Journal of Glaciology, Vol. 11, No. 62, 1972

MOVEMENT OF WATER IN GLACIERS*

By R. L. SHREVE

(University of California, Los Angeles, California 90024, U.S.A.)

ABSTRACT. A network of passages situated along three-grain intersections enables water to percolate through temperate glacier ice. The deformability of the ice allows the passages to expand and contract in response to changes in pressure, and melting of the passage walls by heat generated by viscous dissipation and carried by above-freezing water causes the larger passages gradually to increase in size at the expense of the smaller ones. Thus, the behavior of the passages is primarily the result of three basic characteristics: (1) the capacity of the system continually adjusts, though not instantly, to fluctuations in the supply of melt water; (2) the direction of movement of the water is determined mainly by the ambient pressure in the ice, which in turn is governed primarily by the slope of the ice surface and secondarily by the local topography of the glacier bed; and, most important, (3) the network of passages tends in time to become arborescent, with a superglacial part much like an ordinary river system in a karst region, an englacial part comprised of tree-like systems of passages penetrating the ice from bed to surface, and a subglacial part consisting of tunnels in the ice carrying water and sediment along the glacier bed. These characteristics indicate that a sheet-like basal water layer under a glacier would normally be unstable, the stable form being tunnels; and they explain, among other things, why ice-marginal melt-water streams and lakes are so common, why eskers, which are generally considered to have formed in subglacial passages, trend in the general direction of ice flow with a tendency to follow valley floors and to cross divides at their lowest points, why they are typically discontinuous where they cross ridge crests, why they sometimes contain fragments from bedrock outcrops near the esker but not actually crossed by it, and why they seem to be formed mostly during the later stages of glaciation.

RESUMÉ. *Mouvement de l'eau dans les glaciers*. Un réseau de canaux situés le long des lignes de contact entre trois grains permet à l'eau de percoler à travers la glace des glaciers tempérés. Grâce à la capacité de déformation de la glace, ces canaux peuvent se dilater ou se contracter selon les variations de pression; la chaleur engendrée par l'écoulement visqueux et transportée par l'eau au-dessus de son point de fusion entraine la fusion des parois des canaux et aboutit à accroître les dimensions des plus grands canaux aux dépens des plus petits. Ainsi, le comportement des canaux est en premier lieu le résultat de trois caractéristiques fondamentales: (1) la capacité pour le système de s'ajuster continuellement, quoique non instantanément, aux fluctuations dans les apports d'eau de fusion; (2) la direction du mouvement de l'eau est déterminée surtout par la pression ambiante dans la glace qui à son tour est gouvernée d'abord par la pente de la surface de la glace puis par la topographie locale du lit glaciaire; enfin, et c'est le plus important, (3) le réseau des canaux tend, avec le temps, à devenir arborescent avec une partie superficielle ressemblant beaucoup à un réseau hydrographique dans une région karstique, une partie intraglaciaire composée d'un système de canaux en forme d'arbre pénétrant la glace du fond vers la surface, et une partie sous-glaciaire consistant en tunnels dans la glace charriant de l'eau et des sédiments le long du lit glaciaire. Ces caractéristiques indiquent qu'un niveau d'eau étendu comme un drap sur le lit d'un glacier devrait normalement être une formation instable, la forme stable étant celle des tunnels; ceci explique, entre autres, pourquoi les torrents et lacs d'eau de fusion sont si communs le long des rives des glaciers, pourquoi les eskers, qui sont généralement considérés comme ayant été formés par des canaux sous-glaciaires, tendent à s'aligner dans la direction générale de l'écoulement de la glace avec une tendance à suivre le fond de la vallée et à traverser les obstacles en leur point le plus bas, pourquoi ils sont typiquement interrompus lorsqu'ils traversent la crête d'une ondulation, pourquoi ils contiennent parfois des fragments prélevés sur le lit près de l'esker mais non exactement sur sa trajectoire et pourquoi ils semblent se former surtout au cours des derniers stades d'une glaciation.

ZUSAMMENFASSUNG. *Bewegung von Wasser in Gletschern*. Ein Netzwerk von Durchlässen entlang von Dreifach-Kornverschneidungen ermöglicht das Durchsickern von Wasser durch das Eis temperierter Gletscher. Das Deformationsvermögen des Eises erlaubt in Abhängigkeit von Druckänderungen eine Erweiterung oder Verengung der Durchlässe; Schmelzen der Durchlasswände durch Wärme, die durch viskose Dissipation erzeugt und durch auffrierendes Wasser geleitet wird, verursacht ein stetiges Wachsen der grösseren Durchlässe auf Kosten der kleineren. Somit ist das Verhalten der Durchlässe im wesentlichen durch drei Grundtatsachen bestimmt: (1) Die Kapazität des Systems passt sich ständig, wenn auch nicht sofort, den Schwankungen im Schmelzwassernachschub an; (2) Die Richtung der Wasserbewegung wird vornehmlich durch den umgebenden Druck im Eis bestimmt, der seinerseits primär von der Neigung der Eisoberfläche und sekundär von der lokalen Topographie des Gletscherbettes abhängt; und vor allem (3) Das Netzwerk der Durchlässe neigt dazu, sich mit der Zeit baumartig zu verzweigen, mit einem Teil an der Gletscheroberfläche, der sehr einem gewöhnlichen Flusssystem in einer Karstregion ähnelt, einem innerglazialen Teil, der baumartige Durchlasssysteme enthält, die das Eis vom Grund bis zur Oberfläche durchsetzen, und einem subglazialen Teil, bestehend aus Tunneln im Eis, in denen Wasser und Sedimente am Gletscherbett entlanggeführt werden. Diese Tatsachen zeigen, dass eine blattförmige Wasserschicht am Grunde eines Gletschers normalerweise instabil ist; die stabile Form sind Tunnels. Darüber hinaus

* Publication 956 of the Institute of Geophysics and Planetary Physics, University of California, Los Angeles.

2

https://doi.org/10.3189/S002214300002219X Published online by Cambridge University Press

erklären sie neben anderen Erscheinungen, warum Schmelzwasserströme und Seen am Gletscherrand so häufig auftreten; warum Esker, deren Entstehung allgemein in subglazialen Kanälen angenommen wird, in der Hauptfliessrichtung des Eises verlaufen mit einer Tendenz, Talböden zu folgen und Wasserscheiden an ihren niedrigsten Stellen zu überqueren; warum sie beim Queren von Rücken so typisch unzusammenhängend sind; warum sie manchmal Bruchstücke von Gestein des Untergrundes enthalten, das zwar nahe am Esker ansteht, aber nicht von ihm erfasst wird, und warum sie sich anscheinend meistens während der Spätstadien einer Vergletscherung gebildet haben.

Mechanics of water passages

The water in temperate glaciers comes from a variety of sources. Melt water produced at the surface is by far the most important; its volume is ordinarily orders of magnitude greater than the others. Melt water produced within the ice by heat generated by the internal deformation and at the bed by geothermal heat from below and by heat conducted down the pressure-melting gradient from above is also significant, and is typically of the order of 10 mm of water per year per unit area of bed. Other very minor sources of water are melting due to increases in concentration of solutes, such as air or salts, in the glacier, to increases in pressure, and to viscous dissipation in the layer of melt water at the bed.

Recent observations (Nye and Frank, in press) prove the existence of a network of water-filled passages situated along the three-grain intersections in polycrystalline ice under hydrostatic stress. In nonhydrostatically stressed ice, however, the conditions for existence of the passages are considerably more restricted (Lliboutry, 1971, p. 18–20; Nye and Mae, 1972). Nevertheless, considering the multitude of orientations and interconnections of its three-grain intersections, typical temperate glacier ice almost certainly is at least slightly permeable to the flow of water. If so, then, taking account also of the larger passages through the ice, such as moulins and glacier tunnels, water can move through a glacier in a manner somewhat analogous to the movement of ground water through permeable cavernous limestone.

Ice is far more deformable than limestone, however, so that passages in it can expand and contract significantly in response to the normal increases and decreases in water pressure on their walls relative to the ambient pressure in the enclosing ice, as originally pointed out by Glen (1954) and recognized by Nye and Frank (in press). The two pressures therefore tend to equalize, and so Nye and Frank considered them to be approximately equal. In reality, however, melting of the passage walls by heat generated by viscous dissipation or carried by above-freezing water (from the surface, for example) continually tends to enlarge the passages. This in turn tends to lower the pressure in them, and is therefore compensated by inward flowage of the ice. Moreover, it causes the larger passages in the network gradually to increase in size at the expense of the smaller ones, because more heat relative to wall area is carried by above-freezing water and generated by viscous dissipation in the larger passages than in the smaller ones. Thus, the behavior of the water passages is primarily the result of three basic characteristics: (1) the capacity of the system of passages continually adjusts, though not instantly, to increases or decreases in the supply of melt water; (2) in a steady state the pressure, and hence the movement, of water in them is governed primarily by the ambient pressure in the enclosing ice and secondarily by the rate of melting of the passage walls; and, most important, (3) because of the differential growth of the larger passages, the network of passages tends in time to become arborescent, with tributaries joining into ever-larger trunk passages, like a three-dimensional river network. As will be seen, these three characteristics readily explain many puzzling features of glacier hydraulics and glacial deposits.

Movement of water. These considerations can readily be made more quantitative. Assume as a rough approximation that the passages have circular cross-sections, which is the stable form for a contracting hole (Nye, 1965). Let p_w and p_i be the water pressure in the passage and the ambient pressure in the enclosing ice at the same level; let a and $\dot{a}$ be the radius of the passage and its rate of increase; let M be the rate of melting of the passage wall (that is, thickness melted away per unit time); and, following standard practice (Nye, 1953, p. 478–

80), assume that to good approximation the ice is isotropic and incompressible and deforms according to the flow law

$$\dot{\epsilon} = (\tau/A)^n, \tag{1}$$

in which n and A are constants and $\dot{\epsilon}$ and τ are the effective strain-rate and effective shear stress. Then straightforward dimensional analysis shows that, provided the passage remains circular and variations in surface energy are negligible,

$$p_{\mathrm{w}} = p_{\mathrm{i}} + KnA \left| \frac{\dot{a} - M}{a} \right|^{1/n} \operatorname{sign}\,(\dot{a} - M), \tag{2}$$

where K is a function of the general rate of deformation of the ice beyond the influence of the passage. If there is no general deformation, then $K = 1$ (Nye, 1953, p. 482). In the majority of cases p_{i} is to good approximation equal to the pressure due to the weight of the overlying ice, that is,

$$p_{\mathrm{i}} = \rho_{\mathrm{i}} g (H - z), \tag{3}$$

where ρ_{i} is the density of ice, g is the acceleration due to gravity, H is the elevation of the ice surface, and z is the elevation of the point considered.

The water tends to move through the network of passages in the direction of the negative of the gradient of the potential Φ, which is given by

$$\Phi = \Phi_0 + p_{\mathrm{w}} + \rho_{\mathrm{w}} g z, \tag{4}$$

or, combining with Equations (2) and (3), by

$$\Phi = \Phi_0 + \rho_{\mathrm{i}} g H + (\rho_{\mathrm{w}} - \rho_{\mathrm{i}}) g z + KnA \left| \frac{\dot{a} - M}{a} \right|^{1/n} \operatorname{sign}\,(\dot{a} - M), \tag{5}$$

in which ρ_{w} is the density of water and Φ_0 is an arbitrary constant. This equation is the same as that of Nye and Frank (in press) except for the additional term involving the melting rate. Strictly, Φ is defined only within the water-filled passages, but with suitable caution it is permissible and more convenient for most purposes to treat it as if it were defined throughout the ice.

As remarked by Nye and Frank (in press), the situation is very similar to that of a liquid phase moving through the mantle of the earth (Frank, 1968), except that in the mantle H is essentially constant, ρ_{w} is less than ρ_{i}, and therefore the liquid moves upward, whereas in a glacier H varies with position, ρ_{w} is greater than ρ_{i}, and the water moves steeply downward in the direction of the surface slope.

It is worth emphasizing that it is not high hydrostatic pressure *per se* that causes the water to move, nor is it necessarily a gradient in pressure, even were the ice incapable of responding to changes in discharge, for both high hydrostatic pressure and a gradient of pressure exist in any deep body of standing water. Rather it is the gradient of the excess of pressure over hydrostatic that causes flow.

Differential growth of passages. Consider two passages of radii a and b and the same length and slope connecting the same two nearby points in the ice and jointly carrying a fixed discharge Q. Thus, at corresponding points

$$\Phi_a = \Phi_b \tag{6}$$

and

$$Q_a + Q_b = Q, \tag{7}$$

a constant, where the subscripts refer to the respective passages. Substituting Equation (5) into Equation (6) gives

$$\frac{\dot{a} - M_a}{a} = \frac{\dot{b} - M_b}{b}. \tag{8}$$

https://doi.org/10.3189/S002214300002219X Published online by Cambridge University Press

Then eliminating $\dot{b}$ by means of the relationship

$$\dot{b} = -\dot{a}\,\frac{\partial Q_a/\partial a}{\partial Q_b/\partial b}, \tag{9}$$

obtained by differentiating Equation (7) leads to

$$\dot{a} = \left(\frac{M_a}{a} - \frac{M_b}{b}\right) \bigg/ \left(\frac{1}{a} + \frac{1}{b}\,\frac{\partial Q_a/\partial a}{\partial Q_b/\partial b}\right). \tag{10}$$

If $\dot{a}$ is positive for a greater than b, the larger passage will increase in size at the expense of the smaller, except for the very smallest passages, where (Nye and Frank, in press) this tendency will be opposed by an increase in surface energy.

The problem, therefore, is to find the conditions under which the right-hand side of Equation (10) is positive. As a first step, it can be shown that the denominator is always positive and hence does not affect the sign of $\dot{a}$. Noting that

$$Q_a = \pi a^2 V \tag{11}$$

and rewriting it in terms of the Reynolds number

$$(Re) = 2\rho_w a V/\mu, \tag{12}$$

where V is the mean velocity of water in the passage and μ is its viscosity, gives

$$Q_a = \frac{\pi\mu a (Re)}{2\rho_w}. \tag{13}$$

The Reynolds number (Re), however, is an implicitly known function of a, because the resistance coefficient f, defined by

$$f = -\frac{16\rho_w a^3 \partial\Phi/\partial s}{\mu^2 (Re)^2}, \tag{14}$$

in which s is the distance along the passage in the direction of flow, is an experimentally known function of (Re). Thus, taking logarithms of both sides of Equation (14), differentiating with respect to ln (Re), and using Equation (13) and the fact that $\partial\Phi/\partial s$ is to be treated as constant, and hence Q_a as a function only of a, leads to

$$\frac{\partial Q_a}{\partial a} = \frac{Q}{a}\left(5 + \frac{\partial \ln f}{\partial \ln (Re)}\right) \bigg/ \left(2 + \frac{\partial \ln f}{\partial \ln (Re)}\right), \tag{15}$$

which is positive except if

$$-5 < \frac{\partial \ln f}{\partial \ln (Re)} < -2. \tag{16}$$

But $\partial \ln f/\partial \ln (Re)$ can never be less than -2, because, if it were, the discharge in a pipe of fixed diameter would decrease with increasing pressure gradient. Therefore, $\partial Q_a/\partial a$ is always positive and hence so is the denominator in Equation (10). Thus, for $a > b$, $\dot{a}$ is positive, and the network of passages tends to become arborescent, if $M_a/a > M_b/b$.

If the melting rate is entirely due to local viscous dissipation in the water passage, then

$$M_a = -\frac{Q_a[(1-k)\,\partial\Phi/\partial s + k\rho_w g\,\partial z/\partial s]}{2\pi a \rho_i \lambda}, \tag{17}$$

in which λ is the latent heat of fusion of the ice and $k = \rho_w c_w \gamma$, where c_w is the specific heat capacity of the water and γ is the rate of decrease of melting temperature of ice with pressure. The terms involving k in the numerator account for the heat required to adjust the water to the melting temperature, which in general changes down-stream. As pointed out by Röthlisberger (1972), this term is not negligible, inasmuch as $k = 0.313$. Next, dividing both sides by a, differentiating with respect to a, and using Equation (15) gives

$$\frac{\partial(M_a/a)}{\partial a} = \frac{M_a}{a^2}\left(1 - \frac{\partial \ln f}{\partial \ln (Re)}\right) \bigg/ \left(2 + \frac{\partial \ln f}{\partial \ln (Re)}\right). \tag{18}$$

This is positive if

$$-2 < \frac{\partial \ln f}{\partial \ln (Re)} < 1, \tag{19}$$

that is, if on the usual double-logarithmic plot the curve of resistance coefficient versus Reynolds number has slope less than 1. Comparison with the experimental curves (for example, Rouse, 1938, p. 250, or Schlichting, 1955, p. 418) shows that, unless the relative roughness increases improbably rapidly with increasing Reynolds number, this would always be the case. Thus, $M_a/a > M_b/b$ for $a > b$, and therefore viscous dissipation causes the larger passage to increase in size at the expense of the smaller.

An analysis of the effect of heat carried by above-freezing water from, say, the surface could be carried out by well-known methods (for example, Bird and others, 1960, p. 396–407), but would be much more difficult, because in this case the rate of melting in any given short segment of passage, and hence the rate of temperature drop along it, is dependent on the mean temperature of the water in it, so that the rate of melting at any one point in the system is dependent upon conditions throughout the system. Nevertheless, it may be anticipated that a similar result would be found, inasmuch as more heat relative to wall area is carried in the larger passages just as more heat relative to wall area is generated in them. Moreover, even for temperatures only very slightly above freezing the effect would be much larger. This can be seen by solving for the temperature drop $-\partial\theta/\partial s$ per unit distance in the direction of flow required to make the two melting rates equal. The melting rate due to the heat carried by the water is

$$M_a = -\frac{Q_a \rho_w c\, \partial\theta/\partial s}{2\pi a \rho_i \lambda}, \tag{20}$$

where c is the specific heat capacity of water. Equating to the melting rate due to viscous dissipation given by Equation (17) and solving for $-\partial\theta/\partial s$ gives a temperature drop of 0.2 deg km^{-1}, assuming that the passage is oriented straight down the gradient of Φ, and neglecting the small effect of the melting rate on the gradient of Φ. For a horizontal passage in the direction of a typical surface gradient grad H of 10 m km^{-1}, the drop would be only a tenth as much.

The heat generated by viscous dissipation alone can easily produce significant changes in the network of water passages in the space of a single melt season. The melting rates in the two cases just discussed, for example, would be approximately 150 m a^{-1} and 5 m a^{-1}, assuming a passage 1 m in diameter with a relative roughness of 10^{-2} and using the resistance formula for the completely rough regime (Schlichting, 1955, p. 422).

Direct evidence of significant melting of passage walls and the resultant inward flow of ice is present on many glaciers in the form of "confluent structures" (see Taylor, 1963, p. 744, for excellent photograph), which are usually attributed to collapse of abandoned moulins (Taylor, 1963, p. 746) rather than to flow toward active passages.

Near the glacier surface the melting enlarges moulins so much relative to the inflow of ice that the water falls freely in them to great depth. For the same reason subglacial passages debouching into surface streams are only partially filled by the water flowing in them for some distance above their outlets. For instance, taking the previous example of an englacial passage in which the melting rate is 150 m a^{-1}, making the conservative over-estimate that this rate occurs even in the enlarged upper parts of the moulin, which are partially filled with air at atmospheric pressure, and using Equations (2) and (3) with the values for n and A given by Shreve and Sharp (1970, p. 84), shows that the point at which the diameter narrows to 1 m, and hence the level of the water surface in the moulin, would lie at least 250 m below the ice surface.

The strong diurnal variations in run-off from temperate glaciers (Collier, 1958, p. 353; Meier and Tangborn, 1961, p. B-15–B-16; Mathews, 1964[a], p. 294; Paterson, 1964, p. 279, 282, 283) and the short travel times of dye through them (Meier, 1965, p. 762) agree with the conclusion advanced here that their drainage is through relatively large passages rather than narrow capillaries within the ice or a thin sheet under it, as implied by Meier (1965, p. 762), though it does not prove it, as pointed out by Paterson (1965, p. 875).

Non-steady states. For the sake of simplicity these effects have been discussed as if the rate of supply of melt water were fixed and a steady state attained, whereas on an actual glacier it varies with daily and annual cycles and fluctuates erratically with changes in weather, so that a steady state is never quite attained. The possible non-steady, or transient, states are much more difficult to discuss than the steady state, because the response at any one point depends in part on conditions in the whole system of passages rather than only on those in the immediate vicinity. Perhaps the most important characteristic of the system is the great disparity in response times of the water and the ice. Because the water table in the steady state stands some distance below the ice surface, as pointed out in connection with moulins, changes in water supply from this state will quickly change the level of the water table, which in turn will almost instantaneously be reflected in changes in both pressure and discharge, and hence the melting rate, throughout the system. The maximum changes possible are those that either raise the water table to the glacier surface, as during the peaks of summer melting, or lower it nearly to the glacier bed, as during the depths of winter freezing. The response of the ice, however, will be propagated through the system much less rapidly as kinematic waves (Lighthill and Whitham, 1955), as previously pointed out by Nye and Frank (in press). In this case, however, the relationship between flow and concentration (that is, between discharge and stage, in hydrological terms) will be a partial differential equation instead of a simple functional relationship. Moreover, for large variations, it will be non-linear, owing to the non-linear response of the ice. Broadly speaking, the system of passages will tend to ignore more or less the short-term fluctuations in water supply but to follow closely the long-term ones. The dividing line between the two types of response will be gradational and will depend on such factors as size, positions, and interconnections of the passages, rates of melting of the walls, and magnitude of the change in discharge. An estimate based on the strong diurnal variations in run-off and on the limited data on variations of water table (Shreve and Sharp, 1970, p. 68, 71–73) and water pressure (Mathews, 1964[b], p. 236–40) in glaciers, is that it is at least of the order of a week but probably not of the order of months. Thus, depending on its characteristics, a particular passage in the system will tend to fluctuate about the steady state appropriate to the average water supply of the preceding week or weeks.

Glacial drainage system

As already pointed out, the growth of the larger passages at the expense of the smaller will lead to progressive capture of drainage by the largest passages and hence the gradual development of arborescent networks of passages, much like ordinary river systems. Drainage systems in temperate glaciers during the melt season, however, will consist of three distinct parts, which for convenience may be termed *superglacial*, *englacial*, and *subglacial*. In subpolar glaciers that are at the melting temperature only at or near their beds, or in temperate glaciers during winter, all of the superglacial part and some or all of the englacial part will be missing.

Superglacial part. The superglacial part will be the most like an ordinary river system. It will consist of networks of surface rivulets and channels that eventually enter the englacial system through moulins or crevasses, much like the run-off in a karst region. It will exist only during the transient periods when the capacities of the englacial and subglacial systems

https://doi.org/10.3189/S002214300002219X Published online by Cambridge University Press

are adjusting to increases in melt-water production at the surface. Except on valley glaciers and near the margins of ice sheets, where rock debris is commonly present on the ice surface, it probably will not carry clastic sediments.

Englacial part. The englacial part will consist of literally tree-like systems of passages penetrating the ice from bed to surface. Some of the larger passages will reach all the way to the surface as moulins to receive the superglacial run-off, but most will divide and redivide upward to drain the water percolating downward through the anastomosing three-dimensional network formed by the very smallest passages, namely, the system of capillaries discussed by Nye and Frank (in press). The large cavities that on rare occasions are intersected at depth during drilling operations on glaciers (Savage and Paterson, 1963, p. 4522; Shreve and Sharp, 1970, p. 68) may be trunk passages of the englacial drainage system. The general direction of drainage will be perpendicular to the equipotentials of Φ, which in turn will dip up-glacier at an average angle

$$\alpha = \arctan [\rho_i|\text{grad } H|/(\rho_w - \rho_i)], \tag{21}$$

found from Equation (5); that is, they will slope downward in the up-glacier direction with a gradient approximately 11 times that of the surface.

The average equipotential surfaces are dimpled at the water passages owing to the inward flow of ice required to balance the melting of the passage walls. The argument showing that the larger passages will grow at the expense of the smaller implies also that the larger passages will produce the largest dimples, which is another mechanism tending to divert drainage to them.

Subglacial part. The subglacial part of the drainage system will be more complex and interesting than the other two, because of the presence of a sediment load and because of the influence of bed topography. It is the only part to leave a permanent record after the glacier is gone. It will receive large increments of discharge at points where the trunk passages of the englacial part of the system reach the glacier bed, somewhat as a river system in a karst region is fed by large springs. Much or all of its sediment load will be derived by melting of the debris-laden basal ice and by erosion of the glacier bed.

The subglacial passages will follow the lines of steepest rate of decrease of Φ on the glacier bed (see Weertman, 1966, p. 206–07, for the derivation of a similar result for a thin sheet of water at the glacier bed, except for the term involving melting rate). Thus, on a level bed they will trend in the direction of surface slope, and hence the direction of flow, of the ice. Where the bed is not level, they will still trend in the general direction of ice flow, but will tend to follow valley floors and to cross divides at their lowest points. These inferences exactly describe the behavior of eskers (Flint, [1971], p. 215–16; Charlesworth, 1957, p. 421) and tunnel valleys (Embleton and King, 1968, p. 279–80), the bulk of which are generally considered to have been formed by water flowing in subglacial passages (Flint, [1971], p. 216–18; Embleton and King, 1968, p. 279–80, 369–70). In general, the steeper the ice surface, the less the subglacial passages will be influenced by bed topography.

The equipotentials of Φ near the bed will dip up-glacier in a general way as described by Equations (5) and (21), but will be distorted by the pressures induced in the ice by flow over subglacial topography. The pressure will be elevated on the up-glacier side of a hill or transverse ridge, for example, and therefore the equipotentials will be depressed there. Analysis of this and other effects of the large-scale bed topography ought to be possible by an approach similar to that recently used by Collins (1968), Nye (1969), and Budd (1970[a], [b]) to analyze ice flow over non-planar beds.

Even without detailed analysis, however, it is easy to see that the spacing of equipotentials is closest, and hence the transporting capacity of subglacial passages will be largest, near transverse ridge crests, in agreement with the commonly observed fact that eskers are likely to be discontinuous where they cross divides (Flint, [1971], p. 215).

As another example, it is easy to see that the typical outward slope of the ice surface near glacier margins results in a gradient of Φ that drives the englacial and subglacial waters outward, except in those special situations where the equipotentials dip less steeply than the valley wall. Such situations are favored by low marginal ice slopes, by steep valley walls, and by bedrock spurs or bosses that depress the equipotentials. This explains why marginal melt-water streams and shallow lakes are so common (Flint, [1971], p. 228–32; Charlesworth, 1957, p. 460–61; Embleton and King, 1968, p. 272–73), especially against gently sloping hillsides (Embleton and King, 1968, p. 272), and are most typical of active glaciers (Embleton and King, 1968, p. 273), which will tend to have steeper marginal slopes.

Subglacial lakes (Charlesworth, 1957, p. 455; Embleton and King, 1968, p. 425) will form in closed areas of lower Φ, just as ordinary lakes form in closed areas of lower elevation, and like ordinary lakes they should be sites of deltaic and relatively quiet-water sedimentation. Deposits answering this description occur in esker systems (Flint, [1971], p. 215, 218; Charlesworth, 1957, p. 421), but have generally been interpreted as proglacial rather than subglacial, in part at least because subglacial origin usually has not even been considered as a possibility.

Because the presence of a subglacial lake significantly influences the flow of the nearby ice, the position of the lake and the shape of its ice roof are not easy to deduce quantitatively. A qualitative guess is that the lake would be displaced somewhat down-glacier and its roof would dip up-glacier less steeply at the up-glacier and down-glacier edges and more steeply in the middle than the highest closed equipotential surface of Φ calculated as if the lake were not present.

The shape of subglacial passages carrying water and sediment, such as those in which apparently most tunnel valleys were carved and most eskers were deposited, presents a somewhat different problem. In this case the general flow of the ice is not much affected, and the important processes are flow toward the passage and melting of the walls. Without melting, the stable form of the passage is broad and flat, owing to the greater density of water compared to ice. This may be the situation in which tunnel valleys are produced, and it is essentially the situation envisaged by Weertman (1966) in his analysis of the effect a basal water layer will have on an ice sheet. With melting, on the other hand, the stable form is tunnel-like, owing to the higher melting rate in the larger parts of the passage. In this case the sheet-like water layer envisaged by Weertman is unstable, and it would exist only transiently during periods of rapid increase in discharge or decrease in melting rate in the passages. Thus, a relatively high rate of melting, and probably, therefore, a substantial supply of surface melt water, appears to be necessary for the formation of eskers. This might account for the observation (Charlesworth, 1957, p. 423, 428; Wright, unpublished) that tunnel valleys are often early features, perhaps formed when the surface ice was still subfreezing, whereas eskers are usually late features. The argument also shows that a small esker in a wide tunnel valley could have been formed with no decrease in discharge of the subglacial stream, or even with an increase, because of the lower resistance to flow of the tunnel-like shape.

The inflow of debris-laden basal ice in response to the melting of the passage walls would carry sediment to the subglacial stream somewhat as mass movement on slopes carries it to ordinary rivers. This explains why esker sediments are typically closely similar to the till in the vicinity (Flint, [1971], p. 215; Charlesworth, 1957, p. 421; Embleton and King, 1968, p. 378–79) and why they sometimes contain fragments from bedrock outcrops near the esker but not actually crossed by it (Trefethen and Trefethen, 1944, p. 525–27). In addition, the faster inflow of basal debris closer to the subglacial passage might account for some of the marginal troughs that commonly flank eskers (Charlesworth, 1957, p. 420).

The pseudo-anticlinal structure of the bedding in many eskers, which in part may be due to slumping of the side slopes but in at least some cases is clearly primary (Sharp, 1953,

p. 872; Flint, [1971], p. 215–16; Charlesworth, 1957, p. 421; Embleton and King, 1968, p. 369), suggests that during deposition much or all of the surface of the esker in such cases must have been in contact with the water, rather than with the ice walls. This requires secondary currents flowing upward along the flanks of the growing esker, for otherwise sediment would move down the slopes and eventually be deposited in flat beds. Secondary currents are a well known feature of turbulent flow in non-circular pipes. They generally flow toward sharp corners from the central region of the pipe, then outward along the walls (Rouse, 1938, p. 266–68; Schlichting, 1955, p. 414–16). In a subglacial passage of arched cross-section this would mean flow upward along the side walls and inward across the floor of the passage and up the sides of the esker, just as required.

The size of the passage is determined primarily by the gradient of Φ along it and the amount of discharge through it, and secondarily by the roughness of its walls. Assuming reasonable values for these quantities and calculating the size of the corresponding passage by means of the resistance formula for the completely rough regime (Schlichting, 1955, p. 422) gives results comparable to the range of sizes observed in eskers. For example, a passage of semicircular cross-section that contains an esker of isosceles-triangular cross-section as wide as the passage but only half as high would have a width of about 20 m, assuming it carries 500 $m^3 s^{-1}$ of water (comparable to the discharge of an ordinary alluvial river about 250 m wide), lies beneath ice with a surface slope of 5 m km^{-1}, and has an effective wall roughness of 0.05 m. Increasing the discharge, decreasing the surface slope, and increasing the roughness by a factor of 10 would respectively increase this width by factors of 2.4, 1.5, and 1.2. For the opposite changes the factors would be 0.4, 0.7, and 0.9. Differences in the longitudinal gradient of the passage would have less than a tenth the effect of equal differences in surface slope, as may be seen from Equation (5). Thus, the conditions most conducive to the formation of large eskers in subglacial passages appear to be large discharge and low surface gradient, in agreement with the field evidence that they seem to be deposited mostly during the later stages of glaciation, when melting and down-wasting of the ice are greatest (Flint, [1971], p. 216).

Acknowledgements

It is a pleasure to thank the National Science Foundation for financial support (grant GA-30731), John F. Nye and Herbert E. Wright for the copies of their papers in press, and Robert P. Sharp, John B. Southard, and Bernard Hallet for valuable discussion of an earlier version of this paper.

MS. received 5 July 1971 and in revised form 4 October 1971

REFERENCES

Bird, R. B., *and others*. 1960. *Transport phenomena, by R. B. Bird, W. E. Stewart and E. N. Lightfoot*. New York John Wiley and Sons Inc.

Budd, W. F. 1970[a]. Ice flow over bedrock perturbations. *Journal of Glaciology*, Vol. 9, No. 55, p. 29–48.

Budd, W. F. 1970[b]. The longitudinal stress and strain-rate gradients in ice masses. *Journal of Glaciology*, Vol. 9, No. 55, p. 19–27.

Charlesworth, J. K. 1957. *The Quaternary era, with special reference to its glaciation*. London, Edward Arnold. 2 vols.

Collier, E. P. 1958. Glacier variation and trends in run-off in the Canadian Cordillera. *Union Géodésique et Géophysique Internationale. Association Internationale d'Hydrologie Scientifique. Assemblée générale de Toronto, 3–14 sept. 1957*, Tom. 4, p. 344–57.

Collins, I. F. 1968. On the use of the equilibrium equations and flow law in relating the surface and bed topography of glaciers and ice sheets. *Journal of Glaciology*, Vol. 7, No. 50, p. 199–204.

Embleton, C., *and* King, C. A. M. 1968. *Glacial and periglacial geomorphology*. [London], Edward Arnold.

Flint, R. F. [1971.] *Glacial and Quaternary geology*. New York, etc., John Wiley and Sons, Inc.

Frank, F. C. 1968. Two-component flow model for convection in the Earth's upper mantle. *Nature*, Vol. 220, No. 5165, p. 350–52.
Glen, J. W. 1954. The stability of ice-dammed lakes and other water-filled holes in glaciers. *Journal of Glaciology*, Vol. 2, No. 15, p. 316–18.
Lighthill, M. J., *and* Whitham, G. B. 1955. On kinematic waves. I. Flood movement in long rivers. *Proceedings of the Royal Society*, Ser. A, Vol. 229, No. 1178, p. 281–316.
Lliboutry, L. A. 1971. Permeability, brine content and temperature of temperate ice. *Journal of Glaciology*, Vol. 10, No. 58, p. 15–29.
Mathews, W. H. 1964[a]. Discharge of a glacial stream. *Organisation Météorologique Mondiale et Association Internationale d'Hydrologie Scientifique. Symposium. Eaux de surface, tenu à l'occasion de l'assemblée générale de Berkeley de l'Union Géodésique et Géophysique Internationale, 19–8—31–8 1963*, p. 290–300.
Mathews, W. H. 1964[b]. Water pressure under a glacier. *Journal of Glaciology*, Vol. 5, No. 38, p. 235–40.
Meier, M. F. 1965. Comments on Paterson's paper "Variations in velocity of Athabasca Glacier with time". *Journal of Glaciology*, Vol. 5, No. 41, p. 761–62. [Letter.]
Meier, M. F., *and* Tangborn, W. V. 1961. Distinctive characteristics of glacier runoff. *U.S. Geological Survey. Professional Paper* 424-B, p. B-14–B-16.
Nye, J. F. 1953. The flow law of ice from measurements in glacier tunnels, laboratory experiments and the Jungfraufirn borehole experiment. *Proceedings of the Royal Society*, Ser. A, Vol. 219, No. 1139, p. 477–89.
Nye, J. F. 1965. Stability of a circular cylindrical hole in a glacier. *Journal of Glaciology*, Vol. 5, No. 40, p. 505–07.
Nye, J. F. 1969. The effect of longitudinal stress on the shear stress at the base of an ice sheet. *Journal of Glaciology*, Vol. 8, No. 53, p. 207–13.
Nye, J. F., *and* Frank, F. C. In press. The hydrology of the intergranular veins in a temperate glacier. *Union Géodésique et Géophysique Internationale. Association Internationale d'Hydrologie Scientifique. Commission de Neiges et Glaces. Symposium on the hydrology of glaciers, Cambridge, 7–13 September 1969, organized by the Glaciological Society.*
Nye, J. F., *and* Mae, S. 1972. The effect of non-hydrostatic stress on intergranular water veins and lenses in ice. *Journal of Glaciology*, Vol. 11, No. 61, p. 81–101.
Paterson, W. S. B. 1964. Variations in velocity of Athabasca Glacier with time. *Journal of Glaciology*, Vol. 5, No. 39, p. 277–85.
Paterson, W. S. B. 1965. Reply to Meier's letter on "Variations in velocity of Athabasca Glacier with time". *Journal of Glaciology*, Vol. 5, No. 42, p. 875–76. [Letter.]
Röthlisberger, H. 1972. Water pressure in intra- and subglacial channels. *Journal of Glaciology*, Vol. 11, No. 62, p. 177–203.
Rouse, H. 1938. *Fluid mechanics for hydraulic engineers*. New York, McGraw-Hill Book Co., Inc.
Savage, J. C., *and* Paterson, W. S. B. 1963. Borehole measurements in the Athabasca Glacier. *Journal of Geophysical Research*, Vol. 68, No. 15, p. 4521–36.
Schlichting, H. 1955. *Boundary layer theory*. New York, McGraw-Hill Book Co., Inc.
Sharp, R. P. 1953. Glacial features of Cook County, Minnesota. *American Journal of Science*, Vol. 251, No. 12, p. 855–83.
Shreve, R. L., *and* Sharp, R. P. 1970. Internal deformation and thermal anomalies in lower Blue Glacier, Mount Olympus, Washington, U.S.A. *Journal of Glaciology*, Vol. 9, No. 55, p. 65–86.
Taylor, L. D. 1963. Structure and fabric on the Burroughs Glacier, south-east Alaska. *Journal of Glaciology*, Vol. 4, No. 36, p. 731–52.
Trefethen, J. M., *and* Trefethen, H. B. 1944. Lithology of the Kennebec Valley esker. *American Journal of Science*, Vol. 242, No. 10, p. 521–27.
Weertman, J. 1966. Effect of a basal water layer on the dimensions of ice sheets. *Journal of Glaciology*, Vol. 6, No. 44, p. 191–207.
Wright, H. E., *jr*. Unpublished. Tunnel valleys, glacial surges, and the subglacial hydrology of the Superior lobe, Minnesota. [Paper read at Geological Society of America symposium on the Wisconsin Stage, Milwaukee, Wisconsin, 11 November 1970.]