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Moving-Horizon Modulating Functions-Based Algorithm for Online Source Estimation in a First Order Hyperbolic PDE

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ABSTRACT

In this paper, an on-line estimation algorithm of the source term in a first order hyperbolic PDE is proposed. This equation describes heat transport dynamics in concentrated solar collectors where the source term represents the received energy. This energy depends on the solar irradiance intensity and the collector characteristics affected by the environmental changes. Control strategies are usually used to enhance the efficiency of heat production; however, these strategies often depend on the source term which is highly affected by the external working conditions. Hence, efficient source estimation methods are required. The proposed algorithm is based on modulating functions method where a moving horizon strategy is introduced. Numerical results are provided to illustrate the performance of the proposed estimator in open and closed loops.

Nomenclature

Model's parameters:
A_s Cross-sectional area.
c Fluid specific heat capacity.
G Mirror optical aperture.
I Solar irradiance.
L Tube length.
Q Pump volumetric flow rate.
S Effective solar radiation.
T Fluid temperature.
ν_0 Mirror optical efficiency.
ρ Fluid density.
Algorithm's parameters:
J Number of basis.

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l Modulating function order.
M Number of modulating functions.
N_t Time grid size.
q Parameter in the polynomial modulating functions.
$\hat{S}$ Estimated source.
$\hat{S}_k$ Effective estimation of $\hat{S}$ at iteration k.
t_f Final time.
t_{l_k} Window lower bound.
t_{u_k} Window upper bound.
w_k Window.
Δt Sampling time.
ξ Basis function.
τ Window length.
ϕ Modulating function.

1 Introduction

In the last few decades, there has been an increasing number of studies on the efficiency enhancement of systems driven by renewable energy in order to reduce the environmental impact of fossil resources. A particular attention has been given to concentrated distributed technology widely used in solar thermal plants. The distributed solar plants concentrate the received sunlight in order to generate thermal energy based on a heat transport phenomenon along the collector tube. The solar irradiance is concentrated to the central tube using parabolic shaped mirrors to be absorbed and routed by the heat carrier fluid to the collector outlet (see Fig. 1). Continuous investigations have been conducted to design strategies to manage the production of solar collectors. The control problem is approached by forcing the outlet fluid temperature to track a set reference by tuning the fluid flow rate [1]. However, the system dynamics and heat production are affected by environmental disturbances including the solar irradiance and the optical efficiency of the mirrors. The solar irradiance can be measured by pyrheliometers, but these measurements are usually local; hence, extrapolating them in large plants may lead to inaccurate results [2]. Moreover, the cleanness of the mirrors is usually inhomogeneous, unpredictable and subject to external working factors, especially in a humid dusty environment. These factors motivate the soft-sensing of the efficient value of the energy source term resorting to an online estimation algorithm. The estimated value of the source term is then injected in an indirect adaptive controller, considering the tremendous number of studies on control of the solar collector strategies [3–5].

In this study, based on energy conservation laws, a distributed parameter model is used to describe the heat transfer phenomenon in the collector loop. The heat transport is represented by the fluid temperature and described by a first order hyperbolic partial differential equation (PDE) whose source term includes the received solar irradiance and the mirrors efficiency parameters. The control input is given as a space-derivative coefficient for the PDE.

Various methods have been proposed to estimate unknown parameters and source terms in PDEs which can be classified into optimization methods [6] and recursive methods (such as observers) [7]. However, in addition to the computational burden, most of these methods operate off-line where data are collected first and then used to estimate the unknowns. Nevertheless, in many real applications, such as the control of the solar collector as considered in this paper, the estimation of the unknowns during the operation of the system is crucial.

In this paper, an efficient algorithm based on Modulating Functions Method (MFM) is proposed for on-line estimation of the source term of a first order hyperbolic PDE from boundary measurements. The MFM has been introduced in the early fifties [8] for parameters identification of Ordinary Differential Equations (ODEs). In 1966, Perdreauville and Goodson [9] extended the method to the identification of constant and space varying parameters in PDEs using distributed continuous-time measurements. Later, Fairman and Shen [10] modified the approach of Perdreauville and Goodson by using finite difference scheme to approximate the spatial derivatives. In 1997, B. Co and Ungarala [11] adapted the method for real-time parameters identification for ODEs. Recently, the method has been extended to estimate the unknown input and the state variables of a linear dynamic system, as well as the fractional order derivatives of the output [12–14].

The MFM is designed based on the PDE without reliance on model approximation which reduces the information loss. Moreover, it does not require the knowledge of the system's initial conditions (the initial conditions of the PDE), which are usually unknown. Furthermore, approximating the derivatives of the measurements, which are usually noisy, is avoided with this method. In this work, we propose an online estimation strategy based on the MFM, which we refer to Moving-Horizon Modulating Functions-Based Algorithm (MH-MFBA). To illustrate the MH-MFBA, the paper focuses on the estimation of a time-varying source at the boundary which does not answer the question of local measurements, but current studies investigate the general case of time-space varying sources.

The paper is organized as follows. First, some notations and definitions are given in Section 2. Then the problem is formulated in Section 3. Section 4 studies the identifiability of the source term of the first order hyperbolic PDE from some

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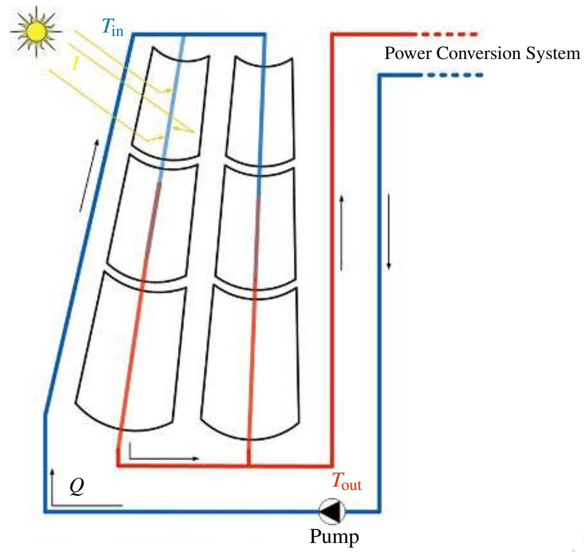

Fig. 1: A schematic diagram of the hydraulic circuit of a distributed solar collector.

boundary measurements. The MH-MFBA is presented in Section 5. In Section 6, some numerical results are shown and discussed. Concluding remarks are drawn in Section 7.

2 Preliminaries

In this section, the definition of modulating functions and the general procedure of the MFM are given. Then, the concept of identifiability study for parameter identification is defined.

2.1 Modulating Functions Method

In this subsection, definitions and concepts about the MFM are recalled.

Definition 1 ([15]). *A function $\phi(t) \neq 0$ is called a modulating function of order l ($l \in \mathbb{N}^*$) on the bounded interval $[0, t_f]$ if it satisfies:*

$$\begin{cases} \phi(t) \in C^l([0,t_f]), & \text{(1a)} \\ \phi^{(p)}(0) = \phi^{(p)}(t_f) = 0, \quad p = 0,1,2,\cdots,l-1, & \text{(1b)} \end{cases}$$

where $t_f > 0$ and p refers to the order of the derivative.

Several types of modulating functions, which satisfy (1), have been proposed and used such as sinusoidal functions [9, 15], Hermit functions [16], spline-type functions [17], Poisson moment functionals [18], and Hartley modulating functions [19].

The following lemma describes a general integration by parts formula; which plays an important role in the MFM.

Lemma 1. *Let $\mathbb{P}^{\bar{n}}$ be a differential operator defined on $C^{\bar{n}}([0,t_f])$ such that $\mathbb{P}^{\bar{n}}u(t) = \sum_{s=1}^{\bar{n}} b_s(t)\frac{\mathrm{d}^s}{\mathrm{d}t^s}u(t)$. If $\phi(t)$ is a modulating function of order l ($l \geq \bar{n}$) defined on a closed bounded interval $[0,t_f]$; then*

$$\int_0^{t_f} \left[\mathbb{P}^{\bar{n}}u(t)\right]\phi(t)\,\mathrm{d}t = \int_0^{t_f} u(t)\left[\mathbb{Q}^{\bar{n}}\phi(t)\right]\mathrm{d}t, \tag{2}$$

where $\mathbb{Q}^{\bar{n}}\phi(t) = \sum_{s=1}^{\bar{n}}(-1)^s\frac{\mathrm{d}^s}{\mathrm{d}t^s}\left[b_s(t)\phi(t)\right]$.

Proof. By applying integration by parts formula to the left hand side of (2), one can obtain

$$\int_0^{t_f} \left[\mathbb{P}^{\bar{n}}u(t)\right]\phi(t)\,\mathrm{d}t = \sum_{s=1}^{\bar{n}} \phi(t)b(t)\frac{\mathrm{d}^{(s-1)}}{\mathrm{d}t^{(s-1)}}u(t)\Bigg|_{t=0}^{t=t_f} - \sum_{s=1}^{\bar{n}}\int_0^{t_f}\frac{\mathrm{d}^{(s-1)}}{\mathrm{d}t^{(s-1)}}u(t)\,\frac{\mathrm{d}}{\mathrm{d}t}\left[b_s(t)\phi(t)\right]\mathrm{d}t. \tag{3}$$

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The first term in the right hand side of (3) is equal to zero from (1b). Applying successive integration by parts to the right hand side of (3) and using the property in (1b) leads to the right hand side of (2). □

The MFM has been used for parameter identification. The general procedure is described below.

Step 1: The differential equation is multiplied by a modulating function.
Step 2: The resulting differential equation is integrated over the time (or space) interval. The integration contributes in filtering and attenuating the noise.
Step 3: Lemma 1 is applied. This step transfers all the temporal (or spatial) derivatives of the measurements, which are usually noisy, to derivatives of the modulating functions, which are usually known analytically, see (2). Consequently, differentiating noisy measurements is avoided. Also, the initial conditions (or boundary conditions), which appear in the integration by parts, are eliminated thanks to the second property of the modulating functions (1b).
Step 4: The obtained equation is formulated into an algebraic linear system.

For more details, the reader can refer to [20] and the references therein.

2.2 Identifiability

Identifiability is a structural property that reflects the ability to estimate unknown parameters from available measurements independently on the used estimation method [21], [22].

Definition 2. *Let $\mathcal{M}$ be a mathematical model in which $f \in F$ and $u_f \in U$ represent the source term and its corresponding model solution, respectively, in appropriate spaces F and U. Then f is said to be identifiable if and only if the mapping $f \to u_f$ is injective; that is*

$$u_{f_1} = u_{f_2} \Rightarrow f_1 = f_2 \qquad \text{for all } f_1, f_2 \in F. \tag{4}$$

3 Problem Formulation

Based on the energy conservation law, the heat transport dynamics in a concentrated distributed solar collector are given by the following first order hyperbolic PDE where the fluid temperature T is the infinite dimensional state vector [23, 24]:

$$\begin{cases} \dfrac{\partial T(x,t)}{\partial t} + \dfrac{Q(t)}{A_s}\dfrac{\partial T(x,t)}{\partial x} = S(t), & \Omega_t := (x,t) \in (0,L] \times (0,\infty) \\ T(0,t) = g(t), & t \in [0,\infty) \\ T(x,0) = h(x), & x \in (0,L] \end{cases} \tag{5}$$

where x refers to the position along the collector, t to the time, and L to the collector length. $T(x,t) \in C^1(\Omega_t)$ denotes the fluid temperature at position x along the tube and at time instant t. The source term $S(t) \in C(\mathbb{R}^+)$ represents the effective energy source where $S(t) = \frac{\nu_0(t)G}{\rho c A_s} I(t)$ such that $I(t)$ is the solar irradiance and $\nu_0(t)$ is the mirrors optical efficiency. The parameters A_s, G, ρ, and c are model parameters represent, respectively, the cross-sectional area, the mirror optical aperture, the fluid density, and the fluid specific heat capacity. $g(t) \in C(\mathbb{R}^+)$ and $h(x) \in C^1(\Omega)$ represent the boundary and initial conditions, respectively. $Q(t)$, which is the pump volumetric flow rate, is the control input to be tuned in order to control the system despite the varying external conditions.

In general, the control objective aims at finding a control law, $Q(t)$, such that the outlet temperature $T(L,t)$ is maintained around a given desired reference, say $Y_r(t)$. However, the system dynamics are constrained by the unevenly variations of the solar irradiance and the cleanness of the collector mirrors. Therefore, in order to achieve the control objectives, soft-sensing of the efficient value of the source term through an on-line source estimation is proposed.

The objective in this work is to estimate the effective energy source $S(t)$ at each time instant $t \in \mathbb{R}^+$ from boundary measurements $Y(t)$, where

$$Y(t) = \begin{bmatrix} T(L,t) \\ \frac{\partial}{\partial x} T(x,t)\Big|_{x=L} \end{bmatrix}. \tag{6}$$

To this end, an adaptation of the MFM is proposed in order to continuously update the controller $Q(t)$ forcing the outlet temperature $T(L,t)$ to track the desired set reference $Y_r(t)$, as an indirect adaptive control, see Fig. 2. Before proceeding to the design of the source term estimation algorithm, the identifiability of the hyperbolic PDE source term from the boundary measurements $Y(t)$ is studied in the next section.

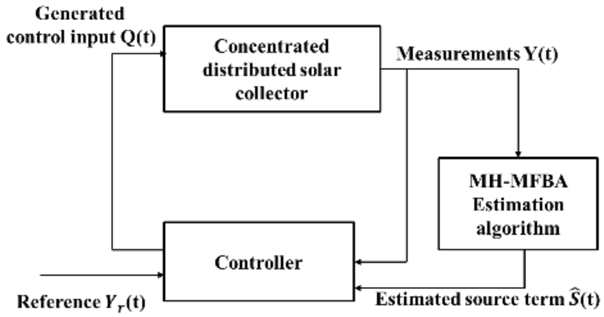

Fig. 2: Indirect adaptive control block diagram

4 Identifiability Analysis

The following lemma studies the identifiability of the source term of the first order hyperbolic PDE from boundary measurements.

Lemma 2. *If $T(L,t)$ and $\frac{\partial}{\partial x}T(x,t)\Big|_{x=L}$ in system (5) are known, then the source term $S(t)$ is identifiable.*

Proof. Let $\left\{T_1(L,t),\, \frac{\partial}{\partial x}T_1(x,t)\Big|_{x=L}\right\}$ and $\left\{T_2(L,t),\, \frac{\partial}{\partial x}T_2(x,t)\Big|_{x=L}\right\}$ be two sets of measurements corresponding to the sources S_1 and S_2, respectively, such that:

$$\begin{cases} T_1 = T_2, \\ \frac{\partial}{\partial x}T_1 = \frac{\partial}{\partial x}T_2. \end{cases} \tag{7}$$

Then from equation (5), one can write the following for T_1 and T_2:

$$\begin{aligned} \frac{\partial}{\partial t}T_1(L,t) + \frac{Q(t)}{A_s}\frac{\partial}{\partial x}T_1(x,t)\Big|_{x=L} &= S_1(t), \\ \frac{\partial}{\partial t}T_2(L,t) + \frac{Q(t)}{A_s}\frac{\partial}{\partial x}T_2(x,t)\Big|_{x=L} &= S_2(t). \end{aligned} \tag{8}$$

Subtracting the two equations in (8) leads to:

$$\frac{\partial}{\partial t}\left[T_1(L,t) - T_2(L,t)\right] = S_1(t) - S_2(t). \tag{9}$$

From (7), the measurements $T_1(L,t)$ and $T_2(L,t)$ are assumed to be equal; hence, $S_1 = S_2$.

To achieve the objective of providing a real time (online) estimation of the effective energy source to the controller, a novel approach based on the MFM is proposed. A moving horizon strategy is proposed to solve the online estimation problem while the solar plant is operating. In the next section, the detailed procedure of the algorithm is presented.

5 Moving-Horizon Modulating Functions-Based Algorithm

The general procedure of the MFM, presented in subsection 2.1, is appropriate for off-line estimation problems; i.e. data are collected first and then the unknowns are estimated. However, for solar plants, the energy source needs to be estimated on-line to continuously update the closed loop controller while the plant is operating. In other words, new data is continuously available during the operation of the system. To solve this on-line estimation problem, a moving horizon strategy is proposed where a time window of a small length that moves one sampling time forward. At each new window, the MFM is applied to estimate the energy source for the current window; then the control $Q(t)$ is updated at a future time instant based on the last estimated efficient value. The procedure for this on-line estimation problem is depicted in Fig. 3 and explained in details in Algorithm 1 and Proposition 1 where w_k, τ, Δt, and $\hat{S}_{w_k}$ refer to the window, the window length, the sampling time, and the estimated source at the window w_k, receptively.

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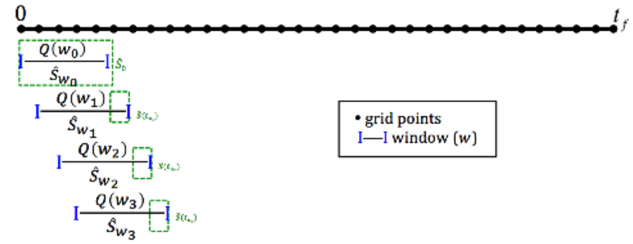

Fig. 3: A schematic diagram for Algorithm 1

Algorithm 1: Moving-Horizon Modulating Functions Based Algorithm

Input: t_f : final time, τ : window length, Q_0: initial profile for Q.

Initialization: $w_0 = [0, \tau]$; $Q(w_0) = Q_0$;

• Estimate $\hat{S}$ at the current window w_0 using Proposition 1; **Output:** $\hat{S}_0$;

$k = 1$;

while $\tau + k\Delta t \leq t_f$ **do**

  $w_k = [k\Delta t, \tau + k\Delta t] = [t_{l_k}, t_{u_k}]$;

  • Control Q at t_{u_k} using the last value of $\hat{S}_{k-1}$; **Output:** $Q(t_{u_k})$;

  • Estimate $\hat{S}$ at the current window w_k using Proposition 1 s.t. $Q = Q(w_k)$; **Output:** $\hat{S}_{w_k}$;

  • Update: $Q_k = \begin{bmatrix} Q_{k-1} & Q(t_{u_k}) \end{bmatrix}$ and $\hat{S}_k = \begin{bmatrix} \hat{S}_{k-1} & \hat{S}_{w_k}(t_{u_k}) \end{bmatrix}$;

  $k = k + 1$;

end

Output: $Q(t)$ and $\hat{S}(t)$

Before stating the proposition, remember that if $\phi(t)$ is a modulating function of order l on the time interval $[0, t_f]$, then $\phi(t)$ and its derivatives, up to $l-1$ derivative order, are zeros at the end points of the interval (see Definition 1). However, here in the proposed on-line estimation, the interval is changing at each iteration. In fact, the time interval at each iteration is the new time window w_k. If $w_k = [t_{l_k}, t_{u_k}]$ such that t_{l_k} and t_{u_k} refer to the window lower and upper bounds, respectively; then the standard modulating functions should be modified to satisfy the following:

$$\begin{cases} \phi(t) \in C^l(w_k), & \text{(10a)} \\ \phi^{(p)}(t_{l_k}) = \phi^{(p)}(t_{u_k}) = 0, \quad p = 0, 1, 2, \cdots, l-1. & \text{(10b)} \end{cases}$$

Proposition 1. *Let $\sum_{j=1}^{J} \gamma_j \xi_j(t)$ be a basis expansion of the unknown source $S(t)$ in (5), where $t \in w_k = [t_{l_k}, t_{u_k}]$; $\xi_j(t)$ and γ_j, for $j = 1, \cdots, J$, are basis functions and basis coefficients, respectively. Let $\{\phi_m(t)\}_{m=1}^{m=M}$ be a class of on-line modulating functions that satisfy (10) with $l \geq 1$ and $M \geq J$. Then, the unknown coefficients γ_j, $j = 1, 2, \cdots, J$, can be estimated by solving the system:*

$$\mathcal{A}\Gamma = K, \tag{11}$$

where the components of the $M \times J$ matrix $\mathcal{A}$ have the form:

$$\mathcal{A}_{mj} = \int_{t_{l_k}}^{t_{u_k}} \phi_m(t) \xi_j(t) \, \mathrm{d}t, \tag{12}$$

for $m = 1, \cdots, M$ and $j = 1, \cdots, J$; the components of the vector $K \in \mathbb{R}^M$ are:

$$K_m = \frac{1}{A_s} \int_{t_{l_k}}^{t_{u_k}} Q(t) \phi_m(t) \frac{\partial}{\partial x} T(x^*, t) \, \mathrm{d}t - \int_{t_{l_k}}^{t_{u_k}} \frac{\mathrm{d}\phi_m(t)}{\mathrm{d}t} T(x^*, t) \mathrm{d}t, \tag{13}$$

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Γ is the vector of unknowns γ_j, $j = 1, \cdots, J$, and x^* refers to a fixed position.

Proof. **STEP 1:** At fixed position x^* in the PDE in (5), multiply the equation by the modulating functions $\phi_m(t)$:

$$\frac{\partial T(x^*,t)}{\partial t}\phi_m(t) + \frac{Q(t)}{A_s}\frac{\partial T(x^*,t)}{\partial x}\phi_m(t) = S(t)\phi_m(t). \tag{14}$$

STEP 2: Integrate over w_k:

$$\int_{t_{l_k}}^{t_{u_k}} \frac{\partial T(x^*,t)}{\partial t}\phi_m(t)\mathrm{d}t + \frac{1}{A_s}\int_{t_{l_k}}^{t_{u_k}} Q(t)\frac{\partial T(x^*,t)}{\partial x}\phi_m(t)\mathrm{d}t = \int_{t_{l_k}}^{t_{u_k}} S(t)\phi_m(t)\mathrm{d}t. \tag{15}$$

STEP 3: Apply Lemma 1 to the first integral in (15):

$$-\int_{t_{l_k}}^{t_{u_k}} T(x^*,t)\frac{\mathrm{d}\phi_m(t)}{\mathrm{d}t}\mathrm{d}t + \frac{1}{A_s}\int_{t_{l_k}}^{t_{u_k}} Q(t)\frac{\partial T(x^*,t)}{\partial x}\phi_m(t)\mathrm{d}t = \int_{t_{l_k}}^{t_{u_k}} S(t)\phi_m(t)\mathrm{d}t. \tag{16}$$

By writing $S(t)$ in its basis expansion, system (11) is obtained with components as in (12) and (13).

Remark 1. *x^* can be chosen arbitrary from the accessible points which depend on the application. In the solar collector considered in this paper, x^* refers to the outlet position, $x = L$ (see Fig. 1).*

Remark 2. *Note that in Algorithm 1, line 6; since the sampling time is very small compared to the system dynamics, substituting the estimated source value at t_{u_k} by the value at the previous time instant will not effect the control result at t_{u_k}.*

6 Numerical Results

To assess the MH-MFBA performance, the estimation algorithm has been tested numerically in both open and closed control loops. The heat transfer phenomenon has been simulated using a semi-discrete approximation of the hyperbolic PDE evaluated at 500 knots. For these numerical tests, the modulating functions have been chosen to be of polynomial-type for its simplicity and efficiency [25]. Its standard form is:

$$\phi_m(t) = t^{q+m}(t_f - t)^{q+M+1-m} \qquad t \in [0, t_f]; \tag{17}$$

where $m = 1, 2, \cdots, M$; M is the number of modulating functions, and $q \in \mathbb{R}^+$ is a degree of freedom which is chosen such that the order l of the modulating functions is greater than or equal to the PDE's order. For example, with the first order PDE considered in this work, $l \geq 1$. For the proposed online estimation algorithm, the function in (17) is modified, to satisfy (10), as follows:

$$\phi_m(t) = (t_{u_k} - t)^{q+m}(t - t_{l_k})^{q+M+1-m} \qquad t \in [t_{l_k}, t_{u_k}]. \tag{18}$$

In addition, polynomial basis for the basis functions ξ_j have been considered. Moreover, it is enough to have one or two basis functions (i.e. $\max J = 2$) because the estimation is done on a small interval w_k, so big variations in the exact source are unexpected. The model parameters are chosen according to the values in Table 1.

6.1 Open Loop Control with Synthetic Data

The first numerical test aims at evaluating the performance of the estimation algorithm in open loop using some synthetic data. Different profiles of the PDE's source term have been considered: constant, linear, and sinusoidal functions. For these simulations, time varying profiles of the solar irradiance $I(t)$ with fixed collectors characteristic parameters following Table 1 have also been considered. The synthetic irradiance profiles are chosen such that the resulting energy source term is within the real range. Also, a fluid flow rate $Q(t)$ varying in time while respecting the physical limitations has been used. It has been chosen between the bounds 0.001 m^3/s and 0.012 m^3/s. The different scenarios are summarized in Table 2. The obtained results for the four cases are presented in Fig. 4, and the corresponding errors are shown in Fig. 5, where the algorithm parameters are set as follows: $q = 200$, $M = J$, $t_f = 50$s, $\Delta t = 0.01$s, and $\tau = 0.5$s. It is clear that the proposed estimation algorithm could reproduce the source term from the boundary measurements. Indeed, the source term has been estimated with good accuracy in all cases.

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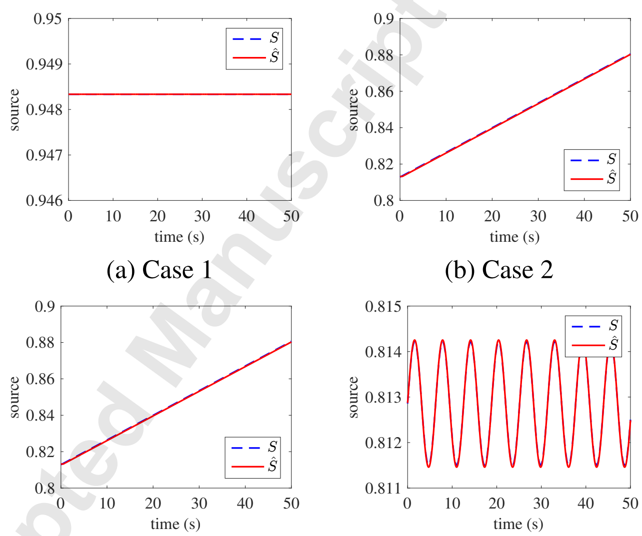
Table 1: Concentrated solar collector model's parameters [1]

Parameter	Value	Unite
ρ	903	kg m^{-3}
c	1820	J C^{-1} kg^{-1}
A_s	0.0006	m^2
ν_0	73%	Unitless
G	1.83	m
L	172	m

Table 2: Different simulation scenarios using synthetic data

	$Q(t)$ [m^3/s]	$I(t)$ [W/m^2]
Case 1	0.001	700
Case 2	0.001	$t+600$
Case 3	$(0.001)(0.01t+1)$	$t+600$
Case 4	$(0.001)(0.01t+1)$	$\sin(t)+600$

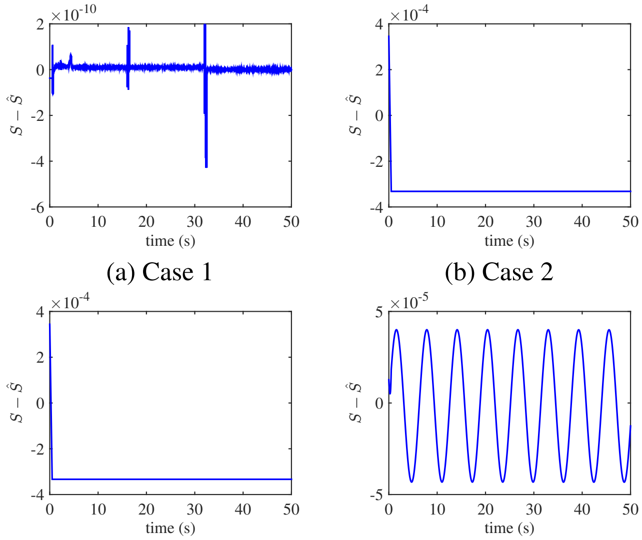


(a) Case 1 (b) Case 2 (c) Case 3 (d) Case 4

Fig. 4: The estimated source $\hat{S}(t)$ using synthetic data. The sub-figures correspond to the cases presented in Table 2

6.2 Open Loop Control Using Real Irradiance Profile

In this subsection, the numerical tests are carried out in order to evaluate the algorithm performance in open control loop using simulated real external working conditions. A varying control input (see Fig. 6) has been considered with a time varying profile of the source term characterized by smooth changes with some abrupt drops in order to emulate the intermittency of the energy source. The source term has been simulated considering a real irradiance profile extracted from real data with constant mirrors characteristic parameters. Fig. 7 and Fig. 8 depict the estimated source term using MH-MFBA and the corresponding estimation error, respectively, where the algorithm parameters are set as follows: $q=5$, $M=1$, $t_f=2$h, $\Delta t=1$s, and $\tau=5$s. The figures show the efficiency of the proposed algorithm and its capability to track



(a) Case 1 (b) Case 2

(c) Case 3 (d) Case 4

Fig. 5: The estimation error, $S-\hat{S}$, for the cases in Fig. 4

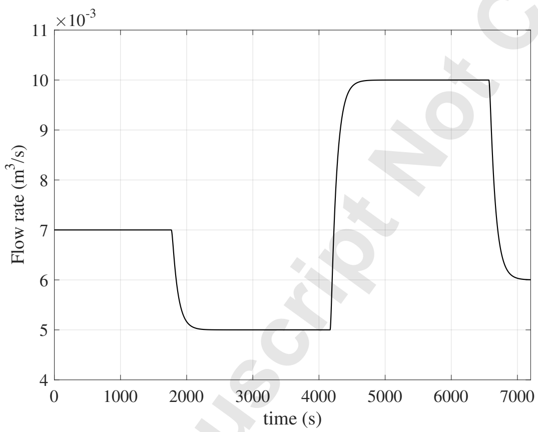


Fig. 6: The control input profile for the open loop test

the sudden variations of the solar irradiance. The corresponding relative errors of the source estimations in all the previous figures are shown in Table 3.

6.3 Closed Loop Control Using Real Irradiance Profile

In the previous subsection, we carried a numerical test simulating real working conditions in order to assess the performance of the estimation algorithm with respect to the intermittent changes of the source term. Thereafter, we will go one step further by numerically testing the system under the same conditions but in closed loop. In this subsection, the nonlinear output control feedback, which was introduced in [26], is considered for the simulations. As mentioned before, the control goal is to force the system outlet temperature $T(L,t)$ to follow a desired temperature $Y_r(t)$. To reach this objective, the control input $Q(t)$ is defined by

$$Q(t) = A_s \left[\frac{\hat{S}(t) - \bar{s}(t)}{\frac{T(L,t)-T(0,t)}{L}} \right] + \bar{Q}, \tag{19}$$

where

$$\frac{\bar{s}(t)}{\bar{Q}} = \frac{Y_r(t) - T(0,t)}{L}, \tag{20}$$

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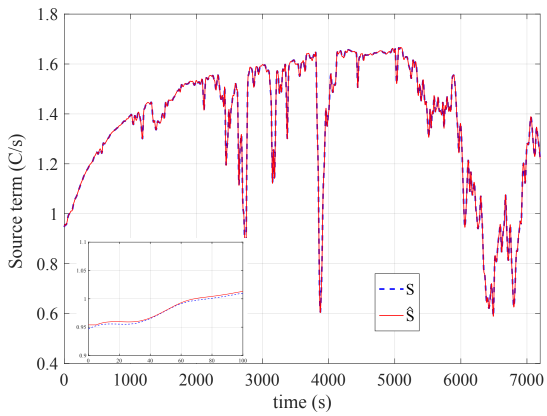

Fig. 7: The estimated source $\hat{S}(t)$ in open loop

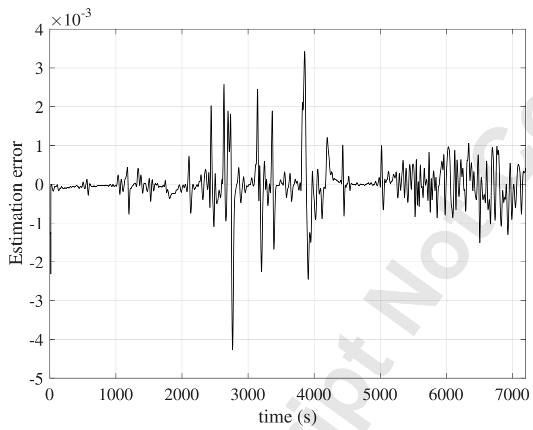

Fig. 8: The estimation error, $S-\hat{S}$, for the result in Fig. 7.

Table 3: The corresponding relative errors for the source estimation in Fig. 4 and Fig. 7

Fig.	Relative error (%)
4-(a)	$2.4688e^{-9}$
4-(b)	0.0391
4-(c)	0.0392
4-(d)	0.0036
7	0.83179

and $\hat{S}$ is the estimated value of the efficient energy source term using the MH-MFBA. The obtained results are shown in Fig. 9 and Fig. 10 highlighting the performance of the estimation algorithm in closed loop. Indeed, the MH-MFBA is able to continuously update the controller in order to ensure the reference tracking efficiently within the desired time. In addition, the estimated value for the source term $\hat{S}(t)$ is almost matching with the used value $S(t)$. Moreover, the generated control input $Q(t)$ ensuring the control objective is within the admissible boundaries (see Fig. 11).

7 Conclusion

This work addressed the problem of estimating the efficient energy source term affecting the heat production in a distributed concentrated solar collector. The paper proposed a new algorithm based on the MFM for an online estimation of

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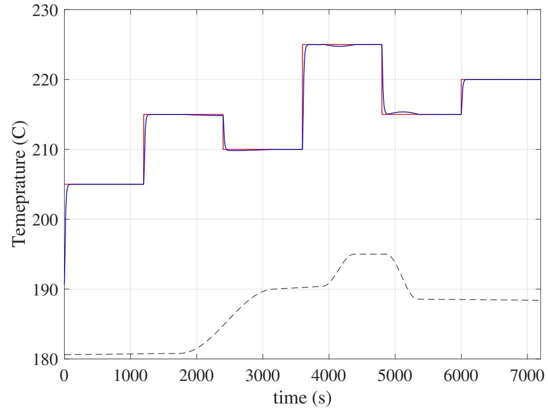

Fig. 9: Results of the indirect adaptive controller in closed loop

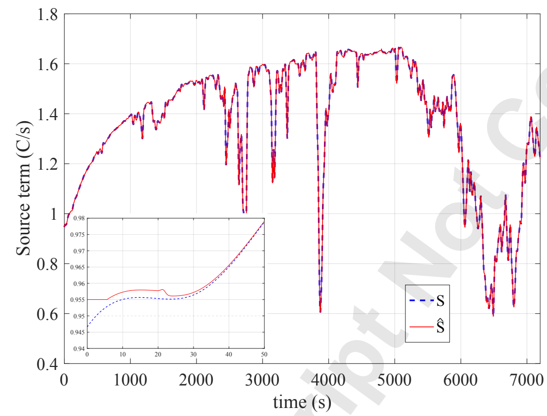

Fig. 10: The estimated source $\hat{S}(t)$ in closed loop

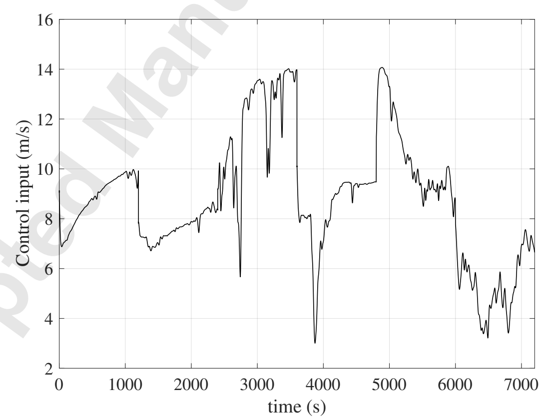

Fig. 11: Generated control input

the effective time-varying energy source to be provided to control the solar closed loop. The effectiveness and efficiency of the proposed algorithm have been demonstrated through several numerical tests using synthetic and real source term profiles.

In future work, the proposed algorithm will be extended to the estimation of distributed source $S(x,t)$ which represents the real profile of energy source in the system taking into consideration the shading affecting the energy production and the inhomogeneous distribution of the optical efficiency parameter.

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