Motion Recovery by Using Stereo Vision

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Abstract. The motion recovery for a class of movements in the space by using stereo vision is considered by observing at least three points in this paper. The considered motion equation can cover a wide class of practical movements in the space. The observability of this class of movement is clarified. The estimations of the motion parameters which are all time-varying are developed in the proposed algorithm based on the second method of Lyapunov. The assumptions about the perspective system are reasonable, and the convergence conditions are intuitive and have apparently physical interpretations. The proposed recursive algorithm requires minor *a priori* knowledge about the system and can alleviate the noises in the image data. Furthermore, the proposed algorithm is modified to deal with the occlusion phenomenon. Simulation results show the proposed algorithm is effective even in the presence of measurement noises.

Keywords: Stereo vision, dynamic vision, motion recovery, occlusion.

1 Introduction

In the study of machine vision, observing the motion and the structure of a moving object in the space by using the image data with the aid of CCD camera(s) has been studied recently. The motion treated in this field is composed of a rotation part and a translation part. A very typical method is the application of the extended Kalman filter (EKF). Numerous successful results have been reported until now where the formulation is based on a discrete expression of the motion, and the observability conditions are derived based on the perspective observations of a group of points [1][4]. Such a recursive algorithm obviously alleviates the noises in the image data in contrast to the non-recursive methods [8] based on solving a set of nonlinear algebraic equations. It should be mentioned that some theoretical convergence conditions of discrete EKF have been established both as observer and filter [10].

The observation problem for continuous time perspective systems has been studied in the point of view of dynamical system theory in [6][9]. A necessary and sufficient condition for the perspective observability is given in [5] for the case that the motion parameters are constants. For the movements with piecewise constant motion parameters, the perspective observability problems are clarified in [12] for the cases of observing one point or a group of points. Furthermore, for the observer design, some simple formulations for observing the position of a moving object are proposed in [2][3][7]. The proposed observers are guaranteed to converge in an arbitrarily large (but bounded) set of initial conditions, and since the convergence is exponential it is believed that the performance of the new observers are reliable, robust and would quickly compute the position on real data.

This paper considers the problem of motion recovery for a class of movements under perspective observation. Naturally, the motions are formulated in continuous-time settings and the so-called motion parameters are assumed to be all time-varying. The motion parameters are estimated by using image data observed through pin-hole camera with constant focal length (normalized to unity). The basic and important idea is to analyze the extent to which we can develop a scheme that is guaranteed to converge by observing minimum number of points. A dynamical systems approach is employed since it provides us with powerful mathematical tools, and a nonlinear observer is developed based on the second method of Lyapunov [11].

The considered motion equation can cover a wide class of practical movements in the space. The observability of this class of movement is clarified by observing three points. The estimation of the motion parameter is developed in this paper. The formulated problem can be converted into the observation of a dynamical system with nonlinearities. It should be noted that smoothened image data instead of the measured one is used in the proposed formulation in order to alleviate the noises in the image data. The assumptions about the perspective system are reasonable, and the convergence conditions are intuitive and have apparently physical interpretations. The attraction of the new method lies in that the algorithm is very simple, easy to be implemented practically. Furthermore, the proposed method requires minor *a priori* knowledge about the system and can cope with a much more general class of perspective systems. It should be noted that the changing of focal length is not considered in this paper. Furthermore, the algorithm is modified to deal with the occlusion phenomenon. Simulation results show the proposed algorithm is effective even in the presence of measurement noises.

2 Problem Statement

Consider the movement of the object described by

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 0 & \omega_1(t) & \omega_2(t) \\ -\omega_1(t) & 0 & \omega_3(t) \\ -\omega_2(t) & -\omega_3(t) & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} b_1(t) \\ b_2(t) \\ b_3(t) \end{bmatrix}, \tag{1}$$

where $x(t) = [x_1, \quad x_2, \quad x_3]^T$ is the position; $\omega_i(t)$ and $b_i(t) (i = 1,2,3)$ are the motion parameters.

It is supposed that the observed position by Camera 1 is defined by

$$y(t) = [y_1(t), \quad y_2(t)] = \left[\frac{x_1}{x_3}, \quad \frac{x_2}{x_3}\right], \tag{2}$$

and the observed position by Camera 2 is defined by

$$y^*(t) = [y_1^*(t), \quad y_2^*(t)] = \left[\frac{x_1 - m}{x_3}, \quad \frac{x_2 - n}{x_3}\right], \tag{3}$$

where m and n are constants. The perspective observations are defined in (2) and (3). The combination of the observations in (2) together with (3) is called "stereo vision".

In this paper, we make the following assumptions.

(A1). m and n are known constants with $m^2 + n^2 \neq 0$.

(A2). The motion parameters $\omega_i(t)$ and $b_i(t)(i = 1,2,3)$ are bounded.

(A3). $x_3(t)$ meets the condition $x_3(t) > \eta > 0$, where η is a constant.

(A4). $y(t)$ and $y^*(t)$ are bounded.

Remark 1. It is easy to see that assumptions (A3) and (A4) are reasonable by referring to the practical systems.

The purpose of this paper is to estimate the motion parameters $\omega_i(t)$ and $b_i(t)(i = 1,2,3)$ by using the perspective observations.

3 Formulation of the Motion Identification

Define

$$y_3(t) = \frac{1}{x_3(t)}. \tag{4}$$

Then, equation (1) can be transformed as

$$\begin{cases} \dot{y}_1(t) = \omega_2 + \omega_1 y_2 + \omega_2 y_1^2 + \omega_3 y_1 y_2 + b_1 y_3 - b_3 y_1 y_3 \\ \dot{y}_2(t) = \omega_3 - \omega_1 y_1 + \omega_2 y_1 y_2 + \omega_3 y_2^2 + b_2 y_3 - b_3 y_2 y_3 \\ \dot{y}_3(t) = \omega_2 y_1 y_3 + \omega_3 y_2 y_3 - b_3 y_3^2 \end{cases} \tag{5}$$

Let

$$\theta(t) = [b_1, b_2, b_3, \omega_1, \omega_2, \omega_3]^T \triangleq [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]^T, \tag{6}$$

and

$$\phi(t)=\begin{bmatrix}\phi_1(t)\\ \phi_2(t)\end{bmatrix}=\begin{bmatrix} y_3 & 0 & -y_1y_3 & y_2 & 1+y_1^2 & y_1y_2\\ 0 & y_3 & -y_2y_3 & -y_1 & y_1y_2 & 1+y_2^2\end{bmatrix}. \tag{7}$$

Thus, the first two equations in (5) can be rewritten as

$$\begin{bmatrix}\dot{y}_1(t)\\ \dot{y}_2(t)\end{bmatrix}=\phi(t)\cdot\theta(t)\,. \tag{8}$$

Similarly, for $y^*(t)$, it gives

$$\begin{bmatrix}\dot{y}_1^*(t)\\ \dot{y}_2^*(t)\end{bmatrix}=\phi^*(t)\cdot\theta(t) \tag{9}$$

with

$$\phi^*(t)=\begin{bmatrix}\phi_1^*(t)\\ \phi_2^*(t)\end{bmatrix}$$

$$=\begin{bmatrix} y_3 & 0 & -y_1^*y_3 & y_2^*+ny_3 & 1+y_1^*(y_1^*+my_3) & y_1^*(y_2^*+ny_3)\\ 0 & y_3 & -y_2^*y_3 & -y_1^*-my_3 & y_2^*(y_1^*+my_3) & 1+y_2^*(y_2^*+my_3)\end{bmatrix} \tag{10}$$

From (2) and (3), $y_3(t)$ can be calculated by the average

$$y_3(t)=m\frac{y_1-y_1^*}{m^2+n^2}+n\frac{y_2-y_2^*}{m^2+n^2}\,. \tag{11}$$

Thus, $\phi(t)$ and $\phi^*(t)$ are available.

In the following, the vectors $\phi(t)\cdot\theta(t)$ and $\phi^*(t)\cdot\theta(t)$ are estimated in section 3.1 by using the perspective observations defined in (2) and (3). Then, the motion parameters $\omega_i(t)$ and $b_i(t)\,(i=1,2,3)$ are estimated in section 3.2 by using the stereo observation of at least three points.

3.1 Identification of $\phi(t)\theta(t)$ and $\phi^*(t)\theta(t)$

In the following, the observer of system (8) is formulated. We consider the system described by

$$\begin{bmatrix}\dot{\hat{y}}_1(t)\\ \dot{\hat{y}}_2(t)\end{bmatrix}=\begin{bmatrix}w_1(t)\\ w_2(t)\end{bmatrix},\ \begin{bmatrix}\hat{y}_1(0)\\ \hat{y}_2(0)\end{bmatrix}=\begin{bmatrix}y_1(0)\\ y_2(0)\end{bmatrix}; \tag{12}$$

$$\dot{w}_i(t)=-(f_i+\alpha_i)w_i(t)+\hat{\lambda}_i(t)\cdot sign(y_i-\hat{y}_i)+f_i\alpha_i(y_i-\hat{y}_i)\,; \tag{13}$$

$$\hat{\lambda}_i(t) = \beta_i\left(\left|y_i - \hat{y}_i\right| + \alpha_i r_i(t)\right); \tag{14}$$

$$\dot{r}_i(t) = \left|y_i - \hat{y}_i\right|, \tag{15}$$

where f_i, α_i, β_i are positive constants, $w_i(0)$ can be any small constants, and $r_i(0)$ is chosen as $r_i(0) = 0$.

Let

$$w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}. \tag{16}$$

The next theorem is obtained.

Theorem 1. All the generated signals in (12)-(15) are uniformly bounded and $w(t)$ is the asymptotic estimate of $\phi(t)\cdot\theta(t)$, i.e.

$$\lim_{t\to\infty}\left(\phi(t)\cdot\theta(t) - w(t)\right) = 0\,. \tag{17}$$

Proof. For simplicity, we only give the proof for i=1. Let

$$e_1(t) = y_1(t) - \hat{y}_1(t)\,. \tag{18}$$

Differentiating $e_1(t)$ yields

$$\dot{e}_1(t) = \phi_1(t)\cdot\theta(t) - w_1(t),\ e_1(0) = 0\,. \tag{19}$$

Now, define

$$r_1(t) = \dot{e}_1(t) + \alpha_1 e_1(t)\,. \tag{20}$$

Differentiating $r(t)$ yields

$$\dot{r}_1(t) = \frac{d}{dt}\left(\phi_1\theta - w_1\right) + \alpha_1\left(\phi_1\theta - w_1\right) = \eta_1(t) - \left(f_1 r_1(t) + \hat{\lambda}_1(t)sign(e_1)\right) \tag{21}$$

with

$$\eta_1(t) = \frac{d}{dt}\left(\phi_1\theta\right) + \left(f_1 + \alpha_1\right)\left(\phi_1\theta\right). \tag{22}$$

The uniformly boundedness of $\eta_1(t)$ and $\dot{\eta}_1(t)$ can be easily derived by using the assumptions. Thus, there exist constants $\lambda_1 > 0$ such that

$$\left|\eta_1\right| + \frac{1}{\alpha_1}\left|\dot{\eta}_1\right| < \lambda_1\,. \tag{23}$$

Now, consider the Lyapunov candidate

$$V(t)=\frac{1}{2}r_1^2(t)+\frac{1}{2\beta_1}\left(\hat{\lambda}_1(t)-\lambda_1\right)^2. \quad \textbf{(24)}$$

Differentiating $V(t)$ yields

$$\begin{aligned}
\dot{V}(t) &= r_1(t)\left(\eta_1(t)-f_1r_1(t)-\hat{\lambda}_1(t)\cdot sign(e_1(t))\right) \\
&\quad+\left(\hat{\lambda}_1(t)-\lambda_1\right)\left(\dot{e}_1(t)\cdot sign(e_1(t))+\alpha_1|e_1(t)|\right) \\
&=-f_1r_1^2(t)+r_1(t)\eta_1(t)-\left(\dot{e}_1(t)+\alpha_1e_1(t)\right)\hat{\lambda}_1(t)\cdot sign(e_1(t)) \\
&\quad+\left(\hat{\lambda}_1(t)-\lambda_1\right)\left(\dot{e}_1(t)\cdot sign(e_1(t))+\alpha_1|e_1(t)|\right) \\
&=-f_1r_1^2(t)+r_1(t)\eta_1(t)-\lambda_1\dot{e}_1(t)\cdot sign(e_1(t))-\alpha_1\lambda_1|e_1(t)| \qquad \textbf{(25)}
\end{aligned}$$

Integrating the both sides of (25) from 0 to t yields

$$\begin{aligned}
V(t) &= V(0)-f_1\int_0^t r_1^2(\tau)d\tau+\int_0^t\left(\dot{e}_1(\tau)+\alpha_1e_1(\tau)\right)\eta_1(\tau)d\tau \\
&\quad-\lambda_1|e_1(t)|-\alpha_1\lambda_1\int_0^t|e_1(\tau)|d\tau \\
&=V(0)-f_1\int_0^t r_1^2(\tau)d\tau+e_1(t)\eta_1(t)-e_1(0)\eta_1(0) \\
&\quad+\int_0^t e_1(\tau)\left(-\dot{\eta}_1(\tau)-\alpha_1\eta_1(\tau)\right)d\tau-\lambda_1|e_1(t)|-\alpha_1\lambda_1\int_0^t|e_1(\tau)|d\tau \\
&\le V(0)-f_1\int_0^t r_1^2(\tau)d\tau+|e_1(t)|\left(|\eta_1(t)|-\lambda_1\right) \\
&\quad-e_1(0)\eta_1(0)+\int_0^t|e_1(\tau)|\left(|\dot{\eta}_1(\tau)|+\alpha_1|\eta_1(\tau)|-\alpha_1\lambda_1(\tau)\right)d\tau \\
&\le V(0)-f_1\int_0^t r_1^2(\tau)d\tau-e_1(0)\eta_1(0) \qquad \textbf{(26)}
\end{aligned}$$

Thus, it can be seen that $V(t)$ and the integral $\int_0^t r_1^2(\tau)d\tau$ are bounded. Therefore, $r_1(t)\to 0$ as $t\to\infty$. By the definition of $r_1(t)$, it gives $e_1(t)\to 0$ and $\dot{e}_1(t)\to 0$ as $t\to\infty$. The theorem is proved.

Similarly to (10), construct the equation

$$\dot{\hat{y}}^*(t)=\begin{bmatrix}\dot{\hat{y}}_1^*(t)\\ \dot{\hat{y}}_2^*(t)\end{bmatrix}=\begin{bmatrix}w_1^*(t)\\ w_2^*(t)\end{bmatrix},\ \begin{bmatrix}\hat{y}_1^*(0)\\ \hat{y}_2^*(0)\end{bmatrix}=\begin{bmatrix}y_1^*(0)\\ y_2^*(0)\end{bmatrix} \quad \textbf{(27)}$$

where $\begin{bmatrix}w_1^*(t)\\ w_2^*(t)\end{bmatrix}\triangleq w^*(t)$ can be defined by referring (13)-(15) by using the obtained image data $y^*(t)$ from Camera 2. Similar to Theorem 1, it can be concluded that $w^*(t)$ is uniformly bounded and

$$\lim_{t\to\infty}\left(\phi^*(t)\cdot\theta(t) - w^*(t)\right) = 0\ , \qquad \textbf{(28)}$$

i.e. $w^*(t)$ is the asymptotic estimate of $\phi^*(t)\theta(t)$.

3.2 Identification of $\theta(t)$

Relations (17) and (28) tell us that, by observing one point via stereo vision, four relations about $\theta(t)$ can be obtained. It can be easily checked that the rank of the matrix $\begin{bmatrix}\phi(t)\\ \phi^*(t)\end{bmatrix}$ is three. It can be argued that the relations about $\theta(t)$ can be increased by increasing the observation points. Since there are six entries in $\theta(t)$, it can be argued that at least two points are needed to get a solution of $\theta(t)$.

Now, suppose p points are observed. For the j-th point, we denote the obtained $\begin{bmatrix}\phi(t)\\ \phi^*(t)\end{bmatrix}$ and $\begin{bmatrix}w(t)\\ w^*(t)\end{bmatrix}$ as $\begin{bmatrix}\phi^{(j)}(t)\\ \phi^{*(j)}(t)\end{bmatrix}$ and $\begin{bmatrix}w^{(j)}(t)\\ w^{*(j)}(t)\end{bmatrix}$, respectively.

Define

$$\Phi(t) = \begin{bmatrix}\phi^{(1)}(t)\\ \phi^{*(1)}(t)\\ \vdots\\ \phi^{(j)}(t)\\ \phi^{*(j)}(t)\\ \vdots\\ \phi^{(p)}(t)\\ \phi^{*(p)}(t)\end{bmatrix},\quad W(t) = \begin{bmatrix}w^{(1)}(t)\\ w^{*(1)}(t)\\ \vdots\\ w^{(j)}(t)\\ w^{*(j)}(t)\\ \vdots\\ w^{(p)}(t)\\ w^{*(p)}(t)\end{bmatrix}. \qquad \textbf{(29)}$$

By Theorem 1, it gives

$$\lim_{t\to\infty}\left(\Phi(t)\cdot\theta(t) - W(t)\right) = 0\ . \qquad \textbf{(30)}$$

About the rank of the matrix $\Phi(t)$, we have the next lemma.

Lemma 1. The matrix $\Phi(t)$ is of full rank if and only if at least three points are not on a same line.

Proof. The proof is omitted.

Lemma 1 means that at least three points are needed in the proposed formulation.

Theorem 2. If at least three observed points are not on a same line, then the motion parameters are observable and it holds

$$\lim_{t\to\infty}\left\{\theta(t) - \left(\Phi^T(t)\Phi(t)\right)^{-1}\Phi^T(t)W(t)\right\} = 0\ , \qquad \textbf{(31)}$$

i.e. $\left(\Phi^T(t)\Phi(t)\right)^{-1}\Phi^T(t)W(t)$ is the asymptotic estimate of the vector $\theta(t)$.

Since the image data is directly used in $\Phi(t)$, the measurement noise will directly influence the accuracy of the estimation. In the practical application of the proposed algorithm, the image data $y^{(j)}(t)$ and $y^{*(j)}(t)$ can be respectively replaced by the generated smooth signals $\hat{y}^{(j)}(t)$ and $\hat{y}^{*(j)}(t)$, since $y^{(j)}(t)-\hat{y}^{(j)}(t)\to 0$ and $y^{*(j)}(t)-\hat{y}^{*(j)}(t)\to 0$. As to the value of $y_3^{(j)}(t)$ in $\Phi(t)$, although it can be calculated in (11) by using the image data, we use a smoothed signal $\hat{y}_3^{(j)}(t)$ to replace it in order to mitigate the influence of measurement noises. The signal $\hat{y}_3^{(j)}(t)$ is generated as follows.

$$\dot{\hat{y}}_3^{(j)} = \hat{\lambda}_3^{(j)}(t)sign\left(m\frac{y_1^{(j)}-y_1^{*(j)}}{m^2+n^2}+n\frac{y_2^{(j)}-y_2^{*(j)}}{m^2+n^2}-\hat{y}_3^{(j)}\right), \quad \textbf{(32)}$$

$$\dot{\hat{\lambda}}_3^{(j)}(t) = \gamma\cdot\left|m\frac{y_1^{(j)}-y_1^{*(j)}}{m^2+n^2}+n\frac{y_2^{(j)}-y_2^{*(j)}}{m^2+n^2}-\hat{y}_3^{(j)}\right|, \quad \textbf{(33)}$$

where γ is a positive constant.

Lemma 2. The generated signals $\hat{y}_3^{(j)}(t)$ and $\hat{\lambda}_3(t)$ are uniformly bounded and

$$\lim_{t\to\infty}\left(y_3^{(j)}(t)-\hat{y}_3^{(j)}(t)\right)=0. \quad \textbf{(34)}$$

Proof. By the assumptions (A2)-(A4), it can be seen that $\omega_2 y_1^{(j)}y_3^{(j)}+\omega_3 y_2^{(j)}y_3^{(j)}-b_3 y_3^{(j)2}$ is uniformly bounded, i.e. there exists a constant $\lambda_3^{(j)}$ such that

$$\left|\omega_2 y_1^{(j)}y_3^{(j)}+\omega_3 y_2^{(j)}y_3^{(j)}-b_3 y_3^{(j)2}\right|<\lambda_3^{(j)}. \quad \textbf{(35)}$$

By considering the Lyapunov candidate

$$L(t)=\left(y_3^{(j)}(t)-\hat{y}_3^{(j)}(t)\right)^2+\tfrac{1}{\gamma}\left(\lambda_3^{(j)}-\hat{\lambda}_3^{(j)}(t)\right)^2 \quad \textbf{(36)}$$

and differentiating it, the lemma can be easily proved.

The recursive algorithms of deriving $\hat{y}^{(j)}(t)$, $\hat{y}^{*(j)}(t)$ and $\hat{y}_3^{(j)}(t)$ obviously alleviate the noises in the image data.

By replacing $y^{(j)}(t)$, $y^{*(j)}(t)$ and $y_3^{(j)}(t)$ in the matrix $\begin{bmatrix}\phi^{(j)}(t)\\ \phi^{*(j)}(t)\end{bmatrix}$ with $\hat{y}^{(j)}(t)$,

$\hat{y}^{*(j)}(t)$ and $\hat{y}_3^{(j)}(t)$ recpectively, we get a matrix $\begin{bmatrix} \hat{\phi}^{(j)}(t) \\ \hat{\phi}^{*(j)}(t) \end{bmatrix}$. By combining the matrices $\begin{bmatrix} \hat{\phi}^{(j)}(t) \\ \hat{\phi}^{*(j)}(t) \end{bmatrix}$ together for all *j*, we get the matrix $\hat{\Phi}(t)$ expressed as

$$\hat{\Phi}(t) = \Big[\left(\hat{\phi}^{(1)}(t)\right)^T, \left(\hat{\phi}^{*(1)}(t)\right)^T, \cdots, \left(\hat{\phi}^{(j)}(t)\right)^T, \left(\hat{\phi}^{*(j)}(t)\right)^T,$$

$$\cdots, \left(\hat{\phi}^{(p)}(t)\right)^T, \left(\hat{\phi}^{*(p)}(t)\right)^T \Big]^T . \qquad \textbf{(37)}$$

If $\Phi(t)$ is of full rank, then $\hat{\Phi}(t)$ is of full rank when *t* is large enough. By using Theorem 1, Theorem 2 and Lemma 2, it can be concluded that

$$\lim_{t\to\infty} \left\{ \theta(t) - \left(\hat{\Phi}^T(t)\hat{\Phi}(t) \right)^{-1} \hat{\Phi}^T(t) W(t) \right\} = 0 . \qquad \textbf{(38)}$$

Thus, the motion parameters can be recovered by observing at least three points.

4 Consideration of Occlusion

In the practical applications, the occurrence of occlusion is inevitable. Thus, the algorithm should be modified in order to cope with this phenomenon. In the occurrence of occlusion, the image data of the observed point is not available. Thus, the image data should be replaced by some other virtual signals.

Suppose the *j*-th point is not visible by the Camera 1 defined in (2). The method formulated in Section 3 of deriving $\hat{y}^{(j)}(t)$ and $\hat{y}_3^{(j)}(t)$ is no longer useful. By referring to the dynamics in (5), the virtual signals for $y^{(j)}(t)$ and $y_3^{(j)}(t)$ are constructed by the following dynamical system

$$\begin{cases} \dot{\hat{\hat{y}}}_1(t) = \hat{\omega}_2(t) + \hat{\omega}_1(t)\hat{\hat{y}}_2(t) + \hat{\omega}_2(t)\hat{\hat{y}}_1{}^2(t) + \hat{\omega}_3(t)\hat{\hat{y}}_1(t)\hat{\hat{y}}_2(t) \\ \qquad + \hat{b}_1(t)\hat{\hat{y}}_3(t) - \hat{b}_3(t)\hat{\hat{y}}_1(t)\hat{\hat{y}}_3(t) \\ \dot{\hat{\hat{y}}}_2(t) = \hat{\omega}_3(t) - \hat{\omega}_1(t)\hat{\hat{y}}_1(t) + \hat{\omega}_2(t)\hat{\hat{y}}_1(t)\hat{\hat{y}}_2(t) + \hat{\omega}_3(t)\hat{\hat{y}}_2{}^2(t) \\ \qquad + \hat{b}_2(t)\hat{\hat{y}}_3(t) - \hat{b}_3(t)\hat{\hat{y}}_2(t)\hat{\hat{y}}_3(t) \\ \dot{\hat{\hat{y}}}_3(t) = \hat{\omega}_2(t)\hat{\hat{y}}_1(t)\hat{\hat{y}}_3(t) + \hat{\omega}_3(t)\hat{\hat{y}}_2(t)\hat{\hat{y}}_3(t) - \hat{b}_3(t)\hat{\hat{y}}_3{}^2(t) \end{cases} \qquad \textbf{(39)}$$

where $\hat{\theta}(t) = \left[\hat{b}_1(t), \hat{b}_2(t), \hat{b}_3(t), \hat{\omega}_1(t), \hat{\omega}_2(t), \hat{\omega}_3(t) \right]^T$ is defined by

$$\hat{\theta}(t) = \left(\hat{\hat{\Phi}}(t)\right)^{+} \hat{W}(t)\,; \tag{40}$$

$\hat{\hat{\Phi}}(t)$ denotes the corresponding matrix of $\hat{\Phi}(t)$ defined in (37) where $\hat{\phi}^{(j)}(t)$ is replaced by $\hat{\hat{\phi}}^{(j)}(t)$ defined by

$$\hat{\hat{\phi}}^{(j)}(t) = \begin{bmatrix} \hat{\hat{y}}_3^{(j)} & 0 & -\hat{\hat{y}}_1^{(j)}\hat{\hat{y}}_3^{(j)} & \hat{\hat{y}}_2^{(j)} & 1+\hat{\hat{y}}_1^{(j)2} & \hat{\hat{y}}_1^{(j)}\hat{\hat{y}}_2^{(j)} \\ 0 & \hat{\hat{y}}_3^{(j)} & -\hat{\hat{y}}_2^{(j)}\hat{\hat{y}}_3^{(j)} & -\hat{\hat{y}}_1^{(j)} & \hat{\hat{y}}_1^{(j)}\hat{\hat{y}}_2^{(j)} & 1+\hat{\hat{y}}_2^{(j)2} \end{bmatrix}; \tag{41}$$

$(A)^{+}$ denotes $\left(A^T A\right)^{-1} A^T$ if A is of full rank, or the pseudo-inverse [13] of it if A is not of full rank; $\hat{W}(t)$ denotes the corresponding vector of $W(t)$ defined in (29) in which $w^{(j)}(t)$ is replaced by $\hat{w}^{(j)}(t)$; $\hat{w}^{(j)}(t)$ can be similarly derived by a procedure defined in (12)-(15), where the corresponding image data should be replaced by the virtual data $\hat{\hat{y}}_1^{(j)}(t)$ and $\hat{\hat{y}}_2^{(j)}(t)$; the initial values at the instant $\tau^{(j)}$ when the j-th point begins to be not visible should be chosen as

$$\hat{\hat{y}}_1(\tau^{(j)}) = \hat{y}_1(\tau^{(j)}-0)\,;\ \hat{\hat{y}}_2(\tau^{(j)}) = \hat{y}_2(\tau^{(j)}-0)\,;\ \hat{\hat{y}}_3(\tau^{(j)}) = \hat{y}_3(\tau^{(j)}-0)\,. \tag{42}$$

If the j-th point is not visible by the Camera 2 defined in (3), then the virtual signal for $y^{*(j)}(t)$ should be similarly derived. Furthermore, if the j-th point is not visible by both of the two cameras, the virtual signals for $y^{(j)}(t)$, $y^{*(j)}(t)$ and $y_3^{(j)}(t)$ should be similarly derived.

The convergence of the computed motion parameters can be assured, if the total length of the intervals on which the data from two cameras is available is much longer than the total length of the intervals on which at least one camera is occluded.

5 Simulation Results

The simulation is done by the software Simulink in Matlab. The sampling period Δ is chosen as $\Delta = 0.02$. The measured image data at the sampling point $k\Delta$ is corrupted by a random noise which is in the range of $0.01y(k\Delta)$ (or correspondingly $0.01y^*(k\Delta)$). Consider the movement of the object described by

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 0 & -4 & 0.5 \\ 4 & 0 & 0.4 \\ -0.5 & -0.4 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \sin t \\ \sin t \end{bmatrix}. \tag{43}$$

Four points starting at $[-1, 1, 1]^T$, $[1, -2, 2]^T$, $[0, 3, 3]^T$ and $[2, 0, 2]^T$ are observed. It is assumed that the third and fourth points are not visible by both of the two cameras during the time period [1, 6].

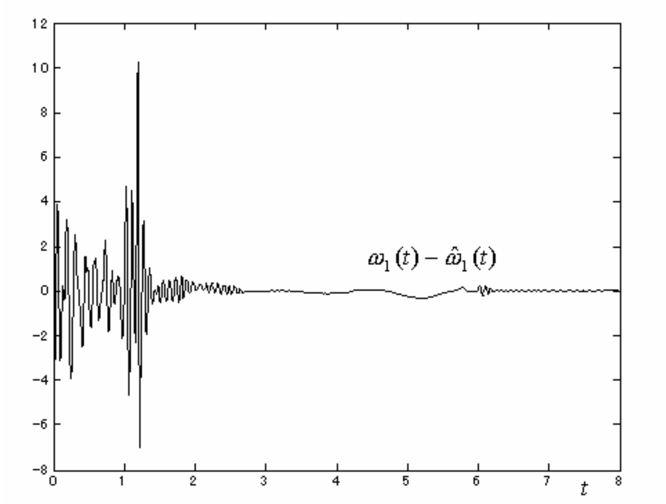

Fig. 1. The difference between $\omega_1(t)$ and $\hat{\omega}_1(t)$.

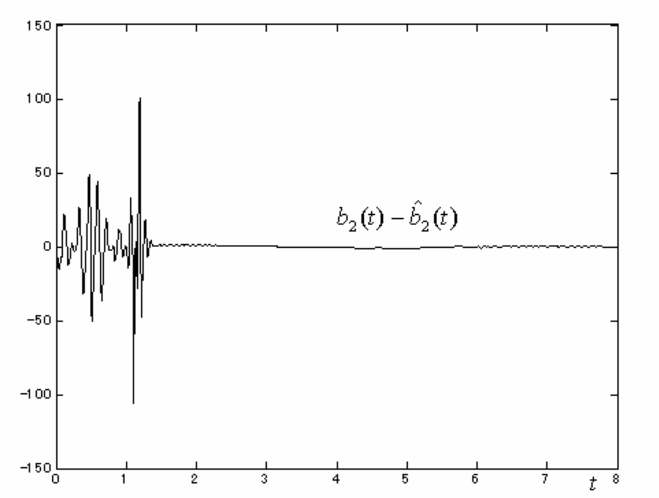

Fig. 2. The difference between $b_2(t)$ and $\hat{b}_2(t)$.

The simulation results are shown in Figures 1-2. The simulation results of the differences $\omega_2(t) - \hat{\omega}_2(t)$ and $\omega_3(t) - \hat{\omega}_3(t)$ are very similar to that in Figure 1. The simulation results of $b_1(t) - \hat{b}_1(t)$ and $b_3(t) - \hat{b}_3(t)$ is very similar to that in Fig. 2. It can be seen that very good estimates for the motion parameters are obtained even in the presence of measurement noises.

6 Conclusions

The motion recovery for a class of movements in the space by using stereo vision has considered by observing multiple (at least three) points in this paper. The considered motion equation can cover a wide class of practical movements in the space. The estimations of the motion parameters which are all time-varying have been developed based on the second method of Lyapunov. The assumptions about the perspective system are reasonable, and the convergence conditions are intuitive and have apparently physical interpretations. The proposed method requires minor *a priori* knowledge about the system and can cope with a much more general class of perspective systems. Furthermore, the algorithm has been modified to deal with the occlusion phenomenon. Simulation results have shown that the proposed algorithm is effective even in the presence of measurement noises.

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