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NUMERICAL SOLUTION OF THE FOKKER-PLANCK-LANDAU EQUATION BY SPECTRAL METHODS*

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Abstract. A new approach for the accurate numerical solution of the Fokker-Planck-Landau (FPL) equation has been presented recently in [1, 2]. The method is based on a fast spectral solver for the efficient solution of the collision operator. The use of a suitable explicit Runge-Kutta solver for the time integration of the collision phase avoids excessive small time steps induced by the stiffness of the diffusive collision operator. Here we present the details of a numerical simulation of the relaxation process in a three-dimensional Coulomb gas.

Key words. Landau-Fokker-Planck equation, spectral methods, Runge-Kutta methods

1. The FPL Equation

In the homogeneous case, the Landau or Fokker-Planck-Landau (FPL) model is described by

$$\begin{aligned}\frac{\partial f}{\partial t} &= \nabla_v \cdot \int_{R^3} \Phi(v-v^*)\left[\nabla_v f(v)f(v^*) - \nabla_{v^*} f(v^*)f(v)\right] dv^* \\ &= Q(f,f), \quad v \in R^3, \end{aligned} \tag{1.1}$$

where the non-negative function $f(t,v)$ depends on time t and velocity v of particles. In the collision operator $Q(f,f)$, Φ is a 3×3 non-negative and symmetric matrix of the form $\Phi(v) = |v|^{\gamma+2}S(v)$, with $S(v) = Id - \frac{v\otimes v}{|v|^2}$. Different values of $\gamma \in R$ lead to the usual classification in hard potentials $\gamma > 0$, Maxwellian molecules $\gamma = 0$, or soft potentials $\gamma < 0$ (which involves the Coulombian case $\gamma = -3$, of primary importance for applications). The structure of the FPL operator is similar to the Boltzmann one; this gives the physical properties of mass, momentum, and energy conservation and the decay of the entropy $H(t) = \int f \ln f \, dv$.

2. The Relaxation Process in 3D

We have considered the relaxation process for a three-dimensional Coulomb gas ($\gamma = -3$). The initial data is chosen as the sum of two symmetric Maxwellian functions

$$f_0(v) = \frac{1}{2}\frac{5}{(2\pi v_{th}^2)^{3/2}}\left[\exp\left(-\frac{|v-v_1|^2}{2v_{th}^2}\right) + \exp\left(-\frac{|v-v_2|^2}{2v_{th}^2}\right)\right],$$

with $v_1 = (1.25, 1.25, 0)$, $v_2 = (-1.25, -1.25, 0)$ and the thermal velocity is $v_{th} = 0.4$. The final time of the simulation is $T_{end} = 80$. The movie reports the time evolution of the level set of the distribution function $f(t, v_x, v_y, v_z) = 0.02$ obtained with $n = 32$

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modes. Initially the level set of the initial data corresponds to two "spheres" in the velocity space. Then, the two distributions start to merge until the stationary state, characterized by a Maxwellian distribution with zero mean velocity, is reached. This is represented by a single centered sphere. We refer to [2, 1] for the details of the numerical method.

REFERENCES

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