over stated BW (SNDR remains unchanged or improves at higher f_{IN} values)

** FOM (SNR) = SNR + 10 log₁₀(BW/Power)

*** FOM (SNDR) = SNDR + 10 log₁₀(BW/Power)

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