

# Motion Without Movement

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## Abstract

We describe a technique for displaying patterns that appear to move continuously without changing their positions. The method uses a quadrature pair of oriented filters to vary the local phase, giving the sensation of motion. We have used this technique in various computer graphic and scientific visualization applications.

**CR Categories:** I.3.3 [Computer Graphics]: Picture/Image Generation; I.3.7 [Computer Graphics]: Graphics and Realism; I.4.9 [Image Processing]: Applications.

**Keywords:** Motion Display, Oriented Filters, Perception.

## 1 Introduction

Motion display is important in scientific visualization and computer graphic displays. One would often like to “paint” motion onto a scene, just as one can paint color or texture. Existing techniques for indicating motion have drawbacks. Superimposed arrows on a static image add clutter and give no sense of motion. If objects in the scene are set into actual motion in a film loop, then they must periodically snap back to their starting positions, giving a jerky display. Other techniques to display motion include the massively parallel simulation of particle systems [10], and pseudo-color techniques such as color table animation [9; 11].

## 2 Creating the motion illusion

We describe a method for assigning perceptual motion to objects that remain in fixed positions, by applying local filters and continuously varying their phase over time. The technique makes use of perceptual phenomena described by Shadlen *et al.* [8] and Anstis [1], where local phase changes are interpreted as global motions. Shadlen *et al.* produce the effect using patch-wise FFT’s and Hilbert transforms. Our new technique is based on recently developed “steerable filters” [3; 4], which provide a convenient and flexible implementation of the illusion.

Consider two filters, identical except shifted in phase from each other by 90 degrees. The filters are called a quadrature pair, and are related by the Hilbert transform [2]. Figure 1 shows such a pair of even and odd phase filters (chosen for orientation properties to be used later), (a) the second derivative of a Gaussian,  $G_2$ , and (b) its Hilbert transform,  $H_2$ . Filters of intermediate phases, shown in

Fig. 1(c), may be constructed as weighted sums of the even and odd phase filters. Note in Fig. 1(c) that the filter ripples move rightward as the phase is shifted. This sequence of phase-shifted filters was computed using the following formula:

$$F(t) = \cos(\omega t)G_2 + \sin(\omega t)H_2 \quad (1)$$

where  $F$  is the phase-shifted filter,  $\omega$  is the rate of phase shift, and  $t$  is time. When one views a phase-shifting stimulus, like the sequence of filters shown in Fig. 1(c), an interesting visual illusion occurs: it appears that the entire pattern (both the ripples and the modulation envelope) is moving.

This visual illusion of motion can be applied to any spatial pattern; an example is shown in Fig 2. Figs 2(a) and (b) show the results of convolving an image of Albert Einstein with a quadrature pair of filters (see [6] for background on image processing and convolution). Fig 2(c) shows the image sequence computed from combining (a) and (b) according to Eq. (1). When viewed as a temporal sequence, the result is a compelling illusion of continual (rightward) motion of a stationary image.

To display motion in different directions, we need to apply the filters at different orientations. For that, we use “steerable filters”, which allow one to synthesize a filter of arbitrary orientation from a linear combination of basis filters [3; 4]. Fig. 3 shows the basis sets for the  $G_2$  and  $H_2$  filters. One can span the space of all rotations of each filter with these basis sets. Introducing the notation  $f^\theta$  to indicate rotation of the function  $f$  through an angle  $\theta$ , we have, for the function  $G_2$ ,

$$G_2^\theta(x, y) = k_1(\theta)G_{2a}(x, y) + k_2(\theta)G_{2b}(x, y) + k_3(\theta)G_{2c}(x, y) \quad (2)$$

where  $k_i(\theta)$  are the interpolation functions, and  $G_{2a, 2b, 2c}(x, y)$  are the basis functions for  $G_2^\theta(x, y)$ . There is an analogous formula for  $H_2^\theta(x, y)$  with four basis functions. (The number of basis filters required is related to the number of angular frequencies in the filter [3; 4]). Tables 1–4 give filter tap values and interpolation functions for computationally efficient x-y separable versions of the  $G_2$  and  $H_2$  steerable filters. The x-y separable basis filters are shown in Figure 6.

To cause an image,  $I(x, y)$ , to appear to move in a different direction,  $\theta(x, y)$ , at every point  $(x, y)$  in the image, we use the even and odd phase images,  $E(x, y)$  and  $O(x, y)$ :

$$E(x, y) = I(x, y) \otimes G_2^{\theta(x, y)} \quad (3)$$

$$O(x, y) = I(x, y) \otimes H_2^{\theta(x, y)} \quad (4)$$

$$D(x, y, t) = \cos(\omega t)E(x, y) + \sin(\omega t)O(x, y) \quad (5)$$

where  $D(x, y, t)$  is the displayed image sequence,  $\omega$  is the temporal frequency of the displayed motion, and  $\otimes$  represents convolution. With steerable filters, this angularly adaptive filtering is simple to perform. The output at each pixel is simply a linear combination of

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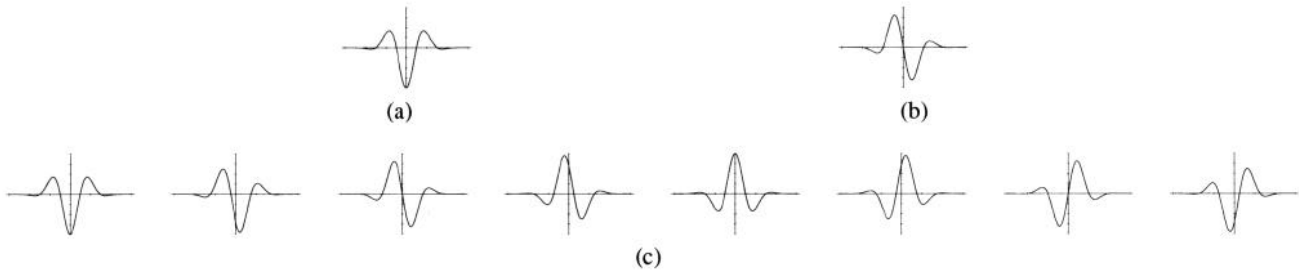


Figure 1: 1-d cross-sections of filters. (a) Even phase ( $G_2$ ). (b) Odd phase ( $H_2$ ). (c) Filters modulated in phase according to Eq. (1). Note the apparent rightward motion of the filter ripples.

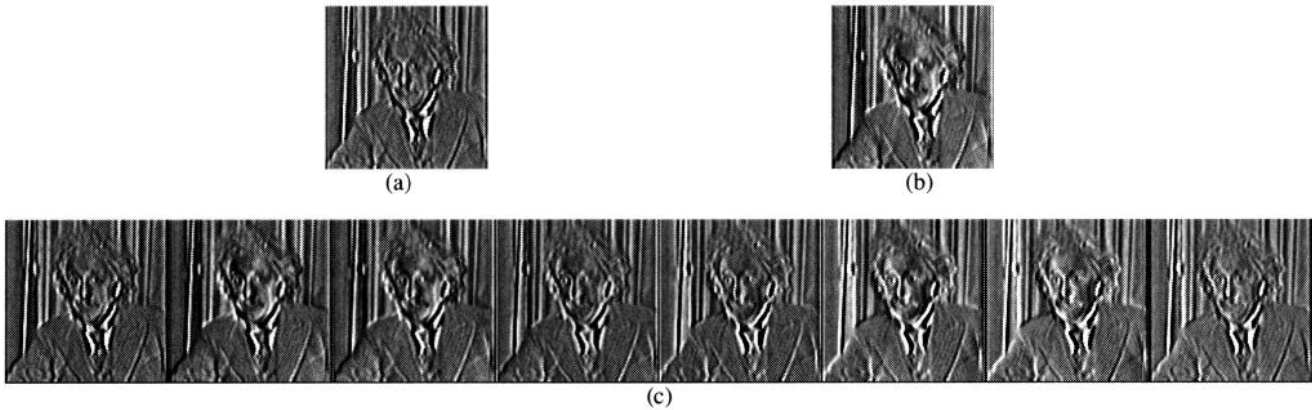


Figure 2: (a) and (b):  $G_2$  and  $H_2$  filters were applied to an image of Einstein. (c) Images modulated as in Eq. (1). When viewed as a temporal sequence, this generates the perception of rightward motion, yet image remains stationary.

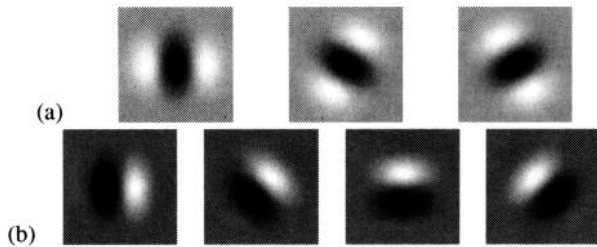


Figure 3: (a)  $G_2$  and (b)  $H_2$  quadrature pair steerable basis filters. The filter sets (a) and (b) span the space of all rotations of their respective filters.

the corresponding pixel values from several basis images. The basis images need to be calculated only once. Motion speed variations may be included by changing  $\omega$  as a function of position (the speed is proportional to  $\omega$ ).

A potential drawback of this technique is that the output images are bandpass filtered along the local direction of motion, and low-pass filtered in the perpendicular direction (reflecting the properties of the oriented filters). To improve the image quality, one can restore some of the frequency components perpendicular to  $\theta(x, y)$  by adding  $-I(x, y) \otimes G_2^{\theta(x, y) + \pi/2}$  to each frame of the processed sequence.

For color images, one can apply the technique to the luminance component of the image, and add in the stationary chrominance components. In the resulting sequence, both the luminance and the chrominance components are perceived to be moving. However, the color gamut is limited to pastelized colors, since there needs to be a positive bias to the luminance, to avoid negative light values.

### 3 Applications

Motion without movement adds a compelling motion cue, with a variety of applications in visualization and computer graphic display. This technique can be used to create a continuous display of instantaneous motion. Figure 4 (a) shows a single frame of a motion sequence of a man catching a teddy bear. From two consecutive frames of the video sequence, instantaneous motion vectors were derived, using the method of Lucas and Kanade [7]. These motion directions, Fig. 4 (b), were used for  $\theta(x, y)$  in Eq. 5 to create the even and odd phase images, Fig. 4 (c) and (d). The resulting sequence, shown in Fig. 4 (e), is a continuous display that corresponds to the motion at one instant. The bear is constantly falling toward the man, who is forever reaching out to grab it.

Figure 5 shows single frames from several other applications of motion without movement. Fig. 5 (a) is a simulated air traffic controller's display, where the airplane velocities are shown by the motion illusion, yet the airplane icons maintain their positions. The sequence of Fig. 5 (b) displays instantaneous velocities in a physical simulation of several objects colliding. Fig. 5 (c) is from a computer art piece entitled "Nude Descending a Staircase" [5], a modern interpretation of Duchamp's painting.

In summary, we have described a flexible technique for indicating motion on graphics display screens. Objects or patterns appear to be continuously moving, yet stay at one position. This compelling motion illusion can be used to continuously display instantaneous motion. We have used this technique in various computer graphic and visualization applications.

### 4 Acknowledgement

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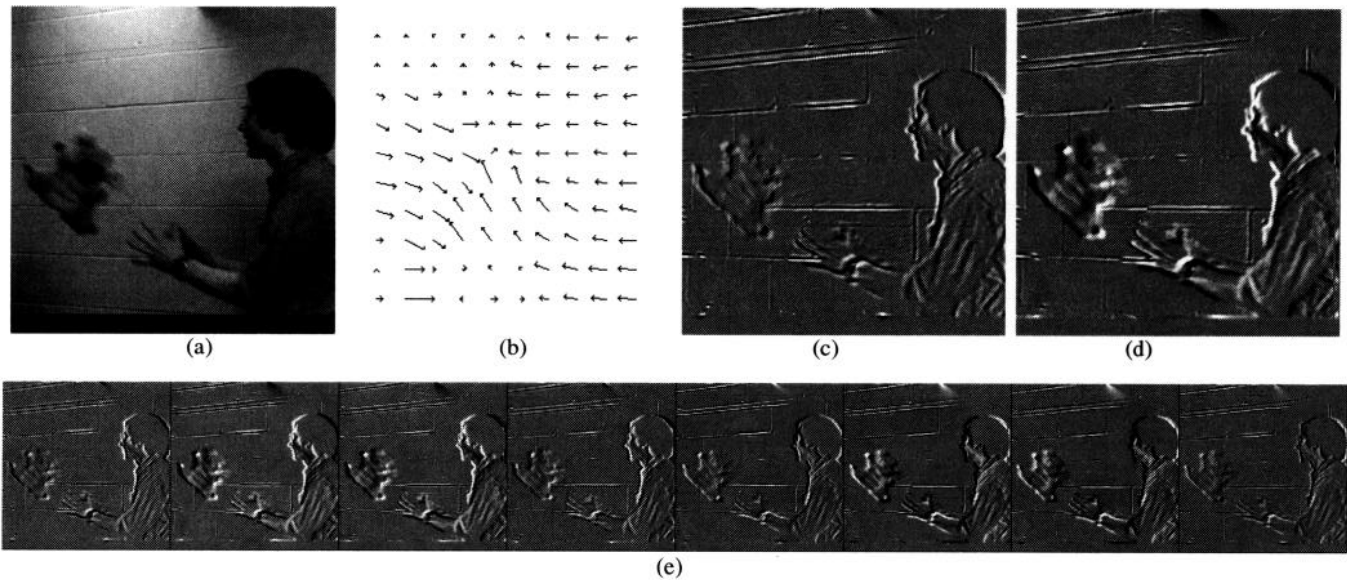


Figure 4: Continuous display of instantaneous motion information. (a) single frame of video sequence. (b) instantaneous motion, obtained from (a) and the frame following it. Even (c) and odd (d) filtered images are combined using Eq. 5 to give the resulting image sequence (e).

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$G_{2a}$	$=$	$0.9213(2x^2 - 1)e^{-(x^2+y^2)}$
$G_{2b}$	$=$	$1.843xye^{-(x^2+y^2)}$
$G_{2c}$	$=$	$0.9213(2y^2 - 1)e^{-(x^2+y^2)}$
$k_a(\theta)$	$=$	$\cos^2(\theta)$
$k_b(\theta)$	$=$	$-2\cos(\theta)\sin(\theta)$
$k_c(\theta)$	$=$	$\sin^2(\theta)$

Table 1:  $X$ - $Y$  separable basis set and interpolation functions for second derivative of Gaussian. To create a second derivative of a Gaussian rotated along to an angle  $\theta$ , use:  $G_2^\theta = k_a(\theta)G_{2a} + k_b(\theta)G_{2b} + k_c(\theta)G_{2c}$ .

tap #	f1	f2	f3
0	-0.9213	1.0	0.0
1	-0.0601	0.6383	0.5806
2	0.3964	0.1660	0.3020
3	0.1148	0.0176	0.0480
4	0.0094	0.0008	0.0028

$G_2$ basis filter	filter in $x$	filter in $y$
$G_{2a}$	f1	f2
$G_{2b}$	f3	f3
$G_{2c}$	f2	f1

Table 2: 9-tap filters for  $x$ - $y$  separable basis set for  $G_2$ . Filters f1 and f2 have even symmetry; f3 has odd symmetry. (The filter tap indices range from  $-4$  to  $4$ . For the even symmetric filters,  $\text{tap}[i] = \text{tap}[-i]$ ; for the odd symmetric filter,  $\text{tap}[i] = -\text{tap}[-i]$ ). These filters were taken from Table 1, with a sample spacing of 0.67. Use the  $k(\theta)$  interpolation functions of Table 1.

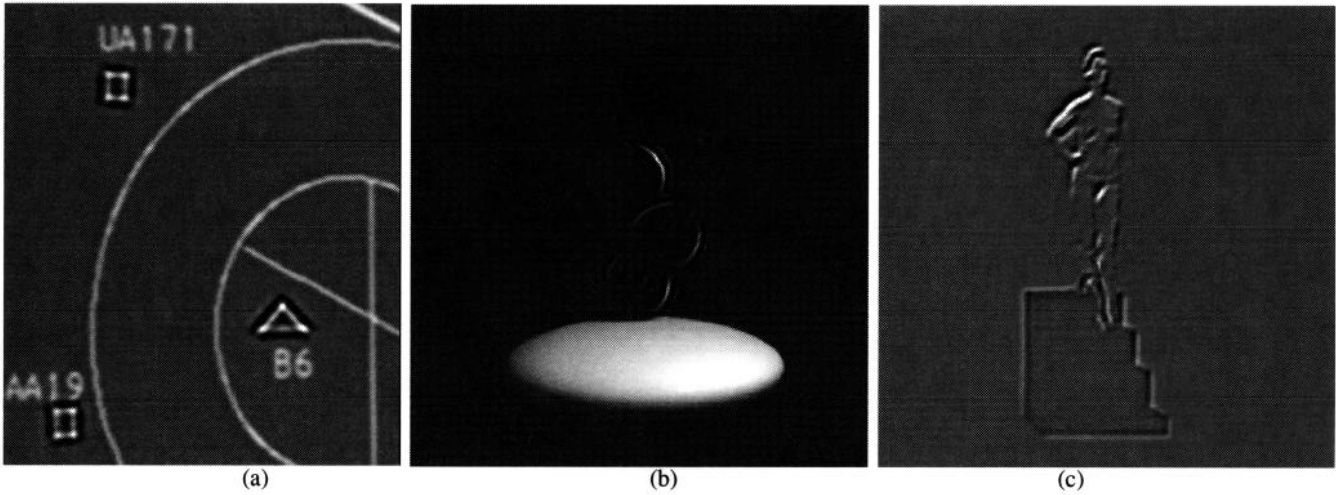


Figure 5: Single frames from several applications of motion without movement. (a) Air traffic controller display: the motion illusion is used to display airplane velocities, while maintaining the proper positions of the airplane symbols. (b) Display of instantaneous velocities of a physical simulation. (c) Computer art piece entitled "Nude Descending a Staircase"[5], after Duchamp.

$H_{2a}$	$= 0.9780(-2.254x + x^3)e^{-(x^2+y^2)}$
$H_{2b}$	$= 0.9780(-.7515 + x^2)(y)e^{-(x^2+y^2)}$
$H_{2c}$	$= 0.9780(-.7515 + y^2)(x)e^{-(x^2+y^2)}$
$H_{2d}$	$= 0.9780(-2.254y + y^3)e^{-(x^2+y^2)}$
$k_a(\theta)$	$= \cos^3(\theta)$
$k_b(\theta)$	$= -3 \cos^2(\theta) \sin(\theta)$
$k_c(\theta)$	$= 3 \cos(\theta) \sin^2(\theta)$
$k_d(\theta)$	$= -\sin^3(\theta)$

Table 3:  $H_2$  basis set:  $x$ - $y$  separable basis set and interpolation functions for fit to Hilbert transform of second derivative of Gaussian. To synthesize a filter oriented along direction  $\theta$ , use:  $H_2^\theta = k_a(\theta)H_{2a} + k_b(\theta)H_{2b} + k_c(\theta)H_{2c} + k_d(\theta)H_{2d}$ .

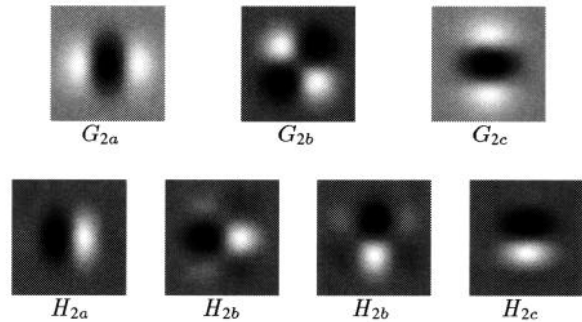


Figure 6: X-Y separable basis filters for  $G_2$ , listed in Tables 1 and 2, and  $H_2$ , listed in Tables 3 and 4.

tap #	f1	f2	f3	f4
0	0.0	1.0	0.0	-0.7349
1	-0.7551	0.6383	0.4277	-0.1889
2	-0.0998	0.1660	0.2225	0.1695
3	0.0618	0.0176	0.0354	0.0566
4	0.0098	0.0008	0.0020	0.0048

$H_2$ basis filter	filter in $x$	filter in $y$
$H_{2a}$	f1	f2
$H_{2b}$	f4	f3
$H_{2c}$	f3	f4
$H_{2d}$	f2	f1

Table 4: 9-tap filters for  $x$ - $y$  separable basis set for  $H_2$ . Filters for which tap 0 is 0.0 have odd symmetry about tap 0; the others have even symmetry. These filters were taken from Table 3, with a sample spacing of 0.67. Use the interpolation functions of Table 3.